A Comment on Galperti and Strulovici (2017)

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1 Introduction

Galperti and Strulovici (2017; GS, henceforth) introduce a novel approach to modelling intergenerational choice in an environment without uncertainty.[1] At the heart of this approach lie two axioms about the preferences of generation 0 (G-0) over consumption streams specifying each generation's consumption. These axioms, GS's Axiom 5 and an axiom which is implicit in GS's analysis and I will call Axiom 5a, are based on the idea that G-0 forms its preference over consumption streams with the same G-0 consumption by aggregating the preferences of generation 1 (G-1), generation 2 (G-2), etc. Axioms 5 and 5a impose conditions on this aggregation exercise that appear normatively and descriptively very appealing.

GS's analysis leads to the following remarkable result: Axioms 5 and 5a, taken together with some standard axioms, imply a failure of dynamic consistency. This poses a significant challenge to the profession. This challenge is all the more unsettling

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[1]The working paper version of GS, Galperti and Strulovici (2014), considers intertemporal choice, i.e, a setting in which consumption over time is done by the same person rather than by different generations. The mathematics is equivalent in the intertemporal and intergenerational choice settings, and the analysis translates between the two settings in a straightforward way. The current comment applies to both settings.

given that, unlike other theories of dynamically inconsistent preferences,[2] it appears to have a strong normative foundation.

The current comment argues that Axioms 5 and 5a are less normatively and descriptively appealing than they seem. Thus, in my view, the challenge to dynamic consistency posed by GS is less convincing than it first appears.

2 Axioms 5 and 5a

The following setup and notation are as in GS. There is an infinite sequence of generations (G-1, G-2, etc.). For simplicity, we can think of each G-t as living and consuming in a single period, period t. Let $c = (c_0, c_1, \ldots)$ denote a consumption stream, where c_t is the consumption of G-t. Let ${}_tc = (c_t, c_{t+1}, \ldots)$ denote c shifted forward by t periods. Let $\succeq$ denote the preference of G-0 over consumption streams. For $t \geq 1$, let $\succeq^t$ denote the preference of G-t over consumption streams that coincide in periods $0, 1, \ldots, t-1$. Let $\sim$ and $\sim^t$ denote the corresponding indifference relations.

Here are the key Axioms 5 and 5a.

Axiom 5. *If* ${}_tc \sim {}_tc'$ *for all* $t \geq 1$, *then* $(c_0, {}_1c) \sim (c_0, {}_1c')$.

Axiom 5a. *For some* c *and* c', *we have* $c_0 = c_0'$, ${}_1c \sim {}_1c'$, *and not* $c \sim c'$.[3]

First, consider Axiom 5. Note that, assuming time invariance (a standard assumption), ${}_tc \sim {}_tc'$ is equivalent to "$(\hat{c}_0, \ldots, \hat{c}_{t-1}, {}_tc) \sim^t (\hat{c}_0, \ldots, \hat{c}_{t-1}, {}_tc')$ for all $\hat{c}$". Thus, Axiom 5 can be stated as follows: if, for each $t \geq 1$, G-t is equally inclined to choose $(\hat{c}_0, \ldots, \hat{c}_{t-1}, {}_tc)$ as $(\hat{c}_0, \ldots, \hat{c}_{t-1}, {}_tc')$ for any $\hat{c}$, then G-0 should be indifferent between $(c_0, {}_1c)$ and $(c_0, {}_1c')$ given that they involve the same period-0 consumption.

Next, consider Axiom 5a. Note that, assuming time invariance, ${}_1c \sim {}_1c'$ is equivalent to $c \sim^1 c'$. Thus, Axiom 5a rules out the possibility that, holding G-0's own consumption fixed and holding G-1 indifferent, G-0 is completely unresponsive to the preferences of G-2, G-3, etc.

[2]Such as (quasi-)hyperbolic discounting (Chung and Herrnstein (1961), Phelps and Pollak (1968), Laibson (1997)) or the model with anticipatory feelings in Loewenstein (1987).

[3]Axiom 5a is not explicitly stated in GS. However, under GS's Axioms 1-5, Axiom 5a is equivalent to the requirement that their function V (see GS's Theorem 2) cannot be written as a function solely of its first two arguments.

Taken together, GS's Axiom 1 (transitivity and completeness), GS's technical Axioms 2-4, Axiom 5, Axiom 5a, and time invariance imply a failure of dynamic consistency.[4,5]

3 Criticisms based on obligation and temptation

It is likely that choices are in part driven by a sense of obligation and the associated feelings of guilt, shame, and pride, as well as by feelings of temptation.[6] Thus, given that the preference of each G-t is about how G-t would choose between consumption streams, each G-t's preference is probably partly based on obligation and temptation. This observation leads to the following criticisms of Axiom 5 and, especially, Axiom 5a.

3.1 Why should G-0 aggregate future generations' preferences?

In GS's setup G-0 makes a full-commitment choice, so that G-1, G-2, etc., will not face any obligation or temptation. Thus, it is unclear, both from a normative and a descriptive point of view, why G-0 forms its preference by aggregating the preferences of G-1, G-2, etc. After all, the preferences of G-1, G-2, etc., are partly based on feelings of obligation and temptation which G-1, G-2, etc., will never face. This undermines the rationale behind Axioms 5 and 5a.[7]

[4] See GS's Theorem 2 and Proposition 4 as well as footnote 3 above.

[5] Axioms 1-5 also feature in GS's utility representation theorems (Theorems 3 and 4). Axiom 5a doesn't explicitly appear in these theorems. However, given that the utility representations in Theorems 3 and 4 imply a V function that cannot be written as a function solely of its first two arguments, these utility representations (and, hence, the explicit axioms in Theorems 3 and 4) imply Axiom 5a. (See footnote 3 above.)

[6] For the revealed-preference implications of some of these feelings, see Gul and Pesendorfer (2001) (for temptation), Dillenberger and Sadowski (2012) (for shame), Yagasaki (2013) (for pride and shame), and Saito (2015) (for shame, pride, and temptation).

[7] This paragraph applies equally to GS's Axioms 6, 7, and 8, which are also about how G-0 aggregates future generations' preferences.

3.2 G-0 could perhaps take into account the preferences of G-2, G-3, etc., indirectly through G-1's preference.

One might insist that, although future generations will never face any obligation or temptation, G-0 should still respect how each future generation would choose if it were given the chance to make a (full-commitment) choice. However, if G-1's preference is based on obligation towards G-2, G-3, etc., G-0 could perhaps indirectly respect the preferences of G-2, G-3, etc., merely by taking into account the preference of G-1. Thus, it is unclear, both from a normative and a descriptive perspective, why G-0 must take into account, as Axiom 5a insists, the preferences of G-2, G-3, etc., over and above the extent to which these preferences are already incorporated in G-1's preference.

3.3 A Model with Obligation and Temptation

According to the previous two subsections, it is unclear why Axioms 5 and 5a should hold in the presence of obligation and temptation. However, it is still possible that, once one explicitly models obligation and temptation, Axioms 5 and 5a happen to hold after all. To explore this possibility, I consider the following model of intergenerational choice that incorporates obligation and temptation.[8]

G-0's overall well-being from choosing consumption stream c from a feasible set of consumption streams C is

$$U(c, C) = H(c) + \theta P(c, C) - \psi T(c, C), \tag{1}$$

where $H(c)$ is what I will call "hedonic well-being", $P(c, C)$ is pride, $\theta \geq 0$ is the weight on pride, $T(c, C)$ is temptation, and $\psi \geq 0$ is the weight on temptation. In anticipation of the fact that $U(c, \{c\}) = H(c)$ will hold given the specification of $P(c, C)$ and $T(c, C)$ below, we shall think of $H(c)$ as the well-being G-0 would derive from c if c were chosen for her and she didn't need to make a choice.

H is assumed to satisfy

[8]This model is inspired by ideas in GS and in Saito (2015).

$$H(c) = u(c_0) + \gamma \sum_{t=1}^{\infty} \alpha^t H({}_tc), \tag{2}$$

where $0 < \alpha < 1$, $0 \leq \gamma < \frac{1-\alpha}{\alpha}$, and $u(c_0)$ is well-being derived from period-0 consumption.[9] The idea behind (2) is that (i) future generations are forward-looking and their hedonic well-being is also captured by H, so that $H({}_tc)$ is the hedonic well-being of G-t, (ii) G-0 derives hedonic well-being both from period-0 consumption as well as from future generations' hedonic well-being (due to intergenerational altruism), (iii) future generations' hedonic well-being is discounted via the discount factor α, and (iv) the weight γ captures how G-0 combines well-being from period-0 consumption with future generations' hedonic well-being to obtain her own hedonic well-being.

Pride is assumed to satisfy

$$P(c, C) = \sum_{t=1}^{\infty} \alpha^t H({}_tc) - \hat{H}(C), \tag{3}$$

where $\hat{H}(C) \in [\min_{c' \in C} \sum_{t=1}^{\infty} \alpha^t H({}_tc'), \max_{c' \in C} \sum_{t=1}^{\infty} \alpha^t H({}_tc')]$ is the level of $\sum_{t=1}^{\infty} \alpha^t H({}_tc)$ that G-0, when choosing a consumption stream from C, is expected by society and G-0's conscience to provide for future generations.[10] Thus, G-0 obtains pride when $\sum_{t=1}^{\infty} \alpha^t H({}_tc)$ exceeds $\hat{H}(C)$ and suffers from shame/guilt otherwise.

Temptation is assumed to satisfy

$$T(c, C) = \max_{c' \in C} u(c'_0) - u(c_0). \tag{4}$$

Thus, G-0 experiences temptation depending on how much lower $u(c_0)$ is than the highest possible well-being from period-0 consumption.[11]

The appendix shows that, using (2), (3), and (4), we can write (1) as

$$U(c, C) = (1 + \psi)u(c_0) + \frac{\theta + \gamma}{1 + \gamma} \sum_{t=1}^{\infty} \alpha^t (1 + \gamma)^t u(c_t) - \theta \hat{H}(C) - \psi \max_{c' \in C} u(c'_0). \tag{5}$$

[9] The requirement $\gamma < \frac{1-\alpha}{\alpha}$ ensures that H is well-defined.

[10] For simplicity, I am assuming the same discount factor α in (2) and (3). Because we can always choose a low value of γ, this does not rule out the possibility that G-0 discounts heavily future generations' hedonic well-being in (2) while society and G-0's conscience do not do so in (3).

[11] I have been implicitly assuming that all sums converge and that all maxima and minima exist.

Given that the last two terms are constant for a fixed C, G-0 can be modelled as choosing a consumption stream that maximises

$$u(c_0) + \frac{\theta + \gamma}{(1+\gamma)(1+\psi)} \sum_{t=1}^{\infty} \alpha^t (1+\gamma)^t u(c_t). \tag{6}$$

When $\frac{\theta+\gamma}{(1+\gamma)(1+\psi)} = 1$, Axiom 5a fails because (6) is discounted utility and discounted utility is recursive. As far as I can tell, the parameter restriction $\frac{\theta+\gamma}{(1+\gamma)(1+\psi)} = 1$ cannot be ruled out on normative or descriptive grounds. Thus, a simple model that doesn't seem normatively reprehensible or descriptively implausible can lead to a violation of Axiom 5a.

In the context of intertemporal (rather than intergenerational) choice (see footnote 1), estimations of the β-δ model suggest $\beta < 1$.[12] Thus, in that context, one could argue that $\frac{\theta+\gamma}{(1+\gamma)(1+\psi)} < 1$ empirically, so that Axiom 5a holds from a descriptive point of view.[13] However, if we need to invoke estimations of the β-δ model in order to argue in favour of Axiom 5a, it is unclear how this axiom is useful in its own right.

4 A further criticism of Axiom 5

Axiom 5 rules out the preference $\succeq$ represented by the utility function $U(c) = \sum_{t=0}^{T} c_t$, where $T \geq 1$ is odd. To see that this preference violates Axiom 5, consider $c = (1, 1, 0, 1, 0, 1, 0, \ldots)$ and $c' = (1, 0, 1, 0, 1, 0, 1, \ldots)$. We have $c_0 = c'_0$ and $U({}_tc) = U({}_tc') = T/2$ for all $t \geq 1$. Yet, $U(c) = T/2 + 1 > T/2 = U(c')$.

To the extent that this preference isn't normatively/descriptively unreasonable, the fact that it violates Axiom 5 undermines this axiom's normative/descriptive appeal.

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[12] See Cohen et al. (forthcoming).

[13] The appendix shows that, under some technical assumptions, Axiom 5a holds whenever $\frac{\theta+\gamma}{(1+\gamma)(1+\psi)} \neq 1$.

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5 Appendix

Claim 1. *Given (2), (3), and (4), we can write (1) as (5).*

Proof:

When $\gamma = 0$, the proof is straightforward.

Assume $\gamma > 0$. The proof of Corollary 4 in GS with H replacing U, establishes that $H(c)$ can be written as:

$$H(c) = u(c_0) + \frac{\gamma}{1+\gamma} \sum_{t=1}^{\infty} \alpha^t (1+\gamma)^t u(c_t). \tag{7}$$

Expressions (2) and (7) imply $\sum\limits_{t=1}^{\infty} \alpha^t H({}_t c) = \frac{1}{1+\gamma} \sum\limits_{t=1}^{\infty} \alpha^t (1+\gamma)^t u(c_t)$. Thus, (3) can be written as:

$$P(c, C) = \frac{1}{1+\gamma} \sum_{t=1}^{\infty} \alpha^t (1+\gamma)^t u(c_t) - \hat{H}(C). \tag{8}$$

Using (7), (8), and (4) to plug into (1), proves the claim. Q.E.D.

Claim 2. *Suppose that (i) the set of possible consumption streams is $X^{\{0,1,2,\ldots\}}$, where X is a connected topological space, (ii) $u : X \to \mathbb{R}$ is continuous and nonconstant, and (iii) $\frac{\theta+\gamma}{(1+\gamma)(1+\psi)} \neq 1$. Then $\succeq$ induced by (6) satisfies Axiom 5a.*

Proof:

Let $\beta = \frac{\theta+\gamma}{(1+\gamma)(1+\psi)}$ and $\delta = \alpha(1+\gamma)$. Take c and c', such that $c_0 = c'_0$, $u(c_1) \neq u(c'_1)$, and $u(c_1) + \beta \sum_{t=1}^{\infty} \delta^t u(c_{t+1}) = u(c'_1) + \beta \sum_{t=1}^{\infty} \delta^t u(c'_{t+1})$ (i.e., ${}_1c \sim {}_1c'$).[14] Note that the last equality implies

$$\beta \sum_{t=1}^{\infty} \delta^t (u(c_{t+1}) - u(c'_{t+1})) = -(u(c_1) - u(c'_1)) \tag{9}$$

We have

[14] For example, we could pick c and c' as follows. Let $a, b \in X$ be such that $u(a) - u(b) > 0$. Let c_1 and c'_1 be such that $0 < \frac{u(c'_1)-u(c_1)}{\beta\delta} \leq u(a) - u(b)$. Let c_2 and c'_2 be such that $u(c_2) - u(c'_2) = \frac{u(c'_1)-u(c_1)}{\beta\delta}$. Finally, let $c_t = c'_t$ for all $t \geq 3$. (That a, b, c_1, c'_1, c_2, and c'_2 can be picked in this way follows from assumptions (i) and (ii) in the claim as well as the intermediate value theorem.)

$$
\begin{aligned}
u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t) - u(c'_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c'_t) &= \\
\beta \sum_{t=1}^{\infty} \delta^t (u(c_t) - u(c'_t)) &= \\
\beta\delta \sum_{t=1}^{\infty} \delta^{t-1} (u(c_t) - u(c'_t)) &= \\
\beta\delta \sum_{t=0}^{\infty} \delta^{t} (u(c_{t+1}) - u(c'_{t+1})) &= \\
\beta\delta (u(c_1) - u(c'_1)) + \beta\delta \sum_{t=1}^{\infty} \delta^{t} (u(c_{t+1}) - u(c'_{t+1})) &= \\
(\beta - 1)\delta(u(c_1) - u(c'_1)) &\neq 0,
\end{aligned}
$$

where the last equality uses (9). Thus, $c \sim c'$ does not hold. Q.E.D.

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