Unemployment, Firm Dynamics, and the Business Cycle

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Abstract

We formulate and estimate a business cycle model which can account for key business cycle properties of labor market variables and other aggregates. Three features distinguish our model from the standard model with Search And Matching (SAM) frictions in the labor market: frictional firm entry, endogenous product variety, and investment in two assets: stocks and physical capital. Our model with firm dynamics displays an endogenous form of wage moderation. Thanks to the latter, it outperforms the SAM framework augmented with exogenous real wage rigidities.

Key words: Entry, Unemployment, Bayesian Analysis, Search and Matching.
JEL codes: C5, E32

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1 Introduction

Business cycles are characterized by sizeable investment dynamics in the extensive margin of investment, that is in entry and exit of firms. Fluctuations in entry and exit account for a sizeable portion of job creation and job destruction and, thus, for fluctuations in vacancies and unemployment. Davis and Haltiwanger (1990), on the basis of U.S. manufacturing data between 1972 and 1986, report that 25% of annual gross job destruction can be attributed to establishment deaths, while 20% of annual gross job creation to the birth of new establishments. More recently, Jaimovich and Floetotto (2008) argue that entry and exit explain U.S. job flows at a higher frequency. Using the Business Employment Data (BED) from the third quarter of 1992 to the second quarter of 2005, they find that the average fraction of quarterly gross job-gains (losses) that can be explained by the opening (closing) of establishments is about 20%. Around a third of the cyclical volatility of the job-gains (losses) comes from opening (closing) establishments. These figures are not confined to a specific sector, but are consistent across U.S. industries.[1] Chatterjee and Cooper (1993), first pointed out the strong procyclicality of net business formation and new business incorporations. Similarly, Bergin and Corsetti (2008) argue that business cycles are characterized by sizeable dynamics in the extensive margin of investment.

We seek to understand the contributions of the extensive margin for business cycle fluctuations in unemployment, vacancies, and hours of work. We provide evidence for the United States concerning the joint response of these variables, and other key macroeconomic aggregates such as GDP and inflation, to shocks to technology, to the price markup and to the relative wage bargaining power of workers. To address the evidence we integrate large-firm models of the labor market with Search and Matching friction (SAM, henceforth) into a framework with endogenous firms entry (E) and monopolistic competition a là Bilbiie *et al.* (2012). We dub the resulting framework as Entry Search And Matching model (ESAM, henceforth). The large-firms assumption allows to separate the firm entry decision from the vacancy creation decision, and hence the dynamics of entry and exit from that of vacancies in response to shocks. ESAM models feature both an intensive and an extensive margin of investment, as well as an intensive and an extensive margin of labor. Households finance the entry of new firms in the market, along with the creation of physical capital, further, if employed, choose how many hours of work to supply. In ESAM, entry is subject to frictions. New firms must pay an entry cost, which is sunk, and it takes one period for new firms to start producing.[2] In our framework, the entry of a new firm corresponds to the creation of a new product variety.[3] For this reason, we could use the terms "firm" and "product" interchangeably, and sunk entry costs can be interpreted as product development costs. As pointed out by Bilbiie *et al.* (2012), this is consistent with the bulk of the macroeconomic literature with monopolistic competition, which similarly uses "firm" to refer to the producer of an individual good. Broda and Weinstein (2010) document a strong procyclicality of product creation, while Bernard *et al.* (2010) show that product creation and destruction account for important shares of overall production.

As in the seminal contribution by Bilbiie *et al.* (2012), firms enter the market up the point where

[1] Jaimovich and Floetotto (2008) argue that changes in the number of establishments or franchises will not be reflected in the data as changes in the number of firms. However, as theirs, our model interprets entry more broadly, and should be seen as analyzing variations in the overall number of competitors, not just in the number of firms.

[2] An incomplete list of models with these assumptions include Bilbiie *et al.* (2012), Bilbiie (2020), Bergin and Corsetti (2008), Etro and Colciago (2010), Colciago and Etro (2010), Bilbiie *et al.* (2014) and Lewis and Poilly (2012). Bilbiie *et al.* (2019) provide a discussion of the distortions and welfare costs in a flexible-price model under general-homothetic preferences for variety.

[3] Empirically, new products are not only introduced by new firms, but also by existing firms As in Bilbiie *et al.* (2012), we take a broad view of producer entry and exit as also incorporating product creation and destruction by existing firms, although our model does not address the determinants of product variety within firms.

the expected discounted value of future profits equals the sunk entry cost. The investment in new productive units is financed by households through the accumulation of shares in the portfolio of firms. The stock market price of this investment fluctuates endogenously in response to shocks. We consider several empirically motivated frictions to make the model estimable, in line with Christiano *et al.* (2005) and Lewis and Poilly (2012). These are internal habit persistence in consumption, physical capital to produce final goods, and adjustment costs in both the intensive and extensive margin of investment. As a result, and in contrast with the standard SAM framework, in ESAM individuals can invest in two assets: stocks and physical capital. The price of both assets fluctuates over the business cycle. Stock market prices, together with the shares' payoff coming from imperfect competition determine the return to entry, while fluctuations in the price of capital and in its marginal product determine investment in physical capital.

We estimate the structural parameters of the theoretical models by matching the impulse responses obtained with a vector autoregressive (VAR) model, that includes labor market variables and firm entry. More precisely, we adopt a Bayesian minimum distance techniques in the spirit of Christiano *et al.* (2010). A minimum distance technique to estimates the parameters of a SAM model is also adopted by Trigari (2009).

In the VAR analysis we identify shocks to technology, to the price markup, and to the wage bargaining power of workers. The VAR-based impulse response functions are identified with sign restrictions. A major challenge in estimating a DSGE model via matching VAR-based impulse responses identified through sign restrictions, is that there is no point estimate that the minimum distance method can take as the empirical reference. To tackle the issue, we follow Hofmann *et al.* (2012). Specifically, we consider a large set of VAR-based impulse response functions fulfilling the sign restrictions, and for each of them, we run a minimum distance estimation with the model-based impulse responses. As a result of the estimation procedure, we obtain moments and quantiles of the implied posterior mode distributions for the estimated parameters. With the estimated parameters in hand, we evaluate the performance of the models in fitting the data on the basis of the marginal likelihood and on the IRFs to three shocks, namely to technology, to price markup, and to wage bargaining power of workers.

How does the performance of estimated ESAM models compare with that of SAM models? To answer to this question, we compare the performances of estimated ESAM models with that of models with a fixed number of varieties, such as the traditional SAM framework with large firms. In the latter, the number of varieties is fixed and the extensive margin of investment is shut off.

The main result of our analysis is that ESAM models account for the response of labor market variables such as wages, unemployment, job vacancies and total hours, and for the response of profits and firm entry to the three shocks we identify. The success of ESAM in replicating the dynamics of those variables is due to a form of endogenous wage moderation in response to technology shocks, that spreads from the extensive margin of investment. In SAM models with a fixed number of varieties the real wage typically displays a sharp response to shocks, that, in the case of technology shocks, leads to counterfactual responses of hours and profits. In contrast, the endogenous wage inertia characterizing ESAM allows replication of the cyclicality of profits and that of hours without resorting to the exogenous assumption of real wage rigidities, that are instead needed in the SAM framework. As we discuss below, the interaction between the asset market and the labor market is at the core of the intuition for our results.

The improvement of ESAM models over SAM ones is not confined to the replication of technology shocks**.** The statistical fit of the various ESAM models we estimate, as measured by the marginal likelihood, is consistently higher than those of SAM models.[4] Our estimates suggest that

[4] The likelihood functions of ESAM and SAM are compared assuming that entry is not in the infomation set. Thus,

structural parameters of the models are generally robust across SAM and ESAM. Both models call for a high outside option value for workers in order to replicate the empirical IRFs. However, since ESAM features investment at both the intensive and the extensive margin, this does not lead to an excessively large steady state labor share of income. Additionally SAM requires a low bargaining power of workers in order to mitigate fluctuations in the real wage in response to shocks. In other words, SAM typically requires a Hagedorn and Manovskii (2008)-type parameterization in order to fit the data. Thanks to endogenous wage inertia, this is not necessary in ESAM, where the estimated value of the wage bargaining power is in line with that used in the bulk of the literature.

To understand how the extensive margin of investment affects the response of the real wage, and the rest of the economy, consider an expansionary technology shock. On impact, the increase in productivity translates into an increase in the real wage and in the return on capital. From the standpoint of the households, this increase in the rate of return translates into an increase in the willingness to invest. Increased productivity stimulates firms' labor demand at both margins. Due to the higher wage, and to the temporarily higher interest rate, households are willing to accommodate some of the increase in demand by working more. In SAM models the increase in labor supply is not enough to avoid a sharp increase in the real wage. As a result, the responses of aggregate hours of work and profits are, under the vast majority of the estimated parameter values, countercyclical.[5]

In ESAM models those described are just partial effects. Households have an additional mean to transfer resources intertemporally, the extensive margin of investment, with respect to what they have in SAM. Indeed, since next period's firms profits are expected to be high, households invests in the creation of new firms. The increase at both margins of investment amplifies the response of output with respect to what is observed in SAM. Since entry is frictional, the number of firms is a state variable, and the increased output is initially produced by incumbent firms. Given incumbents operate under imperfect competition, this fuels their profits. Hours are the only input that firms can adjust on impact, and for this reason firms increase their demand. However, this does not lead to a counterfactual strong procyclical response of the real wage. To make the most of investment opportunities, households further increase their willingness to work. As a result the response of the real wage is milder with respect to what is observed in SAM. This translates into a procyclical response of both aggregate hours and profits together with a countercyclical unemployment rate. Our VAR analysis suggests that a procyclical response of hours and profits in response to a technology shock is the most likely event. Although our models cannot match the magnitude of the empirical response of profits to the shock, they match their cyclicality. The surge in hours, employment and vacancies is sustained over time by the entry of new firms that start producing. Importantly, the ESAM model replicates the VAR-based responses for all the shocks we identify. This motivates why the marginal likelihood of ESAM model is substantially higher than that of SAM ones.

Starting with Hall (2005), the literature has suggested that the wage setting mechanism in SAM models has to be altered, such that the real wage becomes just mildly responsive to technology and other shocks, in order for the model to account for the cyclical properties of unemployment and vacancies. For this reason we compare the performance of our benchmark ESAM model, with no exogenous wage rigidities, to that of a SAM model with real wage inertia in the form of a wage norm, that we dub WSAM. At the posterior mode of the parameters, the estimated ESAM and WSAM models provide very similar performances. Nevertheless, the ESAM model is preferred to WSAM in terms of statistical fit. Additionally, ESAM models explains the cyclicality of the dynamics of

we evaluate the statistical performance of the two models at replicating the dynamics of the same set of variables.

[5] Recall that the estimation procedure delivers a distribution of estimated parameters, and thus we characterize a set of IRFs to each shock.

the number of product/competitors in response to shocks that WSAM, by construction, cannot address.

The benchmark version of ESAM features flexible adjustments both in wages and prices, and, given monopolistic competition, a price markup that does not depend on the extent of competition. We extend the baseline specification to include price rigidities to obtain New Keynesian ESAM models, that we dub NK-ESAM. The inclusion of New Keynesian features further improves the performance of the ESAM model in replicating the dynamics of the main macroeconomic variables over the business cycles, particularly of those for the labor market. Importantly, NK-ESAM models have a substantially higher statistical fit than NK-SAM ones.

As a last extension, we account for the role of market structures by considering oligopolist competition *à la* Bertrand in the market for final goods. In this case, variations in the number of operating firms result in endogenous variations in the markup level along the cycle. Specifically, following Colciago (2016) and Jaimovich and Floetotto (2008), an increase in competition translates into a reduction in the price markup.

Our analysis shows that Bertrand competition fits the data slightly better than monopolistic competition, but modelling the market for final goods as a monopolistically competitive one, as most of the literature does, makes little quantitative difference when it comes to addressing the business cycle properties of the main aggregates. Notice, however, that departing from monopolistic competition could be necessary when trying to address specific aspects of the data, such as the relationship between price markups and the extent of competition.

A recent and growing literature, inspired by the work of Melitz (2003), Bilbiie *et al.* (2012), Jaimovich and Floetotto (2008), Clementi and Palazzo (2016), Rossi (2019) among others, studies how the extensive margin of firm entry and product variety can contribute to understand the business cycle. Bergin and Corsetti (2008) originally studied the monetary transmission mechanism in the presence of an extensive margin of investment in open economies, more recently Lewis and Poilly (2012), Bilbiie (2020), and Colciago and Silvestrini (2020) reconsidered the issue in closed economies. Closer to this paper are contributions by Ambler and Cardia (1998), Colciago and Rossi (2015), Cacciatore and Fiori (2016), Shao and Silos (2013), and Mangin and Sedláček (2018). Ambler and Cardia (1998) consider a general equilibrium model with firms' entry and competition, which features a perfectly competitive labor market. In their setting the number of firms is pinned down by a zero profits condition. Hence, while their model delivers a countercyclical labor share of income, the cyclicality of profits and unemployment cannot be addressed. Colciago and Rossi (2015), Cacciatore and Fiori (2016) and Shao and Silos (2013) consider search and matching models with an extensive margin of investment. Colciago and Rossi (2015) and Mangin and Sedláček (2018) study the role of competition for the response of the labor share of income to technology shocks. Cacciatore and Fiori (2016) consider the macroeconomic effects of deregulating the goods and labor markets. Shao and Silos (2013) find that sunk costs of entry imply a countercyclical net present value of a vacancy, which has implications for the surplus division between firms and workers over the business cycle. With respect to those works, this paper provides both empirical and theoretical contributions. We provide empirical evidence concerning the joint responses of firm entry, investment, unemployment and hours of work to shocks to technology, the price markup and to the relative wage bargaining power of workers. We develop versions of the ESAM model characterized by various empirically relevant frictions. We disentangle the role played by each friction for the transmission of shocks, and establish the statistical fit of each model version.

Our paper is related to the work of Christiano *et al.* (2016), who stress the importance of wage inertia for the dynamics of unemployment, vacancies and inflation in SAM models. Christiano *et al.* (2016) develop a model where wage inertia emerges as the solution to the bargaining problem between firms and workers. Our paper, while maintaining standard Nash bargaining between firms

and workers, obtains wage inertia thanks to the presence of two assets: physical capital and stocks. We endogenize the return of stocks by building a model where profits and firms' entry depend on the conditions of the business cycle. Stock returns affect the willingness to work and saving of households, and through this channel they ultimately affect the equilibrium real wage.

The remainder of the paper is organized as follows. Section 2 introduces the benchmark ESAM model with monopolistic competition. Section 3 extends the baseline framework to nominal rigidities. Section 4 spells out the SAM models that we take as a reference for model comparison, including a version with real wage rigidity. Section 5 outlines the econometric methodology. Section 6 contains the main findings, and Section 7 concludes.

2 The Model Economy: ESAM

In this section we outline ESAM, the benchmark economy with endogenous firm entry and SAM frictions. It embeds firm endogenous entry in a SAM model with large firms. To make the model estimable, following Christiano *et al.* (2005) and Trigari (2009), we consider habit persistence in consumption, physical capital to produce the final goods and adjustment costs at the intensive margin of investment. As in Casares *et al.* (2018) and Lewis and Poilly (2012), we include adjustment costs along the extensive margin of investment.

The economy features a continuum of atomistics sectors, or industries, on the unit interval. Each sector is characterized by different firms producing a good in different varieties, using labor and capital as inputs. The sectoral goods are imperfect substitutes for each other and are aggregated into a final good through a CES aggregator. Households use the final good for consumption and investment purposes. Endogenous firms' entry is modeled at the sectoral level, where firms face search and matching frictions in hiring workers. The benchmark version of the ESAM model features monopolistic competition in the markets for final goods. In the Appendix, we extend the framework to account for strategic interactions between an endogenous number of producers by considering Bertrand Competition. We dub the version of ESAM characterized by Bertrand competition as BESAM.

2.1 Labor and Goods Markets

At the beginning of each period, N^e_{jt} firms enter into sector $j \in (0,1)$, while at the end of the period a fraction $\delta \in (0,1)$ of market participants exits from the market for exogenous reasons. As a result, the number of firms, N_{jt}, in the sector j follows the law of motion:

$$N_{jt+1} = (1-\delta)\left(N_{jt} + N^e_{jt}\right),$$

where N^e_{jt} is the number of new entrants in sector j at time t. Following Bilbiie *et al.* (2012), we assume that new entrants at time t will only start producing at time $t+1$ and that the exit rate, δ, is independent of the period of entry and identical across sectors. The assumption of an exogenous constant exit rate in adopted for tractability, but it also has empirical support. Using U.S. annual data on manufacturing, Lee and Mukoyama (2015) find that, while the entry rate is procyclical, annual exit rates are similar across booms and recessions. Below we describe the entry process in detail. In ESAM models, the markets for final goods are characterized by monopolistic competition. As an extension, we show that the findings are robust to the introduction of oligopolistic competition *à la* Bertrand in the goods market.

The labor market is characterized by search and matching frictions, as in Andolfatto (1996) and Merz (1995). Producers post vacancies in order to hire new workers. Unemployed workers and

vacancies combine according to a constant returns to scale matching function and deliver m_t new hires, or matches, in each period. The matching function reads as:

$$m_t = \gamma_m \left(v_t^{tot}\right)^{1-\gamma} u_t^{\gamma},$$

where γ_m reflects the efficiency of the matching process, v_t^{tot} is the total number of vacancies created at time t and u_t are the workers searching for a job.[6] The probability that a firm fills a vacancy is given by $q_t = \frac{m_t}{v_t^{tot}}$, while the probability to find a job for an unemployed worker reads as $z_t = \frac{m_t}{u_t}$. Firms and individuals take both probabilities as given. Matches become productive in the same period in which they are formed. Each firm separates exogenously from a fraction $1-\varrho$ of existing workers each period, where ϱ is the probability that a worker stays with a firm until the next period. As a result, a worker may separate from a job for two reasons: either because the firm where the job is located exits the market or because the match is destroyed. Since these sources of separation are independent, the evolution of aggregate employment, L_t, is given by:

$$L_t = (1-\delta)\,\varrho L_{t-1} + m_t.$$

2.2 Households

Using the family construct of Merz (1995), the representative household consists of a continuum of individuals of mass one. Members of the household insure each other against the risk of being unemployed. The representative family has lifetime utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln\left(C_t - \vartheta C_{t-1}\right) - \chi L_t \frac{{h_t}^{1+\varphi}}{1+\varphi} \right) \quad \chi, \eta, \varphi \geq 0, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, the variable h_t represents individual hours worked by each member of the household, and C_t is the consumption of the final good. Consumption displays internal habit persistence of degree ϑ. The household is assumed to own physical capital, K_t, which, as in Schmitt-Grohé and Uribe (2005), Christiano *et al.* (2005), Smets and Wouters (2007) and Altig *et al.* (2011), accumulates according to the following law of motion:

$$K_{t+1} = (1-\delta_k)K_t + \left(1 - S\left(\frac{I_t^k}{I_{t-1}^k}\right)\right) I_t^k, \tag{2}$$

where I_t^k denotes gross investment in physical capital, and δ_k is a parameter denoting the rate of depreciation. The function S introduces investment adjustment costs and it is assumed to satisfy $S(1) = S\prime(1) = 0$ and $S''(1) > 0$. These assumptions imply the absence of adjustment costs up to first order in the vicinity of the deterministic steady state.

The representative agent enjoys capital, dividend, and, if employed, labor income. Markets are complete. Unemployed individuals receive a real unemployment benefit b, hence the overall benefit for the household is $b\,(1-L_t)$. This is financed through lump sum taxation by the Government. Notice that the household recognizes that employment is determined by the flows of its members into and out of employment according to

$$L_t = (1-\delta)\,\varrho L_{t-1} + z_t u_t. \tag{3}$$

[6] Given that population is normalized to one, the number of unemployed workers and the unemployment rate are identical.

Timing of investment in the stock market is as in Bilbiie *et al.* (2012) and Chugh and Ghironi (2011). At the beginning of period t, the household owns x_t shares of a mutual fund of the N_t firms that produce in period t, each of which pays a dividend d_t. Denoting the value of a firm with V_t, it follows that the value of the portfolio held by the household is $x_t V_t N_t$. During period t, the household purchases x_{t+1} shares in a fund of these N_t firms as well as of the N_t^e new firms created during period t. Total stock market purchases to be carried into period $t+1$ are thus $x_{t+1} V_t (N_t + N_t^e)$. At the very end of period t, a fraction of these firms disappears from the market.[7] Following the production and sales of the N_t varieties in the imperfectly competitive goods markets, firms distribute the dividend d_t to households. The household's total dividend income is thus $D_t = x_t d_t N_t$. The family receives real labor income $w_t h_t L_t$, where w_t is the real wage. Households rent the capital stock to firms at the real rental rate r_t^k per unit of capital. Thus, total income stemming from the rental of capital is given by $r_t^k K_t$. The household chooses how much to save in bonds, in physical capital and in the creation of new firms through the stock market according to standard Euler and asset pricing equations. The first order condition (FOC) with respect to employment, L_t, is:

$$\Gamma_t = \lambda_t w_t h_t - \eta \frac{h_t^{1+1/\varphi}}{1+1/\varphi} - b\lambda_t + \beta (1-\delta) \varrho E_t \left[z_{t+1} \Gamma_{t+1} \right], \tag{4}$$

where Γ_t is the marginal value to the household of having one member employed rather than unemployed, and λ_t is the marginal utility of consumption. Equation (4) indicates that the household's shadow value of one additional employed member (the left hand side) has four components: first, the increase in utility generated by having an additional member employed, given by the real wage expressed in utils; second, the decrease in utility due to more hours dedicated to work, given by the marginal disutility of employment; third the foregone utility value of the unemployment benefit $b\lambda_t$; fourth, the continuation utility value, given by the contribution of a current match to next period household's employment.

2.3 Firms and Technology

As in Jaimovich and Floetotto (2008), the final good is produced aggregating a continuum of measure one of sectoral goods according to the function

$$Y_t = \left(\int_0^1 \ln Y_{jt}^{\frac{\omega-1}{\omega}} dj \right)^{\frac{\omega}{\omega-1}}, \tag{5}$$

where Y_{jt} denotes output of sector j and ω is the elasticity of substitution between any two different sectoral goods. The final good producer behaves competitively and, solving a static optimization problem, demands the following quantity of sectoral good for each sector j,

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\omega} Y_t, \tag{6}$$

where P_{jt} is the price index of sector j in period t and P_t is the price of the final good in period t,

$$P_t = \left[\int_0^1 P_{jt}^{1-\omega} dj \right]^{\frac{1}{1-\omega}}. \tag{7}$$

[7] Due to the Poisson nature of exit shocks, the household does not know which firms will disappear from the market, so it finances the continued operations of all incumbent firms as well as those of the new entrants.

As in Etro and Colciago (2010), we assumed a unit elasticity of substitution between goods belonging to different sectors. This is done for simplicity, but notice that Jaimovich and Floetotto (2008) estimate a value of the intersectoral elasticity of substitution essentially equal to 1 using US data. This allows realistically separate limited substitutability at the aggregate level, and high substitutability at the disaggregate level. In each sector j, there are $N_{jt} > 1$ firms producing differentiated goods that are aggregated into a sectoral good by a CES aggregating function defined as:

$$Y_{jt} = N_{jt}^{\frac{1}{\varepsilon_t - 1}} \left(\sum_{z=1}^{N_{jt}} y_{jt}(z)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} \right)^{\frac{\varepsilon_t}{\varepsilon_t - 1}}, \tag{8}$$

where $y_{jt}(z)$ is the production of good z in sector j, $\varepsilon_t > 1$ is the elasticity of substitution between sectoral goods.[8] The latter is assumed to follow an AR(1) process with coefficient ρ_ε. Each firm z in sector j produces a differentiated good with the following production function

$$y_{jt}(z) = A_t \left[n_{jt}(z) \, h_{jt}(z) \right]^{1-\alpha} k_{jt-1}^{\alpha}(z), \tag{9}$$

where A_t represents technology which is common across sectors and evolves exogenously over time following an AR (1) process with persistency ρ_a and standard deviation σ_a.Variable $n_{jt}(z)$ is firm z's time t workforce, $h_{jt}(z)$ represents hours per employee, and $k_{t-1}(z)$ is the stock of capital used by firm z at time t. Real profits of a firm at time t are defined as

$$\pi_{jt}(z) = \frac{p_{jt}(z)}{P_t} y_{jt}(z) - w_t h_{jt}(z) \, n_{jt}(z) - r_t^k k_{jt-1}(z) - \kappa v_{jt}(z), \tag{10}$$

where $w_{jt}(z)$ is the real wage paid by firm z, $v_{jt}(z)$ represents the number of vacancies posted at time t and κ is the output cost of keeping a vacancy open. The value of a firm is the expected discounted value of its future profits

$$V_{jt}(z) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} \pi_{js}(z), \tag{11}$$

where $\Lambda_{t,t+1} = (1-\delta)\beta \frac{\lambda_{t+1}}{\lambda_t}$ is the households' stochastic discount factor which takes into account that firms' survival probability is $1-\delta$. Firms which do not exit the market have a time t individual workforce given by

$$n_{jt}(z) = \varrho n_{jt-1}(z) + v_{jt}(z) \, q_t. \tag{12}$$

The unit intersectoral elasticity of substitution implies that the nominal expenditure, EXP_t, is identical across sectors. Thus, the final producer's demand for each sectoral good is

$$P_{jt} Y_{jt} = P_t Y_t = EXP_t. \tag{13}$$

where P_{jt} is the price index of sector j and P_t is the price of the final good at period t. Denoting with $P_{jt}(z)$ the nominal price of good z in sector j, the demand faced by the producer of each variant is

[8] The term $N_{jt}^{-\frac{1}{\varepsilon - 1}}$ implies that there is no variety effect in the model.

$$y_{jt}(z) = \left(\frac{P_{jt}(z)}{P_{jt}}\right)^{-\varepsilon_t} \frac{Y_{jt}}{N_{jt}}, \tag{14}$$

where P_{jt} is defined as

$$P_{jt} = N_{jt}^{\frac{1}{\varepsilon_t - 1}} \left[\sum_{z=1}^{N_{jt}} (P_{jt}(z))^{1-\varepsilon_t}\right]^{\frac{1}{1-\varepsilon_t}}. \tag{15}$$

Using (14) and (6), the individual demand of good z can be written as a function of aggregate expenditure,

$$y_{jt}(z) = \frac{(P_{jt}(z))^{-\varepsilon_t}}{(P_{jt})^{1-\varepsilon_t}} \frac{P_t Y_t}{N_{jt}} = \frac{(P_{jt}(z))^{-\varepsilon_t}}{(P_{jt})^{1-\varepsilon_t}} \frac{EXP_t}{N_{jt}}. \tag{16}$$

As the technology, the entry cost and the exit probability are identical across sectors, we can drop the index j and refer to a representative sector with

$$N_{jt} = N_t,\ P_{jt} = P_t,\ n_{jt}(z) = n_t(z),\ h_{jt}(z) = h_t(z),\ k_{jt}(z) = k_t(z),\ v_{jt}(z) = v_t(z)$$

and

$$p_{jt}(z) = p_t(z),\ \pi_{jt}(z) = \pi_t(z),\ V_{jt}(z) = V_t(z)$$

2.4 Pricing and Job creation

In what follows producers are distinguished according to their period of entry. New firms are those producing units which entered the market in period $t-1$ and in period t produce for the first time. New firms are thereby the fraction of time $t-1$ entrants which survived to the next period.[9] We define incumbent producers entering the market in period $t-2$ or prior. The distinction is relevant because new firms have no beginning of period workforce. Nevertheless, all producing firms, independently of the period of entry, have in equilibrium the same size, impose the same markup over a common marginal cost, and have the same individual level of production. For this reason in what follows we drop the index z denoting variables relative to the individual firm. Optimal pricing implies that the relative price chosen by firms is

$$P_t = \mu_t MC_t, \tag{17}$$

where MC_t are nominal marginal costs, and μ_t defines the price markup. To maintain comparability with the bulk of the literature, the ESAM model features monopolistic competition *à la* Dixit and Stiglitz (1977). In this case, the price markup assumes the traditional form

$$\mu_t = \frac{\varepsilon_t}{\varepsilon_t - 1}, \tag{18}$$

As well known, the price markup, μ_t, is decreasing in the degree of substitutability between products, ε_t belonging to the same sector. We assume that the latter follows an AR (1) process with persistency ρ_ε and standard deviation σ_ε. A firm will hire workers up to the point where the value

[9] Notice that N_{t-1}^e are the entrants at time $t-1$, and that just a fraction $(1-\delta)$ of time $t-1$ entrants start producing in period t. We define these firms as new firms.

of the marginal worker, defined as ϕ_t, equals its marginal cost, that is when

$$\phi_t = \left((1-\alpha)(mc_t A_t) \left(\frac{k_{t-1}}{n_t h_t} \right)^{\alpha} h_t - w_t h_t \right) + (1-\delta)\, \varrho E_t \Lambda_{t,t+1} \phi_{t+1}. \tag{19}$$

Condition (19) implies that the value of the marginal worker, ϕ_t, is represented by the profits associated to the additional worker, the term in brackets, plus the continuation value. Next period, with probability ϱ, the match is not severed. In this event the firm obtains the future expected value of a job. Similarly, a firm will post vacancies such that the value of the marginal worker, ϕ_t, equals to the expected cost of hiring the worker, $\frac{\kappa}{q_t}$:

$$\phi_t = \frac{\kappa}{q_t}, \tag{20}$$

where κ defines the cost of opening a vacant position in term of the final good. Combining the latter two equations delivers the Job Creation Condition (JCC)

$$\frac{\kappa}{q_t} = \left((1-\alpha) \left(\frac{A_t}{\mu_t^M} \right) \left(\frac{k_{t-1}}{n_t h_t} \right)^{\alpha} h_t - w_t h_t \right) + (1-\delta)\, \varrho \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{q_{t+1}} \right), \tag{21}$$

where the pricing condition is used to substitute for the real marginal cost, namely $mc_t \equiv \frac{MC_t}{p_t} = \frac{1}{\mu_t}$.

The approach featuring monopolistic competition *à la* Dixit and Stiglitz (1977) neglects the role of strategic interactions between firms belonging to the same sector, and the impact of entry on the same strategic interactions. To quantitatively assess the importance of strategic interactions for the propagation of aggregate shocks, in the Appendix we consider Bertrand competition among an endogenous number of producers. In that case, as shown by Etro and Colciago (2010) and Jaimovich and Floetotto (2008), the markup function depends on the extent of competition. Specifically, it is decreasing in the degree of sustitutability between goods, ε_t, and in the number of firms in the market N_t. We dub the version of the ESAM model characterized by Bertrand competition as BESAM. In the empirical analysis we estimate both versions of the model.

2.5 Hiring policy

Let π_t^{new} and v_t^{new} be, respectively, the real profits and the number of vacancies posted by a new firm. Symmetrically, π_t and v_t define the individual profits and vacancies posted by an incumbent producer. New firms and incumbent firms are characterized by the same size, n_t. Thus, the optimal hiring policy of new firms, which have no initial workforce, consists in posting at time t as many vacancies as required to hire n_t workers. As a result $v_t^{new} = \frac{n_t}{q_t}$. Since $n_t = \varrho n_{t-1} + v_t q_t$, it has to be the case that

$$v_t^{new} = v_t + \frac{\varrho n_{t-1}}{q_t}. \tag{22}$$

Hence, a new firm posts more vacancies than an incumbent producer. For this reason, and given vacancy posting is costly, the profits of new firms are lower than those of incumbent firms. To see this, notice that

$$\pi_t^{new} = y_t - w_t h_t n_t - r_t^k k_{t-1} - \kappa v_t^{new}. \tag{23}$$

Substituting equation (22) in the latter delivers

$$\pi_t^{new} = \left(y_t - w_t h_t n_t - r_t^k k_{t-1} - \kappa v_t \right) - \kappa \frac{\varrho n_{t-1}}{q_t} = \pi_t - \kappa \frac{\varrho n_{t-1}}{q_t}. \tag{24}$$

The last equality follows from the fact that the term in the round bracket represents the profits of an incumbent producer, π_t. Consistently with the U.S. empirical evidence in Haltiwanger *et al.* (2013) and Cooley and Quadrini (2001), a young firm creates on average more new jobs than a mature firm and distributes lower dividends.

2.6 Endogenous Entry

In each period the level of entry is determined endogenously to equate the value of a new entrant, V_t^e, to the entry cost

$$V_t^e = \psi_t. \tag{25}$$

The latter is composed by a constant term, ψ_0, and by a term which is related to market congestion externalities, $\psi_1 \left(\frac{N_t^e}{N_t}\right)^{\varsigma}$, as in Casares *et al.* (2018). In formula, entry costs reads as

$$\psi_t = \psi_0 + \psi_1 \left(\frac{N_t^e}{N_t}\right)^{\varsigma}. \tag{26}$$

A higher rate of entry, $\frac{N_t^e}{N_t}$, implies an increase in the costs of creating a new firm. This non-constant, state dependent term in the entry cost function can be interpreted as an adjustment cost to extensive margin of investment akin to the cost of adjusting investment in physical capital.

Notice that perspective new entrants have lower value than incumbents because they will have, in case they do not exit the market before starting production, to set up a workforce in their first period of activity. The difference in the value between a firm which is already producing and a perspective entrant is, in fact, the discounted value of the higher vacancy posting cost that the latter will suffer, with respect to the former, in the first period of activity. Formally,

$$V_t = V_t^e + \kappa \varrho E_t \Lambda_{t,t+1} \frac{n_t}{q_{t+1}}, \tag{27}$$

where V_t is the time t value of a producing firm, independently of the period of entry.

2.7 Bargaining over Wages and Hours

As in Trigari (2009), individual bargaining takes place along two dimensions: the real wage and hours of work. We assume Nash bargaining. That is, the firm and the worker choose the wage w_t and the hours of work h_t to maximize the Nash product

$$(\phi_t)^{1-\eta_t} \left(\frac{\Gamma_t}{\lambda_t}\right)^{\eta_t}, \tag{28}$$

where ϕ_t is firm value of having an additional worker, while Γ_t/λ_t is the household's surplus expressed in units of consumption. The parameter η_t reflects the parties' relative bargaining power. We assume that the latter follows an AR(1) process with persistency ρ_η, and standard deviation σ_η. The FOC with respect to the real wage is

$$\eta_t \phi_t = (1 - \eta_t) \frac{\Gamma_t}{\lambda_t}. \tag{29}$$

Using the definition of ϕ_t in equation (19), and that of Γ_t in equation (4), after some manipu-

lations it yields the wage equation

$$w_t h_t = \eta_t (1-\alpha) \left(\frac{A_t}{\mu_t^M}\right) \left(\frac{k_{t-1}}{n_t h_t}\right)^{\alpha} h_t + (1-\eta_t) \left(\frac{\chi}{\lambda_t} \frac{h_t^{1+\varphi}}{1+\varphi} + b\right) + \eta_t \kappa \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \theta_{t+1}\right), \tag{30}$$

where $\theta_t = \frac{z_t}{q_t}$ measures the tightness in the labor market. The wage shares costs and benefits associated to the match according to the extent of the bargaining power, as measured by η_t. The worker is rewarded for a fraction η_t of the firm's revenues and savings of hiring costs, and compensated for a fraction $1-\eta_t$ of the disutility he suffers from supplying labor and the foregone unemployment benefits. Individual hours, h_t, are such that

$$\frac{\chi}{\lambda_t} h_t^{\varphi} = (1-\alpha)^2 \left(\frac{A_t}{\mu_t^M}\right) \left(\frac{k_{t-1}}{n_t h_t}\right)^{\alpha}. \tag{31}$$

Because the firm and the worker bargain simultaneously about wages and hours, the outcome is (privately) efficient and the wage does not play an allocational role for hours.[10]

2.8 Aggregation and Market Clearing

Considering that sectors are symmetric and have a unit mass, the sectoral number of firms and new entrants also represents their aggregate counterpart. Thus, the dynamics of the aggregate number of firms is

$$N_{t+1} = (1-\delta)(N_t + N_t^e). \tag{32}$$

The firms' individual workforce, n_t, is identical across producers, hence $L_t = N_t n_t$. The aggregate production function is:

$$Y_t = N_t y_t = A_t (L_t h_t)^{1-\alpha} K_{t-1}^{\alpha}. \tag{33}$$

Total vacancies posted in period t are $v_t^{tot} = (1-\delta) N_{t-1} v_t + (1-\delta) N_{t-1}^e v_{t-1}^{new}$, where $(1-\delta) N_{t-1}$ is the number of incumbent producers, and $(1-\delta) N_{t-1}^e$ is the number of new firms. Aggregating the budget constraints of households the implied aggregate resource constraint of the economy is

$$C_t + \psi_t N_t^e + I_t^k = w_t h_t L_t + r_t^k K_{t-1} + PRO_t, \tag{34}$$

which states that the sum of consumption, extensive investment and intensive investment must equal the sum between labor income, capital income and aggregate profits, PRO_t, distributed to households at time t. Aggregate profits are defined as

$$PRO_t = (1-\delta) N_{t-1} \pi_t + (1-\delta) N_{t-1}^e \pi_t^{new}. \tag{35}$$

Goods' market clearing requires

$$Y_t = C_t + \psi_t N_t^e + I_t^k + \kappa v_t^{tot}. \tag{36}$$

The GDP is therefore defined as the total output net of the vacancy costs, namely

$$GDP_t = Y_t - \kappa v_t^{tot}. \tag{37}$$

[10] Notice that we ruled out the possibility of a hiring externality. This simplifies the derivation of the wage equation. Ebell and Haefke (2009) show that the quantitative effect of overhiring is minor.

Finally, the dynamics of aggregate employment reads as

$$L_t = (1-\delta)\,\varrho L_{t-1} + q_t v_t^{tot} \tag{38}$$

which shows that workers employed by a firm which exits the market join the mass of unemployed. The list of the full set of equilibrium conditions of the economy is in the Technical Appendix.

3 Nominal Rigidities: NK-ESAM

This section describes ESAM models with nominal price rigidities, that we define NK-ESAM. The price-setting mechanism follows Rotemberg (1982b), where firms face a quadratic cost, $pac_t(z)$, of adjusting nominal prices. The latter is measured in terms of the final good and it is defined as:

$$pac_t(z) = \frac{\phi_P}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 \frac{P_t(z)}{P_t} y_t(z), \tag{39}$$

where $\phi_P > 0$ determines the degree of nominal price rigidity, and $P_t(z)$ is the nominal price of firm z at time t.[11] The price adjustment costs can be interpreted as the amount of marketing materials that a firm must purchase when implementing a price change. We follow Bilbiie *et al.* (2007) and interpret the time $t-1$ price in the expression of (39) as the notional price that the firm would have set at time $t-1$ if it had been producing in that period. All firms suffer the marketing cost in equation (39) when implementing a price decision. As argued by Bilbiie *et al.* (2007), this assumption is consistent with the original Rotemberg (1982b) setup and with the time to build a firm assumption. Specifically, as in Rotemberg (1982b)'s framework, the initial condition for the individual price is dictated by nature. The assumption that a new entrant, at the time of its first price setting decision, knows the average product price last period is consistent with the timing assumption that an entrant starts producing only one period after entering. Hence, an entrant can learn the average product price during the entry period. Optimal pricing is still defined by equation 17. However, nominal rigidities affect the definition of the price markup. Specifically, we obtain

$$\mu_t^{NK-ESAM} = \frac{\varepsilon_t}{(\varepsilon_t - 1)\,\Upsilon_1^{NK-ESAM} + \Upsilon_2^{NK-ESAM}}, \tag{40}$$

where

$$\Upsilon_1^{NK-ESAM} \equiv 1 - \frac{\phi_P}{2}(\Pi_t - 1)^2, \tag{41}$$

and

$$\Upsilon_2^{NK-ESAM} \equiv \phi_P(\Pi_t - 1)\Pi_t - \beta(1-\delta) E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\phi_P(\Pi_{t+1} - 1)\Pi_{t+1}\frac{N_{t+1}}{N_t}\frac{Y_{t+1}}{Y_t}\right], \tag{42}$$

The variable $\Pi_t = \frac{P_t}{P_{t-1}}$ denotes gross price inflation. In the Appendix A, following Etro and Rossi (2015), we derive the NK Phillips Curve spreading from the NK-ESAM model. Also in the Appendix A, we derive the NK Phillips curve when we extend the NK-ESAM model to account for strategic interactions a là Bertrand, namely in NK-BESAM. We show that the NK Phillips Curve implied by the NK-BESAM model is flatter the higher the extent of competition, i.e. the higher the number of competitors in the market. In the empirical analysis we assess whether this is a quantitatively relevant aspect.

The monetary authority is assumed to use the short-term nominal interest rate as the policy

[11] Notice we have already imposed symmetry across sectors.

instrument. The gross nominal interest rate, R_t, follows a Taylor-type rule as

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}\right)^{1-\phi_R},$$

where ϕ_R measures the degree of interest rate smoothing, while ϕ_Π and ϕ_Y are the response coefficients to inflation and output. Variables without a time subscript denote steady state values.

4 The Standard Search and Matching Model: SAM

This section describes a SAM model with fixed variety that we take as the reference to evaluate the role of the extensive margin of investment for the cyclicality of labor market variables. This version of the model is well established in the literature. It can be regarded as a medium scale version of the search and matching model described, *inter alia*, by Trigari (2009).

The key differences with respect to the ESAM model are that there are no entry frictions and the number of varieties is fixed. For this reason, there are no product development costs. As a result, in equilibrium households will invest uniquely in physical capital. In this case, the aggregate resource constraint of the economy reduces to

$$C_t + I_t^k = w_t h_t L_t + r_t^k K_{t-1} + PRO_t, \tag{43}$$

and the dynamics of employment reads as

$$L_t = \varrho L_{t-1} + q_t v_t^{tot}.$$

The rest of the model equations is analogous to the ESAM framework and is reported in the Technical Appendix.

4.1 Nominal rigidities: NK-SAM

We also estimated versions of the SAM model with nominal price rigidities that we dub NK-SAM. Pricing in NK-SAM is again defined by equation 17 where the markup function, μ_t^{NK-SAM}, differs from $\mu_t^{NK-ESAM}$ since it does not depend on the expected dynamics of the stock of firms. We report μ_t^{NK-SAM} in the Technical Appendix. The monetary policy rule is unchanged with respect to that specified above.

4.2 Real Wage rigidities: WSAM

Finally, we estimated a version of SAM with real wage rigidities. Starting with Hall (2005), the literature pointed out that in order for the SAM model to account for the cyclical properties of unemployment and vacancies the real wage should not display sharp changes in response to shocks. For this reason, Hall (2005) augments the SAM framework with a wage norm that dampens fluctuations in the real wage. Following that approach, we model real wage rigidity in the form of a backward-looking social norm:

$$w_t = w_{t-1}^{\phi_w} \left(w_t^*\right)^{1-\phi_w}, \tag{44}$$

where ϕ_w is a parameter reflecting the degree of real wage rigidity and w_t^* is the wage obtained under the Nash bargaining between firms and workers, namely that in equation 30. Notice that ϕ_w implies a fixed real wage, while $\phi_w = 0$ corresponds to the case of Nash bargaining analyzed

above. As observed by Blanchard and Galí (2007), equation 44, though admittedly ad-hoc, is a parsimonious way of introducing a slow adjustment of real wages to labor market conditions. We define the versions of the SAM model augmented with exogenous wage rigidity as WSAM, and its sticky prices counterpart as NK-WSAM.

5 Econometric Methodology

The econometric technique that is particularly suited for our shock-based analysis is one that matches impulse response functions estimated by a vector autoregressive model (VAR) with the corresponding objects in the models. As a first step, we estimate a Bayesian structural vector autoregressive (BVAR) model to identify three shocks: a shock to aggregate productivity, a shock to price markup, a shock to workers' wage bargaining power. Second, we estimate a set of structural parameters for the DSGE models that we illustrated above, by matching theory-based impulse response functions (DSGE-IRFs) with the empirical ones (VAR-IRFs). With the estimated parameters in hand, we assess the relative empirical performance of the alternative models we consider by comparing their marginal likelihoods and by comparing their ability at replicating the VAR-IRFs.

In the remainder of the section, we describe each step of the methodology. Section 5.1 introduces the BVAR estimation and the identification strategy of the three structural shocks. Section 5.2 outlines the Bayesian minimum distance procedure we follow to estimate the structural parameters of DSGE models. Finally, Section 5.3 describes the calibration for those parameters that are not estimated.

5.1 VAR estimation and shocks identification

The empirical counterpart of our analysis is derived from a VAR model estimated through Bayesian techniques. We assume Gaussian-inverse Wishart priors for the reduced-form VAR parameters. Endogenous variables in the VAR consist of $n = 11$ U.S. quarterly series: real GDP, real wages, real profits, total hours worked, unemployment rate, vacancies, inflation rate, labor productivity, firm entry, real consumption, and real investment in physical capital. Details about data sources and definitions are provided in the Technical Appendix.

All series are considered in annual terms and, for those in levels, in per capita. Since the DSGE models we describe are stationary, we take deviations of the non-stationary time series from their respective trend by applying a one-sided Hodrick-Prescott filter to the logarithms of the series.[12] In the benchmark specification for the VAR, we consider the interval 1960:Q1 to 2016:Q2 as the sample period, and 2 as the autoregressive order, as suggested by both Akaike and Bayesian information criterion. Besides the benchmark specification, we run a battery of robustness checks for the VAR model that differ over the data samples and the filters considered. We outline the results of the robustness analysis in the Technical Appendix.

The identification of the structural shocks is achieved by imposing sign restrictions on the impulse response functions, that is on VAR-IRFs. Specifically, we implement the QR decomposition procedure proposed by Rubio-Ramirez *et al.* (2010). The details regarding the identification procedure are left to the Technical Appendix. Table 1 summarizes the set of restrictions we impose to identify the structural shocks. Shocks are meant to increase the aggregate productivity, and to reduce the price markup and the workers bargaining power. All restrictions, but those imposed on labor productivity and inflation to technology shocks, are imposed just on impact. The responses of

[12] As stressed by Born and Pfeifer (2014), using a one-sided, i.e. "causal" filter in Stock and Watson (1999), guarantees that the time ordering of the data is not disturbed and the autoregressive structure is preserved.

labor productivity and inflation to the technology shock are instead imposed for 20 and 4 periods, respectively, after the shock. The length of these restrictions is consistent with that imposed in the related literature, such as Peersman (2005) and Hofmann *et al.* (2012). Specifically, for the response of the labor productivity, the length of the restriction is in line with that considered by Dedola and Neri (2007) and Fujita (2011).[13]

	Shock		
Variable	*Technology*	*Price markup*	*Workers bargaining power*
Real GDP		> 0	> 0
Real wage	> 0	> 0	< 0
Real profits	> 0	< 0	> 0
Inflation	< 0	< 0	< 0
Labor productivity	> 0	< 0	< 0

Table 1: Sign restrictions for the identification of structural shocks in the VAR model. All restrictions last for the impact period, but for the labor productivity (20 periods) and inflation (4 periods) to technology shocks,

A shock that increases the efficiency of production, that is an expansionary technology shock, leads to an increase in wages, profits, the productivity of labor, and to a reduction in inflation. Shocks that weaken the relative bargaining power of workers with respect to that of firms are expansionary since they reduce labor market distortions. We distinguish them from expansionary technology shocks imposing that they lead to a reduction in the real wage and in the productivity of labor. The reduction in the real wage is assumed to result in lower inflation. Finally, shocks that weaken the ability of firms to price above marginal costs, i.e. markup shocks, expand output by reducing distortions in the product markets. A negative price markup shock, is distinguished from an expansionary technology shock by assuming that it affects negatively both profits and labor productivity, while it is distinguished from a bargaining power shock by assuming that it has a positive impact on real wages and a negative one on profits. These restrictions are consistent with those imposed by, *inter alia*, Canova and Paustian (2011). The restrictions imposed on labor productivity are derived from the responses implied by our models.[14]

We leave unconstrained the responses of those variables that represent the main interest of our analysis: the unemployment rate, total hours, vacancies, firm entry, consumption, and investment.

Shaded areas Figures 2-9 correspond to 68% probability credible intervals of the VAR-IRFs to the three identified structural shocks. Though no restrictions are imposed on the responses of consumption and investment, they are procyclical to all three shocks. Total hours and firm entry are procyclical as well, although there is some uncertainty in their impact responses in the case of technology and markup shocks. After very few periods, however, the uncertainty dissipates and credible intervals lie entirely above the zero line. Unemployment is countercyclical to all shocks.

In the Technical Appendix, we show that the empirical findings are robust to different VAR specifications. Robustness checks are carried out along two lines. First, we change the length of the sample period to exclude, in one case, the Great Recession, and in the other, the interval before

[13] Reducing the lenght of the restriction imposed on the response of labor producivity to technology shocks does not alter our findings. We verified this by imposing both an impact restriction and a restriction implying a positive response for 4 periods.

[14] Notice that the response of labor productivity to the shocks we consider does not differ across models we compare, namely ESAM and SAM, neither when prices are flexible in the models nor when they are sticky. For this reason, restrictions imposed on labor productivity are not meant to favour the implications of one model over those of others.

the Great Moderation. Second, we detrend the data using alternative filters, namely by applying a two-sided Hodrick-Prescott filter to the logarithms of the series, and by using linear and quadratic trends.

5.2 Bayesian minimum distance estimation

The different DSGE models we study in the paper are estimated via Bayesian minimum distance techniques in the spirit of Christiano *et al.* (2010). Differently from the aforementioned authors, the VAR-IRFs are identified with sign restrictions. A major challenge in estimating a DSGE model via matching the VAR-IRFs identified through sign restrictions, is that there is no point estimate that the minimum distance method can take as the empirical reference. Imposing indeed sign restrictions to identify the structural shocks in the VAR implies that shocks are only set-identified. Put differently, the identification strategy implies that there is a set of impulse response functions that fulfills the sign restrictions we impose. Any of the VAR-IRFs in that set can be equally taken as the empirical counterpart to perform the estimation. To tackle this issue, we follow Hofmann *et al.* (2012). We take a large set of VAR-IRFs, namely 1000, and for each of them, we run Bayesian minimum distance estimation with the corresponding DSGE-IRFs.[15] The estimation consists in optimizing over the posterior mode of the parameters in θ, the vector containing the parameters we wish to estimate.[16] The procedure delivers 1000 vectors of posterior modes for the structural parameters in θ, that is one for any of the VAR-IRFs that we take as the empirical counterpart in the estimation.

Here, we refrain from delving into all details of the estimation procedure, that are left to the Technical Appendix, and outline the key steps of the analysis. Once endowed with the vectors of posterior modes, we evaluate the statistical fit of the models we estimate through the marginal likelihood, and compute DSGE-IRFs to the three shocks of interest. The marginal likelihood is computed using a Laplace approximation around the posterior mode.[17] As a result of the estimation procedure, for each model we consider, we obtain three distributions, namely: i) the distribution of posterior modes of the structural parameters; the ii) the distribution of marginal likelihoods, and iii) the distribution of DSGE-IRFs. In the remainder, we use distributions i)-iii) to asses the relative performance of the alternative models considered in the analysis.

Notice that the set of VAR-IRFs used for minimum distance estimation depends on the features of the model under scrutiny. In the case of flexible prices models, the matching is carried out over the dynamic responses of the following variables: real GDP, real wages, real profits, total hours worked, unemployment rate, vacancies, real consumption, real investment and labor productivity. In the case of sticky prices models, we add to those variables the VAR-IRFs of the inflation rate. Since SAM does not feature the extensive margin of investment, we never use the VAR-IRFs of new-entrants in the estimation procedure.

[15] Any of the VAR-irfs and DSGE-irfs are stacked vectors, which in our case have dimension 15, the impulse responses horizon, times 3, i.e. the number of identified structural shocks, times the number of endogenous variables we match.

[16] The optimization is run using Dynare 4.4.3, and Chris Sims' *csminwel* as maximization routine. Our programming codes modify the codes used in Christiano *et al.* (2010). We are grateful to Mathias Trabandt for sharing with us the original codes.

[17] Inoue and Shintani (2018) establish the consistency of the model selection criterion based on the marginal likelihood obtained from Laplace-type estimators. Methods like Laplace approximation and Geweke (1999)'s modified harmonic mean procedure are widely used in the literature to calculate the marginal likelihood. However, the former has a large advantage over the latter in terms of computational costs. This is so since in order to compute the marginal likelihood it requires only the posterior mode, and not a Metropolis-Hastings-based sample of the posterior distribution. For this reason, we follow Smets and Wouters (2007) and compare the alternative models we consider using the marginal likelihood computed with a Laplace approximation method.

The structural parameters we estimate, i.e. the elements of θ, are listed in Table 2, along with the prior distributions. Parameters common across model specifications are: the persistence parameters of the shocks ρ_a, ρ_ε, ρ_η, the standard deviations of the shocks σ_a, σ_ε, σ_η, the elasticity of the marginal disutility of labor, φ, the degree of internal habit in consumption, ϑ, the elasticity of the matching function, γ, the steady state value of the wage bargaining power of workers, η, the implied steady state replacement ratio, $rr \equiv \left(\frac{\chi}{\lambda}\frac{h^{1+\varphi}}{1+\varphi} + b\right)\frac{1}{w}$, the steady state value of the elasticity of substitution in the goods market, ε, and the quadratic investment adjustment cost parameter, ϕ_I. We assume a Beta distribution with mean 0.01 for the standard deviation of the shocks, and an Inverse Gamma with mean 0.5 for the autoregressive parameters. These are identical across the exogenous processes. The prior mean for the elasticity of the marginal disutility of labor is 2, while that for the degree of habit persistence is 0.6, in line, among others, with Boldrin *et al.* (2001). Following the standard parameterization strategy of SAM models, we set the prior means of the elasticity of the matching function and the steady state value of the workers' bargaining power to 0.5. In our model the replacement ratio includes, both, the pecuniary unemployment benefit and the utility value of leisure. For this reason we set its prior mean to 0.8, a value which is high compared to those adopted in models where the ratio is affected just by the pecuniary unemployment benefit, as in Christiano *et al.* (2016). The prior mean for the elasticity of substitution among goods is set to 4.3, following the calibration strategy in Ghironi and Melitz (2005) and Bilbiie *et al.* (2012). The investment adjustment cost is set to 4, consistently with Smets and Wouters (2007). In the case of ESAM models, θ also includes the elasticity of entry cost to congestion esternalities, ς. We set its prior mean to 2 following Casares *et al.* (2018). In the case of models augmented with real wage rigidities, θ includes the persistence parameter characterizing the wage norm, γ_w. Finally, in the case of models augmented by nominal price rigidity, θ includes the probability of not-resetting price for a firm in a given period, θ_P,[18] and the parameters in the monetary policy rule, ϕ_R, ϕ_Π, and ϕ_Y. Prior values for these parameters lay within intervals that are regarded as standard in the literature.

Parameter	**Density**	**Mean**	**Std**	**Parameter**	**Density**	**Mean**	**Std**
ρ_a	*Beta*	0.8	0.1	rr	*Beta*	0.7	0.1
ρ_ε	*Beta*	0.8	0.1	ε	*Gamma*	4.3	0.75
ρ_η	*Beta*	0.8	0.1	ϕ_I	*Gamma*	4	0.75
σ_a	*Inv.Gamma*	0.01	0.05	ς	*Gamma*	2	0.2
σ_ε	*Inv.Gamma*	0.01	0.05	γ_w	*Gamma*	0.8	0.1
σ_η	*Inv.Gamma*	0.01	0.05	θ_P	*Beta*	0.45	0.05
φ	*Gamma*	2	0.4	ϕ_R	*Beta*	0.8	0.2
ϑ	*Beta*	0.6	0.2	ϕ_Π	*Gamma*	1.5	0.2
η	*Beta*	0.5	0.1	ϕ_Y	*Gamma*	0.125	0.2
γ	*Beta*	0.5	0.1				

Table 2: Prior distributions for DSGE structural parameters

[18] Notice that, though we assume a pricing sheme *à la* Rotemberg (1982a) in those models embedding nominal frictions in price adjustment, we choose not to estimate the price adjustment parameter, ϕ_P, but we estimate the probability of not-resetting prices in a given period, θ_P, of an equivalent pricing adjustment scheme *à la* Calvo (1983). As standard in the related literature, we recover ϕ_P implictly, by equating the log-linearized versions of the NKPC delivered by the two pricing schemes.

5.3 Calibrated parameters

A subset of the structural parameters is not estimated, but kept constant across the different model specifications. These parameters are calibrated on a quarterly basis following Shimer (2005) and Blanchard and Galí (2010), among others. We take the ESAM framework as the benchmark. The discount factor, β, is set to 0.99, and the capital share, α, to 1/3. The rate of business destruction, δ, equals 0.025 to match the U.S. empirical level of 10 per cent business destruction a year reported by Bilbiie *et al.* (2012). The constant part of the entry cost, ψ_0, is set to 1, which leads to a ratio of firm investment to output close to 15 per cent, as in Bilbiie *et al.* (2012). Without loss of generality, the labor disutility parameter, χ, is set such that steady state hours supply per worker equals 1. We set the steady state value of technology, A, equal to 1.

Next, we turn to parameters that are specific to the search and matching framework. The total separation rate, $1-(1-\delta)\varrho$, is set to 0.1, as suggested by the estimates provided by Hall (2005) and Davis and Haltiwanger (1990). We set the steady state job market tightness to target an average job finding rate, z, equal to 0.7 as in Blanchard and Galí (2010). This amounts to a monthly rate of 0.3, consistent with U.S. evidence.

The vacancy filling rate, q, equals 0.9 as in Andolfatto (1996) and Den Haan *et al.* (2000). The cost of posting a vacancy κ is implied endogenously. The steady state rate of unemployment reads as $u = \frac{(1-(1-\delta)\varrho)}{(1-(1-\delta)\varrho)+q\theta}$, which is increasing in both the firm-level job separation rate, ϱ, and in the rate of business destruction, δ. As expected, the unemployment rate is decreasing in the job filling probability, q. The endogenous steady state rate of unemployment is higher than the one observed in the U.S. However, this is justified by interpreting the unmatched workers in the model as being both unemployed and partly out of the labor force. As argued by Trigari (2009), this interpretation is consistent with the abstraction in the model from labor force participation choices.[19] The steady state ratio between jobs created by new firms (JC^{new}) and total job creation (JC) is given by

$$\frac{JC^{new}}{JC} = \frac{(1-\delta)\, N^e v^{new} q}{v^{tot} q} = \frac{\delta}{\theta q}\frac{(1-u)}{u} = 0.25$$

The calibration implies that job creation by new producers account for about 25 per cent of total (gross) job creation, close to the quarterly U.S. average of 20 per cent reported by Jaimovich and Floetotto (2008). Finally, the ratio between workers employed by first period incumbent firms (L^{new}) and total employment (L) is given by

$$\frac{L^{new}}{L} = \frac{(1-\delta)\, N^e \frac{L}{N}}{L} = \delta \ .$$

Since we set $\delta = 0.025$, this implies that new firms account for about 2.5 per cent of total employment, slightly lower than the 3 per cent reported by Haltiwanger *et al.* (2013) as the average value for the U.S. between 1976 and 2005. In the ESAM framework new entrants create on average a relevant fractions of new jobs, while accounting just for a small share of overall employment, in line with U.S. data.

[19] Krause and Lubik (2007) calibrate their model to deliver an unemployment rate of 12 per cent on the basis of this motivation. Many studies in the search and matching literature feature much higher unemployment rates. For example, Andolfatto (1996)'s model features a steady state unemployment rate of 58 per cent, while Trigari (2009) is characterized by an unemployment rate equal to 25 per cent.

6 Findings

We arrange our findings in two sections. Each of them is devoted to understanding the role played by firms dynamics for the propagation of shocks to labor market variables, and to other aggregates of interest, in the various versions of the models that we spelled out above. In each section we compare models that have been estimated using the same information. Specifically, the VAR-IRFs of firm entry is never included among the observables in the minimum distance estimation procedure. VAR-IRFs of the inflation rate are included among the observables only for the estimation of the models with sticky prices. For this reason, Section 6.1 is dedicated to models with flexible prices, while Section 6.2 to models with nominal price rigidities.

6.1 Flex Prices: ESAM, BESAM, SAM and WSAM

6.1.1 Parameters estimation

In this section, we compare the estimation results across the flexible prices models we outlined earlier. Table 3 reports values of the posterior distribution of the structural parameters. As posterior values, we show the mean of the estimated parameter modes, along with the median and the values at the 5% and 95% tails.[20] Columns (1) and (2) of Table 3 report results for the posterior modes in ESAM and SAM models, respectively. Column (3) refers to the WSAM model, that is the SAM model augment with real wage rigidities, while Column (4) displays results for the BESAM model, that is the ESAM model augmented with Bertrand Competition.

In order to assess the contribution of firm dynamics to shaping the economy dynamics, we start by comparing the predictions of the ESAM model with those of the SAM model. The sole difference between these two models is the presence of endogenous variety associated to frictional firm dynamics. All other features—frictions, parameters, and information used in the estimation—are kept unchanged. Parameter values are consistent across models. In particular, both frameworks require a high value of the replacement ratio, the parameter rr, to match the empirical IRFs. One relevant difference between the two models is the value assumed by the mean of the posterior modes for the bargaining power of workers, η, which is lower under SAM. This suggests that SAM needs, to be consistent with the evidence, a low bargaining power of workers in order to dampen the response of the real wage to shocks. On the contrary, the ESAM model calls for a value of the bargaining power in line with that used by the bulk of the literature. This intuition is confirmed by the estimation results relative to the WSAM model that we report in column 3 of the Table 3. Once the SAM model is augmented with real wage rigidities, as is WSAM, a low bargaining power of workers is no longer required to replicate the empirical evidence. Indeed, the value of η estimated under WSAM gets much closer to that obtained in ESAM. This suggests that the extensive margin of investment delivers an endogenous form of wage moderation that is absent in the SAM model, which we discuss further below.

Column 4 of the table refers to the BESAM model, that is the ESAM model augmented with Bertrand competition in the market for final goods. Estimated parameters are essentially identical to those obtained under ESAM, which features monopolistic competition. This suggests that the form of competition between firms does not matter much for the dynamics of the model.

[20] The mean here is to be intended as the mean of the 1000 posterior modes obtained from the minimum distance estimation.

	Posterior distribution											
Parameter	**(1)** *ESAM*			**(2)** *SAM*			**(3)** *WSAM*			**(4)** *BESAM*		
	Mean	**Std.**	**0.05-0.5-0.95**	**Mean**	**Std.**	**0.05-0.5-0.95**	**Mean**	**Std.**	**0.05-0.5-0.95**	**Mean**	**Std.**	**0.05-0.5-0.95**
ρ_a	**0.756**	0.055	0.652-0.764-0.833	**0.748**	0.076	0.617-0.753-0.864	**0.742**	0.063	0.627-0.747-0.832	**0.755**	0.054	0.650-0.761-0.831
ρ_ε	**0.779**	0.112	0.565-0.803-0.925	**0.795**	0.098	0.619-0.810-0.931	**0.779**	0.102	0.572-0.798-0.916	**0.778**	0.114	0.560-0.802-0.929
ρ_η	**0.801**	0.085	0.641-0.808-0.924	**0.718**	0.109	0.539-0.723-0.892	**0.772**	0.093	0.595-0.780-0.916	**0.801**	0.087	0.636-0.809-0.924
σ_a	**0.002**	0.001	0.002-0.002-0.003	**0.002**	0.001	0.001-0.002-0.003	**0.002**	0.000	0.002-0.002-0.003	**0.002**	0.001	0.002-0.002-0.003
σ_ε	**0.008**	0.005	0.003-0.006-0.017	**0.009**	0.007	0.002-0.006-0.023	**0.009**	0.007	0.003-0.006-0.023	**0.007**	0.004	0.003-0.006-0.016
σ_η	**0.037**	0.036	0.004-0.025-0.114	**0.103**	0.156	0.005-0.034-0.436	**0.058**	0.065	0.004-0.032-0.192	**0.036**	0.035	0.004-0.025-0.111
φ	**2.222**	0.336	1.770-2.192-2.800	**2.318**	0.562	1.444-2.267-3.292	**2.336**	0.465	1.614-2.301-3.063	**2.225**	0.341	1.765-2.193-2.806
ϑ	**0.700**	0.118	0.494-0.703-0.875	**0.677**	0.093	0.533-0.673-0.827	**0.676**	0.088	0.545-0.672-0.822	**0.699**	0.119	0.484-0.701-0.874
η	**0.566**	0.128	0.341-0.579-0.784	**0.354**	0.171	0.135-0.322-0.681	**0.449**	0.143	0.211-0.435-0.741	**0.563**	0.128	0.334-0.574-0.777
γ	**0.608**	0.113	0.410-0.622-0.765	**0.593**	0.119	0.395-0.606-0.763	**0.608**	0.114	0.420-0.626-0.772	**0.608**	0.112	0.410-0.621-0.764
rr	**0.884**	0.130	0.634-0.956-0.995	**0.830**	0.170	0.524-0.895-0.996	**0.863**	0.154	0.552-0.949-0.996	**0.884**	0.132	0.631-0.955-0.995
ε	**4.710**	0.749	3.712-4.550-6.022	**4.605**	1.017	3.008-4.521-6.277	**4.606**	0.996	3.102-4.518-6.185	**4.781**	0.746	3.798-4.623-6.083
ϕ_I	**3.925**	0.677	2.706-3.998-5.015	**3.518**	0.702	2.309-3.544-4.639	**3.682**	0.703	2.400-3.767-4.813	**3.937**	0.681	2.677-3.998-5.029
ς	**1.984**	0.168	1.766-1.970-2.328							**1.989**	0.170	1.769-1.985-2.331
γ_w							**0.496**	0.170	0.265-0.471-0.800			

Table 3: Estimated parameters: ESAM, SAM, WSAM, BESAM

Model	(1) Mean	(2) 0.05-0.5-0.95	(3) % of wins for ESAM
ESAM	343	293; 355; 397	-
SAM	322	265; 333; 382	86%
BESAM	344	294; 356; 398	42%
WSAM	334	283; 344; 390	86%

Table 4: Laplace approximation for marginal likelihood over different DSGE specifications. Values are in log points.

6.1.2 Statistical fit

In this section, we compare the statistical fit of the estimated flexible prices models. The metric adopted for the comparison is the log marginal likelihood. The latter is computed using a Laplace approximation around the posterior modes of the estimated parameters. Since the minimum distance estimation provides us with a set of vectors of posterior modes, one for any of the 1000 VAR-IRFs, we also obtain 1000 values of the marginal likelihood for each of the models we consider. At each estimation round, so taking a specific VAR-IRFs as reference, we subtract from the log marginal likelihood delivered by ESAM that obtained from the competing alternative. Panels a)-c) of Figure 1 display the distribution of those differences when the competing alternative are SAM, WSAM and BESAM respectively. On the horizontal axis of each panel we measure the log points of difference between the marginal likelihoods. Positive (negative) values refer to log points of difference in favor of the ESAM model (the competing model). The star (circle) indicates the median value of the gaps in favor of the ESAM model (the competing model). The distribution is skewed positively when the marginal likelihoods of ESAM are compared to those implied by both SAM and WSAM. Additionally, the median values of the positive gaps, are, in absolute value, larger than the median values of the negative ones.[21] This means that, considering separately the subsets of VAR-IRFs at which one model prevails over the other in terms of marginal likelihood, ESAM is the model whose relative performance is, in median, the strongest. Notice that the distribution of gaps is slightly negatively skewed when ESAM is compared to BESAM, in panel c).

Table 4 provides summary statistics concerning the comparison of the models over the marginal likelihood. In Column (3), we report the fraction of runs in which ESAM delivers a higher value of the marginal likelihood with respect to the competing model (% of wins for ESAM). ESAM displays a higher marginal likelihood than SAM and WSAM in the vast majority of cases, while outperforming BESAM only 42% of the times. Taking instead the full distribution of marginal likelihoods across the 1000 estimation runs for each model, Column (1) displays the mean value, while column (2) reports its median, and the values assumed at the 5% and 95% per centile. Considering mean values, the marginal likelihood of ESAM is about 20 log points higher than that of SAM, and 10 points higher than that of WSAM. BESAM and ESAM essentially display the same mean value of the log marginal likelihood. For this reason, we argue that the form of competition between firms does play a relevant role when it comes to the statistical fit of the models.

The previous analysis leads to three main conclusions: (i) as argued by Christiano *et al.* (2016), models featuring a form of wage moderation are those which provide a better fit to the data, the statistical fit of both ESAM and WSAM is indeed higher than that of SAM; (ii) the endogenous wage moderation implied by ESAM fits the data better than the exogenous one featured in WSAM; (iii) imperfect competition is a necessary ingredient to replicate business cycle fluctuations. With

[21] The medians of the gaps of marginal likelihoods in favor of ESAM over SAM and WSAM are respectively, 25 and 11 log points. The median of the gaps in favor of SAM over ESAM is 16 log-points, and of WSAM over ESAM is 3 log-points.

regard to the last point, we show that augmenting the model with imperfect competition is key to replicating the transmission of shocks to profits, as it will become clearer from the impulse response analysis below, and thus for the incentive to enter the market for new firms. However, the exact form of competition does not matter much to replicate the empirical IRFs of the main macroeconomic variables. Our analysis shows that Bertrand competition fits the data slightly better than monopolistic competition, but modelling the market for final goods as a monopolistically competitive one, as in the bulk of the literature, makes little quantitative difference. Notice, however, that departing from monopolistic competition could be necessary when trying to address specific aspects of the data, such as the relationship between price markups and the extent of competition.

6.1.3 IRFs analysis

This section is dedicated to the analysis of the IRFs in ESAM, SAM and WSAM. We leave the DSGE-IRFs of the BESAM model to the Technical Appendix since they are essentially identical to those obtained from ESAM. We trace out DSGE-IRFs to shocks to aggregate productivity, the price markup and to the workers' bargaining power. We simulate the shocks in the models by setting the values of the parameters in θ to the posterior modes obtained in the estimation procedure, while other parameters according to the calibration strategy described in Section (5.3).

Figures 2 and 3 compare the IRFs delivered by ESAM and SAM to the empirical ones. Figures 4 and 5 run a comparison between ESAM and WSAM. Figures 2 and 4 refer to labor market variables, while Figure 3 and 5 to other aggregates of interest. Shaded areas refer to the 68% probability density intervals of the VAR-IRFs. Solid lines embrace the 68% probability density intervals of the IRFs of the ESAM model, dashed refer to the SAM model, and dotted lines to the WSAM model. The horizontal axis measures time in quarters from impact, while the vertical axis represents the responses in per cent.

Both ESAM and SAM match VAR-IRFs fairly well. There are, however, some differences. We start by considering a positive technology shock. Figure 2 shows that ESAM is characterized by larger fluctuations in vacancies and unemployment with respect to SAM. We argue that this outcome is due to a form of wage moderation characterizing the ESAM framework. The interaction between the asset market and the labor market is at the core of this result. In ESAM, households have an additional means to transfer resources intertemporally, that is the extensive margin of investment, with respect to what they have in SAM. Since future firms profits are expected to be high, households invest in the creation of new firms, leading to an increase in entry, as shown in Figure 3. The increase at both margins of investment amplifies the response of output with respect to what is observed in SAM. Furthermore, since entry is frictional, the number of firms is a state variable, and the additional output is initially produced by incumbent firms. Thanks to imperfect competition, this fuels their profits. Hours are the only input that firms can adjust on impact, and for this reason firms increase the demand of hours. However, this does not lead to a counterfactual strong procyclical response of the real wage. There are two reasons at the basis of this wage moderation. The first one is that in order to enjoy investment opportunities, households further increase their willingness to work. The second one is that investment in the extensive margin, contrary to investment in physical capital, does not contribute to the marginal productivity of labor. As a result the response of the real wage is milder with respect to what is observed in SAM. This translates into a procyclical response of both aggregate hours and profits together with a countercyclical unemployment rate. Our VAR analysis suggests that a procyclical response of these variables in response to a technology shock is the most likely event. Our models cannot match the magnitude of the empirical response of profits to the shock. For this reason,

the panels displaying profits in Figure 3 and 5 measure the VAR-IRFs on the left vertical axis, while the DSGE-IRFs are on the right vertical axis. ESAM matches the procyclicality of profits in response to the shock, whereas in SAM a large fraction of the dynamic responses is countercyclical. The surge in hours, employment and vacancies is sustained over time by the entry of new firms that start producing.

Differences across the ESAM and SAM models are less pronounced in response to the other shocks, nevertheless, as shown above, the statistical fit of the ESAM, as measured by the marginal likelihood, is significantly higher than that of the SAM.

The important role of the endogenous wage moderation that we emphasized in the previous discussion is confirmed by the analysis of Figures 4 and 5, which display a comparison between the ESAM model and the WSAM model. Recall that the latter is a SAM model augmented with a wage norm that leads to real wage rigidity. The response of the real wage to a technology shock is, by construction, dampened in WSAM with respect to SAM. In this case, as displayed in Figure 5, the WSAM model matches the cyclicality of the empirical response of profits, and leads to more sizeable fluctuations in unemployment and vacancies with respect to those observed in the baseline SAM model. The role of entry can be appreciated by considering the response, output and hours of work. In ESAM both are in line with the empirical ones. The marginal log-likelihoods reported in Table 3 show that augmenting the SAM model with wage rigidities improves its empirical performance, in line with the finding of the literature. Nevertheless, ESAM still displays a non-negligible advantage with respect to WSAM in terms of likelihood. We read this result as suggesting that the extensive margin of investment is a relevant amplification channel of shock in business cycle models characterized by search and matching frictions in the labor market.

As mentioned above, the IRFs of the BESAM model are essentially identical to those obtained from ESAM. This suggests that, from a quantitative point of view, the exact form of competition does not matter much to replicate the empirical IRFs of the main macroeconomic variables.

6.2 Sticky Prices: NK-ESAM, NK-SAM, NK-BESAM

6.2.1 Parameters estimation

In this section we consider models featuring nominal rigidities. Recall that in this case the VAR-IRFs of inflation is included in the set of observables when estimating the parameters of the model. Table 5 reports the same information described in Table 3. In this case, Columns (1) and (2) of the table report parameters estimated for the NK-ESAM and the NK-SAM models, respectively. Columns (3) refers to the NK-WSAM model, that is the NK-SAM model augment with real wage rigidities, while column (4) displays results for the NK-BESAM model, that is the NK-ESAM model augmented with Bertrand competition.

The observations drawn in the case of flexible prices extend to the case of sticky prices. Specifically, parameters values are consistent across models, and models with a fixed number of varieties, such as NK-SAM, require a low value of the bargaining power of workers in order to match the IRFs. Once augmented with real wage rigidities, the model with a fixed number of varieties, that is NK-WSAM, features an estimated degree of real wage rigidity quite higher than its flexible prices counterpart, that is WSAM. A higher exogenous inertia in real wage is thus required to replicate the empirical IRFs. The estimated degree of price rigidities is slightly lower in models with endogenous variety. In the NK-BESAM framework, the average estimated duration of prices is lower than two quarters, in line with the average price duration suggested by the micro-evidence provide, *inter alia*, by Nakamura and Steinsson (2008). This suggests that strategic interactions in pricing help match the dynamics of inflation without resorting to degrees of price rigidities that are inconsistent

with the evidence. Turning to the estimation of the parameters in the interest rate rule, all models feature an inflation response coefficient larger than one, together with a small output response coefficient. Models with endogenous product variety call for a lower degree of interest rate inertia. Thus, ESAM models require a lower degree of exogenous rigidities to match the empirical evidence, which suggests that an internal propagation mechanism is at work.

Model	**(1) Mean**	**(2) 0.05-0.5-0.95**	**(3) % of wins for ESAM**
NK-ESAM	457	393; 466; 520	-
NK-SAM	384	295; 399; 469	96%
NK-BESAM	450	391; 461; 517	28%
NK-WSAM	400	321; 416; 481	92%

Table 6: Laplace approximation for marginal likelihood over different DSGE specifications. Values are in log-points. Median target refers to values for matching with the closest VAR-based impulse response function to the pointwise median (Fry and Pagan 2011)

6.2.2 Statistical fit

Table 6 summarizes the statistical fit of the models with sticky prices. Adding the VAR-IRFs of inflation as an observable in the minimum distance estimation has the implication of raising the fitting of all the models. The more so, however, for models with an endogenous number of varieties. As reported in Column (1), the mean value of the marginal likelihood of NK-ESAM is 73 and 57 log-points higher than those characterizing NK-ESAM and NK-WSAM, respectively. Similar differences hold considering median values in Column (2). This suggests that the endogenous form of wage moderation that characterizes mode with endogenous varieties constitutes a relevant propagation mechanism even in the case in which models are extend to account for nominal price rigidities. Additionally, in contrast with the case of flexible prices, the marginal likelihood of the NK-ESAM model is on average higher than that of the NK-BESAM model.

Also, as we did in the case of flexible prices, we consider any of the VAR-IRFs as reference and subtract from the log marginal likelihood of NK-ESAM that obtained from the competiting models. Panels d)-f) of Figure 1 display the distribution of these (log-points) differences between marginal likelihoods when the competiting alternative are NK-SAM, NK-WSAM and NK-BESAM respectively. The skewness of the distributions of the gaps is enhanced with respect to what is observed in the case of flexible prices. Column (3) of Table 6 reports the fraction of runs in which ESAM delivers a higher value of the marginal likelihood with respect to the competing model (% of wins for NK-ESAM). The per centage of wins for NK-ESAM with respect to either NK-SAM or NK-WSAM is overwhelming, as It exceeds 90%. Moreover, considering separately the subsets of VAR-IRFs at which one model prevails over others in terms of marginal likelihood, the median values, in absolute terms, of the gaps in favor of NK-ESAM -stars in panels d) and e) of Figure 1- exceeds the ones in favour of either NK-SAM or NK-WSAM -circles in panels d) and e). of Figure 1-.[22] The per centage of cases where NK-ESAM has a higher marginal likelihood with respect to NK-BESAM is just 28%. However, taking the full distribution of marginal likelihoods across the 1000 estimation runs for NK-ESAM and NK-BESAM, the differences for the median values are quantitatively negligible and NK-ESAM features a higher mean value of marginal likelihood than NK-BESAM.

[22] The medians of the gaps of marginal likelihoods in favor of NK-ESAM over NK-SAM and NK-WSAM are respectively, 56 and 39 log points. The median of the gaps in favor of NK-SAM over NK-ESAM is 30 log points, and of NK-WSAM over NK-ESAM is 27 log points.

Parameter	Posterior distribution											
	(1) *NK-ESAM*			(2) *NK-SAM*			(3) *NK-WSAM*			(4) *NK-BESAM*		
	Mean	**Std**	**.05;.5;.95**	**Mean**	**Std**	**.05;.5;.95**	**Mean**	**Std**	**.05;.5;.95**	**Mean**	**Std**	**.05;.5;.95**
ρ_a	**0.818**	0.047	0.728-0.824-0.874	**0.837**	0.041	0.768-0.837-0.903	**0.829**	0.046	0.745-0.835-0.896	**0.816**	0.047	0.715-0.825-0.872
ρ_ε	**0.803**	0.092	0.610-0.811-0.923	**0.818**	0.055	0.743-0.803-0.913	**0.813**	0.062	0.711-0.803-0.911	**0.801**	0.095	0.600-0.811-0.919
ρ_η	**0.805**	0.078	0.662-0.808-0.916	**0.779**	0.071	0.619-0.800-0.867	**0.784**	0.064	0.650-0.800-0.872	**0.804**	0.083	0.661-0.807-0.918
σ_a	**0.002**	0.000	0.001-0.002-0.003	**0.001**	0.000	0.001-0.001-0.002	**0.001**	0.000	0.001-0.001-0.002	**0.002**	0.000	0.001-0.002-0.003
σ_ε	**0.007**	0.003	0.003-0.006-0.013	**0.007**	0.004	0.003-0.006-0.013	**0.007**	0.004	0.003-0.006-0.014	**0.006**	0.003	0.003-0.006-0.012
σ_η	**0.029**	0.034	0.005-0.019-0.092	**0.030**	0.061	0.004-0.018-0.089	**0.029**	0.037	0.004-0.020-0.111	**0.029**	0.035	0.005-0.019-0.096
φ	**2.099**	0.247	1.882-2.000-2.667	**1.974**	0.196	1.704-2.000-2.093	**1.966**	0.207	1.548-2.000-2.135	**2.101**	0.255	1.875-2.000-2.694
ϑ	**0.557**	0.146	0.302-0.573-0.827	**0.553**	0.083	0.356-0.584-0.605	**0.525**	0.101	0.328-0.570-0.605	**0.562**	0.146	0.305-0.577-0.835
η	**0.567**	0.114	0.395-0.547-0.763	**0.486**	0.092	0.298-0.500-0.590	**0.496**	0.084	0.363-0.501-0.623	**0.564**	0.115	0.381-0.552-0.765
γ	**0.535**	0.127	0.335-0.500-0.752	**0.503**	0.083	0.385-0.499-0.706	**0.510**	0.095	0.368-0.499-0.729	**0.535**	0.130	0.336-0.501-0.753
rr	**0.839**	0.136	0.636-0.846-0.996	**0.739**	0.114	0.614-0.701-0.993	**0.764**	0.125	0.636-0.703-0.995	**0.839**	0.138	0.616-0.851-0.996
ε	**4.341**	0.516	3.503-4.300-5.174	**4.361**	0.452	4.239-4.300-5.070	**4.341**	0.476	3.916-4.300-5.032	**4.393**	0.494	3.798-4.300-5.305
ϕ_I	**3.870**	0.472	2.850-3.998-4.327	**3.897**	0.389	3.188-4.000-4.000	**3.884**	0.385	2.975-3.999-4.005	**3.888**	0.454	2.864-3.998-4.326
ς	**2.144**	0.252	1.967-2.026-2.782							**2.150**	0.259	1.965-2.029-2.770
γ_w							**0.695**	0.155	0.353-0.780-0.801			
θ_P	**0.487**	0.054	0.427-0.473-0.588	**0.510**	0.080	0.440-0.478-0.669	**0.518**	0.077	0.440-0.496-0.655	**0.483**	0.055	0.422-0.469-0.588
ϕ_R	**0.520**	0.214	0.185-0.523-0.800	**0.699**	0.162	0.295-0.775-0.802	**0.657**	0.183	0.264-0.742-0.802	**0.518**	0.213	0.187-0.516-0.800
ϕ_Π	**1.575**	0.171	1.245-1.541-1.865	**1.564**	0.140	1.497-1.504-1.897	**1.601**	0.176	1.498-1.513-1.990	**1.574**	0.162	1.313-1.543-1.850
ϕ_Y	**0.066**	0.069	0.000-0.054-0.194	**0.104**	0.064	0.000-0.129-0.194	**0.102**	0.075	0.000-0.128-0.210	**0.064**	0.069	0.000-0.051-0.186

Table 5: Estimated parameters: NK-ESAM, NK-SAM, NK-WSAM, NK-BESAM

6.2.3 IRFs analysis

Figures 6-9 display DSGE-IRFs obtained for our sticky prices models together with the empirical ones. Solid lines refer to NK-ESAM, i.e. the model with endogenous variety and sticky prices, and include the 68% probability density intervals. Dashed lines in Figures 6 and 7 refer to the NK-SAM model, dotted lines in Figures 8 and 9 to the NK-WSAM model. In the figures, the horizontal axis measures time in quarters from impact. The vertical axis represents the responses in per cent. Figures 6 and 8 refer to labor market variables, while Figures 7 and 9 to other key aggregates.

Both the NK-ESAM and NK-SAM models display dynamic responses in line with the empirical ones. Starting again by considering a positive technology shock, Figure 6 shows that the NK-ESAM model is characterized by larger fluctuations in hours, vacancies and unemployment with respect to the NK-SAM model. Under sticky prices, the differences between the two models in the response of labor market variables and output are amplified with respect to those we observed in the flexible prices case. The increase in the extensive margin of investment induce a large output response in NK-ESAM, as reported in Figure 7. Firms that cannot change their prices adjust labor demand at both margins. In NK-ESAM this does not translate in a sharp wage response due to the increased willingness to work by households. Figures 8 and 9 show that augmenting the NK-SAM framework with real wage rigidities, to deliver the NK-WSAM model, reduces the differences in the fluctuations in labor market variables from the NK-ESAM model. As above, we did not display the dynamic responses of the NK-BESAM model, i.e. the model with Bertrand competition, as they are essentially identical to those obtained from its monopolistically competitive counterpart.

In a nutshell, including inflation between the observables leaves our earlier conclusion unchanged: thanks to the extensive margin of investment, ESAM models quantitatively account for key business cycle properties of macroeconomic aggregates.

7 Conclusions

This paper formulates and estimates an equilibrium business cycle model which can account for the response of the U.S. economy to neutral technology shocks, to markup shocks and to shocks to the bargaining power of workers. The focus of our analysis is on how labor markets respond to these shocks. Three features distinguish our model from the standard search and matching model of the labor market: frictional firm entry, endogenous product variety, and investment in stocks beside that in physical capital. Investment in new productive units, the extensive margin of investment, is financed by households through the accumulation of shares in the portfolio of firms, which have a market price that fluctuates endogenously in response to shocks. An expansionary shock creates expectations of future profits. This provides incentives to households to invest in the creation of new firms, besides accumulating physical capital. The increase at both margins of investment amplifies the response of output with respect to what observed in the traditional search and matching model. Additionally, to make the most of investment opportunities, households increase their willingness to work. Together with the fact that investment in stocks does not affect the marginal productivity of labor, this leads to an endogenous form of wage moderation that is at the basis of the success of our model with entry at replicating the dynamics of labor market variables. The statistical fit of our model with firms' dynamics at replicating the US business cycle is substantially higher than that of a baseline search and matching framework enriched with exogenous wage rigidities. Microeconomic data suggest a pervasive heterogeneity in term of size and productivity among active firms. The interplay between firms dynamics and aggregate shocks determines the composition of active product lines and thus the aggregate level of labor productivity. Identifying empirically the interaction between the composition of the pool of producers and the propagation of shocks to the

labor market is a promising avenue for future research.

A Oligopolistic competition

In this appendix, following Etro and Rossi (2015), the ESAM framework is extended to consider Bertrand competition between an endogenous number of producers. Under Bertrand competition and Rotemberg (1982a)'s price stickiness the price markup function reads as

$$\mu_t^{NK-BESAM} = \frac{\left(\varepsilon_t \left(N_t - 1\right) + 1\right)/N_t}{\left(\varepsilon_t - 1\right)\Upsilon_1^{BESAM} + \Upsilon_2^{BESAM}}, \tag{45}$$

where

$$\Upsilon_1^{BESAM} \equiv \left(\frac{N_t - 1}{N_t}\right) - \frac{\phi_P}{2}\left(\Pi_t - 1\right)^2 + \frac{\phi_P}{2}\frac{\left(\Pi_t - 1\right)^2}{N_t}, \tag{46}$$

and

$$\Upsilon_2^{BESAM} \equiv \phi_P \left(\Pi_t - 1\right)\Pi_t - \beta\left(1 - \delta\right) E_t \left[\frac{\lambda_{t+1}}{\lambda_t}\phi_P \left(\Pi_{t+1} - 1\right)\Pi_{t+1}\frac{N_{t+1}}{N_t}\frac{Y_{t+1}}{Y_t}\right]. \tag{47}$$

Notice that the markup function, $\mu_t^{NK-BESAM}$, depends on the stock of firms both at time t and at time $t+1$. To understand the difference between NK-ESAM and NK-BESAM for inflation dynamics, consider the log-linear approximation of the implied New Keynesian Phillips Curve (NKPC) under monopolistic and Bertrand competition when the elasticity of substitution among goods variety, ε, is assumed to be constant over time. Under monopolistic competition, the log-linearized NKPC is

$$\widehat{\pi}_t = \beta\left(1 - \delta\right) E_t \widehat{\pi}_{t+1} + \frac{\left(\varepsilon - 1\right)}{\phi_p}\widehat{mc}_t, \tag{48}$$

where $\widehat{\pi}_t \equiv \log\left(\frac{\Pi_t}{\Pi}\right)$ is the log-deviation of gross PPI inflation from its steady state value, Π. Under Bertrand competition it, instead, reads as

$$\widehat{\pi}_t = \beta\left(1 - \delta\right) E_t \widehat{\pi}_{t+1} + \frac{\left(\varepsilon - 1\right)\left(N - 1\right)}{N\phi_p}\widehat{mc}_t - \frac{\left(\varepsilon - 1\right)}{\left(1 + \varepsilon\left(N - 1\right)\right)\phi_P}\hat{N}_t. \tag{49}$$

Notice that as $N \to \infty$ equation (49) reduces to equation (48). In that case, $\frac{(\varepsilon-1)(N-1)}{N\phi_p} \to \frac{(\varepsilon-1)}{\phi_p}$ and $-\frac{(\varepsilon-1)}{(1+\varepsilon(N-1))\phi_P} \to 0$. As usual both equations (48) and (49) imply that the current inflation rate depends on expected inflation and by real marginal costs, $\widehat{mc}_t$. In (49), the inflation rate also depends on the extent of competition, as measured by the number of firms in the market, $\hat{N}_t$. Notice that the coefficient attached to $\widehat{mc}_t$ is lower under Bertrand competition with respect to that under monopolistic competition. *Ceteris paribus*, this implies that the response of current inflation to deviations in real marginal costs is weaker under Bertrand competition than under monopolistic competition. In other words the NKPC is flatter under Bertrand competition. In both (48) and (49), expected inflation is discounted by $(1-\delta)\beta$. This differentiates the ESAM framework from the SAM one, where expected inflation in the NKPC is discounted only by β and it implies that in ESAM future expected inflation has lower weight in the determination of today's inflation. This occurs because firms take into account the probability of exit. The higher the probability of exit the higher the weight of current profits with respect to that of future profits in the firm's profit maximization problem. This implies that the higher δ the closer is the optimal price to the flexible prices solution. Indeed, in the limiting case of $\delta = 1$, the optimal price problem collapses to the

solution obtained under flexible prices.

References

Altig, David, Lawrence J Christiano, Martin Eichenbaum and Jesper Linde, 2011, Firm-specific capital, nominal rigidities and the business cycle, *Review of Economic dynamics* **14**(2), 225–247.

Ambler, Steve and Emanuela Cardia, 1998, The cyclical behaviour of wages and profits under imperfect competition, *Canadian journal of economics* pp. 148–164.

Andolfatto, David, 1996, Business cycles and labor-market search, *The american economic review* pp. 112–132.

Bergin, Paul R and Giancarlo Corsetti, 2008, The extensive margin and monetary policy, *Journal of Monetary Economics* **55**(7), 1222–1237.

Bernard, Andrew B, Stephen J Redding and Peter K Schott, 2010, Multiple-product firms and product switching, *American Economic Review* **100**(1), 70–97.

Bilbiie, Florin O, 2020, Monetary neutrality with sticky prices and free entry, *Review of Economics and Statistics* pp. 1–42.

Bilbiie, Florin O, Fabio Ghironi and Marc J Melitz, 2012, Endogenous entry, product variety, and business cycles, *Journal of Political Economy* **120**(2), 304–345.

Bilbiie, Florin O, Fabio Ghironi, Marc J Melitz, Virgiliu Midrigan and Julio J Rotemberg, 2007, Monetary policy and business cycles with endogenous entry and product variety [with comments and discussion], *NBER Macroeconomics Annual* **22**, 299–379.

Bilbiie, Florin O, Ippei Fujiwara and Fabio Ghironi, 2014, Optimal monetary policy with endogenous entry and product variety, *Journal of Monetary Economics* **64**, 1–20.

Bilbiie, Florin Ovidiu, Fabio Ghironi and Marc J Melitz, 2019, Monopoly Power and Endogenous Product Variety: Distortions and Remedies, *American Economic Journal: Macroeconomics (Forthcoming)* .

Blanchard, Olivier and Jordi Galí, 2007, Real wage rigidities and the New Keynesian model, *Journal of money, credit and banking* **39**, 35–65.

Blanchard, Olivier and Jordi Galí, 2010, Labor markets and monetary policy: A New Keynesian model with unemployment, *American economic journal: macroeconomics* **2**(2), 1–30.

Boldrin, Michele, Lawrence J Christiano and Jonas DM Fisher, 2001, Habit persistence, asset returns, and the business cycle, *American Economic Review* **91**(1), 149–166.

Born, Benjamin and Johannes Pfeifer, 2014, Policy risk and the business cycle, *Journal of Monetary Economics* **68**, 68–85.

Broda, Christian and David E Weinstein, 2010, Product creation and destruction: Evidence and price implications, *American Economic Review* **100**(3), 691–723.

Cacciatore, Matteo and Giuseppe Fiori, 2016, The macroeconomic effects of goods and labor markets deregulation, *Review of Economic Dynamics* **20**, 1–24.

Calvo, Guillermo A., 1983, Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* **12**(3), 383–398.

Canova, Fabio and Matthias Paustian, 2011, Business cycle measurement with some theory, *Journal of Monetary Economics* **58**(4), 345–361.

Casares, Miguel, Hashmat Khan and Jean-Christophe Poutineau, 2018, A structural analysis of US entry and exit dynamics, *Technical report*.

Chatterjee, Satyajit and Russell Cooper, 1993, Entry and exit, product variety and the business cycle, *Technical report*, National Bureau of Economic Research.

Christiano, Lawrence J, Martin Eichenbaum and Charles L Evans, 2005, Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of political Economy* **113**(1), 1–45.

Christiano, Lawrence J, Martin S Eichenbaum and Mathias Trabandt, 2016, Unemployment and business cycles, *Econometrica* **84**(4), 1523–1569.

Christiano, Lawrence J, Mathias Trabandt and Karl Walentin, 2010, DSGE models for monetary policy analysis, *Handbook of monetary economics*, Vol. 3, Elsevier, pp. 285–367.

Chugh, Sanjay K. and Fabio Ghironi, 2011, Optimal Fiscal Policy with Endogenous Product Variety, *NBER Working Paper 17319*, National Bureau of Economic Research.

Clementi, Gian Luca and Berardino Palazzo, 2016, Entry, exit, firm dynamics, and aggregate fluctuations, *American Economic Journal: Macroeconomics* **8**(3), 1–41.

Colciago, Andrea, 2016, Endogenous market structures and optimal taxation, *The Economic Journal* **126**(594), 1441–1483.

Colciago, Andrea and Federico Etro, 2010, Real business cycles with Cournot competition and endogenous entry, *Journal of Macroeconomics* **32**(4), 1101–1117.

Colciago, Andrea and Lorenza Rossi, 2015, Firm Dynamics, Endogenous Markups, and the Labor Share of Income, *Macroeconomic Dynamics* **19**(6), 1309–1331.

Colciago, Andrea and Riccardo Silvestrini, 2020, Monetary policy, productivity, and market concentration.

Cooley, Thomas F and Vincenzo Quadrini, 2001, Financial markets and firm dynamics, *American economic review* **91**(5), 1286–1310.

Davis, Steven J and John Haltiwanger, 1990, Gross job creation and destruction: Microeconomic evidence and macroeconomic implications, *NBER macroeconomics annual* **5**, 123–168.

Dedola, Luca and Stefano Neri, 2007, What does a technology shock do? A VAR analysis with model-based sign restrictions, *Journal of Monetary Economics* **54**(2), 512–549.

Den Haan, Wouter J, Garey Ramey and Joel Watson, 2000, Job destruction and propagation of shocks, *American economic review* **90**(3), 482–498.

Dixit, Avinash K and Joseph E Stiglitz, 1977, Monopolistic competition and optimum product diversity, *The American economic review* **67**(3), 297–308.

Ebell, Monique and Christian Haefke, 2009, Product market deregulation and the US employment miracle, *Review of Economic dynamics* **12**(3), 479–504.

Etro, Federico and Andrea Colciago, 2010, Endogenous market structures and the business cycle, *The Economic Journal* **120**(549), 1201–1233.

Etro, Federico and Lorenza Rossi, 2015, New-Keynesian Phillips curve with Bertrand competition and endogenous entry, *Journal of Economic Dynamics and Control* **51**, 318–340.

Fujita, Shigeru, 2011, Dynamics of worker flows and vacancies: evidence from the sign restriction approach, *Journal of Applied Econometrics* **26**(1), 89–121.

Geweke, John, 1999, Using simulation methods for Bayesian econometric models: inference, development, and communication, *Econometric reviews* **18**(1), 1–73.

Ghironi, Fabio and Marc J Melitz, 2005, International trade and macroeconomic dynamics with heterogeneous firms, *The Quarterly Journal of Economics* **120**(3), 865–915.

Hagedorn, Marcus and Iourii Manovskii, 2008, The cyclical behavior of equilibrium unemployment and vacancies revisited, *American Economic Review* **98**(4), 1692–1706.

Hall, Robert E, 2005, Employment fluctuations with equilibrium wage stickiness, *American economic review* **95**(1), 50–65.

Haltiwanger, John, Ron S Jarmin and Javier Miranda, 2013, Who creates jobs? Small versus large versus young, *Review of Economics and Statistics* **95**(2), 347–361.

Hofmann, Boris, Gert Peersman and Roland Straub, 2012, Time variation in US wage dynamics, *Journal of Monetary Economics* **59**(8), 769–783.

Inoue, Atsushi and Mototsugu Shintani, 2018, Quasi-Bayesian model selection, *Quantitative Economics* **9**(3), 1265–1297.

Jaimovich, Nir and Max Floetotto, 2008, Firm dynamics, markup variations, and the business cycle, *Journal of Monetary Economics* **55**(7), 1238–1252.

Krause, Michael U and Thomas A Lubik, 2007, The (ir) relevance of real wage rigidity in the New Keynesian model with search frictions, *Journal of Monetary Economics* **54**(3), 706–727.

Lee, Yoonsoo and Toshihiko Mukoyama, 2015, Entry and exit of manufacturing plants over the business cycle, *European Economic Review* **77**, 20–27.

Lewis, Vivien and Céline Poilly, 2012, Firm entry, markups and the monetary transmission mechanism, *Journal of Monetary Economics* **59**(7), 670–685.

Mangin, Sephorah and Petr Sedláček, 2018, Unemployment and the labor share, *Journal of Monetary Economics* **94**, 41–59.

Melitz, Marc J, 2003, The impact of trade on intra-industry reallocations and aggregate industry productivity, *Econometrica* **71**(6), 1695–1725.

Merz, Monika, 1995, Search in the labor market and the real business cycle, *Journal of monetary Economics* **36**(2), 269–300.

Nakamura, Emi and Jón Steinsson, 2008, Five facts about prices: A reevaluation of menu cost models, *The Quarterly Journal of Economics* **123**(4), 1415–1464.

Peersman, Gert, 2005, What caused the early millennium slowdown? Evidence based on vector autoregressions, *Journal of Applied Econometrics* **20**(2), 185–207.

Rossi, Lorenza, 2019, The overshooting of firms destruction, banks and productivity shocks, *European Economic Review* **113**, 136–155.

Rotemberg, Julio J, 1982a, Monopolistic price adjustment and aggregate output, *The Review of Economic Studies* **49**(4), 517–531.

Rotemberg, Julio J, 1982b, Sticky prices in the United States, *Journal of Political Economy* **90**(6), 1187–1211.

Rubio-Ramirez, Juan F, Daniel F Waggoner and Tao Zha, 2010, Structural vector autoregressions: Theory of identification and algorithms for inference, *The Review of Economic Studies* **77**(2), 665–696.

Schmitt-Grohé, Stephanie and Martin Uribe, 2005, Optimal fiscal and monetary policy in a medium-scale macroeconomic model, *NBER Macroeconomics Annual* **20**, 383–425.

Shao, Enchuan and Pedro Silos, 2013, Entry costs and labor market dynamics, *European Economic Review* **63**, 243–255.

Shimer, Robert, 2005, The cyclical behavior of equilibrium unemployment and vacancies, *American Economic Review* **95**(1), 25–49.

Smets, Frank and Rafael Wouters, 2007, Shocks and frictions in US business cycles: A Bayesian DSGE approach, *American economic review* **97**(3), 586–606.

Stock, James H and Mark W Watson, 1999, Forecasting inflation, *Journal of Monetary Economics* **44**(2), 293–335.

Trigari, Antonella, 2009, Equilibrium unemployment, job flows, and inflation dynamics, *Journal of money, credit and banking* **41**(1), 1–33.

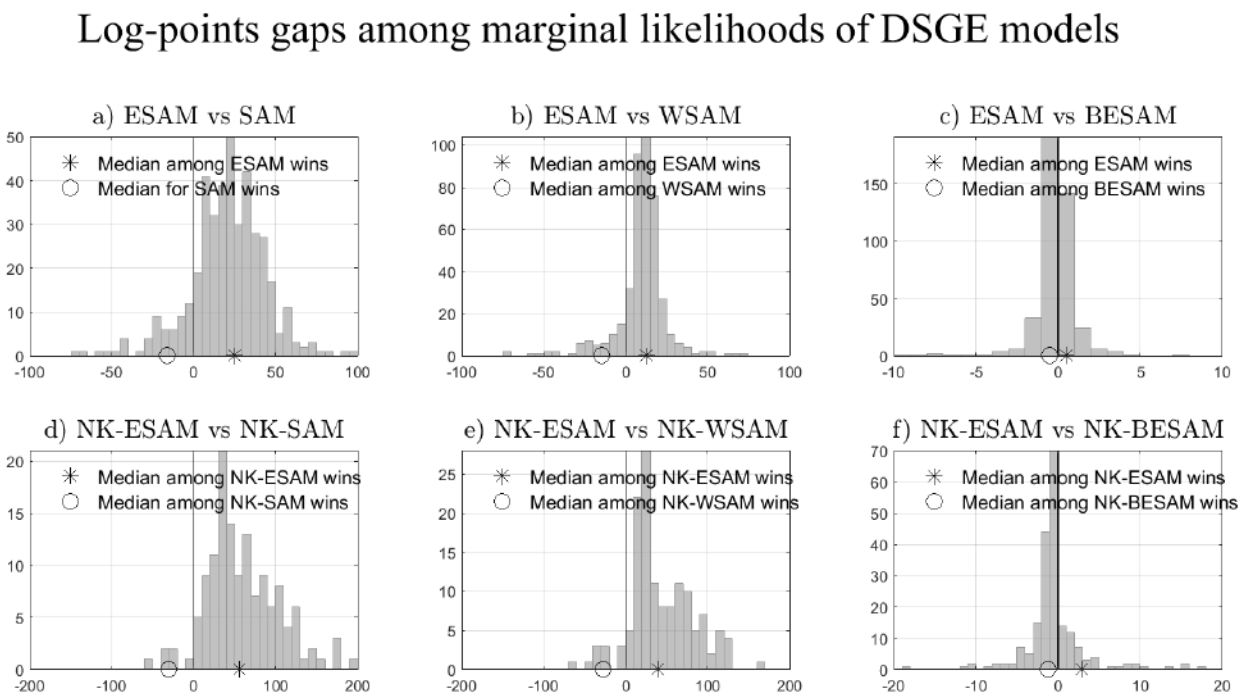
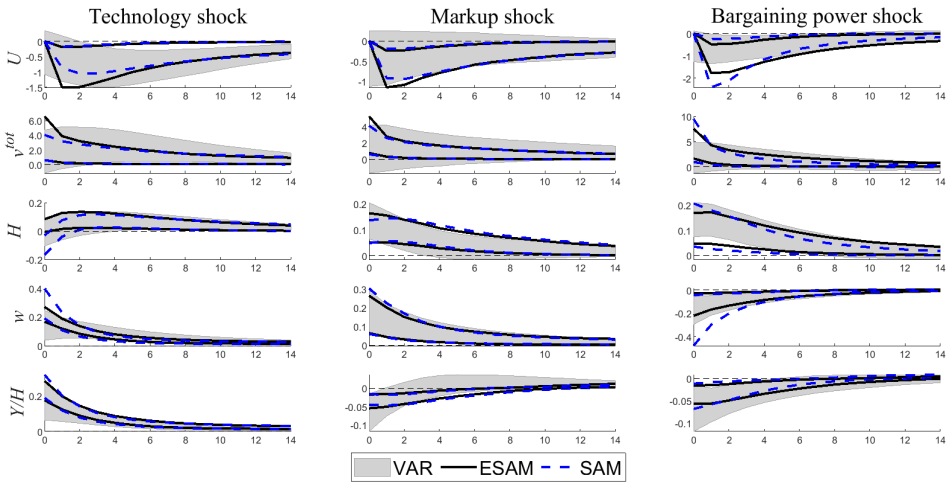


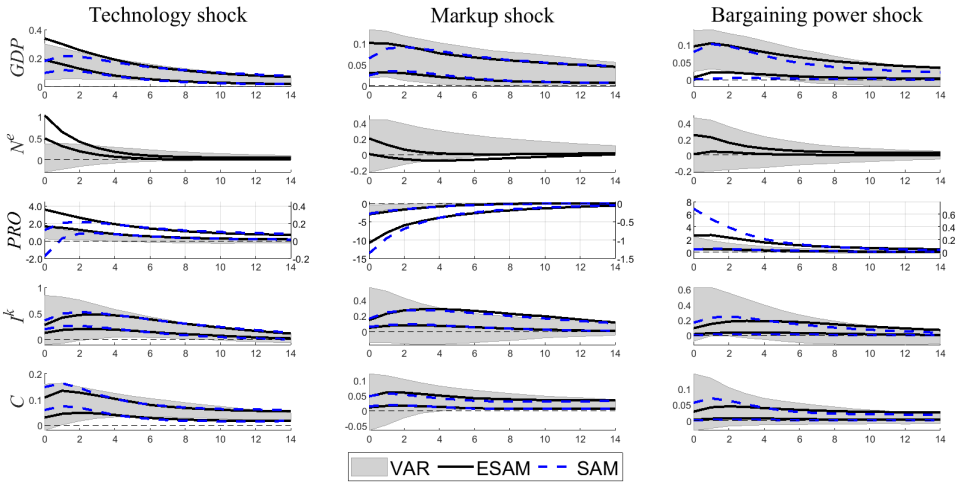
Figure 1: Histograms of differences between DSGE models marginal log-likelihoods taking any of the VAR-IRFs. Positive (negative) values refer to log-points in favor of ESAM (other flexible prices models) in panels a)-c) and in favor of NK-ESAM (other sticky prices models) in panels d)-f) . Stars (circles) refer to the median among the gaps in favor of ESAM or NK-ESAM (other models).

Empirical versus model responses, flexible prices: ESAM vs SAM



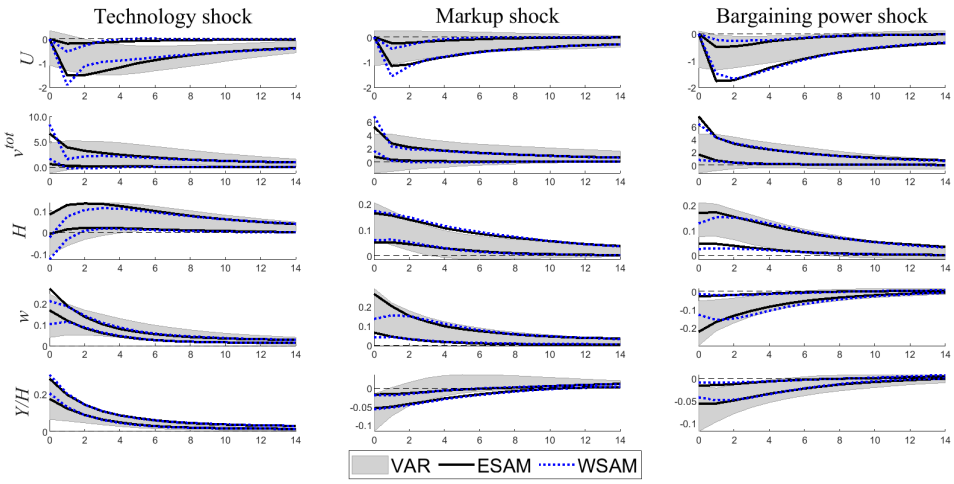
Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: ESAM (black-solid lines) versus SAM (blue dashed-lines).

Empirical versus model responses, flexible prices: ESAM vs SAM



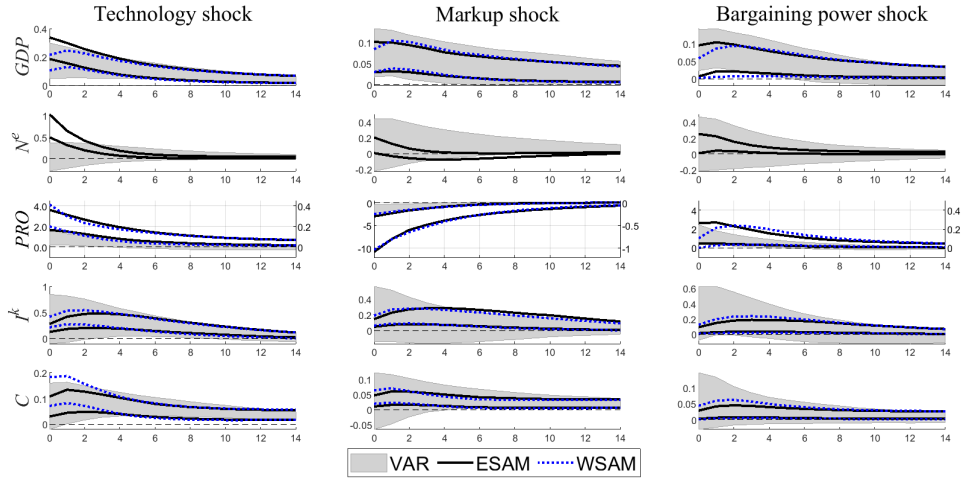
Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: ESAM (black-solid lines) versus SAM (blue dashed-lines). For *PRO* (profits): VAR responses on the left y-axis, DSGE responses on the right y-axis.

Empirical versus model responses, flexible prices: ESAM vs WSAM



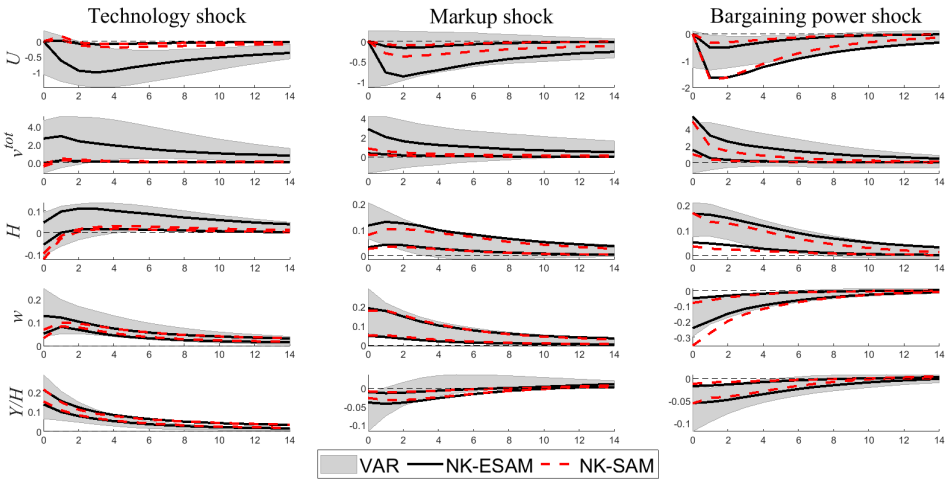
Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: ESAM (black-solid lines) versus WSAM (blue dotted-lines).

Empirical versus model responses, flexible prices: ESAM vs WSAM



Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: ESAM (black-solid lines) versus WSAM (blue dotted-lines). For *PRO* (profits): VAR responses on the left y-axis, DSGE responses on the right y-axis.

Empirical versus model responses, sticky prices: NK-ESAM vs NK-SAM



Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: NK-ESAM (black-solid lines) versus NK-SAM (red dashed-lines).

Empirical versus model responses, sticky prices: NK-ESAM vs NK-SAM

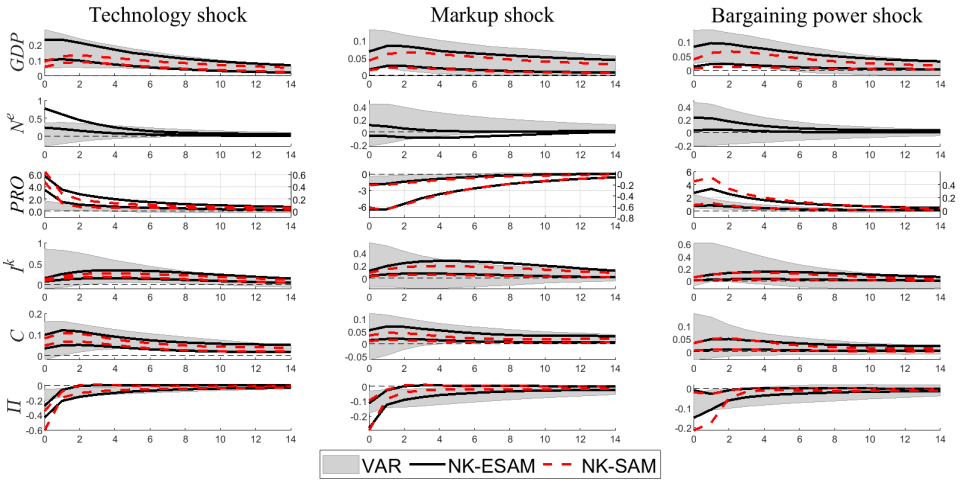
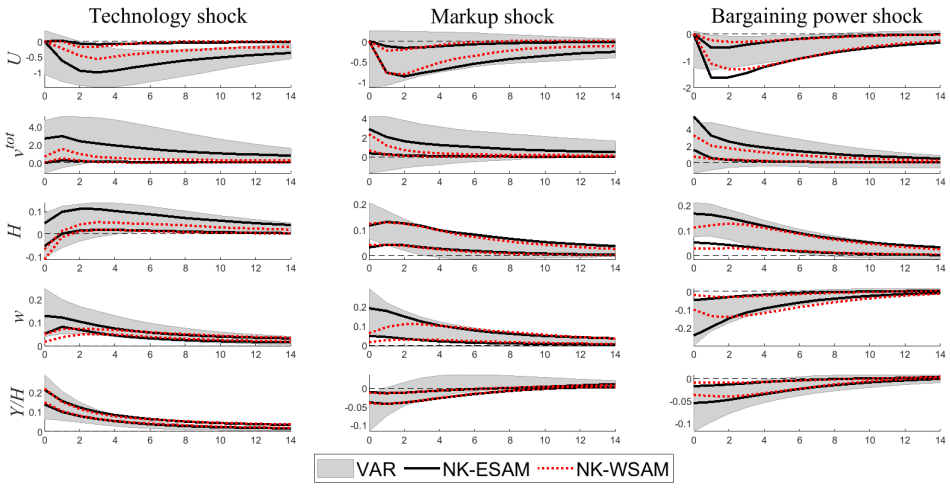


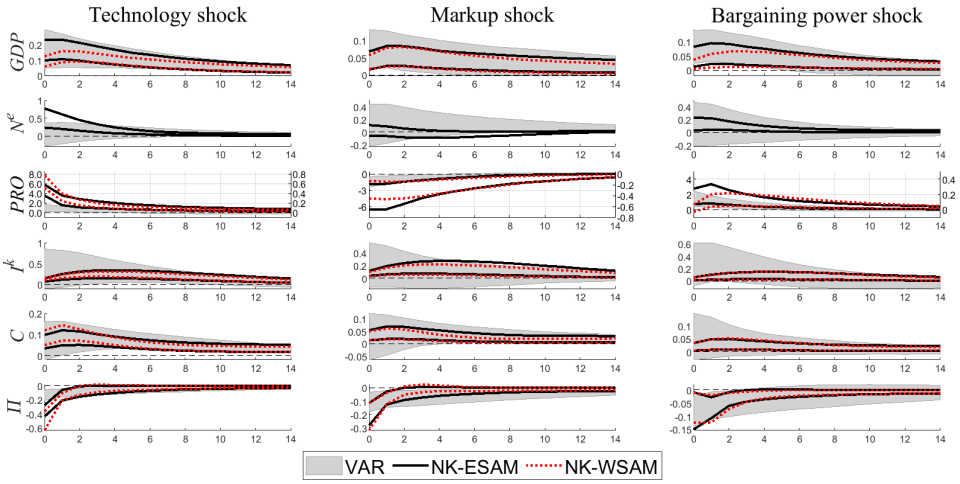
Figure 2: Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: NK-ESAM (black-solid lines) versus NK-SAM (red dashed-lines). For *PRO* (profits): VAR responses on the left y-axis, DSGE responses on the right y-axis.

Empirical versus model responses, sticky prices: NK-ESAM vs NK-WSAM



Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: NK-ESAM (black-solid lines) versus NK-WSAM (red dotted-lines).

Empirical versus model responses, sticky prices: NK-ESAM vs NK-WSAM



Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: NK-ESAM (black-solid lines) versus NK-WSAM (red dotted-lines). For *PRO* (profits): VAR responses on the left y-axis, DSGE responses on the right y-axis.

Technical Appendix: Unemployment, Firms Dynamics, and the Business Cycle

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October 2020

1 VAR estimation

1.1 Empirical model

We estimate a VAR model whose reduced-form is given by,

$$Y_t = B_0 + B_1 Y_{t-1} + ... + B_p Y_{t-p} + \varepsilon_t = X_t' \theta_t + \varepsilon_t \ , \tag{1}$$

for $t = 1, ..., T$. Y_t is a $n \times 1$ vector containing the endogenous variables. $X_t' \equiv I_n \otimes [1, Y_{t-1}', ..., Y_{t-p}']$ is a matrix collecting the first p lags of Y_t. $\theta_t \equiv vec\,(B_{0,t}, B_{1,t}, ..., B_{p,t})$ is a vector stacking the $n \times 1$ vector B_0 and the $n \times n$ matrices $B_{s,t}$, with $s = 1, ..., p$; ε_t is a $n \times 1$ vector of reduced-form VAR residuals, which are assumed independent and identical distributed, as $\varepsilon_t \sim N\,(0_{n\times 1}, \Omega)$, with Ω the positive definitive variance-covariance matrix. We rely on Bayesian techniques to estimate the VAR model. We assume a Gaussian-inverse Wishart prior on the reduced-form VAR parameters. We consider $n = 12$ endogenous variables, namely real GDP, real wages, real profits, total hours worked, unemployment rate, vacancy index, inflation rate, labor productivity, firm entry, labor share, real consumption, real investment in physical capital. We use US quarterly series spanning from 1960:Q1 to 2008:Q2 to exclude the period of the Great Recession.

1.2 Shocks identification

The identification of the structural shocks of the VAR model is achieved through sign restrictions on the VAR-based impulse response functions. To identify structural shocks via sign restrictions, we implement the QR decomposition procedure proposed by Rubio-Ramirez *et al.* (2010). Simulating the posterior of the structural impulse responses requires draws for θ and for the structural impact matrix A_0^{-1}. Let θ^{*r} denotes the rth posterior draw of θ and Ω^{*r} the rth posterior draw for Ω. Then $\tilde{A}_0^{-1} = P^{*r} Q$ where P^{*r} is the lower-triangular Cholesky decomposition of Ω^{*r} such that $P^{*r} P^{*r\prime} = \Omega^{*r}$, and Q is an orthogonal matrix. $\tilde{A}_0^{-1}$ is a potential solution for the unknown structural impact multiplier matrix A_0^{-1} that satisfies $\tilde{A}_0^{-1} \tilde{A}_0^{-1\prime} = P^{*r} Q Q' P^{*r\prime} = \Omega^{*r}$. Following Uhlig (2005), the prior distribution for the matrix Q is postulated to be uniform on the space of orthogonal matrices

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$O(n)$, allowing to simulate the set of potential solutions for $\tilde{A}_0^{-1}$, given θ^{*r} and Ω^{*r}. In practice, once we randomly draw θ^{*r} and Ω^{*r} from the posterior of the reduced form VAR parameters, we compute the lower-triangular Cholesky decomposition $P^{*r} = chol(\Omega^{*r})$. For (θ^{*r}, P^{*r}), we consider random draws of the rotation matrix Q, and for each combination (θ^{*r}, P^{*r}, Q), we compute the set of implied structural impulse responses Φ^{*r}. If Φ^{*r} satisfies the sign restrictions, we store the value of Φ^{*r}. Otherwise we discard Φ^{*r}. We iteratively repeat the procedure to store 1000 structural impulse responses with which we approximate the posterior distribution of the structural impulse responses. This distribution reflects both estimation uncertainty and identification uncertainty.

1.3 Data

Table 1 describes the series and indicates the sources of the data used in the VAR estimation.

Description	Source and series mnemonic
(1): Gross Domestic Product	FRED (BEA), GDPC1
(2): Real Compensation Per Hour	FRED (BLS), COMPRNFB
(3): Corporate Profits After Tax without IVA and CCAdj	FRED (BEA), CP
(4): Hours Worked in nonfarm business sector	BLS (Major Sector Productivity and Costs)
(5): Civilian Unemployment Rate	FRED (BLS), UNRATE
(6): Composite Helped-Wanted Index	Barnichon (2010)
(7): Real Output per hour in nonfarm business sector	FRED (BLS), OPHNFB
(8): Labor Share in nonfarm business sector	FRED (BLS), PRS85006173
(9): Net Business formation	Lewis and Winkler (2017)
(10): Personal Consumption Expenditure	FRED (BEA), PCND, PCDG, PCESV
(11): Gross Private Domestic Investment	FRED (BEA), GPDI
(12): Civilian Noninstitutional Population	FRED (BLS), CNP16OV
(13): Gross Domestic Product: Implicit Price Deflator	FRED (BEA), GDPDEF

Table 1: List of the data used in the VAR model. FRED: Federal Reserve Economic Data, BEA: Bureau of Economic Analysis, BLS: Bureau of Labor Statistics.

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The series used in the estimation of the VAR model are constructed as follows. The series of real GDP is given by (1)/(12), of real wages by (2), of real profits by (3)/((12)x(13)) , of total hours worked by (4)/(12), of unemployment rate by (5), of vacancies by the composite helped-wanted index (6) in Barnichon (2010), of annualized inflation rate by $\Delta \log(10)$x400, of labor productivity by (7), of labor share by (8), of firm entry by the index of net business formation in the BEA's Survey of Current Business as in Lewis and Winkler (2017),[1] of consumption by ((10.PCND)+(10.PCESV))/(12), and investment in physical capital by ((10.PCDG)+(10.GPDI))/(12). All series are considered in annual terms and, for those in levels, in per capita. Series are taken in logs and detrended using the one-sided Hodrick-Prescott filter.

1.4 Robustness checks

In this section, we report the estimated impulse responses for different specifications of the VAR model. For comparison, we plot the responses of the various specifications along with the ones of

[1] We thank Vivien Lewis for sharing with us the data.

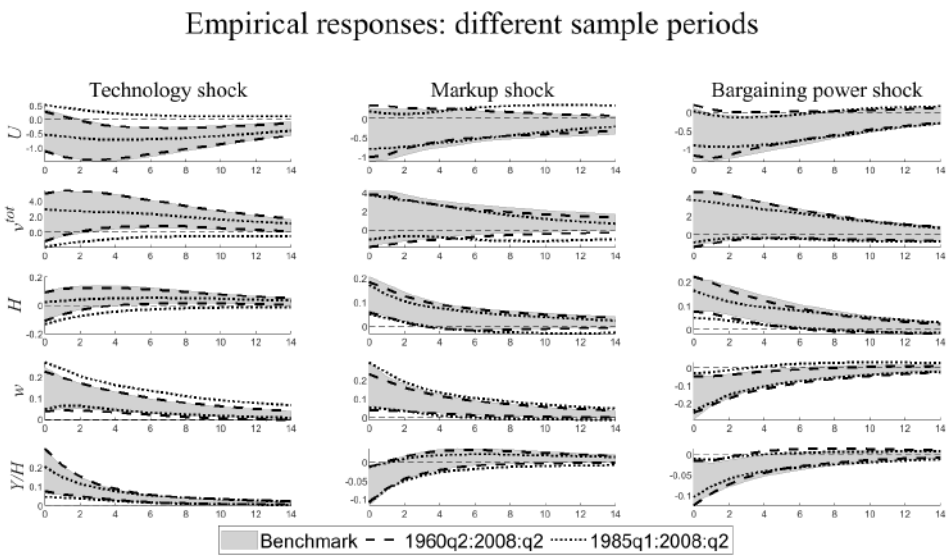


Figure 1: Dynamic responses (68% coverage percentile) to structural shocks in the VAR: benchmark specification in gray areas, with sample period 1960:q1-2008q2 in black-dashed lines, and with sample data 1985q1-2008q2 in black-dotted lines.

the benchmark VAR model introduced in the main text. In the benchmark VAR, all series, but the inflation rate, are taken in logs and detrended using the one-sided Hodrick-Prescott filter, whilst the sample period spans from 1960:Q1 to 2016:Q2.

First, we estimate the same VAR model as the benchmark, but considering two different sample periods. Specifically, we take observations of the series for the intervals 1960:Q1 to 2008:Q2 and 1985:Q1 to 2008:Q2. The former excludes from the sample the period of the Great Recession and that of the following recovery, whilst the latter excludes both the Great Recession and years before the beginning of the Great Moderation. The two additional sub-samples allow to compare the dynamics in the VAR during periods of different uncertainty. At the two extreme, we consider the sample of the benchmark specification, that being the longest includes periods of both espansion and recession for the U.S. economy, and the sub-sample between 1985:Q1 to 2008:Q2, that instead includes only relatively traquil times. Figures 1-2 show the impulse responses for all three VAR specifications to technology shocks, markups shocks, workers' bargaining power. At the first glance, the 68% credible intervals of the dynamic responses to all shocks are similar across the different VAR models. As the pattern of the variables whose responses are left unconstrained is basically preserved from considering different sample periods, this further validates our identification strategy for the structural shocks. Similar responses are especially obtained for those VAR specifications for with the longest sample periods. For the VAR only considering tranquil times, namely the one whose dataset excludes both the Great Moderation and the Great Recession, there is higher uncertainty in the impact response for some variables that left unconstrained, e.g. total hours to technology shocks and consumption to workers' bargaining power shocks. Yet, after few periods, also for those variables the response is consistent across all three VAR specifications.

Second, we estimate a VAR model over the sample period of the benchmark specification, but transforming the original series of the variables by taking the two-sided Hodrick-Prescott filter, instead of the one-sided one. The responses of log-deviations of the variables from their trend are plotted in Figures 3-4. Still for this case, the empirical pattern is shown to be unaffected from the

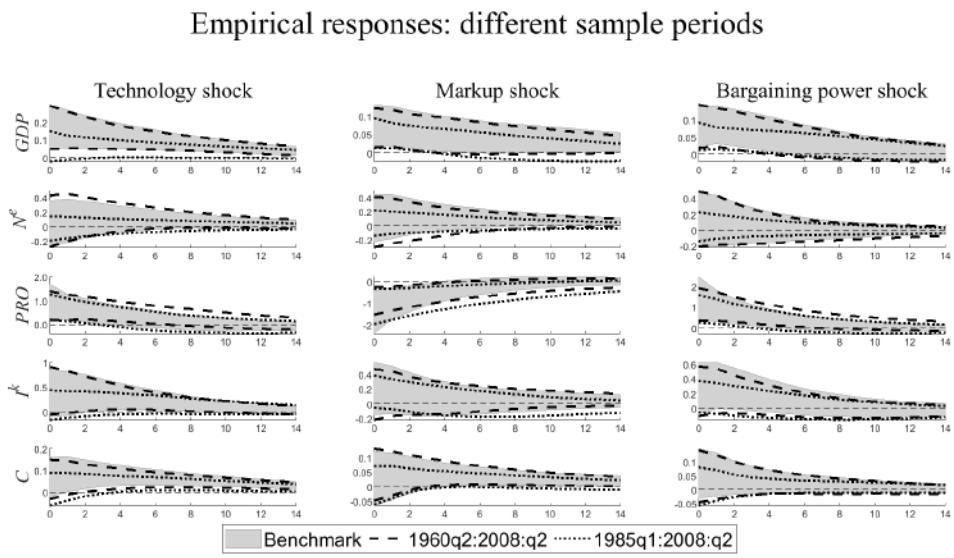


Figure 2: Dynamic responses (68% coverage percentile) to structural shocks in the VAR: benchmark specification in gray areas, with sample period 1960:q1-2008q2 in black-dashed lines, and with sample data 1985q1-2008q2 in black-dotted lines.

transformation of the series.

Third, we estimate a VAR model over the sample period of the benchmark specification, but taking the data in levels, i.e. the natural logarithms of the original series. For this estimation, we deal with the trend in the series by adding a linear trend and a quadratic trend, respectively. Figures 5-6 compare the responses of the benchmark VAR with the responses of the VAR with series in log-levels and respectively, a linear trend term and both a linear and quadratic trend terms. For both cases, the responses at the impact are consistent with the benchmark VAR for all shocks. Yet, the responses for the data in levels is less precisely estimated than of the data in deviations from the trend, as the larger credible intervals for the former show. For most of the variables, however, the higher uncertainty in the estimates arises only after some periods. Overall, the comparison between the impulse responses supports our choice of the VAR with detrended data as the benchmark.

2 DSGE estimation

2.1 Impulse response matching

This section spells out the details of the Bayesian minimum distance estimator we use in our analysis. We strictly follow the approach proposed by Christiano *et al.* (2010).

Consider a DSGE model. Let ψ be the vector of the impulse responses, while θ_0 and ζ_0 be the true value of respectively,.the structural parameters and the parameters of the shocks to be estimated. When the number of observations, T, is large, standard asymptotic theory shows that $\sqrt{T}\left(\hat{\psi} - \psi(\theta_0)\right) \overset{a}{\sim} N(0, W(\theta_0, \zeta_0))$. As a result, the asymptotic distribution of $\hat{\psi}_t$ can be written in the following form:

$$\hat{\psi} \overset{a}{\sim} N(\psi_t(\theta_0), V(\theta_0, \zeta_0, T)) \tag{2}$$

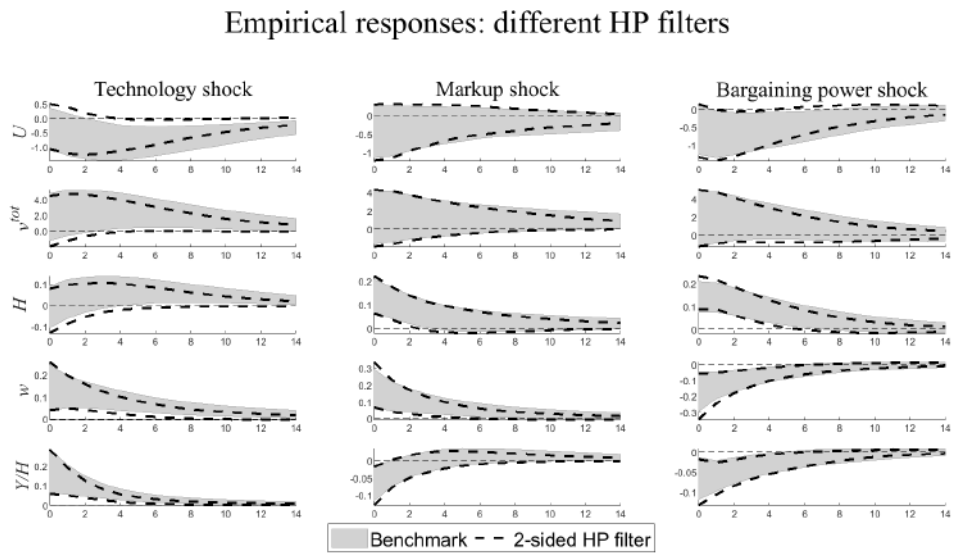


Figure 3: Dynamic responses (68% coverage percentile) to structural shocks in the VAR: benchmark specification in gray areas, log-deviations from the trend filtered with 2-sided Hodrick-Prescott approach in black-dashed lines.

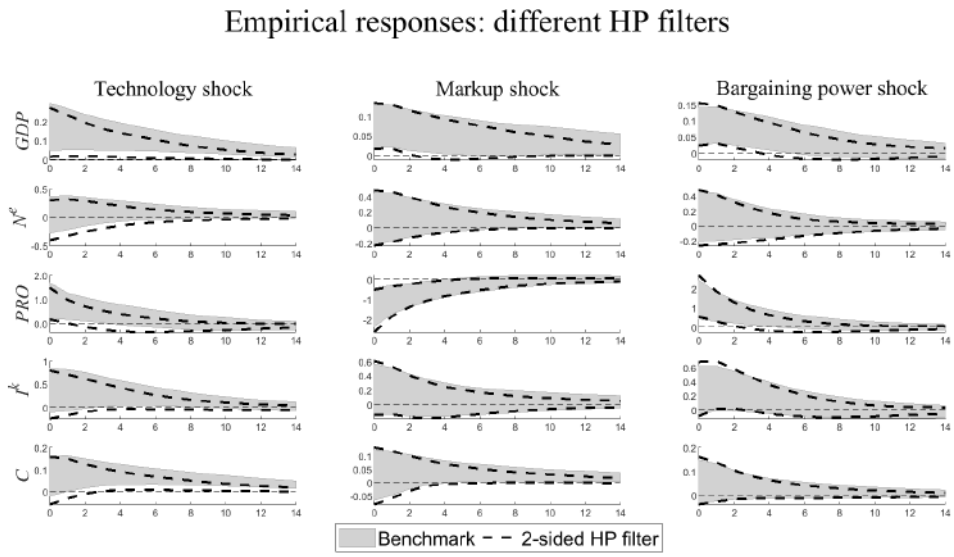


Figure 4: Dynamic responses (68% coverage percentile) to structural shocks in the VAR: benchmark specification in gray areas, log-deviations from the trend filtered with 2-sided Hodrick-Prescott approach in black-dashed lines.

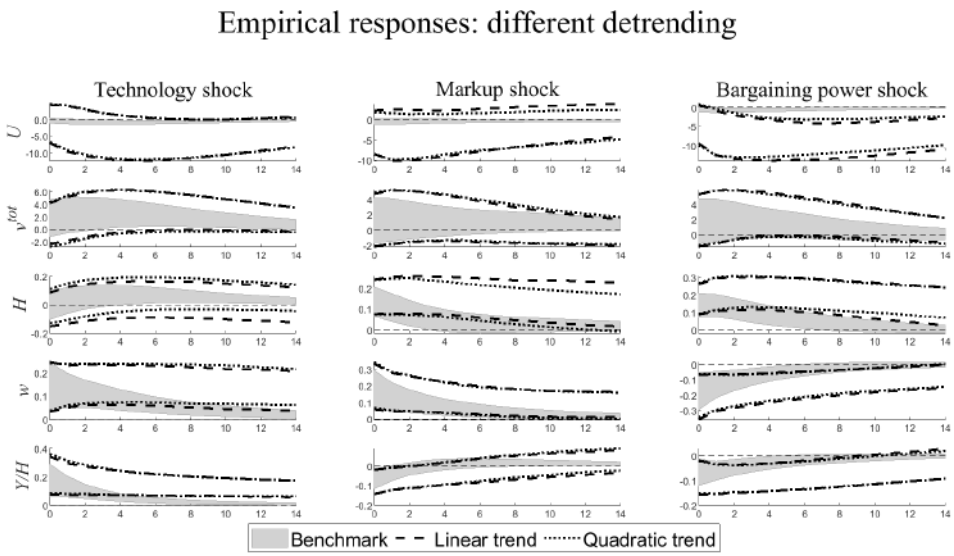


Figure 5: Dynamic responses (68% coverage percentile) to structural shocks in the VAR: benchmark specification in gray areas, with series in log-levels and a linear trend in black-dashed lines, with series in log-levels and a quadratic trend in black-dotted lines.

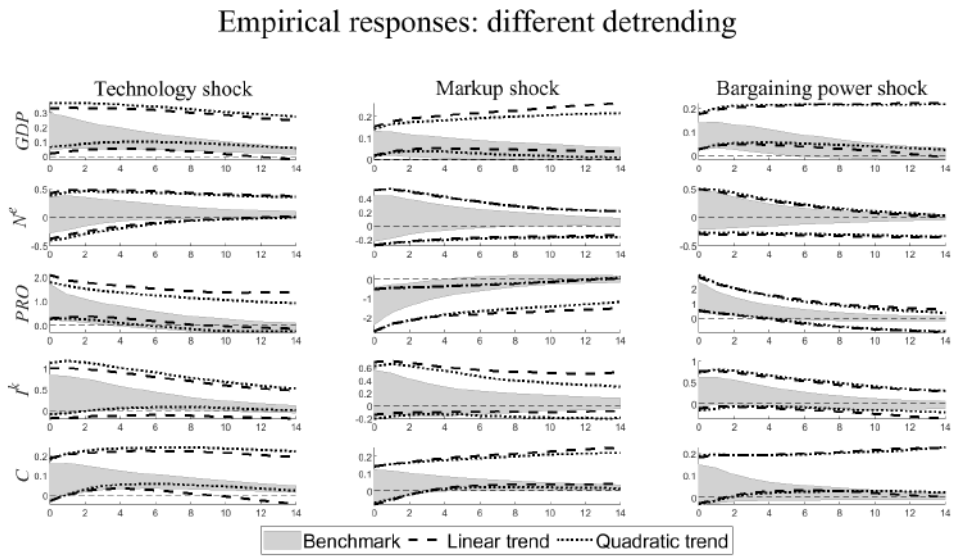


Figure 6: Dynamic responses (68% coverage percentile) to structural shocks in the VAR: benchmark specification in gray areas, with series in log-levels and a linear trend in black-dashed lines, with series in log-levels and a quadratic trend in black-dotted lines.

$$V\left(\theta_0, \zeta_0, T\right) \equiv \frac{W\left(\theta_0, \zeta_0\right)}{T} \tag{3}$$

where $\hat{\psi}_t$ is treated as the data input and the value of θ is chosen to minimize the distance between $\psi\left(\theta_0\right)$ and $\hat{\psi}$. The approximate likelihood of the data, $\hat{\psi}$, is therefore defined as a function of θ:

$$f\left(\hat{\psi}|\theta\right) = \left(\frac{1}{2\pi}\right)^{N/2} V\left(\theta_0, \zeta_0, T\right)^{-1/2} \times \exp\left[-\frac{1}{2}\left(\hat{\psi}_t - \psi_t\left(\theta_0\right)\right)' V\left(\theta_0, \zeta_0, T\right)^{-1}\left(\hat{\psi}_t - \psi_t\left(\theta_0\right)\right)\right] \tag{4}$$

where N denotes the number of elements in $\hat{\psi}$ and $V\left(\theta_0, \zeta_0, T\right)$ is treated as a fixed value. In particular, the weight matrix depends on the second moments of the VAR model-based impulse response functions in each period. The wider the posterior distribution of the empirical impulse responses, the less weight is given to the corresponding observation. As the function f is defined as the likelihood of $\hat{\psi}$, it follows that the Bayesian posterior distribution of θ conditional on $\hat{\psi}$ and $V\left(\theta_0, \zeta_0, T\right)$ can be written as

$$f\left(\theta|\hat{\psi}\right) = \frac{f\left(\hat{\psi}|\theta\right) p\left(\theta\right)}{f\left(\hat{\psi}\right)} \tag{5}$$

where $p\left(\theta\right)$ denotes the priors on θ and $f\left(\hat{\psi}\right)$ is the marginal density of $\hat{\psi}$. Since the denominator of (5) is only a function of $\hat{\psi}$, the mode of the posterior distribution of θ is computed by maximizing the value of the numerator of (5).

2.2 List of Equations in the ESAM model

	Description	Equations
1)	Marginal utility of consumption	$\lambda_t = (C_t - \vartheta C_{t-1})^{-\sigma} - \vartheta\beta E_t\left[(C_{t+1} - \vartheta C_t)^{-\sigma}\right],$
2)	Euler equation	$\lambda_t = \beta E_t\left[\lambda_{t+1}(1+r_t)\right],$
3)	Euler equation for incumbent firm	$V_t = \beta\left(1-\delta\right)E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(V_{t+1}+\pi_{t+1}\right)\right],$
4)	Euler equation for entrant firm	$\psi_t = \beta\left(1-\delta\right)E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(V_{t+1}+\pi_{t+1}^{new}\right)\right],$
5)	Euler equation for capital	$q_t^I = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(r_{t+1}^K + q_{t+1}^I\left(1-\delta_K\right)\right)\right],$
6)	Euler equation for investments	$1 = q_t^I\left(1-\frac{\phi_I}{2}\left(\frac{I_t^k}{I_{t-1}^k}-1\right)^2 - \phi_I\left(\frac{I_t^k}{I_{t-1}^k}-1\right)\frac{I_t^k}{I_{t-1}^k}\right) + \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}q_{t+1}^I\phi_I\left(\frac{I_{t+1}^k}{I_t^k}-1\right)\left(\frac{I_{t+1}^k}{I_t^k}\right)^2\right],$
7)	Law of motion of capital	$K_t = (1-\delta^K)K_{t-1} + \left[1-\frac{\phi_I}{2}\left(\frac{I_t^k}{I_{t-1}^k}-1\right)^2\right]I_t,$
8)	Firm entry costs	$\psi_t = \psi_0 + \psi_1\left(\frac{N_t^e}{N_t}\right)^{\varsigma},$
9)	Capital demand	$r_t^K = \alpha\frac{A_t}{\mu_t^M}\left(\frac{L_t}{K_{t-1}}\right)^{1-\alpha},$
10)	Law of motion of firms	$N_{t+1} = (1-\eta_t)\left(N_t + N_t^e\right),$
11)	Law of motion of employment	$L_t = (1-\delta)\,\varrho L_{t-1} + q_t v_t^{tot},$
12)	Production function	$Y_t = A_t H_t^{1-\alpha}K_{t-1}^{\alpha},$
13)	Wage schedule	$w_t h_t = \eta_t\left(1-\alpha\right)\left(\frac{A_t}{\mu_t^M}\right)\left(\frac{K_{t-1}}{L_t}\right)^{\alpha}h_t + (1-\eta_t)\left(\frac{\chi}{\lambda_t}\frac{h_t^{1+\varphi}}{1+\varphi}+b\right) + \eta_t\kappa\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\theta_{t+1}\right],$
14)	Individual hours schedule	$\frac{\chi}{\lambda_t}h_t^{\varphi} = (1-\alpha)^2\left(\frac{A_t}{\mu_t^M}\right)\left(\frac{K_{t-1}}{L_t}\right)^{\alpha},$
15)	Job creation condition	$\frac{\kappa}{q_t} = (1-\alpha)\left(\frac{A_t}{\mu_t^M}\right)\left(\frac{K_{t-1}}{L_t}\right)^{\alpha}h_t - w_t h_t + (1-\delta)\,\varrho\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa}{q_{t+1}}\right],$
16)	Vacancy for first period producing firm	$v_t^{new} = \frac{L_t}{q_t N_t},$
17)	Aggregate vacancies	$v_t^{tot} = (1-\delta)\,N_{t-1}v_t + (1-\delta)\,N_{t-1}^e v_t^{new},$
18)	Profit of incumbent firm	$\pi_t = \frac{Y_t}{N_t} - w_t\frac{H_t}{N_t} - r_t^K\frac{K_{t-1}}{N_t} - \frac{\kappa}{q_t}\left(\frac{L_t}{N_t} - \varrho\frac{L_{t-1}}{N_{t-1}}\right),$
19)	Aggregate profits	$PRO_t = (1-\delta)\,N_{t-1}\pi_t + (1-\delta)\,N_{t-1}^e\pi_t^{new},$
20)	Resource constraint	$C_t + \psi_t N_t^e + I_t^k = w_t H_t + r_t^k K_{t-1} + PRO_t,$
21)	Market clearing condition	$Y_t = C_t + \psi_t N_t^e + I_t^k + \kappa v_t^{tot} = GDP_t + \kappa v_t^{tot},$
22)	Total hours	$H_t = h_t L_t,$
23)	Labor market tightness	$\theta_t = \frac{v_t^{tot}}{u_t},$
24)	Probability of filling a vacancy	$q_t = \gamma_m\theta_t^{-\gamma},$
25)	Probability of finding a job	$z_t = \gamma_m\theta_t^{1-\gamma},$
26)	Labor productivity	$\frac{Y_t}{H_t},$
27)	Labor share	$\frac{w_t H_t}{GDP_t},$
28)	Profit share	$\frac{F_t}{GDP_t},$
29)	Unemployment rate	$U_t = 1 - N_t,$

Table 2: System of non-linear equations

2.2.1 Exogenous processes

Description	Equations
Technology shock	$\ln\left(\frac{A_t}{A}\right) = \rho_a \ln\left(\frac{A_{t-1}}{A}\right) + \sigma_a \epsilon_{a,t},$
markup shock	$\ln\left(\frac{\varepsilon_t}{\varepsilon}\right) = \rho_\varepsilon \ln\left(\frac{\varepsilon_{t-1}}{\varepsilon}\right) - \sigma_\varepsilon \epsilon_{\varepsilon,t},$
Bargaining power shock	$\ln\left(\frac{\eta_t}{\eta}\right) = \rho_\eta \ln\left(\frac{\eta_{t-1}}{\eta}\right) - \sigma_\eta \epsilon_{\eta,t},$

Table 3: Exogenous processes common to all DSGE models

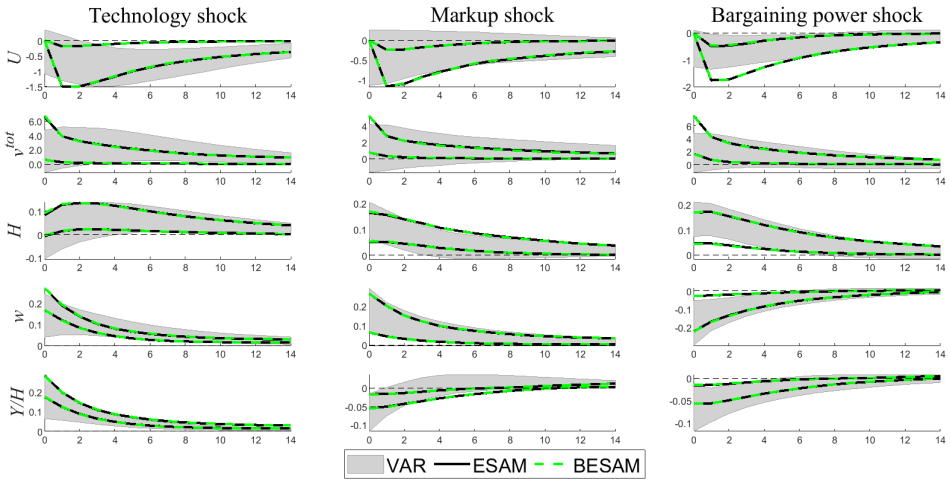
2.2.2 Price markup in DSGE specifications

Model	Equations
ESAM, SAM	$\mu_t^M = \frac{\varepsilon_t}{\varepsilon_t - 1}$
BESAM	$\mu_t^B = \frac{\varepsilon_t(N_t-1)+1}{(\varepsilon_t-1)(N_t-1)}$
CESAM	$\mu_t^C = \frac{\varepsilon_t N_t}{(\varepsilon_t-1)(N_t-1)}$
NK-SAM	$\mu_t^{NK-SAM} = \frac{\varepsilon_t}{(\varepsilon_t-1)\Upsilon_1^{SAM}+\Upsilon_2^{SAM}}$ $\Upsilon_1^{SAM} \equiv 1 - \frac{\phi_P}{2}\left(\Pi_t - 1\right)^2,$ $\Upsilon_2^{SAM} \equiv \phi_P\left(\Pi_t - 1\right)\Pi_t - \beta\left(1-\delta\right)E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\phi_P\left(\Pi_{t+1}-1\right)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right],$
NK-SAM	$\mu_t^{NK-ESAM} = \frac{\varepsilon_t}{(\varepsilon_t-1)\Upsilon_1^{ESAM}+\Upsilon_2^{ESAM}}$ $\Upsilon_1^{ESAM} \equiv 1 - \frac{\phi_P}{2}\left(\Pi_t - 1\right)^2,$ $\Upsilon_2^{ESAM} \equiv \phi_P\left(\Pi_t - 1\right)\Pi_t - \beta\left(1-\delta\right)E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\phi_P\left(\Pi_{t+1}-1\right)\Pi_{t+1}\frac{N_{t+1}}{N_t}\frac{Y_{t+1}}{Y_t}\right],$
NK-BESAM	$\mu_t^{NK-BESAM} = \frac{(\varepsilon_t(N_t-1)+1)/N_t}{(\varepsilon_t-1)\Upsilon_1^{BESAM}+\Upsilon_2^{BESAM}}$ $\Upsilon_1^{BESAM} \equiv \left(\frac{N_t-1}{N_t}\right) - \frac{\phi_P}{2}\left(\Pi_t - 1\right)^2 + \frac{\phi_P}{2}\frac{\left(\Pi_t-1\right)^2}{N_t},$ $\Upsilon_2^{BESAM} \equiv \phi_P\left(\Pi_t - 1\right)\Pi_t - \beta\left(1-\delta\right)E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\phi_P\left(\Pi_{t+1}-1\right)\Pi_{t+1}\frac{N_{t+1}}{N_t}\frac{Y_{t+1}}{Y_t}\right]$

Table 4: Price mark-up in different DSGE models

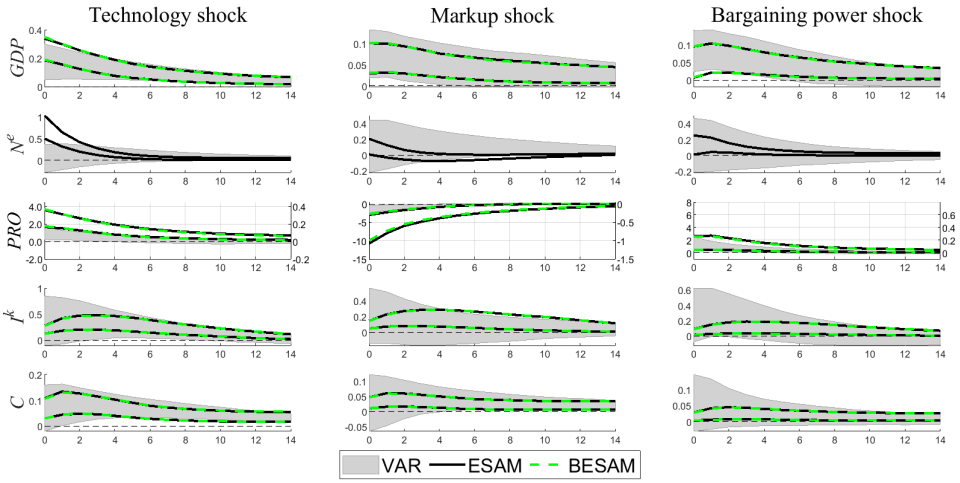
2.3 Impulse responses for BESAM and NK-BESAM

Empirical versus model responses, flexible prices: ESAM vs BESAM



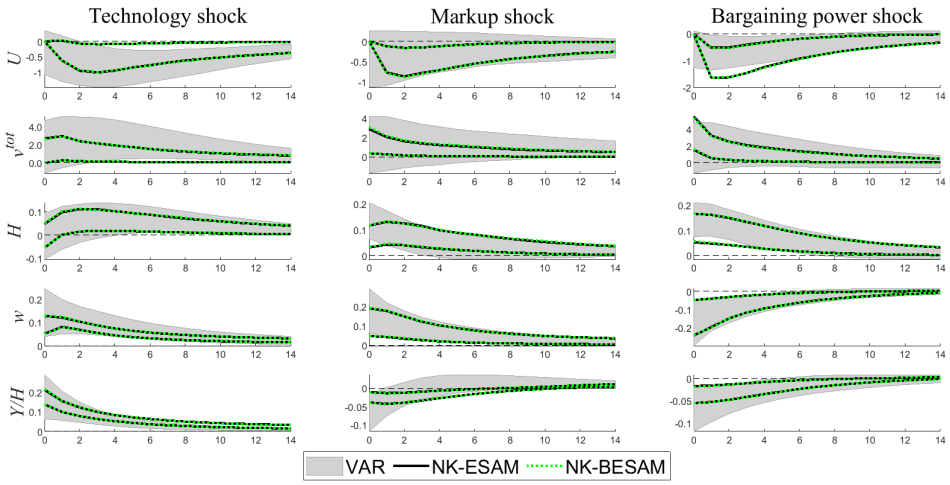
Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: ESAM (black-solid lines) versus BESAM (green dashed-lines).

Empirical versus model responses, flexible prices: ESAM vs BESAM



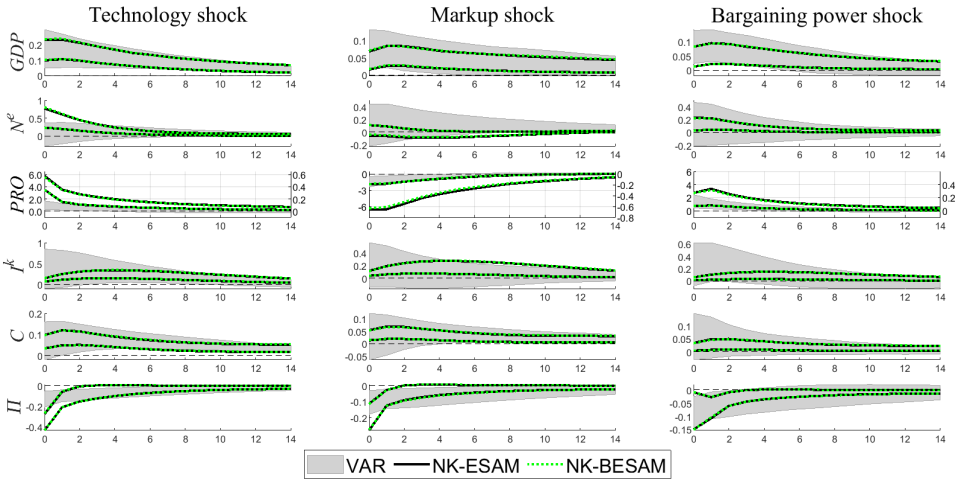
Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: ESAM (black-solid lines) versus BESAM (green dashed-lines). For *PRO* (profits): VAR responses on the left y-axis, DSGE responses on the right y-axis.

Empirical versus model responses, sticky prices: NK-ESAM vs NK-BESAM



Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: NK-ESAM (black-solid lines) versus NK-BESAM (green dotted-lines).

Empirical versus model responses, sticky prices: NK-ESAM vs NK-BESAM



Dynamic responses (68% percentile coverage) to structural shocks in VAR (gray area) and in DSGE: NK-ESAM (black-solid lines) versus NK-BESAM (green dotted-lines). For *PRO* (profits): VAR responses on the left y-axis, DSGE responses on the right y-axis.

References

Barnichon, Regis, 2010, Building a composite help-wanted index, *Economics Letters* **109**(3), 175–178.

Christiano, Lawrence J, Mathias Trabandt and Karl Walentin, 2010, DSGE models for monetary policy analysis, *Handbook of monetary economics*, Vol. 3, Elsevier, pp. 285–367.

Lewis, Vivien and Roland Winkler, 2017, Government Spending, Entry, And The Consumption Crowding-In Puzzle, *International Economic Review* **58**(3), 943–972.

Rubio-Ramirez, Juan F, Daniel F Waggoner and Tao Zha, 2010, Structural vector autoregressions: Theory of identification and algorithms for inference, *The Review of Economic Studies* **77**(2), 665–696.

Uhlig, Harald, 2005, What are the effects of monetary policy on output? Results from an agnostic identification procedure, *Journal of Monetary Economics* **52**(2), 381–419.

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