Discrete Spacetime and Fluctuating Lambda

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Quantum Gravity: we must be both radical and conservative

The Quantum/Gravity boundary: physics is at its least unified. Also physics at its most unified (black hole thermodynamics)!

In working towards quantum gravity, science will have to "make the leap to a new conception". To arrive at a theory of quantum gravity we must make a creative leap, from a position that is grounded in current knowledge. Identifying that "grounded position" is itself a creative act: at any stage in the evolution to a new theory, there will be diverse views on which aspects of our current understanding will survive and which will turn out to be "excess baggage".

The **causal set approach** to quantum gravity claims that certain aspects of General Relativity and quantum theory will have direct counterparts in quantum gravity: the **spacetime causal order** from General Relativity and the **path integral** from quantum theory. It makes one main new hypothesis about the nature of the physical world: **fundamental discreteness** of spacetime.

Causal structure at the heart of General Relativity

Spacetime in General Relativity has a causal order. That causal order is fundamental to understanding the theory e.g. the ubiquity of Penrose diagrams. Another example: the **epitome of GR** is a Black Hole and our understanding is in 4-dimensional, spacetime, causal terms. There is prima facie evidence that:

- Black hole thermodynamics is due to the **causal** nature of the horizon. In particular the Generalised Second Law: Hawking's area theorem; violations of GSL without relativistic causal structure **(Eling, Foster, Jacobson & Wall)**; violation of GSL semiclassically for apparent or dynamical horizons **(Wall)**; a proof scheme for the GSL in full quantum gravity **(Sorkin)**
- **All causal horizons** obey the laws of thermodynamics where a causal horizon is the boundary of the causal past of a future inextendible timelike curve. e.g. Black hole horizon, DeSitter horizon, Rindler horizon. We should think fundamentally about "causal horizon entropy" not black hole entropy **(Jacobson & Parentani).**

Causal structure is a unifying concept

This causal order **unifies within itself** the topology (inc. dimension), differentiable structure and 9/10 of the metric of a Lorentzian manifold (in 4d). **(Robb, Alexandrov, Zeeman, Penrose, Kronheimer, Hawking, Malament, Levichev, Parrikar&Surya)**

In the case of strongly causal spacetimes this follows from knowledge and theorems from the "Classical Period" of GR:

- Causal structure gives chronological structure;
- Chronological structure gives topology;
- Local causal structure gives local null geodesics;
- Local null geodesics give differentiable structure;
- Lightcones in tangent space give metric up to a conformal factor.

So, spacetime in General Relativity almost **is** a causal order. The missing tenth of 4-D spacetime geometry, not given by causal order, is local physical **scale.** (c.f. Twistor theory)

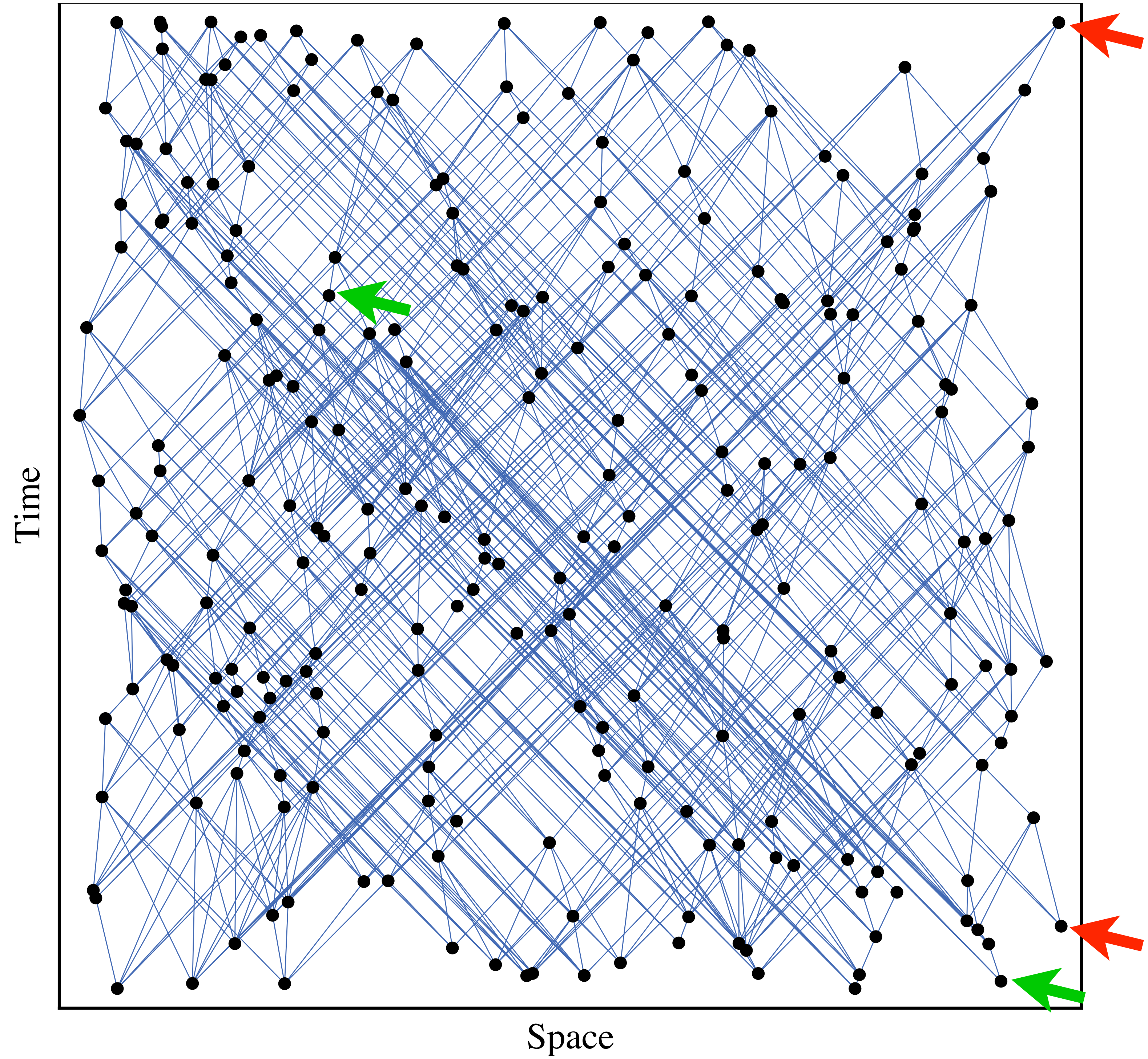
Causal sets: the marriage of causality and atomicity

It is widely expected that spacetime will not be well-described by a differentiable Lorentzian manifold close to the Planck scale. Most strikingly, semiclassical explorations of black hole entropy suggest that physics needs a short-scale cutoff in order to be consistent with the known value **(Sorkin)**. Fundamental spacetime atomicity is perhaps the simplest way to realise this expectation. It happens to be **exactly** what is necessary if one is drawn to conceive of spacetime as "pure causal order".

Atomicity does the job of providing the physical scale missing from pure causal order in the continuum because we can **count**: the number of spacetime atoms corresponds to the volume of spacetime (in some fundamental units). A **causal set** is a **discrete causal order**:

Order + Number = Geometry

A causal set that is well approximated by 2d Minkowski space



- Hasse
- Lorentz invariant
- relational
- austere
- nonlocal (&causal)
- **statistical** N~V

On Planckian scales, this is what Minkowski space is
For any causal Lorentzian space-time M $\exists$ causal sets approximated by M
Unification: (Manifold, metric) is replaced by Causal Set

A Causal Set is a discrete order

A causal set (or causet) is a set, C, with a relation, $\preccurlyeq$

* **Transitive**: if $x \preccurlyeq y$ and $y \preccurlyeq z$ then $x \preccurlyeq z$, $\quad \forall x, y, z \in C$;
* **Acyclic**: if $x \preccurlyeq y$ and $y \preccurlyeq x$ then $x = y$, $\quad \forall x, y \in C$;
* **Locally finite**: for any ordered pair of elements x and z of C, the cardinality of the set $\{y \mid x \preccurlyeq y \preccurlyeq z\}$ is finite

The first two axioms say that C is a **partial order**. The third axiom is what makes the set **discrete**. The elements of C are the **atoms of spacetime**

> The deep structure of spacetime is a causal set

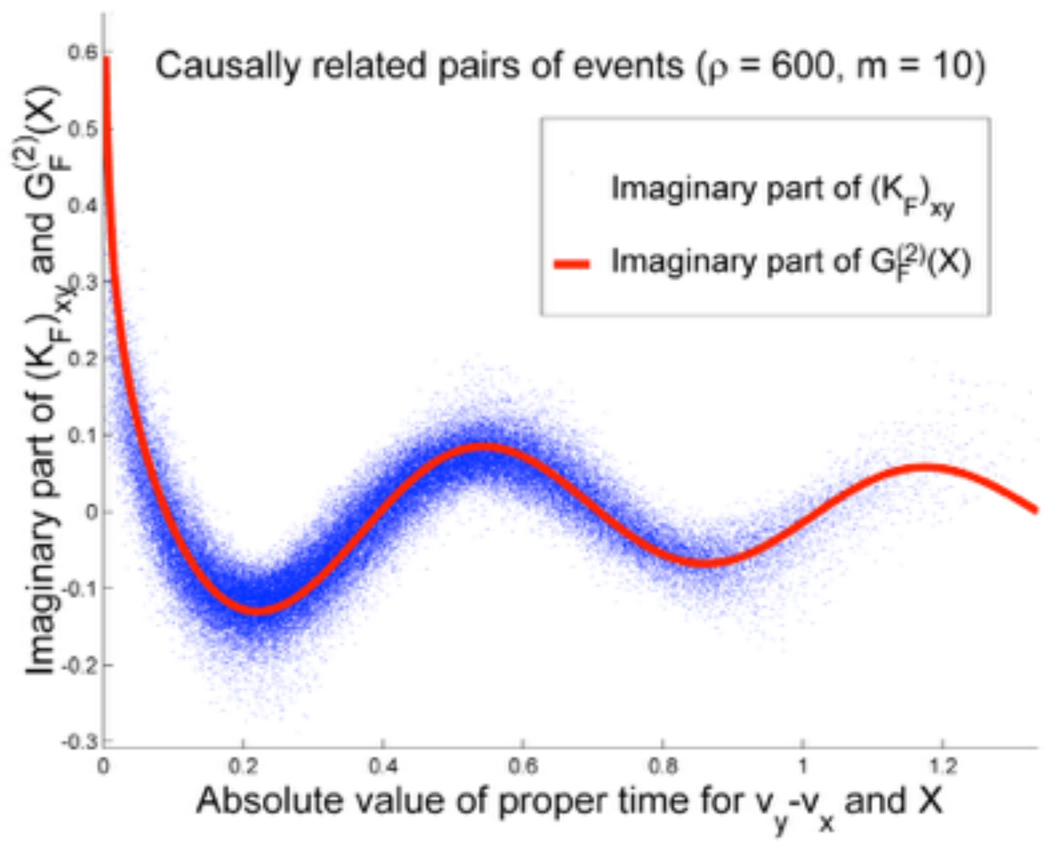
At macroscopic scales, the order $\preccurlyeq$ gives rise to the spacetime causal order and the number of elements gives the spacetime Volume: Number $\approx$ Volume

('tHooft; Myrheim; Bombelli, Lee, Meyer, Sorkin)

Evidence that Lorentzian geometry is encoded in Causet

Technique: Poisson sampling of causal structure, called sprinkling

Flat space:
Dimension (Myrheim; Meyer)
Timelike geodesic distance (Brightwell,Gregory)
Spatial topology (homology) (Major, Rideout, Surya)
Wightman function of free scalar field in 2 and 4 dims (Johnston).



Curved space:
Scalar curvature & certain components of Ricci tensor (Benincasa, FD, Glaser): remain to be tested.

Causal set quantum gravity

The quantum theory of causal sets will be based on the path integral: causal sets are inimical to a canonical approach. Path integral quantum gravity:

$$Z(V) = \sum_M \int \mathcal{D}g \; e^{iS[g]}$$

just a "statement of intent" **(Loll)**. Its fundamental meaning is hypothesised to be:

$$Z(N) = \sum_{\text{causal sets of cardinality N}} e^{iS(C)}$$

Note:
* no continuum limit is taken in the fundamental theory: discreteness is physical
* many dimensions and topologies are summed over

If the causets that dominate this sum are (upon coarse graining) manifold-like, there will be a continuum regime. Otherwise, it fails:

"The primal struggle between Action and Entropy" **(Ambjorn, Loll, Sorkin)**

The one successful Quantum Gravity prediction to date

- In the late 1980's/early 1990's Rafael Sorkin predicted the value of the Cosmological "Constant" using a heuristic argument based on expectations of quantum causal set theory.
- Since Number ~ Volume, it is natural in a path integral for causal set quantum gravity to fix N for the causal sets summed over (Note: a justification for "unimodular" gravity c.f. Weinberg)
- Because the Number/Volume relationship is statistical, fixing N means there are fluctuations in V of order $\Delta V \sim \sqrt{N} \sim \sqrt{V}$
- V and Lambda are canonically conjugate (they are Time and Energy) so

$$\Delta V \Delta \Lambda \sim 1$$

$$\Delta \Lambda \sim \frac{1}{\Delta V} \sim H^2 \sim 10^{-120}$$

If Lambda fluctuates about zero, then what we see is only the fluctuation

Everpresent Lambda (Ahmed, Dodelson, Greene & Sorkin; Ahmed & Sorkin)

In a homogeneous cosmological model realising Sorkin's idea — a stochastic model, incompatible with GR — the value of Lambda fluctuates, on a Hubble timescale, between positive and negative values that are of the same order of magnitude as the ambient matter density (it is "everpresent"). Data consistent with **constant** Lambda back to redshifts of a few would pretty much rule out this idea.

So what do the data tell us? One relevant dataset is from BOSS, the Baryon Oscillation Spectroscopic Survey, part of the Sloan Digital Sky Survey, a long term observational cosmology project, it was being talked about already when I was a student and being planned when I was a postdoc in the Fermilab astro group — "we're going to measure a million redshifts".

https://www.youtube.com/watch?v=rOjrImaPh80

BOSS

BOSS uses the sound horizon in the cosmic plasma at recombination as a "standard ruler". The sound horizon is a physical scale at which the matter correlation function will exhibit a peak because the expansion — at the speed of sound — of the baryonic component of a compression in the plasma stalls at this radius as the photons decouple and free-stream out: the BAO peak.

BOSS analyses the distribution of matter by measuring the spectral "Lyman-alpha forest" of light from very distant quasars. This traces the distribution of clouds of neutral hydrogen at high redshift in which the BOSS team identified the BAO peak. So, we can find out how a physical length scale is changing with z and that gives us the Hubble parameter H(z)

A window of opportunity

"Quasar-Lyman-alpha Forest Cross-Correlation from BOSS DR11: Baryon Acoustic Oscillations" Andreu Font-Ribera et al, arXiv:1311.1767
and
"Baryon Acoustic Oscillations in the Lyα forest of BOSS DR11 quasars" Timothée Delubac et al, arXiv:1404.1801

both report consistent results. From the latter:

Requirements for more general forms of dark energy can be found by considering our measurement of $H(z)$, which, combined with the Friedman equation, determines the density of dark energy $\rho_{\rm de}(z)$. Assuming space to be flat and matter to be conserved, and neglecting the radiation density, we have

$$\frac{8\pi G}{3}\rho_{\rm de}(z) = H^2(z) - H_0^2\Omega_M(1+z)^3 \; . \tag{24}$$

$$\frac{\rho_{\rm de}(z=2.34)}{\rho_{\rm de}(z=0)} = -1.2 \pm 0.8 \; . \tag{26}$$

<u>not constant</u>
<u>at 2.5 (?) sigma</u>

There’s a lot of tension right now

e.g.

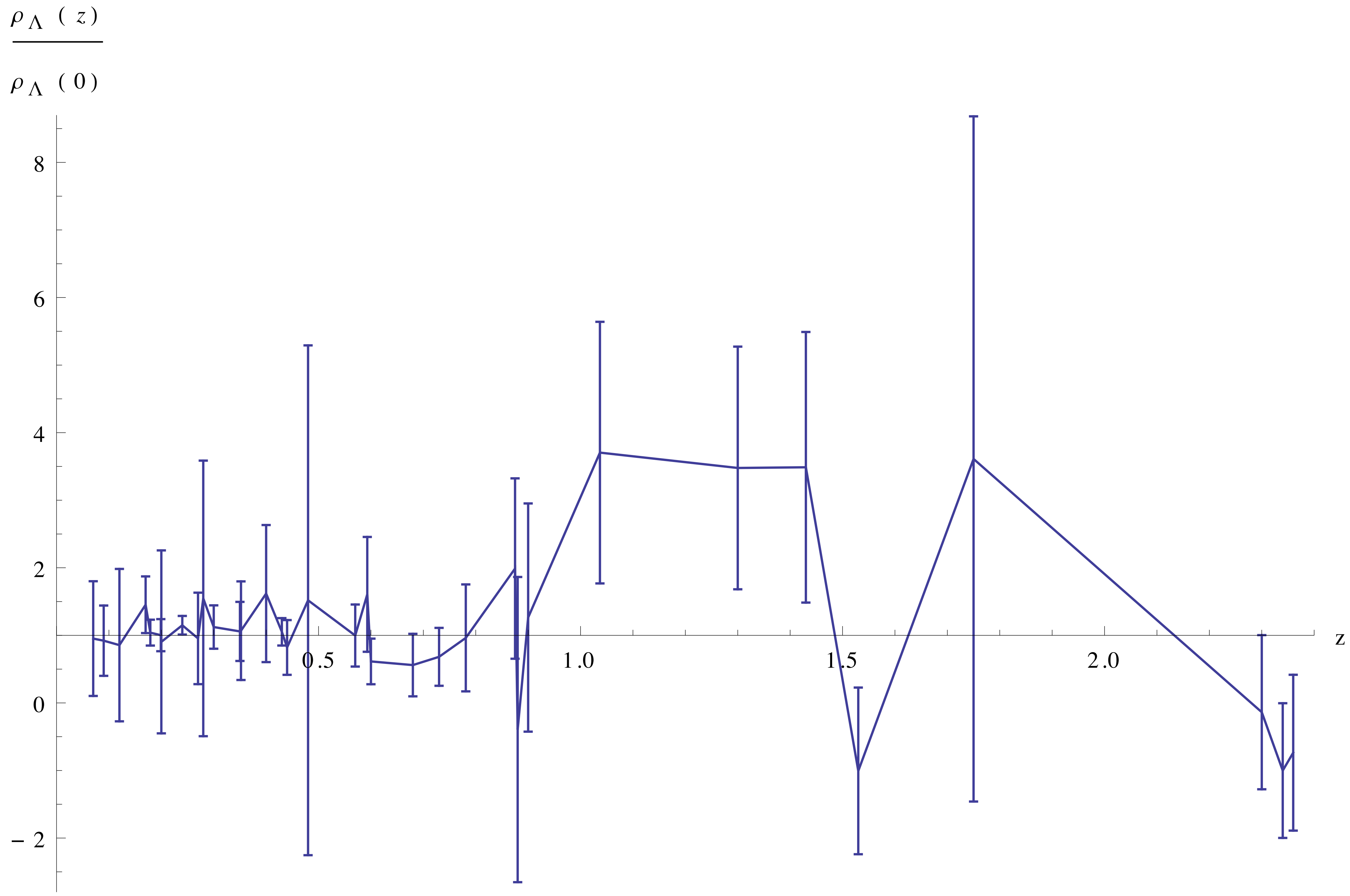
CDM: Neutrinos help reconcile Planck with the Local Universe,
Wyman, Rudd, Ali Vanderveld, & Hu
arXiv:1307.7715 : 19 occurrences of “tension” in 5 pages

“*Model independent evidence for dark energy evolution from baryon acoustic oscillations*” Sahni, Shafiello & Starobinski
arXiv:1406.2209 : 11 occurrences of “tension” in 4 pages

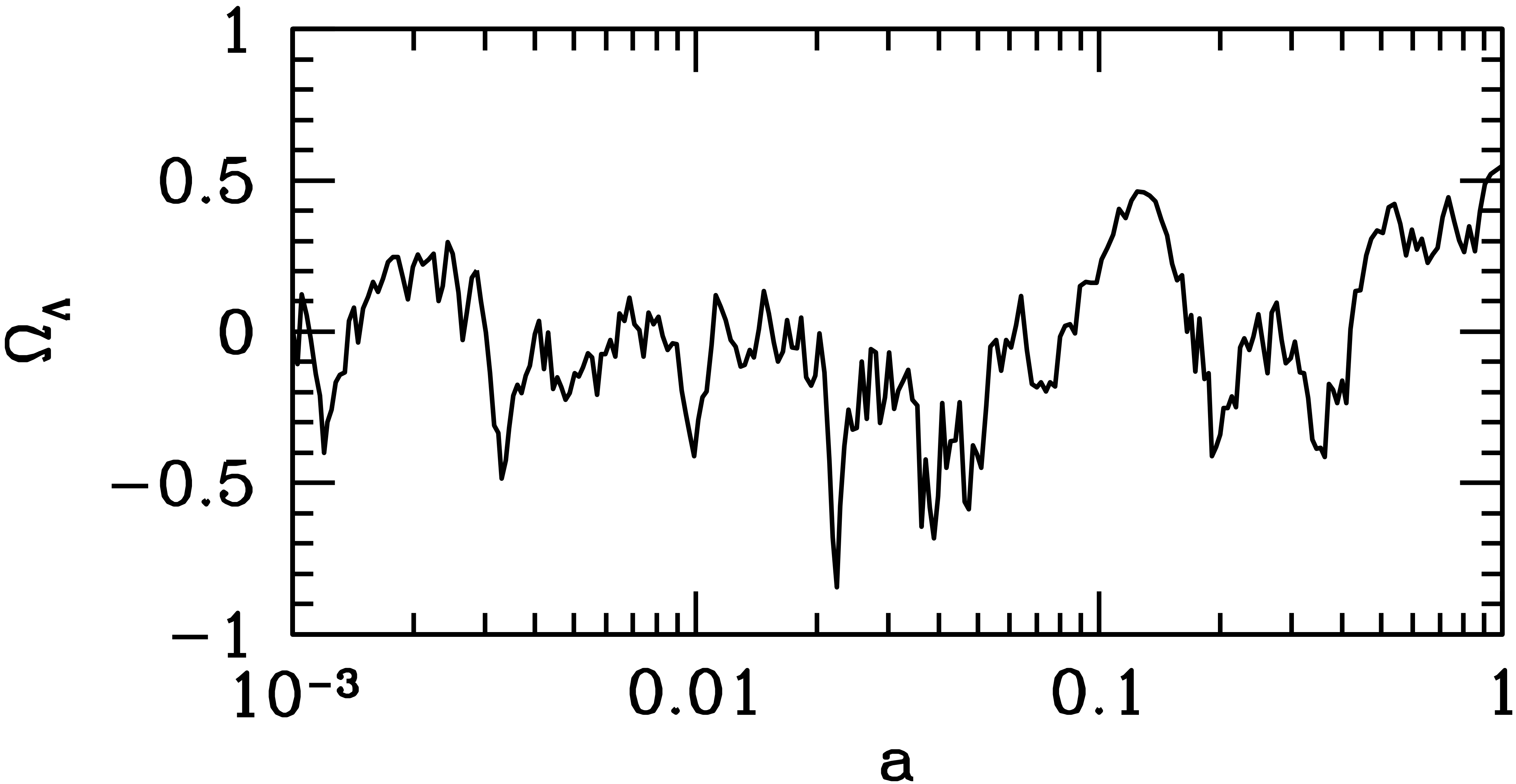
It’s a tension between early and late measurements of the Hubble parameter and the Lambda-CDM model. Is this the beginning of a period similar to that of the early 1990’s when the CDM(Lambda=0) cosmological model was crumbling?

Evidence of Fluctuating Lambda?

Data on H(z) collected from several studies in “Parameters of cosmological models and recent astronomic observations” **Sharov and Vorontsova arXiv:1407.5405** produce a plot like this

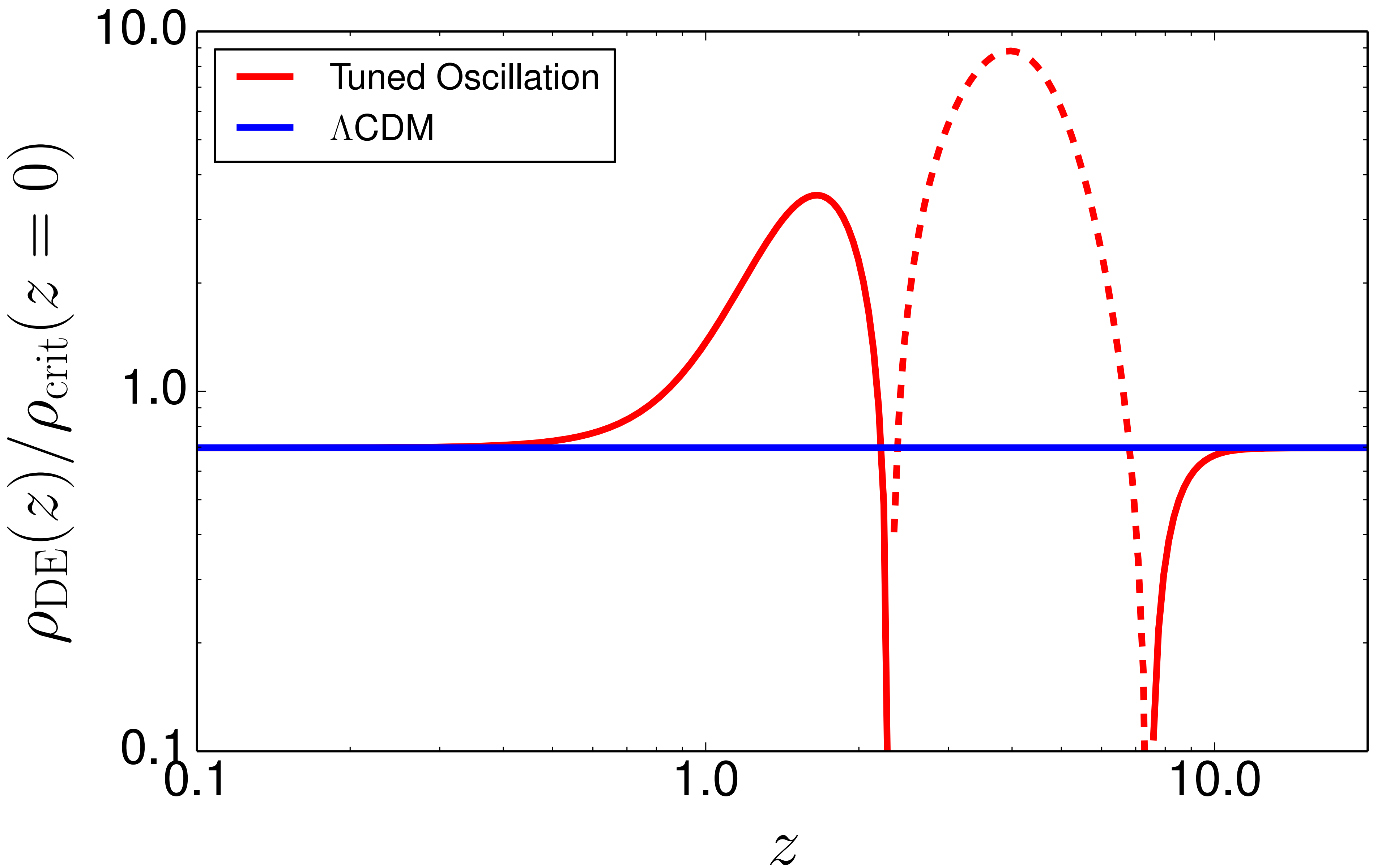


From the Everpresent Lambda paper



Fluctuating Lambda model (of a strange sort) in BOSS paper

"Cosmological implications of baryon acoustic oscillation (BAO) measurements"
http://arxiv.org/abs/1411.1074



Summary

- Causal sets are discrete spacetimes that are Lorentz invariant and **nonlocal** and provide a different cosmological paradigm to "add-a-scalar-field-with-such-and-such-a-potential" modelling.
- A large/small (dimensionless) number like Lambda(today) can only be **explained** by relating it to some other large/small number. Either a parameter, or a number that is **historically** large e.g. the age of the universe. The Causal set Lambda prediction is of the latter sort.
- The observational discovery of Lambda(today) ~ 10^{-120} verified this quantum gravity prediction: "Quantum Gravity bats 1000" (Sorkin).
- It's a nonlocal form of "modified gravity": inconsistent with GR on large scales.
- We may be entering a similar period of transition to a new "standard cosmological model" in which Lambda is not constant and was actually negative in the past. Then Quantum Gravity would maintain its perfect average!
- Need model independent observational results: H(z)
- Many challenges. Inhomogeneous model (Barrow, Zuntz pointed out pitfalls)? Can model be consistent with all data: nucleosynthesis, CMB, structure formation…….?