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Abstract

This paper extends the Bayesian proxy SVAR model (BP-SVAR) of Caldara and Herbst (2019) to examine changes in the transmission of structural shocks in the presence of regime shifts in an economy. I provide a Metropolis-within-Gibbs sampling algorithm to approximate the posterior distribution of model parameters. The model is then used to examine the role of credit spreads on the transmission of monetary policy shocks in the United States between 1994-2007, where identification is achieved using a proxy constructed from high-frequency financial data. The main finding is that the effect of credit spreads differs across regime. Credit spreads significantly change the transmission of monetary policy shocks from 2000-2007 supporting Caldara and Herbst (2019), although, their inclusion appears to only alter the response of industrial production in the short-term with no other significant changes to the rest of the economy during the mid to late 1990s. This result highlights the empirical relevance of accounting for regime changes when assessing the impact of economic shocks.

Key words: Markov-Switching, External Instruments, Proxy BVAR, Monetary Policy shocks

JEL Classification: C2, C11, E3

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1 Introduction

Regime changes in any economy can be prevalent, even during samples of less than a decade. The most prominent recent example has been the global financial crisis of 2008, which triggered changes in both monetary and fiscal policy alongside, shifts in the level of economic policy uncertainty. The phenomenon of regime change can also be observed before the crisis in the times series of an array of macroeconomic variables such as price inflation, wage growth, interest rates and government spending. Understanding the effect of these shifts on the economy is crucial for policymakers, one potential effect may be changes in how an economy responds to exogenous shocks. Although, reduced-form models are able to detect regime changes, examining the effects of regime changes on the transmission of specific economic shocks requires imposing identification restrictions on the responses of endogenous variables that are often controversial, as they are difficult to test.

A recent strand of literature uses external information to identify shocks in a Structural Vector Autoregression (SVAR). The proxy SVAR framework developed by Mertens and Ravn (2013) and Stock and Watson (2008) differs from existing SVAR identification strategies as responses to shocks are estimated using a proxy for structural shocks as an instrumental variable. The proxy series is usually constructed by a narrative approach following the seminal paper of Romer and Romer (2004). In order to identify structural shocks of interest, the proxy must satisfy exogeneity and relevance conditions of an instrumental variable. The use of proxies reduces the number of restrictions imposed on the endogenous variables to identify the impact and propagation of structural shocks. A further advantage of this method is that it addresses the problem of measurement error by treating the proxy as an imperfect measure of the structural shocks of interest. Further developments in Arias, Rubio-Ramírez and Waggoner (2018), Caldara and Herbst (2019), Drautzburg (2016) and Rodgers et al. (2016) extend the proxy SVAR to be estimated using Bayesian methods. The Bayesian approach allows for the estimation of larger models, in addition to estimation across relatively shorter samples that proxy series tend to be available for.

This paper attempts to apply this form of identification to an SVAR that allows for regime changes in the economy and provides an estimation procedure that applies a Metropolis-within-Gibbs algorithm to approximate posterior distributions of parameters. Following Caldara and Herbst (2019), the likelihood of the model is augmented with a measurement equation that relates the proxy to the unobserved structural shock of interest and the model is then estimated using Bayesian techniques. The algorithm combines the approach of Caldara and Herbst (2019) with the Gibbs sampling algorithm developed in Albert and Chib (2003) to estimate a Markov-switching proxy SVAR. The procedure is tested using a simulation exercise, with results suggesting that the algorithm is capable of retrieving changes in the transmission of shocks when the data generating process (DGP) contains switches in coefficients and residual covariances. The procedure is flexible and encompasses a range of models that differ in the parameters that can switch and the number of regimes allowed for.

The proposed model is used to revisit the application of Caldara and Herbst (2019) in a Markov-switching framework. The application assesses the impact of credit spreads on the transmission of monetary policy shocks that are identified using a proxy constructed from high-frequency financial data. The estimation results suggest evidence of a structural break in the U.S. economy during 2001. The economy moves to a regime characterised by relatively higher mean levels of inflation compared to the previous regime that is found to be in place during the mid to late 1990s. The main finding is that the effect of credit spreads differs across regime. Credit spreads significantly change the transmission of monetary policy shocks from 2000-2007 supporting Caldara and Herbst (2019), however, between the mid to late 1990s their inclusion appears to have only short-lived effects on industrial production and no other significant economic effects. This result highlights the empirical relevance of accounting for regime changes when assessing the impact of economic shocks.

This paper is organised as follows. Section 2 presents a brief review of the literature. Section 3 introduces the Markov-switching proxy Bayesian SVAR model. Section 4 discusses the Gibbs sampling algorithm and presents the results of a Monte Carlo simulation exercise. In section 5, the algorithm is applied to assess the impact of credit spreads on the transmission of monetary policy shocks across volatility regimes. Finally, section 6 discusses further applications and concludes.

2 Existing literature

This paper proposes a Markov-switching Bayesian proxy SVAR model (MS-BP-SVAR) to bridge the gap in the literature on identifying structural shocks in the presence of regime shifts and employing external information to identify structural shocks. This section provides a brief review of these two strands of literature.

Since the seminal paper of Sims and Zha (2006), a literature has developed around examining the changes in the transmission of structural shocks across monetary policy and shock volatility regimes. Sims and Zha (2006) apply a Markov-switching SVAR (MS-SVAR) that allows both the contemporaneous impact and persistence of structural shocks to alternate between regimes. The regime in place is governed by an unobserved state variable that follows a Markov process. To uncover the structural representation of the VAR, Sims and Zha (2006) use a combination of sign and zero restrictions to identify structural shocks and find that the best fitting model on U.S. data allows for shock variances to switch between nine regimes between 1959-2003. The advantage of introducing time-variation through regime-switching is that discrete shifts in coefficients can be applied to a range of economic applications, such as monetary and fiscal policy regime changes, and shifts in economic uncertainty and stress. This form of discrete time-variation also allows for larger VARs to be estimated that capture richer macroeconomic dynamics relative to continuous time-variation as in Cogely and Sargent (2005). In addition, compared to other forms of multivariate models that allow for discrete time-variation such as Threshold and Smooth-Transition VARs (TVARs and STVARs respectively), Markov-switching models allow for restrictions to be placed on the probabilities of switching between regimes that can offer a more structured form of time-variation. An empirical application of the structure that can be added to time-variation in this framework is the change-point MS-VAR of Liu et al. (2018) identified by sign restrictions, that distinguishes four macroeconomic regimes in the U.S. from 1960-2011, and prevents an economy from directly re-entering a specific regime.

The MS-SVAR literature adopts a range of alternative identification schemes, that restrict either the sign or the size of structural shocks on the remainder of the economic variables of the VAR. Barnett, Groen and Mumtaz (2011), apply a MS-SVAR identified by sign restrictions to examine changes in the interaction of nominal and real variables with inflation expectations in the United Kingdom.

Nason and Tallman (2013) use an MS-SVAR imposing a recursive identification scheme to assess the role of credit shocks in U.S. financial crises and business cycles from 1890-2010. Hubrich and Tetlow (2015) also adopt a recursive identification strategy to report changes in the transmission of financial shocks in the U.S. during periods of high financial stress. The change-point MS-SVAR model of Liu et al. (2018) is identified by sign restrictions and reports increased importance of shocks to credit spread yields during the global financial crisis. Holm-Hadulla and Hubrich (2017) estimate an MS-VAR with time-varying transition probabilities and observe that oil shocks in the euro area are short-lived during a normal regime and followed by sizeable and sustained macroeconomic fluctuations during an adverse regime. Lhussier and Triper (2016) estimate an MS-DSGE model by matching the impulse responses of an MS-SVAR identified by a combination of sign and zero restrictions to investigate the impact of economic uncertainty shocks. Once solved dynamic stochastic general equilibrium model that allows for regime change models take the form of MS-SVARs in which micro-foundations derived from economic theory are used to impose restrictions. Bianchi and Ilut (2017) place restrictions on the transition matrix to identify alternative regimes consisting of combinations of monetary and fiscal policy in a MS-DSGE. Bianchi (2013), Davig and Doh (2014) and Mumtaz and Liu (2011) estimate MS-DSGE models to investigate the impact of regime changes in monetary policy on the U.S. and U.K., respectively.

Empirical studies that apply alternative models, such as TVARs and STVARs models, that allow for regime change to be triggered by the level of macroeconomic and financial variables relative to a threshold level, also, report changes in the transmission of structural shocks across regimes. Balke (2000) uses a TVAR to find evidence of switching credit regimes in the U.S. and finds that shocks are more potent in the tight-credit regime and that contractionary monetary shocks have a larger effect on output than expansionary shocks. Alessandri and Mumtaz (2017) estimate a TVAR on U.S. data and apply a recursive identification scheme to report a deeper and more abrupt fall of output and prices in response to a contractionary monetary policy shock in a crisis regime relative to normal times. Cheng and Chiu (2019) estimate a STVAR model to examine non-linearities in mortgage spread-shocks and report a more severe impact and protracted reduction of real activity, CPI and house prices during recessionary regimes using a recursive identification scheme. In addition, estimation of DSGE models also provides evidence of changes in the transmission of shocks. Jensen et al (2019) find that financially-driven expansions lead to deeper contractions, as compared with equally-sized non-financial expansions.

In contrast to paper that use sign restrictions for identification, Mertens and Ravn (2012) use external proxies to identify shocks. They find large short-run effects of tax shocks on U.S. output and apply a measure of unexpected changes in tax from narrative accounts to proxy a tax shock. Stock and Watson (2012) attempt to disentangle the channels of the global financial crisis and identify six shocks with seventeen measures that act as instrumental variables, applying the methodology developed in Olea et al. (2012). Peersman (2019) constructs a measure of global food harvest shocks to investigate whether the effect of food price shocks can explain the dynamics of euro area inflation after the global financial crisis in 2008. Further developments in Arias, Rubio-Ramírez and Waggoner (2018), Caldara and Herbst (2019), Drautzburg (2016) and Rodgers et al. (2016) extend the proxy SVAR by using Bayesian methods that allow for the estimation of larger models over the relatively short samples for which many proxy series are available.

The proposed model in this paper is related to Mumtaz and Petrova (2018) who extend the approach of Bayesian proxy SVAR of Caldara and Herbst (2019) to estimate a proxy VAR with continuous time-variation. Mumtaz and Petrova (2018) find evidence of time-variation in the responses of the U.S. and U.K. to tax shocks and report a decline in the effect of the shock on output growth.

The approach of using a proxy in a Bayesian VAR that allows for changes in model parameters is also related to recent contribution by Paul (2017) who incorporates proxies as exogenous variables in a TVP-VAR. He shows that this VARX approach leads to a consistent estimator of the relative or normalised impulse response. This approach could also be extended to model with regime change and while the approach in Paul (2017) is attractive due to its simplicity, the model proposed in this paper has two advantages. First, the proxy is used as an instrument, therefore, I can estimate reliability statistics and provide evidence on instrument relevance. Secondly, the procedure in the paper can easily accommodate missing values in the instrument series, and thus deals with an issue that is common in the existing literature.

This paper applies a Markov-Switching proxy Bayesian VAR to revisit the application and Caldara and Herbst (2019) who find that the inclusion of credit spreads affects the transmission of monetary policy shocks. The main finding is that the effect of credit spreads differs across regimes. Credit spreads significantly change the transmission of monetary policy shocks from 2000-2007 supporting Caldara and Herbst (2019), however, their inclusion appears to have only a short-lived amplification effect on real activity between the mid to late 1990s. This result highlights the empirical relevance of accounting for regime changes when assessing the impact of economic shocks.

3 A Markov-switching proxy Bayesian VAR

This section introduces a benchmark reduced-form Markov-switching Vector Autoregression (MS-VAR) and subsequently describes the identification of structural shocks using external information in the form of a proxy extending the approach of Caldara and Herbst (2019) to identify shocks in this non-linear framework.

3.1 A reduced-form Markov-switching VAR

I consider the following VAR with regime-dependent parameters

$$Y_t = \Phi_{s_t} X_t + u_t,\ u_t = \Sigma_{s_t}\varepsilon_t,\ \varepsilon_t \sim \mathcal{N}(0, I_N), s_t \in \{1,2\} \tag{1}$$

$$s_t = P s_{t-1} \tag{2}$$

$$P = \begin{pmatrix} p_{11} & p_{21} & \cdots & p_{M1} \\ p_{12} & p_{22} & \cdots & p_{M2} \\ \vdots & \vdots & \cdots & \vdots \\ p_{1M} & p_{2M} & \cdots & p_{MM} \end{pmatrix}$$

$$p_{ij} = \Pr(s_{t+1} = j \mid s_{t+1} = i) \text{ for } i,\, j - 1, \ldots, M.$$

where Y_t is a $N \times 1$ matrix of endogenous variables, $X_t = [Y'_{t-1}, ..., Y'_{t-p}, 1],$' is $(NP+1) \times 1$ matrix of regressors, s_t denotes an unobserved state variable and Φ_{s_t} denotes the $N \times (NP+1)$ coefficients matrix which is regime-dependent.

The covariance matrix of the reduced-form residuals u_t is Σ_{s_t} and is also regime-dependent governed by s_t. The Markov-switching specification is centered around the state variables s_t, which determine the regime in place and is modeled as a stationary, time homogeneous, first-order, M-state Markov chain. This assumption implies that the s_t takes on M discrete values, $s_t = 1...M$. A different coefficient matrix Φ_{s_t} and covariance matrix Σ_{s_t} are associated with each possible realisation of state s_t. Equation 2 determines the law of motion for s_t, where P represents the transition probability matrix that determines the frequency of regime change and expected duration of each regime.

3.2 Identification of shocks

The time-varying covariance matrix of the reduced-form residuals Σ_{s_t} can be written as

$$\Sigma_{s_t} = (A_{s_t} q)(A_{s_t} q)'$$

where A_{s_t} is a lower triangular matrix that is also regime-dependent, and q is an element of the family of orthogonal matrices of size N, satisfying $q'q = I_N$. By considering all possible values of q, the matrix $A_{s_t} q$ spans the space of all possible contemporaneous matrices.

The structural shocks of the VAR model ε_t are defined as

$$\varepsilon_t = A_{0s_t}^{-1} u_t \tag{3}$$

where $A_{0s_t} = A_{s_t} q$. The contemporaneous effects of structural shocks are contained in A_{0s_t}, however, in practice identifying this matrix requires imposing restrictions on the sign and timing of responses to shocks based on economic theory. Alternatively, external data in the form of proxies for structural shocks of interest, can be used to inform identification. The proposed algorithm can be used to incorporate a number of proxies in order to identify the same number of shocks. For ease of exposition, I will concentrate on one shock of interest.

Assume the structural shock of interest is ε_{1t} and first in the $N \times 1$ vector of structural shocks $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{.t}]$, where $\varepsilon_{.t}$ contains the remaining $N-1$ elements in ε_t. The proxy m_t by name is not a perfect measure of the unobserved structural shock and is related to ε_{1t} as follows

$$m_t = \beta \varepsilon_{1t} + \sigma_v v_t, \;\; v_t \sim \mathcal{N}(0,1) \tag{4}$$

where v_t represents an iid measurement error. The correlation between m_t and ε_{1t} determines the strength of the external information as a proxy for the structural shock of interest and is given by

$$\rho = \frac{\beta^2}{\beta^2 + \sigma_v^2}.$$

Following Mertens and Ravn (2012) and Caldara and Herbst (2019) the correlation measure is applied as a reliability statistic. To be of use the proxy must be correlated with ε_{1t} implying $\beta \neq 0$, orthogonal to other structural shocks in the VAR, $\mathbb{E}(m_t \varepsilon_{.t}) = 0$. In addition, the measurement error must be orthogonal to structural shocks, $\mathbb{E}(v_t \varepsilon_t) = 0$. As mentioned in Caldara and Herbst (2019) the first two conditions are the relevance and exogeneity conditions required of an instrument in an instrumental variables regression but differ as their validity depends on the specification of the model used to generate the unobserved structural shocks, ε_{1t}. It is important to point out that the parameters of equation 4 are linear contrary to the regime-dependent parameters of the MS-VAR in equation 1. The procedure can be adjusted to allow the parameters β and σ_v also to be regime-dependent, implying that the relevance of the proxy can change. However, for the application considered there is little economic motivation for this adjustment. In the case of monetary policy shocks, allowing for independent changes in the relevance of the instrument may be useful when the sample contains a period where the interest rate reaches the zero lower bound and unconventional measures are used.

To examine how the proxy interacts with the MS-VAR in equation 1, it is useful to look at the covariance between the reduced-form residuals and the proxy. Following Mumtaz and Petrova (2019) the covariance matrix is defined by:

$$\begin{pmatrix} u_t \\ m_t \end{pmatrix} \mid Q_{s_t} \sim \mathcal{N}(0, L_{s_t} L_{s_t}'), \;\; L(s_t) = \begin{pmatrix} A_{s_t} q & 0 \\ b & \sigma_v \end{pmatrix} \tag{5}$$

where $\bar{b} = \begin{bmatrix} \beta & ... & 0 \end{bmatrix}$,

$$\begin{pmatrix} v_t \\ m_t \end{pmatrix} = L_{s_t} \begin{pmatrix} \varepsilon_t \\ v_t \end{pmatrix}$$

The matrix $L_{s_t} L_{s_t}'$ can be interpreted as the covariance matrix of the MS-VAR augmented with the proxy and is used when evaluating the likelihood of the model. This relationship can also be used to fill in missing observations of the proxy m_t.

The likelihood of the model can be written to highlight the identification strategy following Caldara and Herbst (2019):

$$p(Y_{1:T}, m_{1:T} \mid s_{1:T}, \mathit{\Sigma}_{1:M}, \Phi_{1:M}, P, \beta, \sigma_v) = \tag{6}$$

$$p(Y_{1:T} \mid s_{1:T}, \varSigma_{1:M}, \Phi_{1:M}, P) p(m_{1:T} \mid Y_{1:T}, s_{1:T}, \varSigma_{1:M}, \Phi_{1:M}, q, \beta, P)$$

where $Y_{1:T} = [y_1, \ldots, y_T]'$ and $\varSigma_{1:M} = \varSigma_1, \ldots, \varSigma_M$. The first term on the right hand side of equation 6 is the likelihood of the VAR data $Y_{1:T}$ and is dependent on reduced-form parameters $\Phi_{1:M}$, $\Sigma_{1:M}$, the sample history of regimes $s_{1:T}$ and the matrix of transition probabilities P. The second term is the conditional likelihood of the proxy $m_{1:T}$ given the VAR data $Y_{1:T}$. The likelihood of the model differs from the BP-SVAR of Caldara and Herbst (2019) as there are now parameters for each regime and requires conditioning on the history of regimes and transition probabilities. This difference is highlighted in the conditional likelihood:

$$p(m_{1:T} \mid Y_{1:T}, s_{1:T}, A_{1:m}, \Phi_{1:m}, q, \beta, P) \sim N(\mu_{s_t}, \sigma_v^2), \qquad (7)$$
$$\mu_{s_t} = \beta q'_{\varepsilon 1} A_{s_t}^{-1} u_t$$

where q_{ε_1} is the first column of q and μ_{s_t} can be interpreted as a linear combination of the orthogonalised residuals $A_{s_t}^{-1} u_t$. Equation 7 reiterates the importance of the parameters of the proxy equation β and σ_v for identifying the coefficients of the MS-SVAR. If $\beta = 0$, m_t is simply noise and provides no information about ε_{1t}, as in Caldara and Herbst (2019).

The key contribution of this paper is that it extends the BP-SVAR of Caldara and Herbst (2019) to a model with regime-switching coefficients. The reduced-form parameters $\Phi_{1:M}$ and $\Sigma_{1:M}$ allow for changes in impact and propagation of shocks identified by the proxy across regimes. The draws of q_{ε_1} ensure that the A_{0s_t} accounts for the conditional likelihood of the proxy m_t. The framework also accommodates missing values in the proxy series.

As mentioned previously and noted in Mumtaz and Petrova (2018), extending the fixed coefficient Bayesian proxy SVAR approach of Caldara and Herbst (2019) to a time-varying model allows the relevance of the proxy to remain constant. This formulation enables the VAR coefficients to solely account for changes in the transmission of shocks. This feature is needed for separating the effects of regimes changes in the data from changes in the relevance of the proxy and occurs as the likelihood of the regime-switching model is incorporated with the measurement equation of the proxy in equation 4. Alternative methods of estimating Bayesian proxy VARs described in Rodgers et al. (2016) and Drautzburg (2016) incorporate the information of the proxy in ways that make this separation relatively more difficult. Specifically, Rodgers et al. (2016) and Drautzburg (2016) link the instrument m_t to the reduced form residuals u_t and use their covariance to back out the implied normalised impulse response vector.

4 Estimation

The proposed model is estimated using Bayesian methods, with a Metropolis-within-Gibbs algorithm. This section presents the main steps of the algorithm and further details on the prior and posterior distribution are provided in the appendix. The procedure builds on the Gibbs sampling algorithm developed by Albert and Chib (1993) and in Kim and Nelson (1999) to estimate MS-VARs.[1] To incorporate the information of proxies for structural shocks, a metropolis step following the Bayesian proxy SVAR procedure developed in Caldara and Herbst (2019) is used.[2]

4.1 Priors and starting values

Following Liu et al.(2018) priors of the MS-VAR(P) coefficients $\Phi_{1:M}$ and the error covariance matrices $\Sigma_{1:M}$ are set using a Minnesota prior implemented via the dummy observations method of Banbura et al. (2007). Therefore, $p(\Phi_{s_t}) \sim N(B_0, H), p(\Sigma_{s_t}) \sim IW(T_0, \sigma)$ for each value of s_t. Caldara and Herbst (2019) also apply the method to set priors for the fixed VAR coefficients of their Bayesian proxy SVAR (BP-SVAR). A training sample of T_0 observations is used to set the scaling factors that determine the prior mean B_0 and variance H for the VAR coefficients Φ_{s_t} using OLS estimates of an AR(1) model for each endogenous variable. It is worth mentioning that the priors are not regime-dependent and allow for the data to distinguish the features of each regime.

The prior for the elements of the transition probability matrix p_{ij} follows a Dirichlet distribution as in Barnett, Groen and Mumtaz (2009), where the mean is set to reflect the beliefs on the duration of

[1] The reader is referred to paper four in the handbook of Blake and Mumtaz (2017) for a detailed and practical exposition to estimating Markov-switching models in a Bayesian framework.

[2] Caldara and Herbst (2019) provide a detailed description of their BP-SVAR framework, the authors have kindly provided code for their procedure to the public.

each regime. Following Caldara and Herbst (2019), the prior for q is uniform and the parameters β and σ_v^2 of equation 4 follow normal and inverse gamma priors.

The starting values of the history of regimes $s_{1:T}$ are obtained by estimating the MS-BP-SVAR with a Maximum Likelihood procedure applying the CSMINWEL optimisation algorithm of Sims (2001).[3] The likelihood of the MS-BP-SVAR is computed using the Hamilton filter described in Kim and Nelson (1999) and incorporates the information of the proxy, through equation 5. The subsequent estimate of $s_{1:T}$ is used to separate the sample into the regimes in which OLS estimates of VAR coefficients are computed to form initial values of $\Phi_{1:M}$ and $\Sigma_{1:M}$. Initial values of transition probabilities are set to 0.95 that imply an average duration of 20 periods for each regime. Initial values of β and σ_v^2 are set to 0.1 and 0.01 respectively.

To avoid the possibility of 'label switching' associated within Bayesian inference of Markov-switching models, I impose normalisation conditions that take the form of inequality restrictions on the mean level of a chosen variable across each regime following with Barnett, Groen and Mumtaz (2009).[4] Specifically, I use rejection sampling to ensure normalisation conditions are imposed.[5]

4.2 Metropolis-within-Gibbs sampling algorithm

The main difference in this algorithm compared to the fixed parameter BP-SVAR algorithm of Caldara and Herbst (2019), is the presence of regime-switching in the VAR coefficients and error residual matrices. However, once the history of regimes $s_{1:T}$ and the transition probability matrix P are sampled, the following steps are almost identical to those developed in Caldara and Herbst (2019). The difference from the conventional MS-VARs estimation algorithms is that the information of the proxy measure now enters the model likelihood function as shown in equation 6.

The following steps are repeated for each Gibbs sampling iteration, indexed by i.

Step 1. Sampling $s_{1:T}$ - History of regimes

Following Kim and Nelson (1999), the history of regimes $s_{1:T}$ is sampled with a multi-move Gibbs sampling draw from the joint conditional density $p(s_t|Y_t, \Phi_{1:M}, \Sigma_{1:M}, \beta, \sigma_v^2, P, m_t)$. This step uses the Hamilton filter to obtain filter probabilities of each regime, therefore the conventional MS-VAR system is augmented with the proxy information as follows [6]

$$\begin{pmatrix} Y_t \\ m_t \end{pmatrix} = \begin{pmatrix} I_N \otimes X_t' \\ 0 \end{pmatrix} \Phi_{s_t} + \begin{pmatrix} u_t \\ m_t \end{pmatrix} \tag{8}$$

where the conditional covariance matrix of the equation 8 is:

$$cov \left(\begin{matrix} u_t \\ m_t \end{matrix} |\Theta \right) = \begin{pmatrix} \Sigma_{s_t} & A_{s_t} q_{\varepsilon_1}' \beta \\ \beta q_{\varepsilon_1} A_{s_t}' & \beta^2 + \sigma_v^2 \end{pmatrix} \tag{9}$$

Step 2. Sampling P - Transition probability matrix

Given the state variable s_t, the transition probabilities have a Dirichlet posterior and are independent of $Y_{1:T}$ and the other parameters of the model.

Step 3. Sampling $\Phi_{1:M}$ and $\Sigma_{1:M}$ - reduced-form regime-dependent VAR coefficient and residual covariance matrices

Conditional on the history of regimes the data Y_t can be split into the subsamples $Y_{s_t=1}, Y_{s_t=2}, \dots, Y_{s_t=M}$ for $t = 1 : T$ and the VAR coefficients and residual matrices of each individual regime can be drawn using a separate Independence Metropolis-Hastings step. Given the history of regimes the model is conditionally

[3] When maximizing the likelihood, transformations to constrain values of p_{ij}, β and σ_v^2 to feasible values are applied following Kim and Nelson (2009). Specifically, $0 < p_{ij} < 1$, $0 < \beta < 1$ and $\sigma_v^2 > 0$.

[4] Normalisation conditions are placed to pin down a label to each regime ensuring for each draw the property of each regime is consistent with its label. Normalisation conditions are imposed for statistical inference, in the absence of these conditions the posterior distribution of parameters could be symmetric with multiple modes.

[5] In the simulation exercise of Section 4.3 this normalisation condition is that the mean of the first variable is lower in regime 1. The mean is calculated using the VAR coefficients, specifically $\tau_{st} = (I - \Phi_{st}^z)^{-1} \Phi_{st}^c$, where τ_{st} represents the regime dependent mean level, Φ_{st}^c represents the elements of the VAR coefficients that represent constant terms and Φ_{st}^zrepresents the remained of the VAR coefficients. However, other types of restrictions can be imposed that place strict inequality restrictions on the relative magnitudes of the selected parameters of the reduced from coefficients $\Phi_{1:M}$ or residual covariance $\Sigma_{1:M}$ or a combination of both.

[6] The Hamilton filter algorithm is described in the appendix.

linear and can be treated as a series of linear VARs. Therefore, the procedure of Caldara and Herbst (2019) can be applied to draw the coefficients of each regime.

Given that normalisation conditions are satisfied, consider this step for the VAR parameters of regime one, Φ_1 and Σ_1.[7] As in Caldara and Herbst (2019) the proposal distribution for Σ_1 is a mixture of a known posterior distribution under the data in regime one, Y_1, and an inverse Wishart distribution with a scaling matrix of the previous draw Σ_1^{i-1}.

- Draw Σ_1^* from $p(\Sigma_1|Y_{s_t=1}, s_{1:T}^i, P^i, \Sigma_1^{i-1}, \Phi_1^{i-1}, q_{\varepsilon_1}^{i-1}, \beta^{i-1}, \sigma_v^{i-1}, m_{s_t=1})$.
- Draw Φ_1^* from $p(\Phi_1|Y_{s_t=1}, s_{1:T}^i, P^i, \Sigma_1^i, q_{\varepsilon_1}^{i-1}, \beta^{i-1}, \sigma_v^{i-1}, m_{s_t=1})$, using the algorithm described in Carter and Kohn (1993) to incorporate the information of proxy series using the model formulation in equations 8 and 9 as conditioning on being in regime one, the system is a linear VAR model. For drawing Φ_{s_t} the data on the proxy m_t is also split into regimes following the same notation.
- With probability α, set Σ_1^i=Σ_1^* and $\Phi_1^i = \Phi_1^*$, otherwise set Σ_1^i=Σ_1^{i-1} and $\Phi_1^i = \Phi_1^{i-1}$. With α defined as

$$\alpha = min\left\{\frac{p(m_{s_t=1}, Y_{s_t=1}, \Sigma_1^*, \Phi_1^*, q_{\varepsilon_1}^{i-1}, \beta^{i-1}, \sigma_v^{i-1})}{p(m_{s_t=1}, Y_{s_t=1}, \Sigma_1^{i-1}, \Phi_1^{i-1}, q_{\varepsilon_1}^{i-1}, \beta^{i-1}, \sigma_v^{i-1})}, 1\right\}$$

Repeat for each regime $s_t = 2, \dots M$.

Step 4. Sample q_{ε_1} using an Independence Metropolis-Hasting step

Following Caldara and Herbst (2019) draw $q_{\varepsilon_1}^*$ from $p(q_{\varepsilon_1}|Y_{1:T}, s_{1:T}, \Sigma_{1:M}^i, \Phi_{1:M}^i, \beta^{i-1}, \sigma_v^{i-1})$.

With probability α, set $q_{\varepsilon_1}^i = q_{\varepsilon_1}^*$, otherwise $q_{\varepsilon_1}^i = q_{\varepsilon_1}^{i-1}$. With α defined as

$$\alpha = min\left\{\frac{p(m_{1:T}|Y_{1:T}, s_{1:T}, \Sigma_{1:M}^i, \Phi_{1:M}^i, q_{\varepsilon_1}^*, \beta^{i-1}, \sigma_v^{i-1})}{p(m_{1:T}|Y_{1:T}, s_{1:T}, \Sigma_{1:M}^i, \Phi_{1:M}^i, q_{\varepsilon_1}^{i-1}, \beta^{i-1}, \sigma_v^{i-1})}, 1\right\}$$

Step 5. Sample β and σ_v - Proxy equation parameters

To draw these parameters the structural shock of interest ε_{1t} can be calculated as $\varepsilon_{1t} = A_{s_t} q_{\varepsilon_1} u_t$. Then draw$\beta^i$ from the known normal conditional posterior distribution associated with β. Respectively, draw σ_v^i from the known inverse gamma conditional posterior distribution associated with σ_v.

4.3 Simulation evidence

The estimation procedure is tested by attempting to recover impulse responses from data generated from a Markov-switching VAR with two regimes. An artificial proxy to a structural shock is generated using the relation to VAR residuals described in equation 5.

The following DGP is used to generate artificial data:

$$Y_t = \Phi_{s_t} X_t + u_t, \ u_t = \Sigma_{s_t} \varepsilon_t, \ \varepsilon_t \sim \mathcal{N}(0, I_N), s_t \in \{1, 2\} \tag{10}$$

Where Y_t is 3×1, $q'q = I_3$. The coefficient of the VAR Φ_{s_t} and residual covariance matrices Σ_{s_t} are set using values of an estimated MS-VAR as in equation 12 on U.S. data consisting of CPI inflation, real GDP growth and the Federal funds rates from 1965-2007. The regimes are defined by the magnitude of the implied mean of the first variable, where regime 1 has a lower implied mean and the diagonal elements of the covariance matrix of the second regime are larger in magnitude.

The history of regimes $s_{1:T}$ follows a two-state first-order Markov process. The law of motion for s_t is

$$s_t = P s_{t-1}$$

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix}$$

where $p_{11} = 0.98$ and $p_{22} = 0.95$ implying regime one is more persistent with an average duration of 50 periods.

[7] I draw the matrices $\Phi_{1:M}^*$ $\Sigma_{1:M}^*$ and employ rejection sampling to impose normalisation conditions before computing acceptance probabilities of each regime.

The history of states is generated by a two-state first-order Markov-chain given the values implied by the transition probabilities.[8]

The proxy m_t is generated via:

$$m_t = \beta\varepsilon_{1t} + \sigma_v v_t,\ v_t \sim \mathcal{N}(0,1) \tag{11}$$

where ε_{1t} is the structural shock of interest and $\beta = 0.2$ and $\sigma_v = 0.1^{1/2}$. The covariance matrix and the proxy m_t are set according to the relationship between the proxy and the data, where Ξ represents all model parameters $\Sigma_1, \Sigma_2, B_1, B_2, P, \beta, \sigma_v$ and states

$$cov\begin{pmatrix} u_t & \\ m_t & \end{pmatrix} | \, \Xi \Bigg) = \begin{pmatrix} A_{s_t}q(A_{s_t}q)' & A_{s_t}q'_{\varepsilon_1}\beta \\ \beta q_{\varepsilon_1}A'_{s_t} & \beta^2+\sigma_v^2 \end{pmatrix}.$$

Following Mumtaz and Petrova (2018), I generate 320 observations and discard the first 100 as an initialisation period. A training sample of 20 periods is then used to inform the prior distributions of $\Sigma_1, \Sigma_2, B_1, B_2$, which leaves 200 observations for estimation. The simulation experiment is repeated 500 times with the history of regimes and parameters kept constant for every simulation.

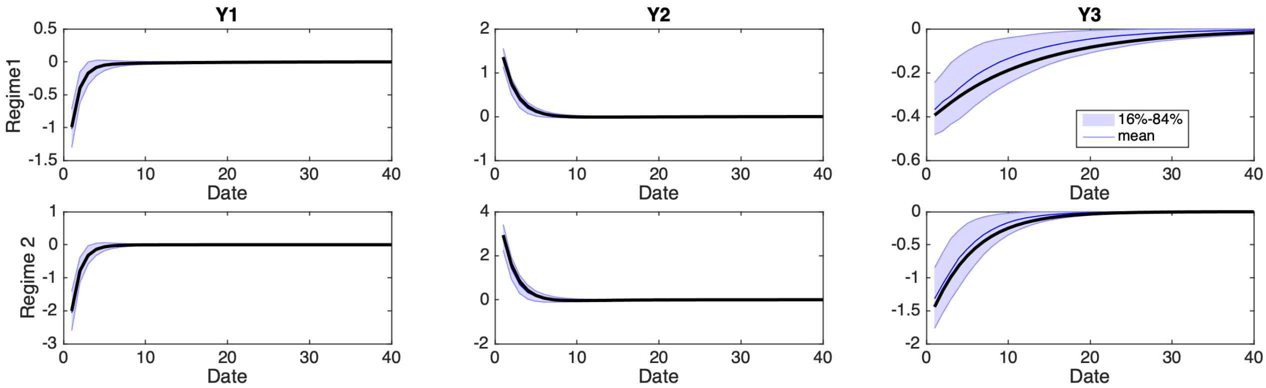
The estimation of the Markov-switching proxy VAR for each simulated data set uses 5000 iterations of the Metropolis-within-Gibbs algorithm. The last 2000 iterations are kept to approximate the posterior of the model parameters and impulse responses.

Figure 1 presents the true and estimated impulses responses to the structural shock of interest. The true values are always within the one-standard deviation error bands of the estimated responses; this implies that the algorithm is able to pick up the changes in the propagation and shifts in the impact of the structural shock of interest. In addition, the procedure accurately picks up the timing of shifts in the DGP as displayed by the true history of regimes and the estimated filter probabilities in figure 2. [9]

[8] paper4 of the handbook of Blake and Mumtaz (2017) gives practical examples of generating data from Markov-switching models.

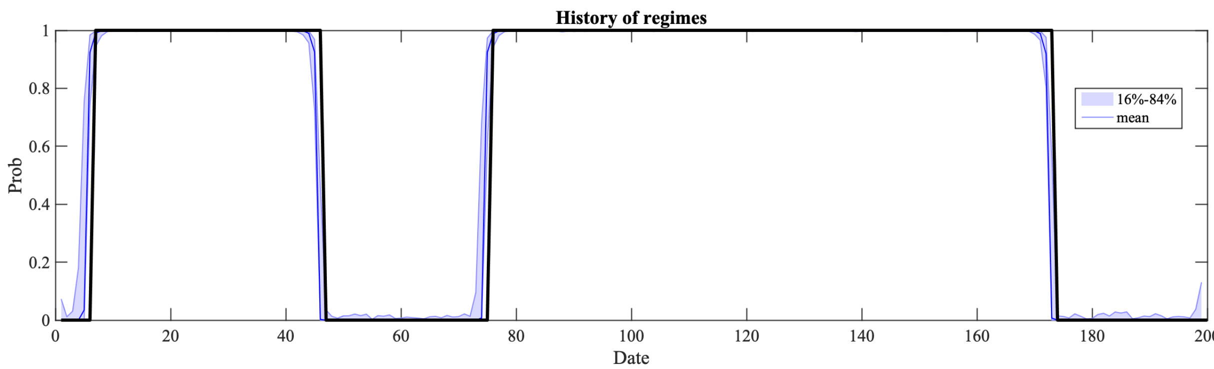
[9] Actual impulse responses normalised to represent a one-unit increase of the variable related to the structural shock of interest are also closely tracked for each regime and presented in the appendix. To assess the robustness of the algorithm, a number of similar simulation exercises have been conducted to include increases in the lag order to four, alternative DGPs and switching solely in the residual covariance matrix. The algorithm is able to recover the impulse responses of the data generating process for these alternative models and results are available on request.

Figure 1: Impulse responses to a one-standard deviation negative shock to the first equation



Note: The blue line and shaded area represent the median and one-standard deviation error bands, while the thick black line shows the true impulse responses.

Figure 2: Actual and estimated history of regimes



Note: The blue line and shaded area represent the median and one-standard deviation error band of the filter probabilities of regime 1 while the thick black line shows the true history of regimes.

5 Monetary policy, real activity and credit spreads in a regime-switching model

To display the empirical relevance of the MS-BP-SVAR, this section extends the work of Caldara and Herbst (2019) to incorporate the possibility of regime shifts in the economy.

Caldara and Herbst (2019) examine the transmission of monetary policy shocks to real activity in the U.S. and use a series of monetary policy surprises associated with FOMC announcements to proxy for monetary policy shocks. The proxy is constructed using high-frequency financial data on the price of federal funds futures contracts. Their sample is from 1994-2007 and characterises the Great Moderation period associated with a historical reduction in the volatility of key macroeconomic indicators such as real activity and inflation. In addition, 1994 is the year that the FOMC started releasing statements immediately after each meeting.[10]

They find a persistent decline in real activity when including credit spreads as a variable in their Bayesian proxy SVAR. Revisiting this question with a proxy VAR that allows for regime-switching may be of interest as a number studies that estimate MS-SVARs discover evidence of discrete time-variation during this period. Hubrich and Tetlow (2015) provide evidence of regime change in the coefficients and volatility of VARs estimated on U.S. data during this period. Sims and Zha (2006) and Nason and Tallman (2015) find evidence of switching across volatility regimes.

5.1 Empirical model, data and priors

The model considered given the data and sample size is a two-regime Markov-switching proxy VAR(1) which takes the following form:

$$Y_t = \Phi_{s_t} X_t + u_t,\ u_t = \Sigma_{s_t}\varepsilon_t,\ \varepsilon_t \sim \mathcal{N}(0, I_N), s_t \in \{1,2\} \tag{12}$$

where X_t contains one and an intercept $X_t = [Y_{t-1}, 1]'$. The elements of coefficients are allowed to alternate simultaneously between two sets of values governed by the latent variable s_t, specifically

$$\begin{matrix} Regime\,1 \\ Regime\,2 \end{matrix} \left\{ \begin{matrix} s_t = 1 & \quad \Phi_1, \Sigma_1 \\ s_t = 2 & \quad \Phi_2, \Sigma_2. \end{matrix} \right.$$

The series of FOMC monetary policy surprises constructed in Caldara and Herbst (2019) are denoted by m_t and employed to proxy monetary policy shocks.[11] The coefficients β and σ that determine the signal and noise of the proxy series of monetary policy surprises are kept constant for reasons highlighted in section 3.2.

$$m_t = \beta\varepsilon_{MP,t} + \sigma_v v_t, v_t \sim N(0,1)\, and\, v_t \perp \varepsilon_t \tag{13}$$

$$Model\ 1 : Y_t = \{FFR_t, \triangle IP_t, U_t, \pi_{PPI,t}\}, m_t = \{FOMC\,Suprises\}$$
$$Model\ 2 : Y_t = \{FFR_t, \triangle IP_t, U_t, \pi_{PPI,t}, Baa\,Spread_t\}, m_t = \{FOMC\,Suprises\}$$

Following Caldara and Herbst (2019), to highlight the impact of credit spreads, two models are considered that differ in the set of endogenous variables included. The first model is a 4-equation MS-BP-SVAR that consists of the federal funds rate (FFR_t); the yearly growth rate of manufacturing industrial production ($\triangle IP_t$); the unemployment rate (U_t) and the annual rate of price inflation calculated using the producer price index for finished goods ($\pi_{PPI,t}$). The second model is a 5-equation MS-BP-SVAR that includes a measure of credit spreads given by the difference of the Moody's seasoned Baa corporate bond yield relative to the yield on 10-year Treasury bonds with constant maturity ($Baa\,Spread_t$). The dataset is that of Caldara and Herbst (2019) and is of a monthly frequency; the estimation sample is from January 1994 to June 2007. Following Caldara and Herbst (2019) a training sample from January 1990 to December 1993 is used to set priors for the VAR coefficients Φ_1, Φ_2 and residual error covariance matrix Σ_1, Σ_2. Prior beliefs are identical across regime. The priors on the proxy equation are set loosely, specifically: $p(\beta) \sim N(0,1), p(\sigma_v^2) \sim IG^*(0.02, 1)$, where IG^* is an inverse gamma density,

[10] Before 1994, changes in the target interest rate had to be inferred by the size and type of open market operation. As mentioned in Caldara and Herbst (2019) the introduction of after meeting announcements may have altered the transmission of policy surprises as a result of increased transparency.

[11] The reader is referred to Caldara and Herbst (2019) for a detailed description of the construction of the FOMC monetary surprises.

reparameterised in terms of the mean 0.05 and variance 1.[12] As mentioned in Caldara and Herbst (2019), the prior on the variance of the measurement error σ_v^2 is important in determining the usefulness of the proxy and for this reason they experiment with two types of prior beliefs: a non-informative and a high-relevance prior. The high-relevance prior allows for adjustments in the relevance of the proxy. However, this paper imposes a non-informative prior, letting the data decide this value.[13] The prior for the elements of the transition probability matrix p_{ij} follows a Dirichlet distribution with a prior mean implying expected duration of regimes is 20 periods as in Barnett, Groen and Mumtaz (2009).

Asides from allowing for the possibility of regime changes, the specification departs from that of Caldara and Herbst (2019) in two ways. Firstly, yearly differences of industrial production and CPI are used as opposed to log levels due to the relative difficulty of finding initial values for the history of regimes in the latter case that lead to convergence in the algorithm. Secondly, due to the short sample size, I consider specifications with one lag following Holm-Hadulla and Hubrich (2017).[14] The main results of Caldara and Herbst (2019) are maintained when the fixed coefficient model is estimated using log differences and the choice of one lag and are used as as a comparison against the regime-switching models and also presented separately in the appendix.

Each model is estimated using 100,000 replications with a burn-in period of 95000 following Barnett, Groen and Mumtaz (2009), in addition, every second draw after this period is kept to leave 2500 draws to approximate posterior distributions. To avoid the problem of label-switching I impose the normalisation condition that the mean level of inflation implied by the matrices Φ_1, Φ_2 is greater in the second regime.[15]

5.2 Empirical results

This subsection presents results from extending the study of Caldara and Herbst (2019) to allow for shifts in the U.S. economy. Firstly, I present the results of a model comparison exercise, followed by a description of the timings and nature of the regimes estimated in the four and five equation MS-BP-SVAR models. Subsequently, I analyse the impact of monetary policy shocks across regimes and models. Lastly, this subsection discusses the estimated structural elasticities linked to of the systematic component of monetary policy.

5.2.1 Model comparison

Before examining the results of the MS-BP-VARs, I compare their model-fit of the data with the fixed coefficient BP-VAR. Taking into account the difficulty in the accurate computation of the marginal likelihood for the class of models estimated, the deviance information criterion (DIC) introduced by Spiegelhalter et al. (2002), is used for model comparison. The DIC is a generalisation of the Akaike information criterion, penalising model complexity and emphasising model fit to the data. The DIC is defined as

$$DIC = \bar{D} + p_D$$

where $\bar{D}$ measures model fit and is referred to as the deviance, it is the average of the log likelihood after the evaluated for each MCMC draw and is given as

$$\bar{D} = \frac{1}{N}\sum_{i=1}^{N}(-2\ln(Y_t|\theta^i))$$

$$p_D = \bar{D} - (-2\ln(Y_t|\bar{\theta}))$$

p_D is the effective number of parameters and is defined as the deviance subtracted by the log-likelihood evaluated at the posterior median.

The results of the model comparison exercise are displayed in Table 1 and indicate that the regime switching model improves data fit. This result provides evidence of regime change in the coefficients and volatility of VARs estimated on U.S. data during this period and supports the findings of Liu et al. (2018) and Hubrich and Tetlow (2015).

[12] The priors of the measurement equation are set close to values of Caldara and Herbst (2019) who set $p(\beta) \sim N(0,1), p(\sigma_v^2) \sim IG^*(0.02, 2)$

[13] The high relevance prior of Caldara and Herbst (2019) differs by placing the dogmatic view that only half of the variation in their proxy can be attributed to measurement error. I.e. $\sigma_v = 0.5 \times std(M_{1:T})$ with probability 1.

[14] Holm-Hadulla and Hubrich (2017) estimate a MS-VAR (1) using monthly euro area data from January 2005 to December 2015.

[15] The mean is calculated using the VAR coefficients, specifically $\tau_{st} = (I - \Phi_{st}^z)^{-1}\Phi_{st}^c$, where τ_{st} represents the regime

Table 1: Deviance Information Criterion for each estimated model

	DIC	
	MS-BP-VAR	Fixed Coefficient BP-VAR
4-equation - Without credit spreads	**1185**	1561
5-equation - With credit spreads	**1704**	2059

Note: This table presents the Deviance Information Criterion (DIC) in order to find the best model that describes the US data. The model preferred lowest DIC.

5.2.2 Estimated regimes

Figure 3 and 4 presents the filter probabilities of being in the second regime alongside periods of regime uncertainty in both 4-equation and 5-equation versions of the model. For each version of the model, the sample appears to be roughly split across the two regimes. Regime 1 appears to be in place during the mid to late 1990s and is followed by regime 2 for the remainder of the sample. The sample split supports the narrative described in Liu et al. (2018) and Benati and Goodhart (2010), who detect important changes in the U.S. economy towards the beginning of the 2000s. The shaded areas indicate periods where there is disagreement between how the draws separate the sample before drawing the parameters of each regime.[16] There is uncertainty over when the sample split occurs within the 4-equation model that indicates a later date of 2001 whereas, the 5-equation model indicates 1999. However both breaks are consistent with the findings of Benati and Goodhart (2010) who detect important changes in the response of monetary policy to the 9/11 terrorist attack and the Nasdaq/tech bubble burst in the mid-2000s. An interesting observation is that the inclusion of credit spreads reduces the uncertainty around regime changes.

Table 2 presents the model implied means of the endogenous variables during each regime. The coefficient matrix of regime 1, Φ_1, in both models implies larger mean levels of interest rates, and industrial production, and lower levels of inflation relative to regime 2 in both models. The 5-equation model also associates a lower mean level of credit spreads to regime 1 and a mean level of inflation that is near zero. Moreover, the regimes are further defined by differences in the diagonal elements of the residual covariance matrices Σ_1, Σ_2 across each regime, and are displayed in Table 3. Regime 1, implies a lower variance in the residuals of inflation in both models, with the 5-equation model also suggesting a lower variance in the residuals of the credit spread equation in regime 1. The findings of a higher variance in the residuals of the inflation equation during the 2000s are consistent with the results of Liu et al. (2018).

The estimates of ρ measure the strength of the relationship between the proxy and the identified monetary policy shock. Table 4 displays the estimates of the relevance statistic ρ in equation 4 of the MS-BP-SVAR models alongside the estimates of the fixed coefficient models. For the two MS-BP-SVARs the estimates of ρ are consistent with the fixed coefficient-model and with those reported in Caldara and Herbst (2019), centering around 0.1 under an uninformative prior.[17, 18]

5.2.3 The effect of monetary policy shocks

Before comparing the impulse responses to monetary policy shocks of the regime-switching models it is worth briefly discussing the responses of the fixed coefficient model which are displayed in Figure 5. Unless stated, I discuss the dynamics implied by the median responses. The impulse responses of the fixed coefficient models are consistent with the finding in Caldara and Herbst (2019), despite the use of a lower lag order and log differences of industrial production and inflation. The first row of Figure 5 displays the responses to a monetary policy shock that increases the federal funds rate (FFR) by 1% in the model without credit spreads.[19] The federal funds rate slowly falls, returning to zero after 20 months. There is little evidence of the contractionary effects on real activity usually associated with increases in

dependent mean level, Φ^c_{st} represents the elements of the VAR coefficients that represent constant terms and Φ^z_{st}represents the remained of the VAR coefficients.

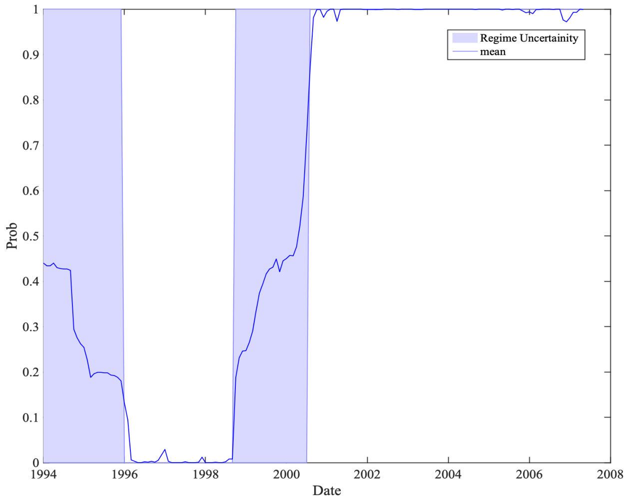
[16]This step refers to step 3 in algorithm 4.2.

[17]This specification differs with Caldara and Herbst (2019) in considering log differences for Industrial Production and inflation in addition, to a lower lag order of 1.

[18]Subsample estimates of the fixed coefficient model around 1994-1999 and 2000-2007 indicate no change in the relevance of the monetary policy surprises and support the modeling assumption in the MS-BP-VAR of fixed relevance.

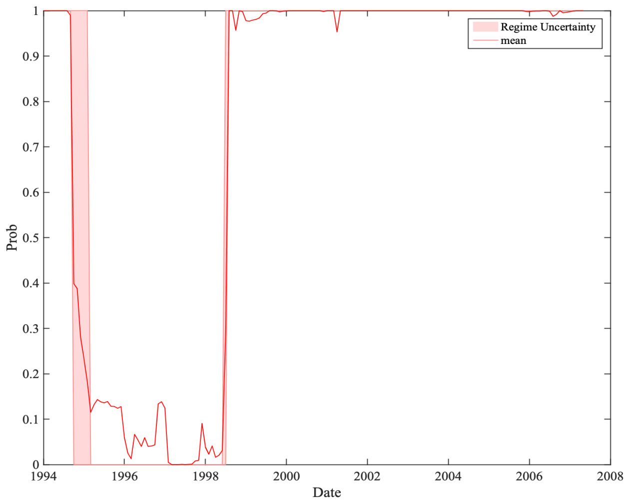
[19]For comparison across the number of models and regimes a 1% increase is the FFR is considered. One-standard deviations shocks are presented on in the appendix.

Figure 3: Filter probabilities of regime 2 and periods of regime uncertainty in model without credit spreads



Note: Figure 3 displays the filter probabilities of regime 2 in the 4 equation MS Bayesian proxy SVAR, respectively, the blue line represents the median filter probability and the shaded interval represents periods of uncertainty in regime determination. Specifically, the shaded error band indicates periods where there is disagreement between how to separate the sample before drawing the parameters of each regime.

Figure 4: Filter probabilities of regime 2 and periods of regime uncertainty in model with credit spreads



Note: Figure 4 displays the filter probabilities of regime 2 in the 5 equation MS Bayesian proxy SVAR, respectively, the red line represents the median filter probability and the shaded interval represents periods of uncertainty in regime determination. Specifically, the shaded error band indicates periods where there is disagreement between how to separate the sample before drawing the parameters of each regime.

Table 2: Implied means by the posterior median estimates of VAR coefficients

	4 equation - without credit spreads			5 equation - with credit spreads		
	Regime 1	Regime 2	Fixed-coefficient	Regime 1	Regime 2	Fixed-coefficient
$\overline{FFR}$	5.59 [5.29,6.04]	3.34 [2.19,4.52]	3.88 [2.6,4.82]	5.56 [5.38,5.75]	3.95 [3.13,4.87]	4.08 [3.35,4.72]
$\overline{\triangle IP}$	4.98 [2.71,6.58]	1.91 [1.29,2.68]	2.69 [1.22,3.75]	5.93 3.76.,8.1]	2.26 [1.41,3.05]	2.82 [1.82,3.65]
$\overline{U}$	4.87 [4.14,5.46]	5.13 [4.77,5.47]	5.03 [4,73,5.32]	4.97 [4.01,6]	4.94 [4.65,5.2]	5.01 [4.76,5.24]
$\overline{\pi_{PPI,t}}$	2.47 [0.44,5.43]	2.64 [1.79,3.36]	2.25 [1.35,3.45]	0.52 [0.2,2.78]	2.39 [1.72,3.01]	2.11 [1.39,2.91]
$\overline{Baa\,Spread}$	-	-		1.68 [1.57,1.83]	2.21 [1.92,2.45]	2.16 [1.95,2.39]

Note: Table 2 presents posterior median estimates of the coefficient implied mean corresponding to each variable from the 4 and 5 equation MS Bayesian proxy VARs and fixed-coefficient Bayesian proxy VARs. The mean is calculated using the VAR coefficients, specifically $\tau_{st} = (I - \Phi^z_{st})^{-1}\Phi^c_{st}$, where τ_{st} represents the regime dependent mean level, Φ^c_{st} represents the elements of the VAR coefficients that represent constant terms and Φ^z_{st}represents the remained of the VAR coefficients. The 16$^{\text{th}}$ and 84$^{\text{th}}$ percentiles are in brackets.

Table 3: Variances of VAR residuals

	4 equation - without credit spreads			5 equation - with credit spreads		
	Regime 1	Regime 2	Fixed-coefficient	Regime 1	Regime 2	Fixed-coefficient
FFR	0.06	0.04	0.05	0.06	0.064	0.04
$\triangle IP$	0.64	0.66	0.65	0.56	0.64	0.64
U	0.02	0.01	0.02	0.03	0.01	0.02
π_{PPI}	0.19	0.79	0.59	0.16	0.71	0.60
$Baa\,Spread$	-	-	-	0.002	0.02	0.01

Note: Table 3 presents posterior median estimates of the diagonal elements of the residual covariance matrix Σ_{s_t} corresponding to each variable from the 4 and 5 equation MS Bayesian proxy VARs and fixed-coefficient Bayesian proxy VARs.

Table 4: Relevance statistics

	MS-BP-SVAR		Fixed-Coefficient	
	4 equation	5 equation	4 equation	5 equation
$\rho = \frac{\beta^2}{\beta^2+\sigma^2}$	0.08 [0.02,0.18]	0.12 [0.04,0.23]	0.09 [0.03,0.17]	0.10 [0.03,0.2]

Note: Table 4 presents posterior median estimates of the relevance statistics with 90 percent intervals in brackets from the 4 and 5 equation MS Bayesian proxy VARs and fixed-coefficient Bayesian proxy VARs.

short-term interest rates. Industrial production falls 50 basis points (bps) and unemployment surprisingly decreases by 10bps, however, it is worth noting that the credible sets indicate no significant effect.

The second row of Figure 5 displays the responses to a monetary policy shock that increases the FFR by 1% in the model with credit spreads. The dynamics are considerably different when credit spreads are included, supporting the findings of Caldara and Herbst (2019). The FFR decreases relatively sharply after the shock and then becomes negative after approximately 8 months. This change in interest rates can be explained by the real and financial effects of the shocks. The effect on real activity is relatively stronger in the first 20 months after the shock. Industrial production falls by around 70 bps on impact but then decreases to around 150 bps 10 months after the shock. The unemployment rate is almost zero on impact and then gradually increases by 20bps around 20 months after the shock. Inflation responses are negative initially but return to zero after the 20 months in both models. In addition, the monetary policy shock causes a long-lasting tightening in financial conditions with the Baa spread increasing by 25bps and remaining above zero for more than two years. The main findings of the fixed-coefficient model are that credit spreads considerably change the transmission of the monetary policy and are in line with the findings of Caldara and Herbst (2019).

The MS-BP-SVAR is well-suited to investigating changes in the impact and propagation of monetary policy shocks implied by the regime-switching coefficients Φ_{s_t} and Σ_{s_t}. Figures 6 and 5.2.3 plot the regime-dependent impulse response functions to a one-percentage point contractionary monetary policy shock alongside those implied by the fixed coefficient models both with and without credit spreads.

Figure 6 plots the responses of the MS-BP-VARs in regime 1 with the first row representing the model without credit spreads. The MS-BP-VARs roughly suggest that regime 1 was in place during 1994-2000 and has considerably different dynamics than the fixed-coefficient model both with and without credit spreads. The FFR returns quickly to zero after 10 months indicating relatively short-term effects of the shock. There is evidence of an initial significant decline in industrial production during regime 1, with a negative growth rate of IP at -170 bp and lasting for 18 months and differs from the considerably smaller effect in the fixed-coefficient model without credit spreads. The median response in regime 1 indicates an increase in the unemployment rate, but this is not significant as the 68% credible intervals include both positive and negative values. The unemployment rate slightly increases on impact around 10 bps. In regime 1, there is a surprising increase of inflation although, this is low in magnitude, around 20bps.

The second row of Figure 6 shows that adding credit spreads does not considerably alter the transmission of monetary policy shocks in regime 1. The FFR falls to zero slightly quicker at around 8 months. The decrease in the growth rate of industrial production is amplified on impact to 250 bps and is sharper in the first 3 months after the shock. In addition, credit spreads increase by 20 bps on impact and this dissipates entirely after 10 months. The unemployment rate has a short-lived increase of 20bps on impact and this is slightly larger than the model without credit spreads. Inflation is marginally negative on impact but is then followed by a surprising increase of inflation. A interesting result is that credit spreads have a relatively short-lived effect and appear to only change the responses of industrial production.

Figure 5.2.3 plots the responses of the MS-BP-VARs in regime 2 with the rows representing the model without/with credit spreads respectively. Regime 2 is present from 2000-2007 and in this regime the responses are quite similar to the fixed-coefficient model which are represented by the dashed line. The change in dynamics when including credit spreads is consistent with the main finding of Caldara and Herbst (2019) and can be similarly explained by examining the real and financial implications of the monetary policy shock in this regime.The responses in the first row without credit spreads show little evidence of a contractionary effect on real activity from a shock that increases interest rates. When including credit spreads the FFR falls relatively quicker in regime 2 and becomes negative, implying a more accommodative monetary policy stance relative to the initial level after 10 months. The effect on real activity is more persistent when including credit spreads in regime 2, with evidence of statistically significant decreases in industrial production after ten months and increases in the unemployment rate after 20 months. The effect on prices is hump-shaped and suggests a significant larger and longer-term decrease in PPI inflation between month 4 until month 26. The shock also causes the Baa spread to increase by 30 bp on impact and significantly persists for 30 months indicating a long-lasting tightening in financial conditions.

Overall, when including credit spreads in regime 2 monetary policy shocks have a significant effect on real activity and the remaining variables in the system, this result is almost entirely consistent with the findings of Caldara and Herbst (2019). The results that a tightening of credit spreads amplifies the reduction of real activity growth from a contractionary monetary policy shocks is consistent with the finding of Balke (2000). However, in regime 1, which is suggested to be place during the mid to late 1990s, the inclusion of the Baa spreads has only a short-lived amplification of the response of industrial production growth to monetary policy shock and no other significant changes to the rest of the economy.

5.2.4 The systematic component of monetary policy

The identification of monetary policy shocks is equivalent to the identification of the systematic component of monetary policy. Following Caldara and Herbst (2019), the estimates of the VAR coefficient matrices $\Phi_1, \Phi_2, \Sigma_1, \Sigma_2$ and $q_{\varepsilon_{MP}}$ can be used to uncover the elasticities of the policy instrument rate at time t to contemporaneous and lagged movements in the model's endogenous variables. The federal funds rates equation of the structural MS-VAR takes the following representation:

$$A_{0,1,s_t} FFR_t = A_{+,1,s_t} X_t + \varepsilon_{MP}$$

where $A_{0,1,s_t}$ and $A_{+,1,s_t}$ are the first row of the structural matrices A_0 and A_+, that are related to the reduced-form parameters of equation 12 through

$$\Sigma_{s_t} = (A_{s_t} q (A_{s_t} q)') = A_{0,s_t} A'_{0,s_t}$$

$$\Phi_{s_t} = A^{-1}_{0,s_t} A_{+,s_t}.$$

Therefore, the federal funds rate in equation 12 can be rewritten as

$$r_t = \sum_{j=2}^{N} y^{'}_{j,t} \psi_{0,j,s_t} + \sum_{l=1}^{P} y^{'}_{t-l} \psi_{l,s_t} + \sigma_{MP,s_t} \varepsilon_{MP,t} \; s_t = \{1, 2\} \tag{14}$$

where $\psi_{0,j,s_t} = -a_{0,1,j,s_t}/a_{0,11,s}, \psi_l = a_{l,1j,s_t}/a_{0,11,s_t}$ and $\sigma_{MP,s_t} = 1/a_{0,11,s_t}$, with a_{l,ij,s_t} denoting the $ijth$ element of A_{l,s_t}. The first two terms of equation 14 describe the systematic component of monetary policy and ψ_{l,j,s_t} represents the elasticity of the federal funds rate to variable j at lag l in regime s_t. Tables 5 and 6 respectively, report the contemporaneous and cumulative elasticities of the federal funds rate to the remaining endogenous variables across 4- and 5-equation models and regimes.

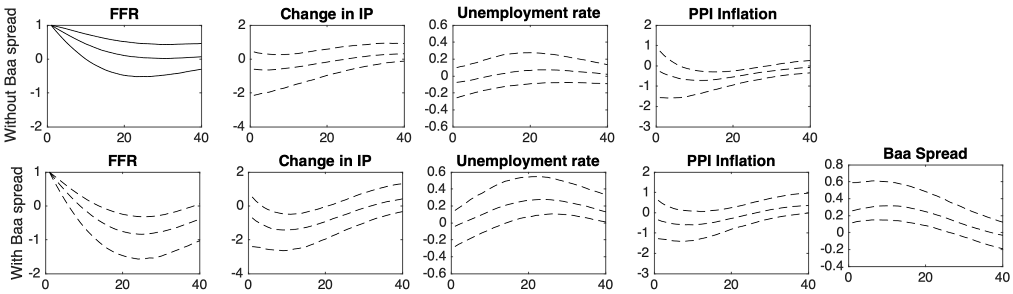
Overall, the uncertainty around the estimates is large, and the majority of 90 percent intervals contain zero.[20] In the 4-equation MS Bayesian proxy SVAR, as shown in Table 5 the median estimates of the contemporaneous elasticity of industrial production across both regimes are positive; this is in line with economic intuition. The contemporaneous estimates of the elasticities of the unemployment rates are also positive across each regime, but the respective elasticities of inflation are economically insignificant. The median cumulative elasticities of the 4-equation model show in table 6 are all close to zero and similar across both regimes except for the FFR in regime 1 with an estimate of 0.79 that indicates a lower level of persistence in the interest rate than in Caldara and Herbst (2019). The relatively lower estimates of the cumulative responses than Caldara and Herbst (2019) are related to only allowing for one lag in the MS Bayesian proxy SVAR specifications. The inclusion of the Baa credit spread leads to positive median estimates of contemporaneous elasticities of inflation across both regimes. However, the magnitude decreases from 0.2 to 0.02 when comparing the estimates of regime 1 and 2 within this model, respectively. The median estimates of the contemporaneous elasticities to credit spreads are negative in both regimes with values of -3.3 in regime 1 and -0.85 in regime 2. The cumulative responses to credit spreads are also negative and move between -0.82 in regime 1 to -0.19 in regime 2. The main difference with cumulative responses when credit spreads are included is that the responses of unemployment become more negative and move between -0.35 in regime 1 to -0.13 in regime 2.

6 Conclusion

This paper proposes a Markov-switching Bayesian proxy SVAR model that can be applied to examine changes in the transmission of structural shocks in the presence of economic regime shifts. I provide a Metropolis-within-Gibbs sampling algorithm to approximate the posterior distribution of parameters. The results of simulation exercises suggest that the estimation procedure is capable of retrieving changes

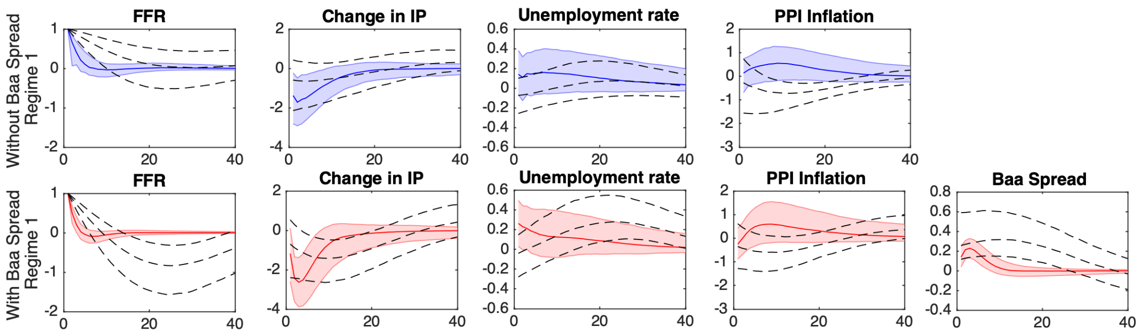
[20] Caldara and Herbst (2019) also find large uncertainty around the estimated elasticities.

Figure 5: Impulse responses to one-percent increase in the Federal Funds rate - Fixed-coefficient proxy VAR



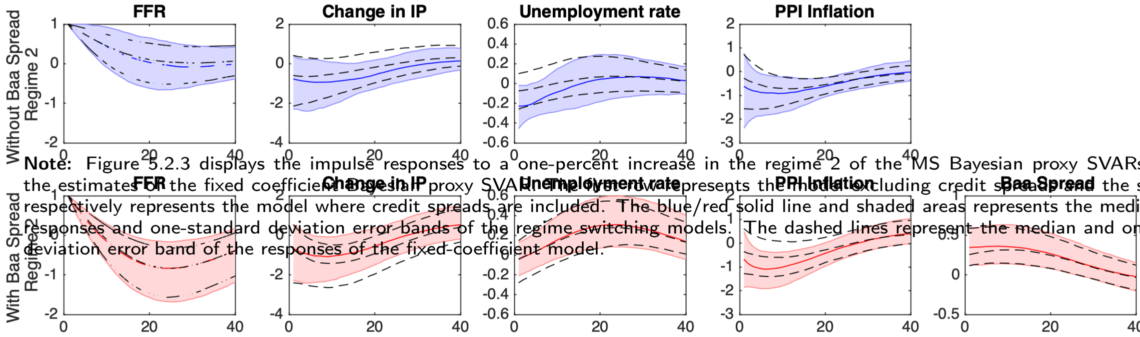
Note: Figure 5 displays the impulse responses to a one-percent increase in the fixed-coefficient Bayesian proxy SVARs. The first row represents the model excluding credit spreads and the second row respectively represents the model where credit spreads are included. The dashed lines represent the median and one-standard deviation error band of the responses of the fixed-coefficient model.

Figure 6: Impulse responses to one-percent increase in the Federal Funds rate - MS-BP-VARs regime 1



Note: Figure 6 displays the impulse responses to a one-percent increase in the regime 1 of the MS Bayesian proxy SVARs alongside the estimates of the fixed coefficient Bayesian proxy SVAR. The first row represents the model excluding credit spreads and the second row respectively represents the model where credit spreads are included. The blue/red solid line and shaded areas represents the median impulse responses and one-standard deviation error bands of the regime switching models. The dashed lines represent the median and one-standard deviation error band of the responses of the fixed-coefficient model.

Figure 7: Impulse responses to one-percent increase in the Federal Funds rate - MS-BP-VARs regime 2



Note: Figure 5.2.3 displays the impulse responses to a one-percent increase in the regime 2 of the MS Bayesian proxy SVARs alongside the estimates of the fixed coefficient Bayesian proxy SVAR. The first row represents the model excluding credit spreads and the second row respectively represents the model where credit spreads are included. The blue/red solid line and shaded areas represents the median impulse responses and one-standard deviation error bands of the regime switching models. The dashed lines represent the median and one-standard deviation error band of the responses of the fixed-coefficient model.

Table 5: Contemporaneous elasticities $\psi_{0,j}$

	4 equation - without credit spreads			5 equation - with credit spreads		
j	Regime 1	Regime 2	Fixed-coefficient	Regime 1	Regime 2	Fixed-coefficient
$\triangle IP$	0.12 $[-0.05,0.37]$	0.10 $[-0.02,0.25]$	0.09 $[-0.03,0.31]$	0.10 $[-0.23,0.73]$	0.05 $[-0.10,0.25]$	0.08 $[-0.1,0.27]$
U	0.43 $[-0.36,1.26]$	0.46 $[-0.70,1.46]$	0.44 $[-0.36,1.67]$	-0.05 $[-2.03,1.33]$	0.12 $[-0.72,0.98]$	0.32 $[-0.69,1.60]$
π_{PPI}	-0.003 $[-0.36,0.42]$	-0.001 $[-0.13,0.16]$	0.02 $[-0.11,0.20]$	0.2 $[-0.38,1.49]$	0.02 $[-0.10,0.16]$	0.03 $[-0.10,0.20]$
$Baa\,Spread$	-	-	-	-3.299 $[-5.90,1.15]$	-0.85 $[-2.24,0.05]$	-0.79 $[-2.90,0.02]$

Note: Table 5 presents posterior median estimates of the contemporaneous elasticities of the federal funds rate with 90 percent intervals in brackets. This table compares the estimates of the 4 and 5 equation MS proxy BayesianVAR alongside the 4 and 5 equation fixed-coefficient Bayesian proxy SVAR.

Table 6: Cumulative elasticities $\psi_{,j}$

	4 equation - without credit spreads			5 equation - with credit spreads		
j	Regime 1	Regime 2	Fixed-coefficient	Regime 1	Regime 2	Fixed-coefficient
$\triangle IP$	0.07 $[0.004,0.18]$	0.06 $[0.02,0.09]$	0.02 $[0.03,0.6]$	0.03 $[-0.24,0.33]$	0.04 $[0.01,0.07]$	0.03 $[-0.001,0.05]$
U	-0.02 $[-0.71,0.3]$	-0.05 $[-0.31,0.24]$	-0.02 $[-0.11,0.06]$	-0.35 $[-2.37,0.45]$	-0.13 $[-0.29,0.01]$	-0.09 $[-0.20,-0.01]$
π_{PPI}	0.05 $[-0.07,0.21]$	-0.01 $[-0.05,0.05]$	0.02 $[0.001,0.05]$	0.08 $[-0.19,0.62]$	-0.02 $[-0.06,0.02]$	0.001 $[-0.02,0.03]$
$Baa\,Spread$	-	-	-	-0.82 $[-4.87,2.17]$	-0.19 $[-0.3,-0.08]$	-0.17 $[-0.31,-0.06]$
FFR	0.79 $[0.2,1.0]$	0.96 $[0.87,1.01]$	0.96 $[0.93,0.99]$	1.01 $[0.34,1.5]$	0.92 $[0.87,0.98]$	0.94 $[0.90,0.99]$

Note: Table 6 presents posterior median estimates of the cumulative elasticities of the federal funds rate with 90 percent intervals in brackets. This table compares the estimates of the 4 and 5 equation MS proxy BayesianVAR alongside the 4 and 5 equation fixed-coefficient Bayesian proxy SVAR.

in the impact and propagation of shocks when the DGP contains switches in coefficients and residual covariances. The model is then used to examine the role of credit spreads in the transmission of monetary policy shocks, where identification is achieved using a proxy constructed from high-frequency financial data. Credit spreads significantly change the transmission of monetary policy shocks from 2000-2007 however, appear to have only a short-lived amplification of the effects on industrial production between the mid to late 1990s.

The procedure developed in this paper is flexible and encompasses a range of models that differ in the parameters that can switch and the number of regimes allowed for. Future applications of this model that may be of interest would be analysing changes in the propagation of economic shocks identified by proxies across different combinations of monetary and fiscal regimes over a historical sample. In addition extensions can be made to allow for time-varying transition probabilities that allow the probability of a regime to be directly affected by endogenous variables.

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7 Appendix

7.1 Prior distributions

The priors for the parameters roughly follow those of Caldara and Herbst (2019). The priors specific to the regime-switching parameters are set following to Liu et al. (2018) and Barnett, Groen and Mumtaz (2009).

The priors for the MS-VAR(P) coefficients $\Phi_1 \dots \Phi_M$ and the error covariance matrices $\Sigma_1 \dots \Sigma_M$ are set the same across regimes and implemented via the dummy observations method applied in Banbura et al. (2007).[21] The Normal Wishart prior is defined as

$$Y_D = \begin{pmatrix} \frac{diag(\gamma_1\sigma_1 \dots \gamma_N\sigma_N}{\tau} \\ 0_{N\times(P-1)\times N} \\ \dots\dots\dots\dots \\ diag(\sigma_1 \\ \dots\dots\dots\dots \\ 0_{1\times N} \end{pmatrix}, and\ X_D = \begin{pmatrix} \frac{J_P \otimes diag(\sigma_1 \dots \sigma_N}{\tau} \ 0_{NP\times 1} \\ 0_{N\times NP}\ 0_{NP\times 1} \\ \dots\dots\dots\dots \\ 0_{NP\times 1}\ c \end{pmatrix}$$

where σ_i for $i = 1, 2, \dots N$ represent scaling factors, γ_i is the prior mean for coefficients on the first lag, τ is the tightness of the prior on the MS-VAR coefficients, c controls the tightness of the prior on the constant. To obtain a value of γ_i, σ_i, I estimate an AR(1) model via OLS following Mumtaz el al. (2018) for each endogenous variable. γ_i is equal to the OLS estimate of the AR(1) coefficient and is the standard deviation of the residual. The matrix J_p is a diagonal matrix with elements $(1, 2, \dots P)$. I set the values of τ to 10 implying a low degree of shrinkage and c is 1/1000 representing loose beliefs on the values of the constant.

The priors for the transition probabilities p_{ij} for $i = 1, \dots, M, j = 1, \dots, M$ are set to follow a Dirichlet distribution

$$p^0_{ij} = D(u_{ij})$$

where $D()$ denotes the Dirichlet distribution and $u_{ij} = 15$ if $i = j$. This pior choice implies a prior mean of that the regimes are fairly persistent, lasting 20 periods. The posterior distribution is

$$p_{ij} = D(u_{ij} + \eta_{ij})$$

where η_{ij} is the number of time regime i is followed by regime j.

7.2 Estimation algorithm

This subsection provides details on the sampling of the history of regimes and how the proxy information is accounted for in this step and also in addition discusses how to impose the normalisation conditions to identify regimes.

Sampling the history of regimes $s_{1:T}$

The history of regimes $s_{1:T}$ can be drawn using a multi-move Gibbs sampling step to draw from the conditional density $f(s_t|Y_t, \Phi_{1:M}, \Sigma_{1:M}, \beta, \sigma^2, P, m_t)$. Given the starting values for the MS-VAR (P) coefficients $\Phi_1 \dots \Phi_M$ and the error covariance matrices $\Sigma_1 \dots \Sigma_M$, transition probabilities p_{ij} and the parameters β, σ that govern the relevance of the proxy information m_t. Starting values can be obtained using maximum likelihood estimation as described in section 4.1.

Kim and Nelson (1999) show that the Markov property of s_t implies that

$$f(s_t|Y_t) = f(s_T|Y_T) \prod_{t=1}^{T-1} f(s_t|s_{t+1}, Y_t)$$

where conditioning arguments are suppressed for ease of exposition. The density is then simulated in two steps:

1. Calculating $f(s_T|Y_T)$: The filter is Hamilton (1989) provides $f(s_t|Y_t), t = 1 \dots T$. The last iteration of the filter provides $f(s_T|Y_T)$.

[21] The reader is referred to the handbook of Blake and Mumtaz (2017) for a details description.

2. Calculating $f(s_t|s_{t+1}, Y_t)$ where Kim and Nelson (1999) show that

$$f(s_t|s_{t+1}, Y_t) \propto f(s_{t+1}|s_t)f(s_t|Y_t)$$

where $f(s_{t+1}|s_t)$ is the transition probability and $f(s_t|Y_t)$ is obtained via the Hamilton filter. Kim and Nelson (1999) show how to sample from the above equation.

To augment the proxy information m_t in the likelihood as in equation 6 the system is rewritten before implementing the Hamilton filter steps as

$$\begin{pmatrix} Y_t \\ m_t \end{pmatrix} = \begin{pmatrix} I_N \otimes X_t' \\ 0 \end{pmatrix} \Phi_{s_t} + \begin{pmatrix} u_t \\ m_t \end{pmatrix}$$

where the conditional covariance matrix is:

$$\Omega_{s_t} = cov \begin{pmatrix} u_t \\ m_t \end{pmatrix} |\Theta \Bigg) = \begin{pmatrix} \Sigma_{s_t} & A_{s_t} q_{\varepsilon_1}' \beta \\ \beta q_{\varepsilon_1} A_{s_t}' & \beta^2 + \sigma^2 \end{pmatrix}.$$

Given initial probabilities $\hat{\xi}_{1|0} = \Pr(s_{t=1} = j|\Theta)$ and conditioning on model parameters Θ of each regimes the likelihood of each regime for each time period is given.

$$\eta_t = \begin{cases} (2\pi)^{-N/2} |\Omega_1|^{-1/2} exp\left[-\frac{1}{2}(\tilde{Y}_t - \Phi_1 \tilde{X}_t)' \Omega_1^{-1} (\tilde{Y}_t - \Phi_1 \tilde{X}_t)\right], & when\, s_t = 1 \\ (2\pi)^{-N/2} |\Omega_2|^{-1/2} exp\left[-\frac{1}{2}(\tilde{Y}_t - \Phi_2 \tilde{X}_t)' \Omega_2^{-1} (\tilde{Y}_t - \Phi_2 \tilde{X}_t)\right], & when\, s_t = 2 \\ \ldots\ldots\ldots\ldots\ldots & \ldots \\ (2\pi)^{-N/2} |\Omega_M|^{-1/2} exp\left[-\frac{1}{2}(\tilde{Y}_t - \Phi_M \tilde{X}_t)' \Omega_M^{-1} (\tilde{Y}_t - \Phi_M \tilde{X}_t)\right], & when\, s_t = M \end{cases}$$

where $\tilde{Y}_t = \begin{pmatrix} Y_t \\ m_t \end{pmatrix}, \tilde{X}_t = \begin{pmatrix} I_N \otimes X_t' \\ 0 \end{pmatrix}$.

The Hamilton filter recursions are then run as follows for $t = 1 \ldots T$.

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t} \odot \eta_t}{i' \hat{\xi}_{t|t} \odot \eta_t},$$

$$\hat{\xi}_{t+1|t} = P \hat{\xi}_{t|t}.$$

Where $\hat{\xi}_{t|t-1} = f(s_t|\tilde{Y}_{t-1}, \Theta)$ represents the filter probabilities of being in regime j at time t conditional on information up to the previous period $t-1$, $\hat{\xi}_{t|t-1} = f(s_t|\tilde{Y}_{t-1}, \Theta)$ represents the filter probabilities of regime j at time t updated with information $\tilde{Y}_t$, i is a $M \times 1$ vector of ones and $\odot$ is the Hadamart product.

The likelihood function is equal to

$$L(\Theta) = \sum_{t=1}^{T} \log f(\tilde{Y}_t|\tilde{Y}_{t-1}, \Theta) = \sum_{t=1}^{T} \log[i'(\hat{\xi}_{t|t-1} \odot \eta_t)]$$

A potential problem when s_t is drawn is that one of the regimes is not visited, implying that the data is not informative for this regime. To deal with this problem I follow Barnett, Groen and Mumtaz (2009) by redrawing $s_{1:T}$ if one regime is in place for less than $(N \times L) + 1$, if this condition is not met after 1000 redraws, the draw is used to sample the transition probabilities p_{ij} and MS-VAR coefficients $\Phi_1 \ldots \Phi_M$ and $\Sigma_1 \ldots \Sigma_M$ but then discarded.

Normalisation conditions

As mentioned in section 4.2 normalisation restrictions must be placed on the draws of the VAR coefficients $\Phi_1 \ldots \Phi_M$ to avoid the problem of 'label switching' which may lead to multi-modal posterior distribution. These restrictions are imposed via rejection sampling following Barnett, Groen and Mumtaz (2009).

Table 7: Relevance statistics of fixed coefficient BP-SVAR - 90 percent intervals

	Fixed Coefficient BP-SVAR	
	4 equation	5 equation
$\rho = \frac{\beta^2}{\beta^2+\sigma^2}$	0.09 [0.03,0.17]	0.10 [0.03,0.2]

7.3 Fixed parameter Bayesian proxy SVAR estimates

This subsection presents the results of estimating a fixed coefficient BP-SVAR with log differences and the choice of one lag in Tables 7, 8, 9.

As mentioned in section 5, the specification estimated departs from that of Caldara and Herbst (2019) in two ways. Firstly, yearly differences of industrial production and CPI are used as opposed to log levels due to the relative difficulty of finding initial values for the history regimes in the latter case that lead to convergence in the algorithm. Secondly, due to the short sample size, I consider specifications with one lag following Holm-Hadulla and Hubrich (2017). To ensure that the results in section 5 are comparable to those of Caldara and Herbst (2019) the following two fixed coefficient specification are estimated.

$$Y_t = \Phi X_{t-1} + (A_t q)\varepsilon_t,\ \varepsilon_t \sim \mathcal{N}(0, I_N), s_t \in \{1, 2\}$$

$$m_t = \beta\varepsilon_{MP,t} + \sigma_v v_t, v_t \sim N(0, 1)\, and\, v_t \perp \varepsilon_t$$

$$Model1 : Y_t = \{FFR_t, \triangle IP_t, U_t, FFR_t, \pi_{PPI,t}\}, m_t = \{FOMC\, Suprises\}$$
$$Model2 : Y_t = \{FFR_t, \triangle IP_t, U_t, FFR_t, \pi_{PPI,t}, Baa\, Spread_t\}, m_t = \{FOMC\, Suprises\}$$

The results of estimation are in line with the results of Caldara and Herbst (2019). The relevance coefficient displayed in table 7 are around the posterior median of 0.1 reported in Caldara and Herbst (2019).

Table 8 and 9 show the contemporaneous and cumulative elasticities of the fixed coefficient BP-SVAR . The sign and magnitude of posterior median estimates of the contemporaneous responses are comparable with those reported in Caldara and Herbst (2012). Due to the reduced lag length the magnitude of posterior median estimates of the cumulative elasticity is lower, however signs are still consistent.

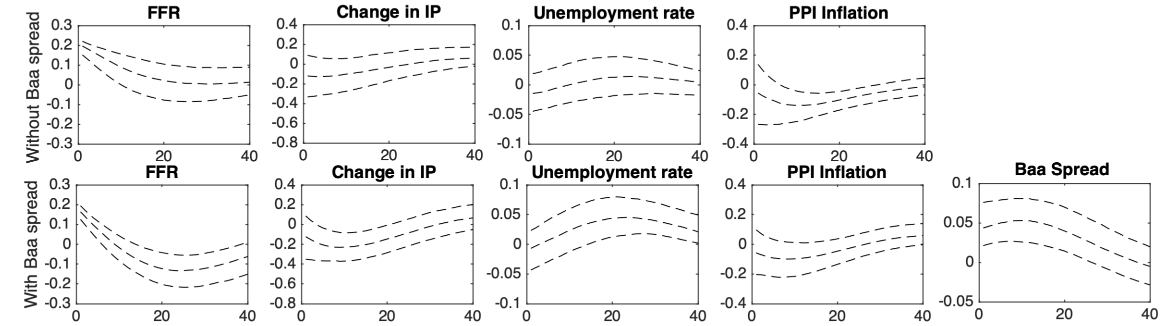
7.4 One-standard deviations IRFS - Fixed coefficient BP-SVAR and MS-BP-SVARs

Impulse responses to a one-standard deviation monetary policy shock compared to those of the MS-BP-SVAR model in Figure 8,9 and 7.4.

7.5 Normalised impulsed responses of the simulation exercise

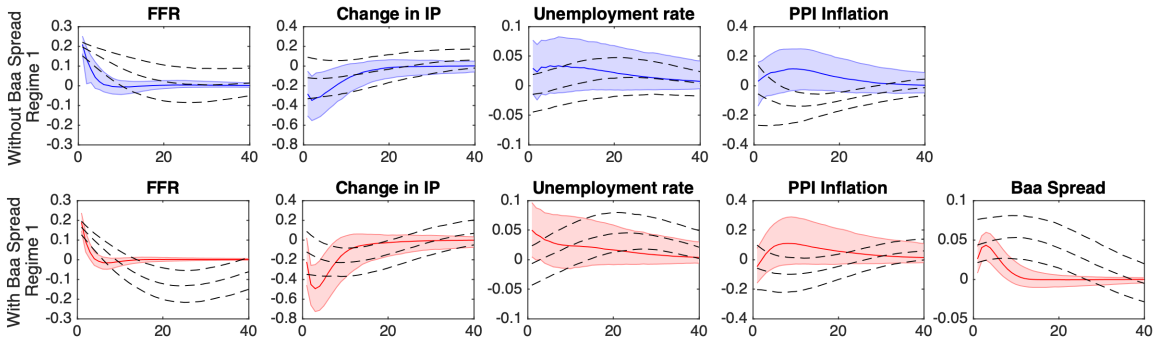
The normalised impulsed responses of the simulation exercise are presented in figure 11 and show the estimation procedure is able to track the actual responses.

Figure 8: Impulse responses to one-percent increase in the Federal Funds rate - Fixed-coefficient proxy VAR



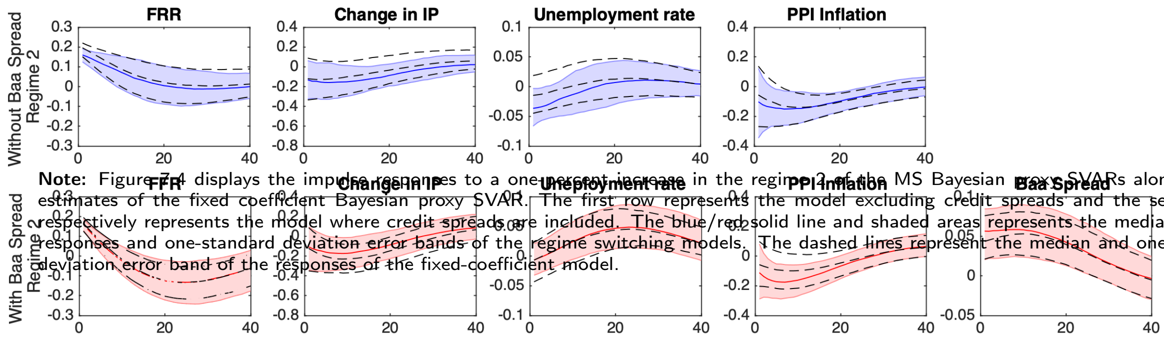
Note: Figure 8 displays the impulse responses to a one-percent increase in the fixed-coefficient Bayesian proxy SVARs. The first row represents the model excluding credit spreads and the second row respectively represents the model where credit spreads are included. The dashed lines represent the median and one-standard deviation error band of the responses of the fixed-coefficient model.

Figure 9: Impulse responses to one-percent increase in the Federal Funds rate - MS-BP-VARs regime 1



Note: Figure 9 displays the impulse responses to a one-percent increase in the regime 1 of the MS Bayesian proxy SVARs alongside the estimates of the fixed coefficient Bayesian proxy SVAR. The first row represents the model excluding credit spreads and the second row respectively represents the model where credit spreads are included. The blue/red solid line and shaded areas represents the median impulse responses and one-standard deviation error bands of the regime switching models. The dashed lines represent the median and one-standard deviation error band of the responses of the fixed-coefficient model.

Figure 10: Impulse responses to one-percent increase in the Federal Funds rate - MS-BP-VARs regime 2



Note: Figure 7.4 displays the impulse responses to a one-percent increase in the regime 2 of the MS Bayesian proxy SVARs alongside the estimates of the fixed coefficient Bayesian proxy SVAR. The first row represents the model excluding credit spreads and the second row respectively represents the model where credit spreads are included. The blue/red solid line and shaded areas represents the median impulse responses and one-standard deviation error bands of the regime switching models. The dashed lines represent the median and one-standard deviation error band of the responses of the fixed-coefficient model.

Table 8: Contemporaneous elasticities of the fixed coefficient Bayesian proxy VAR

	4 equation model	5 equation model
$\triangle IP$	0.09 $[-0.03,0.31]$	0.08 $[-0.1,0.27]$
U	0.44 $[-0.36,1.67]$	0.32 $[-0.69,1.60]$
π_{PPI}	0.02 $[-0.11,0.20]$	0.03 $[-0.10,0.20]$
$Baa\,Spread$	-	-0.79 $[-2.90,0.02]$

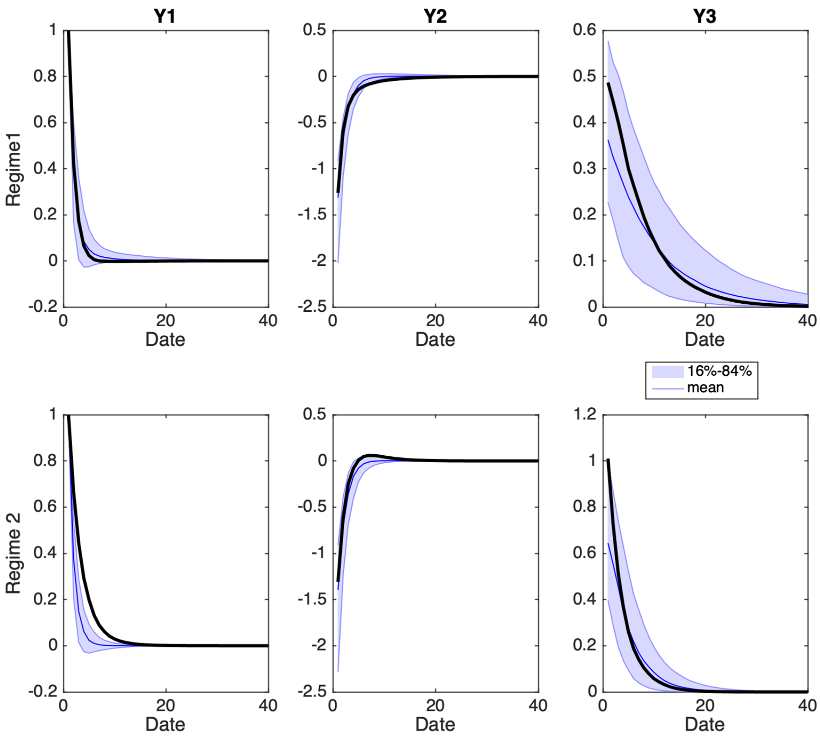
Note: Table 8 presents posterior median estimates of the contemporaneous elasticities of the federal funds rate with 90 percent intervals in brackets

Table 9: Cumulative elasticities of the fixed coefficient Bayesian proxy VAR

	4-equation model	5-equation model
$\triangle IP$	0.02 $[0.03,0.6]$	0.03 $[-0.001,0.05]$
U	-0.02 $[-0.11,0.06]$	-0.09 $[-0.20,-0.01]$
π_{PPI}	0.02 $[0.001,0.05]$	0.001 $[-0.02,0.03]$
$Baa\,Spread$	-	-0.17 $[-0.31,-0.06]$
FFR	0.96 $[0.93,0.99]$	0.94 $[0.90,0.99]$

Note: Table 9 presents posterior median estimates of the cumulative elasticities of the federal funds rate with 90 percent intervals in brackets

Figure 11: Impulse responses normalised to represent a 1 unit increase to the first variable of VAR estimated on artificial data



Note: The blue line and shaded area represents the median and one-standard deviation error band while the thick black line shows the true responses to a 1% percent increase in the variable of the first equation.

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