Dynamic Inconsistency, Commitment, and Welfare in Strategic Settings

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Abstract

The choices of a dynamically inconsistent individual depend on whether she commits to consumption ahead of time or chooses consumption in the moment. In individual-choice settings, it is normatively ambiguous whether such an individual's choices with commitment or in the moment are "better". This impasse can be overcome in settings in which dynamically inconsistent individuals interact strategically. Policy implications are discussed.

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1 Introduction

There is considerable evidence that a large fraction of individuals are dynamically inconsistent, i.e., that their choices between consumption streams depend on the timing of choice. For example, a typical pattern is that an individual choosing between (apple on Tuesday, good health on Wednesday) and (cake on Tuesday, poor health on Wednesday) would choose the former if she commits on Monday and the latter if she makes a choice in the moment on Tuesday.[1]

Dynamic inconsistency raises the following question: Choice under which timing is the "right" one? In individual-choice settings, if we respect the individual's choices, there is simply no answer and we must live with the normative ambiguity (Bernheim and Rangel (2009)).[2] The key insight in the current paper is that this impasse can be overcome once we consider settings in which dynamically inconsistent individuals interact strategically.

The main analysis is based on three games with the following common features: (i) there are two symmetric agents, (ii) in period 1, each agent i makes a costly investment, k_i, in (physical or human) capital, (iii) in period 2, output is produced via a production function that depends on k_1 and k_2, (iv) output is split between agents according to a given rule, (v) no uncertainty (other than uncertainty due to mixed strategies) resolves before period 2, and (vi) agents have β-δ preferences.[3]

The three games differ from each other as follows. In the no-commitment (NC)

[1]Some key references are Read and van Leeuwen (1998), Read et al. (1999), Augenblick et al. (2015), Carrera et al. (2020). For the purposes of the current paper, I take the evidence for dynamic inconsistency in the literature at face value. To be sure, there are methodological issues in this literature such as the presence of uncertainty, possible consumption substitution outside of the experiments, experimenter demand effects, interpretation of the evidence through the lens of a particular model, etc.

[2]Bernheim and Rangel (2009) does discuss possible justifications for dismissing for normative purposes some choices of an individual. However, none of these justifications apply to the contradictory choices considered in the current paper.

[3]The β-δ model was popularised by Laibson (1997) and O'Donoghue and Rabin (1999).

game, each agent i chooses k_i in period 1, i.e., in the moment. In the forced-commitment (FC) game, there is a period 0 in which each agent i must commit to k_i. In the optional-commitment (OC) game, there is also a period 0; each agent i commits in period 0 to a minimum investment (which can be arbitrarily low, so that commitment is effectively optional) and chooses k_i in period 1 subject to this constraint.

The key findings are the following. First, FC can dominate NC in the sense that each agent, both from the perspective of period 0 and from the perspective of period 1, prefers the equilibria of the FC game to the equilibria of the NC game. The reverse cannot occur except in a somewhat special case. FC can dominate NC because agents discount period-2 utility relative to period-1 utility less from the perspective of period 0 than from the perspective of period 1 and this can either (i) counteract inefficiently low investment that is related to a positive externality from how output is split or (ii) if investment is binary (i.e., either "high" or "low"), eliminate the possibility of coordination on an inferior equilibrium.

Second, if agents are sophisticated (i.e., are aware of their dynamic inconsistency), there is a certain equivalence between the FC and OC games (in a sense that will be made precise later) and, as a result, the comparisons between the FC game and the NC game continue to hold if we replace the FC game with the OC game.

The paper proceeds as follows. Section 2 formally introduces the three games and formalises the idea of one game dominating another. Sections 3 and 4 analyse the NC and FC games in the special cases of binary and continuous investments. Section 5 analyses the OC game. Section 6 discusses a crucial assumption in the analysis, namely that no uncertainty (other than uncertainty due to mixed strategies) resolves before period 2. Section 7 discusses policy implications in the context of gambling, recreational-drug use, addiction clinics, social media use, and saving for retirement.

Section 8 concludes.

2 General Set-Up

2.1 The Games

The main analysis is based on three games–the NC game, the FC game, and the OC game. All three games share the following features.

There are two agents, 1 and 2. All games are symmetric. In describing them, and in much of the paper, I will often adopt the perspective of agent 1. Analogous statements apply to agent 2.

Each game has a period 1 and a period 2. The FC and OC games also have a period 0 which will be described later.

In period 1, each agent i makes an investment in physical capital or human capital (such as her health or skills), $k_i \in K \subseteq [0, \omega]$, from her endowment ω and consumes $\omega - k_i$.

In period 2, production takes place via the production function $F : K \times K \to [0, \infty)$ which is symmetric (i.e., $F(k', k'') = F(k'', k')$) and strictly increasing in each argument. Then, if $k_1 \geq k_2$, agent 1 consumes $0.5F(k_2, k_2) + s(F(k_1, k_2) - F(k_2, k_2))$, where $0.5 \leq s \leq 1$. That is, agent 1 obtains half of the output that would have been produced if both agents had invested at the lower level k_2 and, on top of that, receives share s of the surplus output due to her higher investment. On the flip side, if $k_1 < k_2$, agent 1 receives $0.5F(k_1, k_1) + (1 - s)(F(k_1, k_2) - F(k_1, k_1))$. If $s = 1$, the agent making the higher investment reaps the whole surplus due to her higher investment; $s = 1/2$ corresponds to equal sharing of output. The latter case can also be interpreted as one in which the output is a public good produced through the technology $0.5F(k_1, k_2)$.

When output is a private good, the specification for how output is shared is admittedly ad hoc. Nevertheless, this specification accommodates in a simple, albeit crude, way (through the parameter s) a wide range of possibilities that result from different redistributive policies, property rights, market structures, etc.[4]

I allow F and s to be random. Importantly, I assume that any uncertainty about them resolves in period 2 (more on this in section 6). To avoid technical complications, I assume that the possible realisations of F and s are finitely many.

I will use "Self-0" and "Self-1" as shorthand for "agent 1 in period 0" and "agent 1 in period 1", respectively. Self-1 maximises the expectation of

$$U(k_1, k_2, \beta) = \begin{cases} u(\omega - k_1) + \beta\delta u\left(\frac{F(k_1,k_1)}{2} + (1-s)(F(k_1,k_2) - F(k_1,k_1))\right) & \text{if } k_1 < k_2 \\ u(\omega - k_1) + \beta\delta u\left(\frac{F(k_2,k_2)}{2} + s(F(k_1,k_2) - F(k_2,k_2))\right) & \text{if } k_1 \geq k_2 \end{cases}, \tag{1}$$

where $\delta > 0$, $0 < \beta < 1$, and $u : [0, \infty) \to \mathbb{R}$ is strictly increasing. Self-0 maximises the expectation of $U(k_1, k_2, 1)$. E will denote the expectations operator.

The three games differ from each other as follows. In the NC game, Self-1 decides how much to invest in period 1. In the FC game, there is also a period 0 in which Self-0 must commit to period-1 investment. In the FC and NC games, a profile of pure strategies will be denoted (k_1, k_2), where k_i is agent i's investment.

In the OC game, there is also a period 0 in which Self-0 commits to a minimum investment, $\underline{k}_1 \in K$. Self-1 then chooses $k_1 \geq \underline{k}_1$. She does so without any information about agent 2's decisions (more on this in section 5.3).

Note that, in applications, it is usually impractical for an agent to commit to a particular investment, k_i, because she can usually break the commitment in period 1

[4]There is also no compelling justification for $s \leq 1$ (e.g., the agent with the higher k_i might be able to expropriate or enslave the other agent), but allowing for $s > 1$ would complicate the analysis.

by investing more than k_i (unless k_i is the maximum possible investment). Therefore, I view the FC game (in which Self-0 commits to a particular investment) as more of a theoretical construct that will allow us to compare the NC and OC games.

2.2 Comparing Games

The following definition will play a central role.

Definition 1 *Fix an equilibrium concept and let $E^{NC} \neq \emptyset$ and $E^{FC} \neq \emptyset$ be the set of equilibria based on that concept in the NC game and FC game, respectively. FC dominates (respectively, strictly dominates) NC based on the given equilibrium concept if (i) both Self-0 and Self-1 weakly prefer any $e^{FC} \in E^{FC}$ to any $e^{NC} \in E^{NC}$ and (ii) for some (respectively, for any) $e^{FC} \in E^{FC}$ and $e^{NC} \in E^{NC}$, Self-0 or Self-1 strictly prefers e^{FC} to e^{NC}.*

NC dominating or strictly dominating FC is defined analogously. One can also similarly compare the NC and OC games as well as the FC and OC games.

In terms of equilibrium concepts, I will consider symmetric pure-strategy Nash equilibrium (NE), pure-strategy NE, symmetric NE, and NE as well as a refinement of these that will be relevant in the OC game.

3 Binary Investments

3.1 Set-Up

Assume binary investments: $K = \{L, H\}$, where $0 \leq L < H \leq \omega$. I make the normalisations $u(\omega - L) = 0$ and $u(\omega - H) = -1$.

The NC game is illustrated in Figure 1. Rows and columns correspond to the actions of agents 1 and 2, respectively. To save space, only the payoffs (in terms of

	L	H
H	$\beta\delta E\left(u\left(\frac{F(L,L)}{2}+s(F(H,L)-F(L,L))\right)\right)-1$	$\beta\delta E\left(u\left(\frac{F(H,H)}{2}\right)\right)-1$
L	$\beta\delta E\left(u\left(\frac{F(L,L)}{2}\right)\right)$	$\beta\delta E\left(u\left(\frac{F(L,L)}{2}+(1-s)(F(H,L)-F(L,L))\right)\right)$

Figure 1: Payoff matrix for the NC game with binary investments. Rows and columns correspond to the actions of agents 1 and 2, respectively. The entry in each cell is the payoff of agent 1. The payoffs of agent 2 can be inferred based on the symmetry of the game. The FC game is identical, but with 1 replacing β.

expected utility) of agent 1 are shown. The payoffs of agent 2 can be inferred based on the symmetry of the game. The payoff matrix for the FC game is identical to that in Figure 1, but with 1 replacing β.

Both in the NC and FC games, I rule out the knife-edge cases in which agent 1 is indifferent between playing H and L given some pure strategy of agent 2. This reduces significantly the number of cases that need to be considered when characterising and comparing the Nash equilibria of the two games.

3.2 Comparing the FC and NC Games

Proposition 1 *FC dominates NC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, and NE if and only if one of the following cases holds.*[5]

Case a): All of the following hold.

$$E\left(u\left(\frac{F(L,L)}{2}+s(F(H,L)-F(L,L))\right)-u\left(\frac{F(L,L)}{2}\right)\right)<\frac{1}{\delta} \tag{2}$$

$$\frac{1}{\delta}<E\left(u\left(\frac{F(H,H)}{2}\right)-u\left(\frac{F(L,L)}{2}+(1-s)(F(H,L)-F(L,L))\right)\right)<\frac{1}{\beta\delta} \tag{3}$$

$$E\left(u\left(\frac{F(L,L)}{2}+(1-s)(F(H,L)-F(L,L))\right)-u\left(\frac{F(L,L)}{2}\right)\right)\geq\frac{1-\beta}{\beta\delta} \tag{4}$$

[5]There are other cases in which FC dominates NC (sometimes, strictly so) based on one or more of these four equilibrium concepts, but not based on NE. I have not found these cases to be particularly instructive. At any rate, they can be seen in the proof.

Case b): All of the following hold.

$$\frac{1}{\delta} < E\left(u\left(\frac{F(L,L)}{2} + s(F(H,L) - F(L,L))\right) - u\left(\frac{F(L,L)}{2}\right)\right) < \frac{1}{\beta\delta} \tag{5}$$

$$\frac{1}{\delta} < E\left(u\left(\frac{F(H,H)}{2}\right) - u\left(\frac{F(L,L)}{2} + (1-s)(F(H,L) - F(L,L))\right)\right) < \frac{1}{\beta\delta} \tag{6}$$

$$E\left(u\left(\frac{F(H,H)}{2}\right) - u\left(\frac{F(L,L)}{2}\right)\right) \geq \frac{1}{\beta\delta} \tag{7}$$

Case c): All of the following hold.

$$\frac{1}{\delta} < E\left(u\left(\frac{F(L,L)}{2} + s(F(H,L) - F(L,L))\right) - u\left(\frac{F(L,L)}{2}\right)\right) < \frac{1}{\beta\delta} \tag{8}$$

$$E\left(u\left(\frac{F(H,H)}{2}\right) - u\left(\frac{F(L,L)}{2} + (1-s)(F(H,L) - F(L,L))\right)\right) > \frac{1}{\beta\delta} \tag{9}$$

Moreover, each case a)-c) is nonvacuous, i.e., it occurs for some primitives of the model.

Here, I demonstrate, for each case a)-c), that FC dominates NC based on symmetric pure-strategy NE and pure-strategy NE. As far as I can tell, this demonstration contains the key insights from the full proof.[6]

Consider case a). In the NC game, (2) and the right inequality in (3) imply that L is a strictly dominant strategy so that (L,L) is the unique NE. In the FC game, (2) implies that L is the best-response to L and the left inequality in (3) ensures that H is the best-response to H so that (L,L) and (H,H) are the pure-strategy Nash equilibria. Finally, the left inequality in (3) and condition (4) imply $\beta\delta E\left(u\left(\frac{F(H,H)}{2}\right)\right) - 1 > \beta\delta E\left(u\left(\frac{F(L,L)}{2}\right)\right)$,[7] so that that Self-0 and, hence, Self-1 strictly prefer (H,H) to (L,L).

[6] All proofs that are not given in the main text are in the appendix.

[7] To see this, add to each side of $E\left(u\left(\frac{F(H,H)}{2}\right) - u\left(\frac{F(L,L)}{2} + (1-s)(F(H,L) - F(L,L))\right)\right) > \frac{1}{\delta}$ the respective side of $E\left(u\left(\frac{F(L,L)}{2} + (1-s)(F(H,L) - F(L,L))\right) - u\left(\frac{F(L,L)}{2}\right)\right) \geq \frac{1-\beta}{\beta\delta}$.

	NC game	FC game
Case a)	$\{(L, L)\}$	$\{(L, L), (H, H)\}$
Case b)	$\{(L, L)\}$	$\{(H, H)\}$
Case c)	$\{(L, L), (H, H)\}$	$\{(H, H)\}$

Table 1: Pure-strategy Nash equilibria of the NC and FC games in cases a)-c).

Now, let us turn our attention to case b). In the NC game, the right inequalities in (5) and (6) imply that L is a strictly dominant strategy so that (L, L) is the unique NE. In the FC game, the left inequalities in (5) and (6) imply that H is a strictly dominant strategy so that (H, H) is the unique NE. Finally, (7) ensures that Self-0 and, hence, Self-1 prefer (H, H) to (L, L) (Self-0 strictly so).[8]

Next, consider case c). In the NC game, the right inequality in (8) implies that L is the best response to L and (9) implies that H is the best response to H so that the pure-strategy Nash equilibria are (L, L) and (H, H). In the FC game, the left inequality in (8) and condition (9) imply that H is a strictly dominant strategy so that (H, H) is the unique NE. Finally, (9) implies $\beta\delta E\left(u\left(\frac{F(H,H)}{2}\right)\right) - 1 > \beta\delta E\left(u\left(\frac{F(L,L)}{2}\right)\right)$ so that Self-0 and, hence, Self-1 strictly prefer (H, H) to (L, L).

Table 1 summarises the pure-strategy Nash equilibria of the two games.

The previous paragraphs demonstrate in a rather mechanical fashion that, in cases a)-c), FC dominates NC based on symmetric pure-strategy NE and pure-strategy NE. The economic logic is as follows.

First, in the NC game, in all three cases the (L, L) NE is inferior to (H, H) according to both agents in period 1. In case c), this inefficiency occurs because of a coordination failure. In cases a) and b), this inefficiency is related to a positive externality.[9] In particular, these cases imply that Self-1 strictly prefers (L, H) to

[8]In case b), FC *strictly* dominates NC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, and NE.

[9]In the knife-edge subcase of case b) in which (7) binds, agents in period 1 are indifferent between (L, L) and (H, H) so that there is no inefficiency in the NC game.

(H, H) (recall that L is a dominant strategy) and prefers (H, H) to (L, L). Hence, Self-1 strictly prefers (L, H) to (L, L) and $s < 1$ with positive probability. Thus, at the (L, L) NE, Self-1 would experience a positive externality if agent 2 were to increase her investment to H.

Second, given that Self-0 discounts period-2 utility relative to period-1 utility less than Self-1, Self-0 strictly prefers (H, H) to (L, L).

Third, the fact that Self-0 discounts period-2 utility relative to period-1 utility less than Self-1 counteracts the inefficiency in the NC game by (i) making H a strictly dominant strategy in the FC game, thus eliminating the (L, L) NE of the NC game and making (H, H) the unique NE (this occurs in cases b) and c)) or (ii) making H a best response to H (but not to L) in the FC game, thus making (H, H) a NE in addition to the unique (L, L) NE of the NC game (this occurs in case a)).[10]

Can NC dominate FC? The following proposition addresses this question.

Proposition 2

1) *NC never dominates FC based on symmetric pure-strategy NE, pure-strategy NE, or NE.*

2) *NC can strictly dominate FC based on symmetric NE. The following is a necessary condition:*

$$E\left(u\left(F(H, L) - \frac{F(L, L)}{2}\right)\right) > E\left(u\left(\frac{F(H, H)}{2}\right)\right). \tag{10}$$

Observe the following. First, any domination of NC over FC occurs in a weak sense. In particular, we need to disregard asymmetric Nash equilibria (which are all

[10]It would be useful to describe in intuitive terms the restrictions on the primitives of the model imposed by cases a)-c). However, this is difficult. For example, to show that each case a)-c) is nonvacuous, the proof of Proposition 1 considers a special case in which cases a)-c) can be depicted on a two-dimensional graph. Even then, no major insights emerge.

in pure strategies, as it turns out) and the domination occurs based on a comparison of a mixed-strategy NE in one of the games to a (pure-strategy or mixed-strategy) NE in the other game.

Second, condition (10) is ruled out if (i) F is supermodular with certainty[11] or (ii) u is linear and $E\left(F(H,H)-F(L,H)\right) \geq E\left(F(H,L)-F(L,L)\right)$ (i.e., loosely speaking, F is supermodular in expectation). Given that supermodularity is probably the empirically plausible case, condition (10) appears to be somewhat special.

Third, as we will see in the next section, the possibility of NC dominating FC is an artefact of binary investments.

4 Continuous Investments

Some investments may be usefully viewed as binary. Still, in many settings it may be more natural to view investments as continuous. Therefore, I now consider the case with continuous investments.

4.1 Set-Up

I make the following assumptions in addition to those in section 2. First, the set of possible investments is $K = (0, \omega)$.

Second, the production function is of the form $F(k_1, k_2) = f(k_1, k_2)^{\gamma}$. The function f is homogenous of degree 1, symmetric, and continuously differentiable. Its partial with respect to k_i, f_i, is strictly positive and strictly decreasing in k_i and satisfies $\lim_{k_i \downarrow 0} f_i(k_1, k_2) = \infty$. The parameter γ $(0 < \gamma \leq 1)$ captures returns to scale, where $\gamma < 1$ and $\gamma = 1$ correspond to decreasing and constant returns to scale, respectively.[12]

[11] If F is supermodular with probability 1, $F(H,L) - \frac{F(L,L)}{2} \leq \frac{F(H,H)}{2}$ and, hence, $u\left(F(H,L) - \frac{F(L,L)}{2}\right) \leq u\left(\frac{F(H,H)}{2}\right)$ always holds.

[12] I rule out increasing returns to scale ($\gamma > 1$) because this ensures the sufficiency of the first-order conditions employed in the proofs.

Let $A > 0$ be the constant defined by $f(k, k) = Ak$.[13] f (and, hence, A) and γ can be random. I assume that they are independent of s. This will aid the exposition and will also play a role in the proof of part 6) of Lemma 1 below.

Third, u is continuously differentiable on $(0, \infty)$; its derivative, u', is strictly positive and strictly decreasing and satisfies the Inada condition $\lim_{x \downarrow 0} u'(x) = \infty$. Let U_i denote the partial derivative of U with respect to k_i.

4.2 Comparing the FC and NC Games

The following lemma lays the groundwork for the main result in Proposition 3.

Lemma 1

1) *In the NC game, there exists a unique symmetric pure-strategy NE. In it, each agent's investment, k^{NC}, is the unique value of k solving*

$$\beta E(s) = \frac{u'(\omega - k)}{\delta E\left(\frac{\gamma A^{\gamma}}{2k^{1-\gamma}} u'\left(\frac{A^{\gamma} k^{\gamma}}{2}\right)\right)}. \tag{11}$$

2) *In the FC game, there exists a unique symmetric pure-strategy NE. In it, each agent's investment, k^{FC}, is the unique value of k solving*

$$E(s) = \frac{u'(\omega - k)}{\delta E\left(\frac{\gamma A^{\gamma}}{2k^{1-\gamma}} u'\left(\frac{A^{\gamma} k^{\gamma}}{2}\right)\right)}. \tag{12}$$

Moreover, $k^{FC} > k^{NC}$.

3) *$E(U(k, k, \beta))$ is strictly increasing in k on $(0, \hat{k}^{NC}]$, where $\hat{k}^{NC}$ is the unique*

[13] Such a constant exists and is unique given that f is homogenous of degree 1 and strictly increasing.

value of k solving

$$\beta = \frac{u'(\omega - k)}{\delta E\left(\frac{\gamma A^{\gamma}}{2k^{1-\gamma}} u'\left(\frac{A^{\gamma} k^{\gamma}}{2}\right)\right)}. \tag{13}$$

Moreover, $\hat{k}^{NC} \geq k^{NC}$, the inequality being strict if and only if $E(s) < 1$.

4) *$E(U(k,k,1))$ is strictly increasing in k on $(0, \hat{k}^{FC}]$, where $\hat{k}^{FC} \geq k^{FC}$, the inequality being strict if and only if $E(s) < 1$.*

5) *If $E(s) < 1$, then $E(U_2(k,k,\beta)) > 0$ and $E(U_2(k,k,1)) > 0$.*

6) *If f is supermodular with probability 1, there is no asymmetric pure-strategy NE in the FC and NC games.*[14]

The basic logic for part 1) is that any symmetric pure-strategy NE, (k,k), of the NC game must satisfy the first-order condition $E(U_1(k,k,\beta)) = 0$, which is precisely (11). Figure 2 illustrates the determination of k^{NC}.

The logic for part 2) is analogous. That $k^{FC} > k^{NC}$ follows from the fact that the left-hand side of (12) is just a scaled-up (by a factor $1/\beta$) version of the left-hand side of (11). See Figure 2.

Turning to part 3), observe that equation (13) is just the first-order condition for the problem $\max_k U(k,k,\beta)$. That $\hat{k}^{NC} > k^{NC}$ if $E(s) < 1$ and $\hat{k}^{NC} = k^{NC}$ if $E(s) = 1$ follows from the fact that the left-hand side of (13) is just a scaled-up by a factor $1/E(s)$ version of the left-hand side of (11). See Figure 2. The figure is drawn for the case $E(s) < \beta$ in which $k^{FC} < \hat{k}^{NC}$. The logic behind part 4) is similar.

[14] Two examples of a supermodular f (that also satisfies the other assumptions imposed on f further above) are the Cobb-Douglas form $f(k_1,k_2) = A(k_1 k_2)^{1/2}$ and the constant-elasticity-of-substitution form $f(k_1,k_2) = A(k_1^{\rho} + k_2^{\rho})^{1/\rho}$ with $0 \neq \rho < 1$. Supermodularity follows because the derivatives of these functions with respect to k_1 ($0.5Ak_2^{1/2}/k_1^{1/2}$ and $A(k_1^{\rho} + k_2^{\rho})^{1/\rho - 1}/k_1^{1-\rho}$, respectively) are increasing in k_2. Note that f being supermodular is a weaker assumption than F being supermodular because $\frac{\partial F}{\partial k_1}(k_1,k_2) = \frac{\gamma f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}}$ being weakly increasing in k_2 implies that $f_1(k_1,k_2)$ is weakly increasing in k_2.

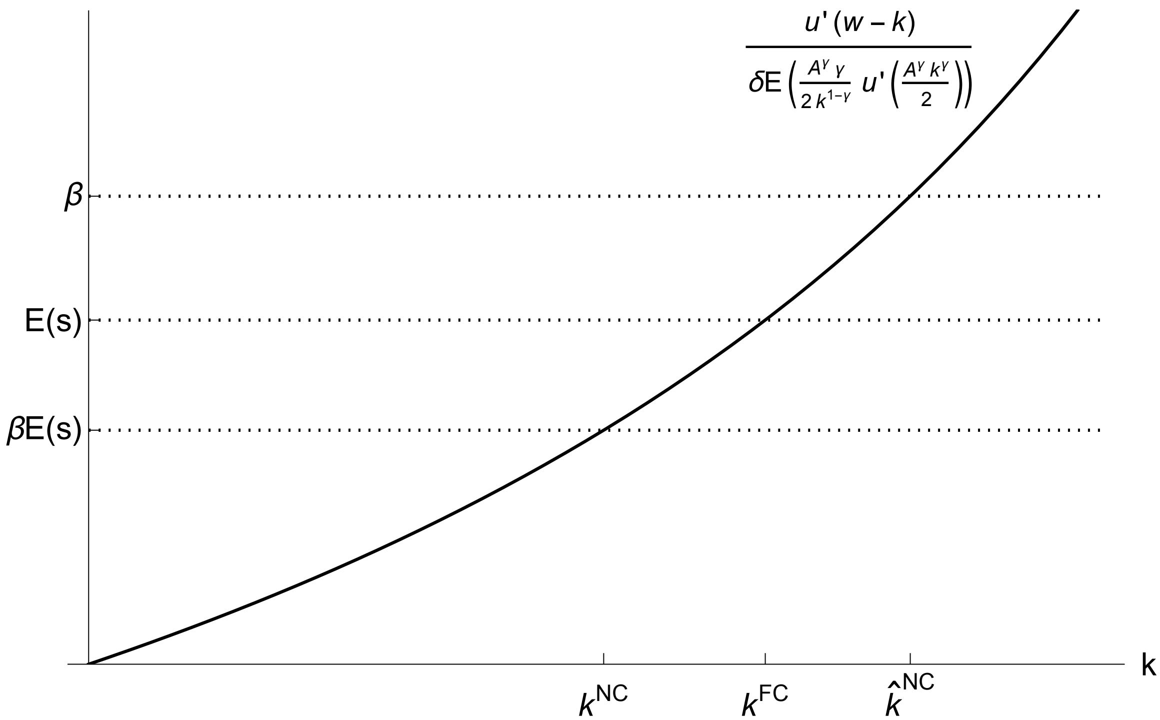

Figure 2: Determination of k^{NC}, k^{FC}, and $\hat{k}^{NC}$. The figure assumes $E(s) < \beta$.

$\hat{k}^{NC} > k^{NC}$ (respectively, $\hat{k}^{FC} > k^{FC}$) when $E(s) < 1$ means that investment in the symmetric pure-strategy NE of the NC (respectively, FC) game is too low according to Self-1 (respectively, Self-0). Part 5) clarifies the reason for this by revealing that, if $E(s) < 1$, agent 1 experiences a positive externality from investment by agent 2 at the symmetric pure-strategy NE of the NC and FC games.

I have no good intuition for part 6).

The main result of this section can now be stated.

Proposition 3

1) $E(U(k^{FC}, k^{FC}, 1)) > E(U(k^{NC}, k^{NC}, 1))$.

2) $E(U(k^{FC}, k^{FC}, \beta)) \geq E(U(k^{NC}, k^{NC}, \beta))$ *if* $\beta \geq E(s)$.

Proof of part 1): Part 1) holds because $k^{NC} < k^{FC} \leq \hat{k}^{FC}$ and $E(U(k, k, 1))$ is strictly increasing in k on $(0, \hat{k}^{FC}]$ (see parts 2) and 4) of Lemma 1). Q.E.D.

The intuition is the following. First, because of the externality, investment in the symmetric pure-strategy NE of the FC game is too low according to Self-0 ($k^{FC} \leq \hat{k}^{FC}$). Second, because of the dynamic inconsistency, investment in the symmetric pure-strategy NE of the NC game is even lower ($k^{NC} < k^{FC}$).

Proof of part 2): Part 2) holds because $k^{NC} < k^{FC}$ (see part 2) of Lemma 1), $k^{FC} \leq \hat{k}^{NC}$ when $\beta \geq E(s)$ (see Figure 2), and $E(U(k, k, \beta))$ is strictly increasing in k on $(0, \hat{k}^{NC}]$ (see part 3) of Lemma 1). Q.E.D.

The intuition is the following. First, because of the externality, investment in the symmetric pure-strategy NE of the NC game is too low according to Self-1 ($k^{NC} < \hat{k}^{NC}$). Second, because of the dynamic inconsistency, investment in the symmetric pure-strategy NE of the FC game is higher ($k^{NC} < k^{FC}$), but not too high according to Self-1 (because, loosely speaking, the dynamic inconsistency is less severe than the externality).

The significance of Proposition 3 is the following. Part 1) means that NC never dominates FC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE. Taken together, parts 1) and 2) mean that FC can strictly dominate NC based on symmetric pure-strategy NE and, if f is supermodular with certainty, based on pure-strategy NE.[15]

[15] Note that $E(s) \leq \beta$ is merely a sufficient condition for $E(U(k^{FC}, k^{FC}, \beta)) \geq E(U(k^{NC}, k^{NC}, \beta))$. I have not been able to characterise the latter inequality through necessary and sufficient conditions.

4.3 Comparing the Cases with Binary and Continuous Investments

Both the case with binary and the case with continuous investments convey the message that FC can dominate NC based on various equilibrium concepts, but not the other way around except as in part 2) of Proposition 2.

A notable difference is that, with continuous investments, $E(s) < 1$ (i.e., the presence of a positive externality) is necessary for FC to dominate NC.[16] In contrast, with binary investments, FC can dominate NC even for $E(s) = 1$ because of a possible coordination failure in the NC game (recall case c)).[17]

5 The OC Game

5.1 Comparing the OC Game to the FC and NC Games

In the FC and NC games, a strategy of agent i is a probability measure, σ_i, on K.

In the OC game, agent i in period 0 and agent i in period 1 are treated as separate players. A strategy of agent i in period 0 is a probability measure, σ_{i0}, on K that specifies how i chooses in period 0 to what minimum period-1 investment to commit. A strategy of agent i in period 1, σ_{i1}, consists of a collection $\{\sigma_{i1\underline{k}}\}_{\underline{k}\in K}$, where $\sigma_{i1\underline{k}}$ is a probability measure on $\{k_i \in K | k_i \geq \underline{k}\}$ that specifies how i in period 1 chooses investment after she committed to minimum investment $\underline{k}$. Given σ_{i0} and σ_{i1}, let $\pi_{\sigma_{i0},\sigma_{i1}}$ denote the probability measure on K (i.e., on agent i's investment) that is induced by σ_{i0} and σ_{i1}.

The analysis of the OC game will be based on the following refinement of NE.

[16] Indeed, when $E(s) = 1$, it follows from (11) and (13) that $k^{NC} = \hat{k}^{NC}$, so that $E(U(k^{NC}, k^{NC}, \beta)) > E(U(k^{FC}, k^{FC}, \beta))$.

[17] The proof of Proposition 1 shows that case c) is nonvacuous even for $E(s) = 1$.

Definition 2 $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$ *is a Perfect NE (PNE) of the OC game if it is a NE such that, for each* $i \in \{1, 2\}$ *and each* $\underline{k} \in K$, $\sigma_{i1\underline{k}}$ *is a best-response for agent* i *in period 1 to* $(\sigma_{j0}, \sigma_{j1})$, *where* $j \neq i$.

This refinement is based on the compelling idea that agent i must have the same beliefs about agent j's behaviour after two different $\underline{k}_i'$ and $\underline{k}_i''$ (i.e., after two different commitments that i made in period 0), even if $\underline{k}_i'$ or $\underline{k}_i''$ is off the equilibrium path. In the FC and NC games, I define PNE to be the same as NE.

Proposition 4 *In the settings with binary and continuous investments in sections 3 and 4, the following hold.*

$$\{(\sigma_1, \sigma_2) | (\sigma_1, \sigma_2) \text{ is a symmetric pure-strategy PNE of the FC game}\} = \{(\pi_{\sigma_{10},\sigma_{11}}, \pi_{\sigma_{20},\sigma_{21}}) | (\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21}) \text{ is a symmetric pure-strategy PNE of the OC game}\} \quad (14)$$

$$\{(\sigma_1, \sigma_2) | (\sigma_1, \sigma_2) \text{ is a pure-strategy PNE of the FC game}\} = \{(\pi_{\sigma_{10},\sigma_{11}}, \pi_{\sigma_{20},\sigma_{21}}) | (\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21}) \text{ is a pure-strategy PNE of the OC game}\}. \quad (15)$$

Moreover, in the settings with binary investments in section 3, each of these equalities continues to hold if we remove "pure-strategy" from both sides.[18]

Thus, there is an equivalence between the FC and OC games in terms of their symmetric pure-strategy perfect Nash equilibria and pure-strategy perfect Nash equilibria as well as, with binary investments, symmetric perfect Nash equilibria and perfect Nash equilibria. This implies that the comparisons between the FC and NC games made in sections 3 and 4 continue to apply if the FC game is replaced with the OC game (and PNE is replaced with NE).

[18] A strategy profile of the OC game, $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$, is symmetric if $\sigma_{10} = \sigma_{20}$ and $\sigma_{11} = \sigma_{21}$.

The intuition for Proposition 4 is the following. In the OC game, given agent 2's behaviour, suppose that some k_1 is an optimal investment for Self-0. In that case, given that Self-0 discounts period-2 utility relative to period-1 utility less than Self-1, any k_1' that is optimal for Self-1 must satisfy $k_1' \leq k_1$. Thus, Self-0 can commit to k_1 as the minimum period-1 investment and be sure that Self-1 will choose at the lower bound of her constraint set, namely k_1. Hence, in a PNE of the OC game, Self-1's behaviour does not impose a constraint on Self-0. This is what makes the OC game similar to the FC game.

5.2 Naiveté

The analysis of the OC game above implicitly assumes that each agent is sophisticated, i.e., that Self-0 correctly anticipates the behaviour of Self-1. If agents are naive, it is likely that they would not commit (or, within the formalism of the OC game, would commit to a very low minimum investment) so that the OC game is likely to end up being equivalent to the NC game. Still, the OC game should compare to the NC game in the same way in which the FC game compares to the NC game as long as, in the OC game, (i) agents do not commit (so that they are effectively playing the NC game) if they are naive but (ii) there is some chance that agents are sophisticated (in which case, the OC game is equivalent in the sense of Proposition 4 to the FC game).[19]

5.3 Information Available to Self-1

In the OC game, Self-1 chooses k_1 without any information about agent 2's decisions. This assumption seems like the most natural benchmark for the following reasons. First, in many applications people make investments repeatedly over long periods of

[19] I am ignoring the more complicated possibilities that (a) one agent is sophisticated and the other is naive or (b) agents are partially naive.

time (see the examples in section 7) in which case it makes sense to view k_i as an individual's cumulative investment (because society's production function presumably depends on cumulative investments). Second, as argued in section 7, it often makes sense to view k_i as an individual's cumulative investment across different domains (and at different times). Third, in reality opportunities to commit and invest may arise at exogenously determined points in time that differ across individuals so that, in the window of time between the opportunity to commit and the opportunity to invest for a given individual, only a small fraction of the population may be facing similar decisions. As a result of all this, an individual at the time of investment (Self-1) is unlikely to know much more than her earlier self at the time of commitment (Self-0) knew about others' overall investment.

6 Uncertainty Resolving between Periods 0 and 1

In the NC, FC, and OC games, there is no uncertainty that resolves between periods 0 and 1 (except, possibly, for uncertainty due to Self-0 playing a mixed strategy in the OC game). In the presence of such uncertainty, the analysis would need to be substantially modified both when state-contingent commitment is and isn't available in period 0.

When state-contingent commitment isn't available, there is an option value to waiting until period 1 to make the investment decision (because one can adjust investment in response to new information). This reduces the scope for FC to dominate NC, expands the scope for NC to dominate FC, and breaks the equivalence (in the sense of Proposition 4) between the FC and OC games.

The current analysis also needs to be modified if state-contingent commitment is available. For example, a (for simplicity, pure-strategy) NE of the FC game would consist of functions $(k_1^{FC}(\theta), k_2^{FC}(\theta))$ that depend on any information, θ, received between periods 0 and 1; given θ, a (for simplicity, pure-strategy) NE of the NC game would be a tuple $(k_1^{NC}(\theta), k_2^{NC}(\theta))$. Thus, any comparison between the FC and NC games in terms of domination based on pure-strategy NE would have to consider how Self-0 and Self-1 evaluate $(k_1^{FC}(\cdot), k_2^{FC}(\cdot))$ and $(k_1^{NC}(\cdot), k_2^{NC}(\cdot))$. Such a comparison is likely to be harder to analyse.

7 Policy Implications

The analysis in the current paper provides an argument in support of the government implementing OC directly, requiring companies to provide OC, or providing the legal framework that enables companies to voluntarily provide OC to customers or employees without fear of litigation from customers or employees who wish to break their commitments. This argument is only relevant in applications in which it is unlikely that substantial uncertainty will resolve between the time of committing and the time of making the actual investment. Promising candidates for such applications are ones in which commitment needs to be made only a short time in advance.[20] With this in mind, consider the following tentative policy proposals.

1) Casinos or sports betting companies are required to maintain and enforce a centralised list of people who are not allowed to gamble.[21] Individuals can put themselves on the list any time they like. They can also remove themselves from the list, but only with, say, a week's notice.[22]

[20] How much time, τ, in advance does commitment need to be made? Through the lens of the β-δ model, we can think of τ as the maximum amount of time such that consumption between the current moment and τ units of time into the future can be counted as current consumption (so that utility from it does not get discounted by β). Empirically, it is unclear what is a good estimate of τ. However, at least in some settings, τ can be very low, on the order of twenty minutes (see McClure et al. (2007)). See section 4.1 in Ericson and Laibson (2018) for further discussion of this issue.

[21] Lists of this sort apparently exist for casinos in some states in the United States.

[22] In light of footnote 20, the suggested week's notice should be viewed merely as tentative. A similar remark applies to the other policy proposals as well.

2) Similarly to the previous example, countries in which recreational drugs are legal can require sellers to maintain and enforce a centralised list of people who are not allowed to buy drugs.

3) Individuals can voluntarily sign themselves into an alcohol or drug addiction clinic. They can leave, but only after, say, a three days' notice. The government provides the legal basis that allows clinics to enforce the notice period.[23]

4) Operating systems are required to provide apps that allow people to voluntarily limit their daily use of social media. The set limit can be changed, but only, say, for the next day. Importantly, the limits should not be easy to override in contrast with the flexible limits in existing tools such as Android's Digital Wellbeing and iOS's Screen Time.[24]

5) Individuals' are allowed to withdraw funds from their defined contribution retirement accounts. However, they are given the option to commit not to do so unless they give, say, a month's notice.

One can also imagine a version of these proposals in which commitment is the default. For example, all individuals are automatically on the list of people who cannot buy recreational drugs and can take themselves off the list with, say, a week's notice. Or, the default is that individuals cannot withdraw funds from their defined contribution retirement accounts, but are allowed to do so, say, with a month's notice. This kind of default seems especially appealing if we think there is a good chance people would not commit because of naiveté.

Some subtleties are worth noting regarding how these applications relate to the theory in the current paper. First, the theory says that OC dominates NC *for some*

[23]The notice period may need to be shorter for first-time enrollees as they may face more uncertainty about the difficulty of withdrawal.

[24]Hunt et al. (2022) show in a field experiment that such limits reduce screen time on smartphones.

primitives of the model. Whether the primitives in any given application or, under the interpretation in the next paragraph, across a range of applications are such that OC dominates NC is probably hard to establish. Having said that, one could argue less formally that, even if we are not certain that OC dominates NC, the fact that OC can dominate NC, but not the other way around (except in a somewhat special case), is sufficient to endorse OC.

Second, the mapping between the theory and individual applications can often be problematic because, in any given application, the connection between one's future consumption and others' investments can be rather loose. For example, the connection is probably quite loose between the future consumption of a gambler and the decisions by other gamblers about how much money to spend on gambling (as opposed to on investments in their physical or human capital). To put it more formally, individual applications may correspond to the special case of the theory in which (i) $s = 1$ and $F(k_1, k_2) = h(k_1) + h(k_2)$ for some $h(\cdot)$ and (ii) as a result, $U(k_1, k_2, \beta)$ and $U(k_1, k_2, 1)$ are independent of k_2 so that we are in effect back to individual choice. Nevertheless, the theory here may still be relevant if we think of k_i as representing an individual's cumulative investment across many different applications so that a substantial connection between one's future consumption and others' investments becomes more plausible.

8 Concluding Remarks

The main message of the paper is that OC can dominate NC based on various equilibrium concepts, but not the other way around except in a somewhat special case. I discussed some tentative policy implications in the context of gambling, recreational-drug use, addiction clinics, social media use, and saving for retirement.

Admittedly, the analysis is highly stylised. Notably, it assumes two agents, dynamic inconsistency, symmetric games, one-shot investment, no uncertainty that resolves between periods 0 and 1, all-or-nothing commitment (as opposed to commitment that can be broken at some cost), and (a nontrivial chance that agents exhibit) sophistication about the dynamic inconsistency.

References

Allcott, Hunt, Matthew Gentzkow, and Lena Song. 2022. “Digital addiction.” American Economic Review 112 (7): 2424-63.

Augenblick, Ned, Muriel Niederle, and Charles Sprenger. 2015. “Working over Time: Dynamic Inconsistency in Real Effort Tasks.” The Quarterly Journal of Economics, 1067-1115.

Bernheim, B. Douglas, and Antonio Rangel. 2009. ”Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics.” The Quarterly Journal of Economics 124, no. 1: 51-104.

Carrera, Mariana, Heather Royer, Mark Stehr, Justin Sydnor, and Dmitry Taubinsky. 2022. “Who chooses commitment? Evidence and welfare implications.” The Review of Economic Studies 89, no. 3: 1205-1244.

Ericson, Keith Marzilli and David Laibson. 2018. “Intertemporal Choice.” NBER working paper 25358.

Laibson, David. 1997 “Golden Eggs and Hyperbolic Discounting.” Qutarterly Journal of Economics, 112, 443-477.

McClure, S.M., Ericson, K.M., Laibson, D.I., Loewenstein, G., Cohen, J.D. 2007. "Time Discounting for Primary Rewards." The Journal of Neuroscience 27 (21), 5796–5804. doi:10.1523/JNEUROSCI.4246-06.2007.

O'Donoghue, Ted and Matthew Rabin. 1999. "Doing It Now or Later." American Economic Review, 89, 103-124.

Read, Daniel, George Loewenstein, and Shobana Kalyanaraman. 1999. "Mixing Virtue and Vice." Journal of Behavioral Decision Making: 257-273.

Read, D., van Leeuwen, B. 1998. "Predicting Hunger: The Effects of Appetite and Delay on Choice." Organizational Behavior & Human Decision Processes, 76 (2), 189–205.

Topkis, Donald M. 1998. "Supermodularity and Complementarity." Princeton University Press.

9 Appendix: Proofs

9.1 Proof of Propositions 1 and 2

Define

$$
\begin{aligned}
u_{LL} &= E\left(u\left(\frac{F(L,L)}{2}\right)\right) \\
u_{LH} &= E\left(u\left(\left(s-\frac{1}{2}\right)F(L,L)+(1-s)F(L,H)\right)\right) \\
u_{HL} &= E\left(u\left(sF(H,L)-\left(s-\frac{1}{2}\right)F(L,L)\right)\right) \\
u_{HH} &= E\left(u\left(\frac{F(H,H)}{2}\right)\right) \\
\Delta_1 &= \delta(u_{HL}-u_{LL}) \\
\Delta_2 &= \delta(u_{HH}-u_{LH}).
\end{aligned}
$$

Note that we must have $\Delta_1 \neq 1/\beta$, $\Delta_2 \neq 1/\beta$, $\Delta_1 \neq 1$, and $\Delta_2 \neq 1$. This is because of the assumption that, both in the NC and FC games, agent 1 is not indifferent between playing H and L given some pure strategy of agent 2.

Let (p_1, p_2) denote a strategy profile in which agents 1 and 2 choose H with probabilities p_1 and p_2, respectively. Note that I will now write, say, $(1,0)$ instead of (H, L).

Let us start with the following preliminary lemma.

Lemma 2

1) The NC game has the following Nash equilibria:

$$\begin{cases} \{(0,0)\} & \textit{if } \Delta_1 < 1/\beta, \Delta_2 < 1/\beta \\ \{(0,0),(1,1),\left(p^{NC},p^{NC}\right)\} & \textit{if } \Delta_1 < 1/\beta < \Delta_2 \\ \{(1,0),(0,1),\left(p^{NC},p^{NC}\right)\} & \textit{if } \Delta_1 > 1/\beta > \Delta_2 \\ \{(1,1)\} & \textit{if } \Delta_1 > 1/\beta, \Delta_2 > 1/\beta \end{cases}, \tag{16}$$

where $p^{NC} = \frac{1/\beta - \Delta_1}{\Delta_2 - \Delta_1}$.

2) In the FC game, the Nash equilibria are as in (16), but with 1 replacing β and $p^{FC} = \frac{1-\Delta_1}{\Delta_2-\Delta_1}$ replacing p^{NC}.

Proof of part 1) of Lemma 2:

It is straightforward to show that the pure-strategy Nash equilibria are as in (16).

Let (p_1, p_2) be a NE in which both agents are mixing.[25] Given p_2, agent 1 must be indifferent between L and H. Thus, we must have

$$(1-p_2)\left(\beta\delta u_{HL}-1\right)+p_2\left(\beta\delta u_{HH}-1\right)=(1-p_2)\beta\delta u_{LL}+p_2\beta\delta u_{LH}$$

This can be rewritten as:

$$(\Delta_2 - \Delta_1)p_2 = 1/\beta - \Delta_1$$

If $\Delta_1 = \Delta_2$, the last equality reduces to $1/\beta = \Delta_1$ which has been ruled out. Thus, we must have:

$$p_2 = \frac{1/\beta - \Delta_1}{\Delta_2 - \Delta_1}$$

[25]If one agent plays a pure strategy, the other has a unique optimal pure strategy. Thus, there cannot be a NE in which only one agent mixes.

Similarly, we can conclude that $p_1 = \frac{1/\beta - \Delta_1}{\Delta_2 - \Delta_1}$ must hold.

$(\frac{1/\beta-\Delta_1}{\Delta_2-\Delta_1}, \frac{1/\beta-\Delta_1}{\Delta_2-\Delta_1})$ is a mixed-strategy NE if and only if $0 < \frac{1/\beta-\Delta_1}{\Delta_2-\Delta_1} < 1$. The latter two inequalities reduce to: $\Delta_1 < 1/\beta < \Delta_2$ or $\Delta_1 > 1/\beta > \Delta_2$. Q.E.D.

Proof of part 2) of Lemma 2:

The proof of part 1) still applies if we replace β with 1. Q.E.D.

Based on Lemma 2, Figure 3 illustrates the Nash equilibria of the NC and FC games (denoted E^{NC} and E^{FC}, respectively) depending on the values of Δ_1 and Δ_2.

Let E_{p_1,p_2} denote an expectations operator that is computed based on the probabilities over pure-strategy profiles (i.e., over the set $\{(0,0),(0,1),(1,0),(1,1)\}$) induced by the profile of mixed strategies (p_1, p_2) as well as based on any uncertainty about F or s.

We are now ready to prove Propositions 1 and 2. We need to consider each of the nine cases in Figure 3 separately.

<u>Case 1: $\Delta_1 < 1$ and $\Delta_2 < 1$.</u>

Both games have the same unique NE, $(0,0)$. Hence, FC cannot dominate NC, and vice versa, based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE.

<u>Case 2: $1 < \Delta_1 < 1/\beta$ and $\Delta_2 < 1$.</u>

The FC game does not have a symmetric pure-strategy NE. Hence, FC cannot dominate NC, and vice versa, based on symmetric pure-strategy NE.

Self-1 strictly prefers $(0,0)$ to $(1,0)$ and Self-0 strictly prefers $(1,0)$ to $(0,0)$. Thus, FC cannot dominate NC, and vice versa, based on pure-strategy NE or NE.

If $s = 1$ with certainty (so that $u_{LL} = u_{LH}$), Self-0 is indifferent between $(0,0)$ and (p^{FC}, p^{FC}) and Self-1 strictly prefers the former (because Self-1 prefers $(0,0)$

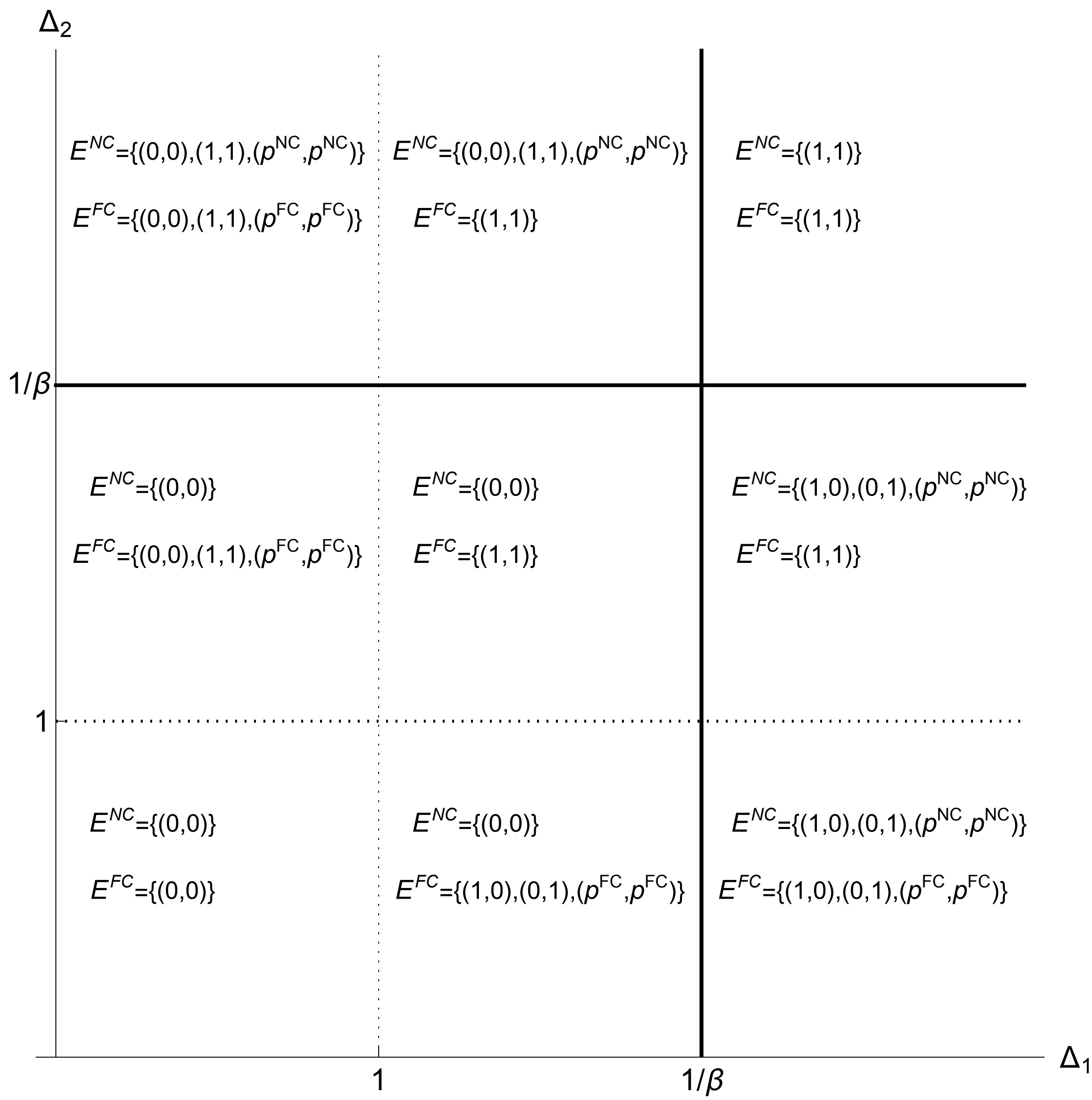

Figure 3: Sets of Nash equilibria of the NC game and FC game (denoted E^{NC} and E^{FC}, respectively) depending on the values of Δ_1 and Δ_2. $p^{NC} = \frac{1/\beta - \Delta_1}{\Delta_2 - \Delta_1}$ and $p^{FC} = \frac{1-\Delta_1}{\Delta_2-\Delta_1}$.

weakly to $(0, 1)$ and strictly to $(1, 0)$ and $(1, 1)$). Thus, NC strictly dominates FC based on symmetric NE. Note that in this case $\Delta_1 > \Delta_2$ is equivalent to condition (10). Note also that $1 < \Delta_1 < 1/\beta$, $\Delta_2 < 1$, and $s = 1$ with certainty is possible. E.g., let $u(x) = \frac{x}{H-L} - \frac{\omega-L}{H-L}$ (this is consistent with the normalisations $u(\omega - L) = 0$

and $u(\omega - H) = -1$), $F(L, L) = 0$ with certainty, and $F(H, L)$ and $F(H, H)$ be deterministic and such that $\frac{H-L}{\delta} < F(H, L) < F(H, H) < \min\{\frac{H-L}{\beta\delta}, 2\frac{H-L}{\delta}\}$. Thus, NC can dominate FC based on symmetric NE.

If $s < 1$ with positive probability (so that $u_{LL} < u_{LH}$), $E(U(L, L, 1)) - E_{p^{FC}, p^{FC}}(U(k_1, k_2, 1)) = \frac{-\delta(u_{LH} - u_{LL})(\Delta_1 - 1)}{\Delta_1 - \Delta_2} < 0$ so that Self-0 strictly prefers (p^{FC}, p^{FC}) to $(0, 0)$. Also, $E(U(L, L, \beta)) - E_{p^{FC}, p^{FC}}(U(k_1, k_2, \beta)) = \frac{(\Delta_1 - 1)(1 - \beta - \beta\delta(u_{LH} - u_{LL}))}{\Delta_1 - \Delta_2}$. Thus, FC strictly dominates NC based on symmetric NE if and only if $\delta(u_{LH} - u_{LL}) \geq \frac{1}{\beta} - 1$.

Case 3: $\Delta_1 > 1/\beta$ and $\Delta_2 < 1$.

Neither the FC game nor the NC game has a symmetric pure-strategy NE. Hence, FC cannot dominate NC, and vice versa, based on symmetric pure-strategy NE.

Both games have the same pure-strategy Nash equilibria, $(1, 0)$ and $(0, 1)$. Hence, FC cannot dominate NC, and vice versa, based on pure-strategy NE.

If $s < 1$ with positive probability (so that $u_{LL} < u_{LH}$), we have (i) $E_{p^{NC}, p^{NC}}(U(k_1, k_2, \beta)) - E(U(L, H, \beta)) = \frac{-\beta\delta(u_{LH} - u_{LL})(1/\beta - \Delta_2)}{\Delta_1 - \Delta_2} < 0$ so that Self-1 strictly prefers $(0, 1)$ to (p^{NC}, p^{NC}) and, hence, NC cannot dominate FC based on NE and (ii) $E_{p^{FC}, p^{FC}}(U(k_1, k_2, 1)) - E(U(L, H, 1)) = \frac{-\delta(u_{LH} - u_{LL})(1 - \Delta_2)}{\Delta_1 - \Delta_2} < 0$ so that Self-0 strictly prefers $(0, 1)$ to (p^{FC}, p^{FC}) and, hence, FC cannot dominate NC based on NE. If $s = 1$ with certainty (so that $u_{LL} = u_{LH}$), we have (i) Self-1 strictly prefers $(1, 0)$ to (p^{NC}, p^{NC}) (because she strictly prefers $(1, 0)$ to $(0, 0)$ and is indifferent between the latter and (p^{NC}, p^{NC})) so that NC cannot dominate FC based on NE and (ii) Self-0 strictly prefers $(1, 0)$ to (p^{FC}, p^{FC}) (because she strictly prefers $(1, 0)$ to $(0, 0)$ and is indifferent between the latter and (p^{FC}, p^{FC})) so that FC cannot dominate NC based on NE. The bottom line is that NC cannot dominate FC, and vice versa, based on NE.

Turning to possible domination based on symmetric NE, straightforward calculations yield:

$$E_{p^{NC},p^{NC}}(U(k_1,k_2,\beta)) - E_{p^{FC},p^{FC}}(U(k_1,k_2,\beta)) = \frac{(1-\beta)(\Delta_1 - \delta(u_{LH}-u_{LL})-1)}{\Delta_1 - \Delta_2}$$
$$E_{p^{NC},p^{NC}}(U(k_1,k_2,1)) - E_{p^{FC},p^{FC}}(U(k_1,k_2,1)) = \frac{(1-\beta)(\beta(\Delta_1 - \delta(u_{LH}-u_{LL}))-1)}{\beta^2(\Delta_1 - \Delta_2)}.$$

Thus, (i) FC strictly dominates NC based on symmetric NE if and only if $\Delta_1 - \delta(u_{LH} - u_{LL}) \leq 1$ and (ii) NC strictly dominates FC based on symmetric NE if and only if $\Delta_1 - \delta(u_{LH} - u_{LL}) \geq 1/\beta$.

Note that in case (ii) condition (10) must hold. To see this, note that $\Delta_2 < 1$ and $\Delta_1 - \delta(u_{LH} - u_{LL}) \geq 1/\beta$ can be written as $u_{HH} - u_{LH} < \frac{1}{\delta}$ and $u_{HL} - u_{LH} \geq \frac{1}{\beta\delta}$, respectively. The latter two inequalities imply $u_{HL} > u_{HH}$ which implies condition (10).

Case 4: $\Delta_1 < 1$ and $1 < \Delta_2 < 1/\beta$.

Self-0 strictly prefers $(1,1)$ to $(0,1)$ and, hence, to $(0,0)$. Thus, NC cannot dominate FC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE.

If $\Delta_2 + \delta(u_{LH} - u_{LL}) < 1/\beta$, Self-1 strictly prefers $(0,0)$ to $(1,1)$ so that FC cannot dominate NC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE.

If $\Delta_2 + \delta(u_{LH} - u_{LL}) \geq 1/\beta$, Self-1 prefers and Self-0 strictly prefers $(1,1)$ to $(0,0)$ so that FC dominates NC based on symmetric pure-strategy NE and pure-strategy NE.

Let us turn to a comparison between (p^{FC}, p^{FC}) and $(0,0)$. We have $E_{p^{FC},p^{FC}}(U(k_1,k_2,1)) - E(U(L,L,1)) = \frac{\delta(u_{LH}-u_{LL})(1-\Delta_1)}{\Delta_2-\Delta_1} \geq 0$, so that Self-0 prefers (p^{FC}, p^{FC}) to $(0,0)$. Also, $E_{p^{FC},p^{FC}}(U(k_1,k_2,\beta)) - E(U(L,L,\beta)) = \frac{(1-\Delta_1)(\beta(1+\delta(u_{LH}-u_{LL}))-1)}{\Delta_2-\Delta_1}$.

If $1 + \delta(u_{LH} - u_{LL})) < 1/\beta$, Self-1 strictly prefers $(0,0)$ to (p^{FC}, p^{FC}) so that FC cannot dominate NC based on symmetric NE or NE.

Now, suppose $1 + \delta(u_{LH} - u_{LL})) \geq 1/\beta$.[26] Self-1 prefers (p^{FC}, p^{FC}) to $(0, 0)$. Because the latter inequality implies $\Delta_2 + \delta(u_{LH} - u_{LL})) > 1/\beta$, Self-1 and Self-0 strictly prefer $(1, 1)$ to $(0, 0)$. Thus, FC dominates NC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, and NE.

Case 5: $1 < \Delta_1 < 1/\beta$ and $1 < \Delta_2 < 1/\beta$.

Self-0 strictly prefers $(1, 1)$ to $(0, 1)$ and, hence, to $(0, 0)$. Thus, NC cannot dominate FC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE.

If $\Delta_2 + \delta(u_{LH} - u_{LL}) < 1/\beta$, Self-1 strictly prefers $(0, 0)$ to $(1, 1)$ so that FC cannot dominate NC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE.

If $\Delta_2 + \delta(u_{LH} - u_{LL}) \geq 1/\beta$, Self-1 prefers and Self-0 strictly prefers $(1, 1)$ to $(0, 0)$ so that FC strictly dominates NC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, and NE.[27]

Case 6: $\Delta_1 > 1/\beta$ and $1 < \Delta_2 < 1/\beta$.

The NC game does not have a symmetric pure-strategy NE. Hence, FC cannot dominate NC, and vice versa, based on symmetric pure-strategy NE.

Self-0 strictly prefers $(1, 1)$ to $(0, 1)$ and Self-1 strictly prefers $(0, 1)$ to $(1, 1)$. Thus, NC cannot dominate FC, and vice versa, based on pure-strategy NE or NE.

Let us turn to a comparison between (p^{NC}, p^{NC}) and $(1, 1)$. We have $E_{p^{NC},p^{NC}}(U(k_1, k_2, \beta)) - E(U(H, H, \beta)) = \frac{\beta(1/\beta - \Delta_2)(\Delta_1 - \Delta_2 - \delta(u_{LH} - u_{LL}))}{\Delta_1 - \Delta_2}$ and
$E_{p^{NC},p^{NC}}(U(k_1, k_2, 1)) - E(U(H, H, 1)) = \frac{(1/\beta - \Delta_2)(-1 + \beta + \beta(\Delta_1 - \Delta_2 - \delta(u_{LH} - u_{LL})))}{\beta(\Delta_1 - \Delta_2)}$.

Thus, (i) FC strictly dominates NC based on symmetric NE if and only if $\Delta_1 \leq$

[26] This inequality is equivalent to (4).

[27] $\Delta_2 + \delta(u_{LH} - u_{LL}) \geq 1/\beta$ is equivalent to (7).

$\Delta_2 + \delta(u_{LH} - u_{LL})$ and (ii) NC strictly dominates FC based on symmetric NE if and only if $\Delta_1 \geq \Delta_2 + \delta(u_{LH} - u_{LL}) + \frac{1}{\beta} - 1$.

Note that in case (ii) condition (10) must hold. To see this, note that the latter inequality can be written as $u_{HL} \geq u_{HH} + \frac{1}{\beta\delta} - \frac{1}{\delta}$. The latter inequality implies $u_{HL} > u_{HH}$ which implies condition (10).

Case 7: $\Delta_1 < 1$ and $\Delta_2 > 1/\beta$.

Both Self-0 and Self-1 strictly prefer $(1,1)$ to $(0,1)$ and, hence, to $(0,0)$. Thus, NC cannot dominate FC, and vice versa, based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE.

Case 8: $1 < \Delta_1 < 1/\beta$ and $\Delta_2 > 1/\beta$.

Both Self-0 and Self-1 strictly prefer $(1,1)$ to $(0,1)$ and, hence, to $(0,0)$. Thus, (i) NC cannot dominate FC based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE and (ii) FC dominates NC based on symmetric pure-strategy NE and pure-strategy NE.

Let us turn to a comparison between (p^{NC}, p^{NC}) and $(1,1)$. We have $E_{p^{NC},p^{NC}}(U(k_1,k_2,\beta)) - E(U(H,H,\beta)) = \frac{-\beta(\Delta_2 - 1/\beta)(\Delta_2 - \Delta_1 + \delta(u_{LH} - u_{LL}))}{\Delta_2 - \Delta_1} \leq 0$ and $E_{p^{NC},p^{NC}}(U(k_1,k_2,1)) - E(U(H,H,1)) = \frac{-(\Delta_2 - 1/\beta)(1 - \beta + \beta(\Delta_2 - \Delta_1 + \delta(u_{LH} - u_{LL})))}{\beta(\Delta_2 - \Delta_1)} \leq 0$. Thus, FC dominates NC based on symmetric NE and NE as well.

Case 9: $\Delta_1 > 1/\beta$ and $\Delta_2 > 1/\beta$.

Both games have the same unique NE, $(1,1)$. Hence, FC cannot dominate NC, and vice versa, based on symmetric pure-strategy NE, pure-strategy NE, symmetric NE, or NE.

It remains to show that each case a)-c) is nonvacuous. To do this, assume that (i) u is linear and (ii) s is uncorrelated with $M = \frac{F(H,L) - F(L,L)}{H - L}$ (i.e., the marginal product of investment when the other agent chooses L). Define $S =$

$\frac{(F(H,H)-F(L,H))-(F(H,L)-F(L,L))}{H-L}$, i.e., S is a measure of the degree of supermodularity (if S is nonnegative) or submodularity (if S is nonpositive) of F. It is straightforward to show that cases a)-c) can be written as follows.

Case a):

$$
\begin{aligned}
&E(M) < \frac{1}{\delta E(s)} \\
&\frac{2}{\delta} - 2E(s)E(M) < E(S) < \frac{2}{\beta\delta} - 2E(s)E(M) \\
&E(M) \geq \frac{1-\beta}{\beta\delta E(1-s)}
\end{aligned}
$$

Case b):

$$
\begin{aligned}
&\frac{1}{\delta E(s)} < E(M) < \frac{1}{\beta\delta E(s)} \\
&\frac{2}{\delta} - 2E(s)E(M) < E(S) < \frac{2}{\beta\delta} - 2E(s)E(M) \\
&E(S) \geq \frac{2}{\beta\delta} - 2E(M)
\end{aligned}
$$

Case c):

$$
\begin{aligned}
&\frac{1}{\delta E(s)} < E(M) < \frac{1}{\beta\delta E(s)} \\
&E(S) > \frac{2}{\beta\delta} - 2E(s)E(M)
\end{aligned}
$$

Figure 4 depicts cases a)-c) in the space of $E(M)$ and $E(S)$. (The top and bottom panels are for the cases when $E(s) < \beta$ and $\beta \leq E(s) \leq 1$, respectively.) It is clear from the figure that each case a)-c) is nonvacuous. Q.E.D.

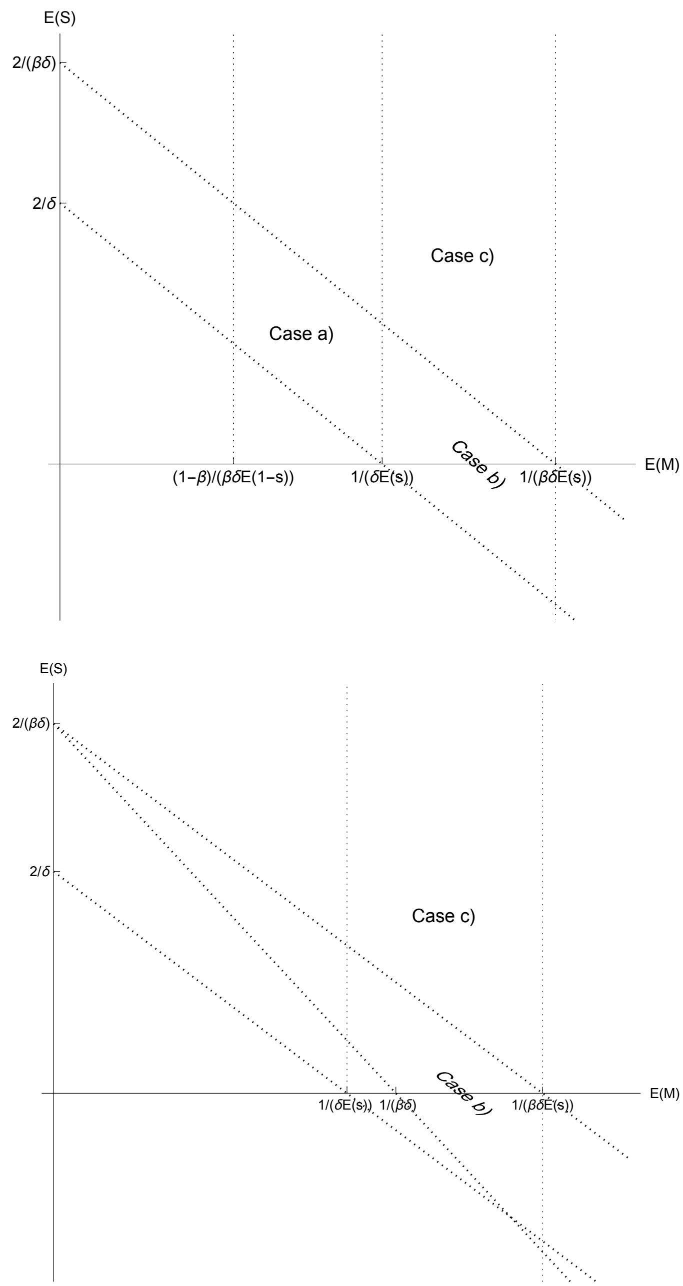

Figure 4: Cases a)-c) when u is linear and s is uncorrelated with M. The top and bottom panels are for $E(s) < \beta$ and $\beta \leq E(s) \leq 1$, respectively. If $\beta \leq E(s) \leq 1$, case a) is not feasible. If $E(s) = 1$, $\frac{1}{\beta\delta} = \frac{1}{\beta\delta E(s)}$ so that the case b) region in the bottom panel disappears. The figure is not explicit about which boundaries (if any) belong to a given region. $S > -M$ must always hold so that some points in the southeast corner of the case b) region may be unattainable.

9.2 Proof of Lemma 1

Observe that $f_i(k,k) = A/2$.[28]

I first state and prove two claims.

Claim 1 *$U(\cdot, k_2, \beta)$ is differentiable and*

$$U_1(k_1,k_2,\beta) =$$
$$\begin{cases} \beta\delta\gamma u'\left((s-0.5)A^\gamma k_1^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\left(\frac{(s-0.5)A^\gamma}{k_1^{1-\gamma}} + \frac{(1-s)f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}}\right) - u'(\omega - k_1) & \textit{if } k_1 < k_2 \\ \beta\delta\gamma s u'\left(sf(k_1,k_2)^\gamma - (s-0.5)A^\gamma k_2^\gamma\right)\frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} - u'(\omega - k_1) & \textit{if } k_1 \geq k_2 \end{cases}. \tag{17}$$

Proof:

We can rewrite (17) as:

$$U_1(k_1,k_2,\beta) =$$
$$\begin{cases} \beta\delta\gamma u'\left((s-0.5)A^\gamma k_1^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\left(\frac{(s-0.5)A^\gamma}{k_1^{1-\gamma}} + \frac{(1-s)f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}}\right) - u'(\omega - k_1) & \text{if } k_1 < k_2 \\ \beta\delta\gamma s u'\left(sf(k_1,k_2)^\gamma - (s-0.5)A^\gamma k_2^\gamma\right)\frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} - u'(\omega - k_1) & \text{if } k_1 = k_2 \\ \beta\delta\gamma s u'\left(sf(k_1,k_2)^\gamma - (s-0.5)A^\gamma k_2^\gamma\right)\frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} - u'(\omega - k_1) & \text{if } k_1 > k_2 \end{cases}. \tag{18}$$

At $k_1 \neq k_2$, (18) can be verified via straightforward differentiation of $U(\cdot, k_2, \beta)$. At $k_1 = k_2$, the left and right derivative of $U(\cdot, k_2, \beta)$ equal the partial derivative of the first and, respectively, second piece in (1) with respect to k_1. It is straightforward to verify that the latter two partial derivatives, when evaluated at $k_1 = k_2$, are both equal to the middle piece in (18). Q.E.D.

[28]This follows because $Ak = f(k,k) = kf_1(k,k) + kf_2(k,k) = 2kf_i(k,k)$, where the second and third equalities follow from Euler's homogeneous function theorem and the symmetry of f, respectively.

Claim 2 *$U_1(\cdot, k_2, \beta)$ is strictly decreasing.*

Proof:

First, let us consider the first piece of (17). $u'\left((s-0.5)A^\gamma k_1^\gamma + (1-s)f(k_1,k_2)^\gamma\right)$ and $-u'(\omega - k_1)$ are strictly decreasing in k_1. Given that $\gamma \leq 1$, $(s-0.5)A^\gamma/k_1^{1-\gamma}$ is weakly decreasing in k_1. Given that $f_1(k_1,k_2)$ is strictly decreasing in k_1 and $f(k_1,k_2)^{1-\gamma}$ is weakly increasing in k_1, $(1-s)f_1(k_1,k_2)/f(k_1,k_2)^{1-\gamma}$ is weakly decreasing in k_1. Thus, $U_1(\cdot, k_2, \beta)$ is strictly decreasing on $(0, k_2)$.

Next, let us consider the second piece of (17). $u'\left(sf(k_1,k_2)^\gamma - (s-0.5)A^\gamma k_2^\gamma\right)$ and $-u'(\omega - k_1)$ are strictly decreasing in k_1. By the same logic as in the previous paragraph, $f_1(k_1,k_2)/f(k_1,k_2)^{1-\gamma}$ is strictly decreasing in k_1. Thus, $U_1(\cdot, k_2, \beta)$ is strictly decreasing on (k_2, ω).

Moreover, taking the limit of the first piece in (18) as $k_1 \uparrow k_2$ and the limit of the third piece in (18) as $k_1 \downarrow k_2$ shows that both of these limits equal the middle piece in (18). Thus, $U_1(\cdot, k_2, \beta)$ is continuous at $k_1 = k_2$. It follows that $U_1(\cdot, k_2, \beta)$ is strictly decreasing on $(0, \omega)$. Q.E.D.

With these two claims in hand, we can now prove the lemma.

Proof of part 1):

Any symmetric equilibrium $k_1 = k_2 = k$ must satisfy the first-order condition $E(U_1(k,k,\beta)) = 0$. Moreover, this first-order condition is sufficient given that $U_1(\cdot, k_2, \beta)$ is strictly decreasing on $(0, \omega)$.

It remains to show that there exists a unique $k \in (0, \omega)$ satisfying $E(U_1(k,k,\beta)) = 0$ or, equivalently, (11). This holds because the left-hand of (11) is a constant, the right-hand-side is continuous and strictly increasing in k, and

$$\lim_{k \downarrow 0} \frac{u'(\omega - k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}} u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)} < \beta E(s) < \lim_{k \uparrow \omega} \frac{u'(\omega - k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}} u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)}$$

given that u satisfies the Inada condition. Q.E.D.

Proof of part 2):

The proofs above of Claims 1 and 2 and of part 1) of Lemma 1 go through if we replace β with 1, any reference to (11) with a reference to (12), and

$$\lim_{k\downarrow 0}\frac{u'(\omega-k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}}u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)}<\beta E(s)<\lim_{k\uparrow \omega}\frac{u'(\omega-k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}}u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)}$$

with

$$\lim_{k\downarrow 0}\frac{u'(\omega-k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}}u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)}<E(s)<\lim_{k\uparrow \omega}\frac{u'(\omega-k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}}u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)}.$$

That $k^{FC}>k^{NC}$ is evident from Figure 2. Q.E.D.

Proof of part 3):

Define $g(k)=E(U(k,k,\beta))=u(\omega-k)+\beta\delta E(u(A^\gamma k^\gamma/2))$. We have $g'(k)=-u'(\omega-k)+\beta\delta E\left(\frac{A^\gamma\gamma u'(A^\gamma k^\gamma/2)}{2k^{1-\gamma}}\right)$, which is strictly decreasing in k. Thus, $g'(k)=0$, or equivalently (13), is necessary and sufficient for g to be strictly increasing on $(0,k)$ and strictly decreasing on (k,ω).

It still needs to be shown that there exists a unique $k\in(0,\omega)$ satisfying (13). This holds because the left-hand of (13) is a constant, the right-hand-side is continuous and strictly increasing in k, and

$$\lim_{k\downarrow 0}\frac{u'(\omega-k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}}u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)}<\beta<\lim_{k\uparrow \omega}\frac{u'(\omega-k)}{\delta E\left(\frac{\gamma A^\gamma}{2k^{1-\gamma}}u'\left(\frac{A^\gamma k^\gamma}{2}\right)\right)}$$

given that u satisfies the Inada condition.

That $\hat{k}^{NC}\geq k^{NC}$, the inequality being strict if and only if $E(s)<1$, is evident from Figure 2. Q.E.D.

Proof of part 4):

The proof of part 3) goes through if we replace β with 1, k^{NC} with k^{FC}, $\hat{k}^{NC}$ with $\hat{k}^{FC}$, and any reference to (13) with a reference to the following equation:

$$1 = \frac{u'(\omega - k)}{\delta E\left(\frac{\gamma A^{\gamma}}{2k^{1-\gamma}} u'\left(\frac{A^{\gamma} k^{\gamma}}{2}\right)\right)}. \tag{19}$$

Q.E.D.

Proof of part 5):

At $k_1 = k_2$, the left and right derivative of $U(k_1, \cdot, \beta)$ equal the partial derivative of the first and, respectively, second piece in (1) with respect to k_2. It is straightforward to verify that the latter two partial derivatives, when evaluated at $k_1 = k_2$, are both equal to $\frac{\beta\delta A^{\gamma}\gamma(1-s)u'(A^{\gamma}k^{\gamma}/2)}{2k^{1-\gamma}}$. Thus, $E(U_2(k,k,\beta)) = \beta\delta(1 - E(s))E\left(\frac{A^{\gamma}\gamma u'(A^{\gamma}k^{\gamma}/2)}{2k^{1-\gamma}}\right)$, which is strictly positive if $E(s) < 1$.

The argument in the previous paragraph continues to hold if we replace β with 1.
Q.E.D.

Proof of part 6):

I prove the claim for the NC game. The proof also applies with $\beta = 1$ and, hence, to the FC game.

Assume that (k_1, k_2) is an asymmetric NE in which (without loss of generality) $k_1 > k_2$. Then the following first-order conditions must hold for agents 1 and 2, respectively.[29]

[29] The first equation below follows from setting the expectation of the second piece of (17) equal to zero. The second equation below follows from setting the expectation of the first piece of (17), adapted to the perspective of agent 2, equal to zero.

$$\beta\delta E(s) E\left(\gamma \frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} u'\left(sf(k_1,k_2)^\gamma - (s-0.5)f(k_2,k_2)^\gamma\right)\right) = u'(\omega - k_1) \tag{20}$$

$$\beta\delta E(1-s) E\left(\gamma \frac{f_2(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right) = \\ u'(\omega - k_2) - \beta\delta E\left(\gamma(s-0.5)\frac{f(k_2,k_2)^\gamma}{k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right). \tag{21}$$

Multiplying each side of (20) by $E(1-s)/E(s)$, we get

$$\beta\delta E(1-s) E\left(\gamma \frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} u'\left(sf(k_1,k_2)^\gamma - (s-0.5)f(k_2,k_2)^\gamma\right)\right) = \frac{E(1-s)}{E(s)} u'(\omega - k_1) \tag{22}$$

$$\beta\delta E(1-s) E\left(\gamma \frac{f_2(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right) = \\ u'(\omega - k_2) - \beta\delta E\left(\gamma(s-0.5)\frac{f(k_2,k_2)^\gamma}{k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right). \tag{23}$$

Note that (i) $f_2(k_1,k_2) = f_1(k_2,k_1) > f_1(k_1,k_1) \geq f_1(k_1,k_2)$,[30] (ii) u' is decreasing, and (iii) agent 1's consumption in period 2, $sf(k_1,k_2)^\gamma - (s-0.5)f(k_2,k_2)^\gamma$, is weakly larger than agent 2's consumption in period 2, $(s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma$. Given (i)-(iii), equalities (22) and (23) above imply

$$\frac{E(1-s)}{E(s)} u'(\omega - k_1) \leq \\ u'(\omega - k_2) - \beta\delta E\left(\gamma(s-0.5)\frac{f(k_2,k_2)^\gamma}{k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right).$$

[30] The equality follows from the symmetry of f. The first inequality holds because $f_1(\cdot, k_1)$ is strictly decreasing. The second inequality holds as a result of the supermodularity of f.

The latter inequality, given that $u'(\omega - k_2) < u'(\omega - k_1)$, implies

$$\frac{E(1-s)}{E(s)} u'(\omega - k_1) <$$
$$u'(\omega - k_1) - \beta\delta E\left(\gamma(s-0.5)\frac{f(k_2,k_2)^\gamma}{k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right),$$

which can be rewritten as

$$\beta\delta(E(s)-0.5)E\left(\gamma\frac{f(k_2,k_2)^\gamma}{k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right) <$$
$$\frac{2(E(s)-0.5)}{E(s)} u'(\omega - k_1).$$

If $E(s) = 0.5$, we've reached a contradiction. Otherwise, the last inequality can be rewritten as

$$\beta\delta E(s) E\left(\gamma\frac{f(k_2,k_2)^\gamma}{2k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right) < u'(\omega - k_1).$$

This inequality and (20) imply

$$E\left(\gamma\frac{f(k_2,k_2)^\gamma}{2k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right)\right) <$$
$$E\left(\gamma\frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} u'\left(sf(k_1,k_2)^\gamma - (s-0.5)f(k_2,k_2)^\gamma\right)\right).$$

This implies that, for some realisation of s, f, and γ, we must have

$$\frac{f(k_2,k_2)^\gamma}{2k_2} u'\left((s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma\right) <$$
$$\frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}} u'\left(sf(k_1,k_2)^\gamma - (s-0.5)f(k_2,k_2)^\gamma\right).$$

This inequality, together with the facts that u' is decreasing and $sf(k_1,k_2)^\gamma - (s -$

$0.5)f(k_2,k_2)^\gamma > (s-0.5)f(k_2,k_2)^\gamma + (1-s)f(k_1,k_2)^\gamma$, implies

$$\frac{f(k_2,k_2)^\gamma}{2k_2} < \frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}}.$$

Given that $f(k_2,k_2) = Ak_2$ and $f_1(k_2,k_2) = A/2$, the latter inequality can be written as

$$\left(\frac{f(k_1,k_2)}{f(k_2,k_2)}\right)^{1-\gamma} < \frac{f_1(k_1,k_2)}{f_1(k_2,k_2)}.$$

The left-hand side is greater than or equal to 1 while the right-hand side, given that $f_1(\cdot,k_2)$ is strictly decreasing, is strictly less than 1. We've reached a contradiction. Q.E.D.

9.3 Proof of Proposition 4

Suppose the setting is either that with binary investments in section 3 or that with continuous investments in section 4.

Given a probability measure, μ, on K, let E_μ denote an expectation that is computed based on agent 2's investment, k_2, being distributed according to μ.[31] Also, if $\mu(\{k\}) = 1$ for some $k \in K$, I will use μ and k interchangeably.

Let us start with the following lemma.

Lemma 3 *Consider a probability measure, μ, on K. If the setting is that with continuous investments in section 4, assume that μ is degenerate, i.e., $\mu(\{k_2\}) = 1$ for some $k_2 \in K$. Then, the following hold.*

1) $\arg\max_{k_1 \in K} E_\mu(U(k_1,k_2,\beta)) \neq \emptyset$ *and* $\arg\max_{k_1 \in K} E_\mu(U(k_1,k_2,1)) \neq \emptyset$.

[31] E_μ also takes into account any uncertainty about F and s.

2) Given $k \in K$, if all elements in $\arg\max_{k_1 \in K} E_\mu(U(k_1, k_2, \beta))$ *are weakly smaller than k, then* $\arg\max_{k_1 \in K, k_1 \geq k} E_\mu(U(k_1, k_2, \beta)) = \{k\}$.

3) $k \in \arg\max_{k_1 \in K} E_\mu(U(k_1, k_2, 1))$ *implies* $\arg\max_{k_1 \in K, k_1 \geq k} E_\mu(U(k_1, k_2, \beta)) = \{k\}$.

Proof of part 1):

With binary investments, the statement is obviously true. Now, consider the case with continuous investments. I drop the μ subscript on the expectations because μ is degenerate.

By Claims 1 and 2 in the proof of Lemma 1, $E(U_1(k_1, k_2, \beta)) = 0$ is sufficient for $k_1 \in \arg\max_{k_1 \in K} E(U(k_1, k_2, \beta))$. Moreover, given (17), $\lim_{k_1 \downarrow 0} f_1(k_1, k_2) = \infty$, and $\lim_{x \downarrow 0} u'(x) = \infty$, we have[32]

$$\lim_{k_1 \downarrow 0} E(U_1(k_1, k_2, \beta)) =$$
$$E\left(\beta\delta\gamma \lim_{k_1 \downarrow 0} u'\left((s-0.5)A^\gamma k_1^\gamma + (1-s)(f(k_1,k_2)^\gamma)\right)\left(\frac{(s-0.5)A^\gamma}{\lim_{k_1 \downarrow 0}(k_1^{1-\gamma})} + (1-s)\lim_{k_1 \downarrow 0} \frac{f_1(k_1,k_2)}{f(k_1,k_2)^{1-\gamma}}\right)\right) - u'(\omega) = \infty, \tag{24}$$

and, given (17) and $\lim_{x \downarrow 0} u'(x) = \infty$, we have

$$\lim_{k_1 \uparrow \omega} E(U_1(k_1, k_2, \beta)) =$$
$$E\left(\beta\delta\gamma s u'\left(s f(\omega, k_2)^\gamma - (s-0.5)A^\gamma k_2^\gamma\right) \frac{f_1(\omega, k_2)}{f(\omega, k_2)^{1-\gamma}}\right) - \lim_{k_1 \uparrow \omega} u'(\omega - k_1) = -\infty. \tag{25}$$

Thus, $E(U_1(k_1, k_2, \beta)) = 0$ has a solution so that $\arg\max_{k_1 \in K} E_\mu(U(k_1, k_2, \beta)) \neq \emptyset$. Analogously, $\arg\max_{k_1 \in K} E(U(k_1, k_2, 1)) \neq \emptyset$. Q.E.D.

Proof of part 2): With binary investments, the statement is obviously true. Now, consider the case with continuous investments. I drop the μ subscript on the expectations

[32] In (24), I'm using $\lim_{k_1 \downarrow 0} f_1(k_1, k_2) = \infty$ for $s < 1$ and $\lim_{x \downarrow 0} u'(x) = \infty$ for $s = 1$.

because μ is degenerate.

By Claim 1 in the proof of Lemma 1, $E(U_1(k_1, k_2, \beta)) = 0$ is necessary for $k_1 \in \arg\max_{k_1 \in K} E(U(k_1, k_2, \beta))$. Moreover, given Claim 2 in the proof of Lemma 1, $E(U_1(\cdot, k_2, \beta))$ is strictly decreasing. Thus, $E(U_1(\cdot, k_2, \beta))$ must be strictly negative on $(k, \omega]$ so that $E(U(\cdot, k_2, \beta))$ must be strictly decreasing on $[k, \omega)$. Q.E.D.

Proof of part 3):

For any $k'' > k'$ (where $k', k'' \in K$), it is straightforward to show that $E_\mu(U(k'', k_2, 1)) - E_\mu(U(k', k_2, 1)) > E_\mu(U(k'', k_2, \beta)) - E_\mu(U(k', k_2, \beta))$ so that $E_\mu(U(\cdot, k_2, \cdot))$ has strictly increasing differences on $K \times \{\beta, 1\}$. Take $k \in \arg\max_{k_1 \in K} E_\mu(U(k_1, k_2, 1))$. By Theorem 2.8.4 in Topkis (1998), we must have that any element in $\arg\max_{k_1 \in K} E_\mu(U(k_1, k_2, \beta))$ is weakly smaller than k. Given part 2), we are done. Q.E.D.

We are now ready to prove Proposition 4.

Suppose that $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$ is a pure-strategy PNE of the OC game and $(\pi_{\sigma_{10},\sigma_{11}}, \pi_{\sigma_{20},\sigma_{21}})$ is not a pure-strategy NE of the FC game. Given that $(\pi_{\sigma_{10},\sigma_{11}}, \pi_{\sigma_{20},\sigma_{21}})$ is clearly a pure-strategy profile, it must not be a NE of the FC game. Then, in the FC game, agent 1 (where the choice of agent 1 is without loss of generality), given part 1) in Lemma 3, has a pure-strategy best response, $k \in K$, that she strictly prefers to $\pi_{\sigma_{10},\sigma_{11}}$ (given $\pi_{\sigma_{20},\sigma_{21}}$). Suppose that, in the OC game, Self-0 commits to minimum investment k. Then, given part 3) in Lemma 3, Self-1 will choose k so that $\pi_{k,\sigma_{11}}(\{k\}) = 1$. But, then, k is a profitable deviation from σ_{10} for Self-0. We've reached a contradiction. Thus, if $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$ is a pure-strategy PNE of the OC game, then $(\pi_{\sigma_{10},\sigma_{11}}, \pi_{\sigma_{20},\sigma_{21}})$ is a pure-strategy NE of the FC game. Note that, if $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$ is a symmetric strategy profile, then $(\pi_{\sigma_{10},\sigma_{11}}, \pi_{\sigma_{20},\sigma_{21}})$ is a symmetric strategy profile.

With binary investments, the argument in the previous paragraph also goes through if we delete from that paragraph the second sentence as well as all instances of "pure-strategy".

In the other direction, suppose that (σ_1, σ_2) is a pure-strategy NE of the FC game. Let $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$ be such that $\sigma_{10} = \sigma_1$, $\sigma_{20} = \sigma_2$, $\sigma_{11\underline{k}}$ is a pure strategy in $\arg\max_{k_1 \in K, k_1 \geq \underline{k}} E_{\sigma_2}(U(k_1, k_2, \beta))$ for each $\underline{k} \in K$, $\sigma_{21\underline{k}}$ is defined analogously for each $\underline{k} \in K$ if $\sigma_1 \neq \sigma_2$, and $\sigma_{21\underline{k}}$ is set equal to $\sigma_{11\underline{k}}$ for each $\underline{k} \in K$ if $\sigma_1 = \sigma_2$.[33] Given part 3) in Lemma 3, we must have, for each agent i, $\sigma_{i1\underline{k}} = \underline{k}$ for any $\underline{k}$ in the support of σ_{i0} so that $\pi_{\sigma_{i0}, \sigma_{i1}} = \sigma_i$. Thus, $\sigma_{i1\underline{k}}$ is a best response for agent i in period 1 to $(\sigma_{j0}, \sigma_{j1})$ (where $j \neq i$) for each $\underline{k} \in K$ and σ_{i0} is a best response for agent i in period 0 to $(\sigma_{j0}, \sigma_{j1}, \sigma_{i1})$ (where $j \neq i$). Thus, $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$ is a pure-strategy PNE of the OC game. Note that, if (σ_1, σ_2) is a symmetric strategy profile, then $(\sigma_{10}, \sigma_{11}, \sigma_{20}, \sigma_{21})$ is a symmetric strategy profile.

With binary investments, the argument in the previous paragraph also goes through if we delete from that paragraph all instances of "pure-strategy" (but not of "pure strategy", i.e., without the hyphen). Q.E.D.

[33] If some element in $\arg\max_{k_1 \in K} E_{\sigma_2}(U(k_1, k_2, \beta))$ is greater than $\underline{k}$, then clearly $\arg\max_{k_1 \in K, k_1 \geq \underline{k}} E_{\sigma_2}(U(k_1, k_2, \beta)) \neq \emptyset$. Otherwise, $\arg\max_{k_1 \in K, k_1 \geq \underline{k}} E_{\sigma_2}(U(k_1, k_2, \beta)) \neq \emptyset$ follows from part 2) in Lemma 3.

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