Monetary policy regimes and inflation persistence in the United Kingdom

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Monetary policy regimes and inflation persistence in the United Kingdom*

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This paper conducts a structural analysis of inflation persistence in the United Kingdom between 1965-2009. I allow for the possibility of shifts in the U.K. economy by estimating open-economy dynamic stochastic general equilibrium models in which parameters of a Taylor-type monetary policy rule, New Keynesian Phillips curve, and volatilities of structural economic shocks, follow Markov processes (Markov-switching DSGEs). The best-fitting model allows for changes in monetary policy and stochastic shock volatility. The first policy regime responds passively to movements in inflation, adjusting the nominal interest rate less than one-for-one and is estimated to be in place from the early 1970s until the late 1980s. The other regime responds actively to inflation and places less weight on exchange rate movements. This regime is present for the rest sample and almost coincides with the period after the Bank of England explicitly adopted an inflation target in 1992. I find a small but insignificant decrease in inflation persistence in the policy regime that responds more actively to inflation.

Abstract

Key words: Markov-Switching, DSGE, Inflation persistence, Bayesian estimation
JEL Classification: C11, E31, E52

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1 Introduction

Understanding changes in the statistical process of price inflation is important for policymakers, especially at central banks. Inflation persistence captures the relationship between inflation and its past levels. Often, inflation is thought of as being a highly persistent variable. However, empirical studies have found lower persistence during monetary policy regimes that have explicitly targeted inflation. A very persistent low rate of inflation can be seen as desirable if the rate is in line with a central banks definition of price stability. Although, if inflation is more persistent the effect of exogenous shocks take a longer amount of time to entirely dissipate and these shocks can leave inflation at high rates that are not desirable for a number reasons. Therefore, given the strong relationship of persistence and predictability, concentrating on the decreasing persistence of deviations of inflation from a long-run level or trend, namely the inflation gap, can be more interesting from a monetary policy perspective.

This paper examines whether changes in monetary policy affected the persistence of the inflation gap and whether this property of inflation changed in the United Kingdom between 1965-2009. During this period, specifically in the late 1980s, inflation amongst other macroeconomic variables experienced a significant decrease in volatility and mean level, an event known as the Great Moderation. In the meantime, U.K. monetary policy experienced fundamental changes, with the adoption of an explicit inflation target in October 1992 and the independence of the Bank of England in May 1997.

Typically, to assess whether the adoption of inflation-targeting has affected persistence, reduced-form models of inflation are estimated across subsamples which split the data into periods pre and post-1992. However, this approach is unable to isolate the effects of monetary policy from other changes that the economy was subject to during this period. For example, one of the prominent explanations of the great moderation was a reduction in the volatility of shocks that hit the economy. This paper contributes to the literature by examining U.K. inflation persistence through estimated small open-economy Markov-switching dynamic stochastic general equilibrium models (MS-DSGE) that allow the structure of the economy to change over time.[1] To identify the changes in the economy that best explain U.K. data, I estimate four MS-DSGE models that allow for stochastic volatility. The first model allows monetary policy to switch between two regimes that differ on the strength placed on pursuing inflation. The policy rule of the second model is constant however, the degree of price stickiness in the economy is allowed to switch between two regimes. The third model enables both monetary policy and price stickiness to switch simultaneously. Finally, the fourth model solely allows for changes in the volatility of economic shocks.

The model preferred by the data identifies two distinct monetary policy regimes. The regime present during the post-1992 period is distinguished by a relatively aggressive stance towards movements in inflation and almost no sensitivity to movements of the exchange rate. To measure the inflation persistence implied by each model, I consider three measures used in both reduced-form and structural analysis that enable regime change to affect persistence directly.

The main finding of this paper is that there is no significant evidence of changes in inflation persistence during the sample. Moreover, the inflation-targeting regime produces a small decrease in persistence relative to a monetary policy regime with a relatively accommodative stance towards inflation; however, this difference is insignificant.

The remainder of this paper is organised as follows. Section 2, presents a brief review of how this paper fits in with the existing literature. Section 3, illustrates the underlying New Keynesian model and introduces subsequent versions of the model that allows for regime change in the economy. Section 4, discusses the estimation strategy. Section 5, presents the estimation results. In section 6, I consider the implications of estimation results for inflation persistence. Section 7, discusses the effect of inflation-targeting monetary policy on the economy. Finally, section 8 concludes.

2 Existing literature

This paper builds on the work of Davig and Doh (2014) by examining U.K. inflation persistence through a Markov-switching DSGE model (MS-DSGE). Davig and Doh (2014), find that the model that best fits U.S. data identifies two monetary policy regimes, that differ on the stance towards movements of inflation. They observe a decrease in inflation persistence when the relatively aggressive inflation-targeting regime

[1] To be consistent with the remainder of the thesis, I follow Maih (2015) and use Markov-Switching DSGE (MS-DSGE) to denote regime-switching DSGE (RS-DSGE) models without state-dependent regime change, in particular, they are RS-DSGE models with constant transition probabilities.

is in place. However, this change although significant is small compared to other empirical studies that focus on the U.S. such as Cogely, Sargent and Premiceri (2011), Kang and Kim (2009) and Levin and Piger (2010).

I extend the analysis of Davig and Doh (2014) to the United Kingdom by estimating a model that allows a role for international trade. This modelling assumption reflects the relative trade position of the U.K. as a small open economy. Also, I consider models that allow for changes in price stickiness to affect inflation persistence following the theoretical work of Fruher (2016). To measure inflation persistence, I use three measures to facilitate comparisons with reduced-form and structural studies that allow for regime changes to affect persistence. In particular, the measures are inflation predictability and the normalised spectrum applied in Cogely, Premiceri and Sargent (2010) and Cogely and Sargent (2005), respectively and finally the moment of autocorrelation proposed by Davig and Doh (2014).

By examining U.K. inflation persistence across monetary policy regimes, this paper is related to the work of Alogoskoufis and Smith (1991) and Benati (2009, 2010 and 2012). Alogoskoufis and Smith (1991) provide both empirical and theoretical evidence that suggests greater monetary accommodation of inflation in addition to exchange-rate accommodation of inflation differentials increase inflation persistence in the U.K and the U.S. The empirical approach of Alogoskoufis and Smith (1991) involves the subsample estimation of expectations-augmented Phillips curves across alternative monetary regimes which represent the departure from the effective gold standard and the gold dollar standard. Benati considers univariate models estimated across subsamples of a historical inflation time-series that represent particular monetary policy regimes and finds that persistence almost completely disappears after the adoption of inflation-targeting. The univariate approach implies inflation is intrinsically persistent and does not inherit any persistence from other macroeconomic variables such as domestic output or interest rates. Moreover, subsample estimation can struggle to isolate the effect of monetary policy on inflation persistence from other changes in the economy that affected inflation across samples.

Levin and Piger (2004) also examine inflation persistence in the U.K. in addition to other countries that adopted explicit inflation-targeting and apply univariate models to test for breakpoints in the inflation process. The authors find that after conditioning on the effect of estimated structural breaks in the early 1990s, inflation persistence significantly decreases.

Contrastingly, Barnett et al. (2010) report estimates of high levels of U.K. inflation persistence throughout 1970 until 2006. The authors apply a Markov-switching VAR and report small changes of inflation persistence across regimes classified by high and low level of mean inflation.

Mumtaz and Surico (2009) and Liu, Matthes and Petrova (2018), estimate U.K. inflation persistence through VARs with time-varying parameters and stochastic volatility. Both studies observe a decrease in persistence after 1992 until the end of the 1990s. However, Liu, Matthes and Petrova (2018) find inflation returns to being highly persistent during 2000 the end of their sample in 2013. These models are multivariate and allow for changes in the volatility of shocks, however, they do not impose enough economic structure to identify precisely the impact of monetary policy regime change on persistence.

Cogely, Premiceri and Sargent (2010) measure inflation gap persistence in the U.S. from 1960-2006 using a time-varying parameter VAR and then attempt to structurally uncover the effects of monetary policy regime change through subsample estimation of a new-Keynesian DSGE model. The authors find a substantial decrease in persistence during the subsample which represents a more aggressive monetary policy stance towards inflation spearheaded by the Fed chairmanship of Paul Volcker in 1979. They also conduct a counterfactual experiment and find that imposing the post-Volcker monetary policy rule during 1960-1979 would result in lower inflation volatility and persistence. However, subsample estimation within a DSGE framework does not take into account the role of agents expectations of regime change and can affect model agents decision and hence, affects the solution of the model and parameter estimates. This criticism is addressed in this paper by applying model solution methods that allow agents to incorporate the probability of regime change when forming expectations.

The main finding of this paper of no change in inflation persistence is somewhat in contrast to U.K. literature. However, the relatively small number of studies that examine U.K. inflation persistence report considerably different movements of inflation persistence. Differences between measures of persistence applied and the ability to isolate the impact of monetary policy away from other factors may explain the variation of findings. It is important to highlight that across the literature during a similar sample in the U.S., there are also different accounts of the dynamics of inflation persistence. Studies such as Cogely, Premiceri and Sargent (2010), Levin and Piger (2014), Mumtaz and Surico (2012), find evidence of a substantial reduction in persistence. In contrast, Pivetta and Reis (2007) observe that U.S. inflation is highly persistent and is unchanged during the post-war period.

This paper is also related to literature that estimates open-economy models with regime-switching on

U.K. data. Liu and Mumtaz (2011) consider a larger-scale model than the one of this paper and find that monetary regime change in addition to switches in the volatility of structural shocks are a crucial feature in explaining U.K. macroeconomic dynamics between 1970-2009. In relation, Chen and MacDonald (2012) and Alstadheim et al. (2013) consider a similar underlying model as this chapter. Chen and MacDonald (2012) derive the implications of monetary policy regime shifts on optimal monetary policy rules. Whilst, Alstadheim et al. (2013), highlight differences in the monetary policy reaction to exchange rate movements of small open-economies after the adoption of inflation-targeting.

3 A small open-economy model of the U.K.

Tracing the sources of inflation persistence to changes in the U.K. economy within a DSGE framework requires introducing a form of time-variation to the parameters of a structural model of the economy. Therefore, I estimate versions of the model considered in Lubik and Schorfheide (2007) that allow for regime-change across different areas of the economy. The small open-economy property of the model reflects this dimension of the U.K. economy. Similar versions that incorporate regime-switching have been applied to U.K. data by Alstadheim et al. (2013) and Chen and MacDonald (2012), which provide a comparison of parameter estimates.

In this section, I briefly describe the log-linearised model and then introduce the subsequent versions that allow for regime changes in monetary policy, the degree of nominal rigidities in prices and the volatility of structural shocks. The reader is referred to the appendix for a detailed description of the underlying non-linear model and to Del Negro and Schorfheide (2009) for details of log-linearisation.

3.1 Benchmark model

The model allows for interaction between a small domestic economy and a continuum of other small open economies representing the rest-of-the-world. In this model the growth of output per capita results from technology growth, therefore, to obtain stationarity, all variables are measured in percentage deviations from a stochastic balanced growth path. The following equations determine the evolution of the small open economy.

The log of the global technology is represented by $\ln Z_t$ and is assumed to follow a random walk process with a drift term γ,

$$\Delta \ln Z_t = \gamma + \hat{z}_t, \hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon^z_t, \varepsilon^z_t \sim NID(0, \sigma^2_z). \tag{1}$$

Where Δ denotes a temporal difference operator, $\hat{z}_t$ is the growth rate of the global technology process Z_t and is treated as unobserved and follows an AR(1) process.

A consumption Euler equation describes the household behaviour in this small open-economy and after substituting equilibrium conditions, can be rewritten as an open-economy IS-curve:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - (\tau + \lambda)(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \mathbb{E}_t \hat{z}_{t+1}) + \alpha(\tau + \lambda)\mathbb{E}_t \Delta \hat{q}_{t+1} + \frac{\lambda}{\tau}\mathbb{E}_t \Delta \hat{y}^*_{t+1}. \tag{2}$$

The output gap of the domestic economy at time period t, $\hat{y}_t$, depends on its expected future value $\mathbb{E}_t \hat{y}_{t+1}$, the real interest rate $(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1})$, the expected growth rate of the technology process $\mathbb{E}_t \hat{z}_{t+1}$, the expected change in the terms of trade $\Delta \hat{q}_t$ and the expected change in world output $\mathbb{E}_t \Delta \hat{y}^*_{t+1}$. The parameter α denotes the share of imports in the domestic economy and controls the degree of openness and takes a range of values between $0 < \alpha < 1$. The model reduces to its closed economy variant when $\alpha = 0$. The intertemporal elasticity substitution is denoted by τ and $\lambda = \alpha(2 - \alpha)(1 - \tau)$. The terms of trade is defined as the the relative price of exports in terms of imports.

The following New Keynesian Phillips curve describes inflation dynamics in this economy and the open-economy variant allows for the effects of terms of trade and the world output gap:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \alpha\beta \mathbb{E}_t \Delta q_{t+1} - \alpha \Delta \hat{q}_t + \frac{\kappa}{(\tau + \lambda)}\hat{y}_t + \frac{\kappa\lambda}{\tau(\tau + \lambda)}\hat{y}^*_t. \tag{3}$$

Domestic firms set prices of intermediate goods and nominal rigidities are introduced *à la* Calvo (1983). In a standard New Keynesian model, κ is the slope coefficient and is related to the degree of price stickiness, the degree of competition and the representative firm's cost function parameters; as $\kappa \to \infty$ the degree of nominal rigidities vanish.

This paper focuses on inflation persistence therefore, it is worth discussing why the Phillips curve in this model does not have a backwards-looking component of inflation expectations that can be introduced by allowing firms to index prices on the previous level of inflation. Fruher and Moore (1994) and Fruher (2016) show that New Keynesian DSGE models do not generate enough persistence in the response of inflation to exogenous shocks when compared to the hump shape responses found commonly in Vector Auto-regressions (VARs). Introducing a backwards-looking Phillips curve is one way of introducing additional persistence as described in Fruher (2016). However, empirical studies on the U.K., estimate relatively small values for this parameter. Liu and Mumtaz (2011) estimate an MS-DSGE model with a hybrid Phillips curve that contains both expectations and lagged values of inflation and find considerable uncertainty around the parameter on lagged inflation with credible intervals including the possibility of insignificance. Also, the authors find little evidence of systematic change in this parameter. Also, Davig and Doh (2014) find that adding a backwards-looking component of inflation expectations does not change results with regards to their estimates of inflation persistence or model fit. Therefore rather than introduce a backwards-looking element I consider the original model specification which allows for more direct comparison of model estimates with other studies.

The monetary authority conducts policy by setting the nominal interest rate $\hat{R}_t$ and follows a systematic rule along the lines of those described in Taylor (1993) but adjusted for an open-economy following the findings of Lubik and Schorfhiede (2007).

$$\hat{R}_t = \rho_r(\hat{R}_{t-1}) + (1-\rho_r)[\psi_\pi(\hat{\pi}_t) + \psi_y(\Delta\hat{y}_t + \hat{z}_t) + \psi_e e_t + \varepsilon_t^R], \varepsilon_t^R \sim NID(0, \sigma_R^2). \tag{4}$$

The Taylor rule in this model places emphasis on smooth movements of the interest rate relative to its level in the previous period $\hat{R}_{t-1}$, deviations of inflation away from target level $\hat{\pi}_t$, output growth $(\Delta\hat{y}_t + \hat{z}_t)$ and the nominal exchange rate e_t. Movements in the interest rate that are not explained by the rule are captured in the monetary policy shock ε_t^R. Since $\hat{y}_t$ measures percentage deviations from the stochastic growth trend induced by the productivity process Z_t, output growth deviations from the mean γ are given by $\Delta\hat{y}_t + \hat{z}_t$ following Del Negro and Schorfheide (2009). The parameter ρ_r represents the degree of interest-smoothing and parameters ψ_π, ψ_y, ψ_e, represent the elasticities of the monetary policy instrument with respect to movements in inflation, output growth and the exchange rate, respectively.

The definition of CPI introduces the nominal exchange rate $\hat{e}_t$

$$\Delta\hat{e}_t = \hat{\pi}_t - (1-\alpha)\Delta\hat{q}_t - \hat{\pi}_t^*, \tag{5}$$

where $\hat{\pi}_t^*$ captures a shock to world inflation and is treated as unobservable. It can also be interpreted as capturing any deviations from PPI. Since the other variables are observable, this relaxes the potentially tight cross-equation restrictions embedded in the model.

$$\Delta\hat{q}_t = \rho_{\Delta q}\Delta\hat{q}_{t-1} + \varepsilon_t^q, \varepsilon_t^q \sim NID(0, \sigma_q^2). \tag{6}$$

Following Lubik and Schorfheide (2007), changes in terms of trade are assumed to follow an AR(1) process.[2] Rest-of-the-world output and inflation are treated as unobservable and evolve according to a univariate AR(1) process.[3]

$$\hat{y}_t^* = \rho_{y^*}\hat{y}_{t-1}^* + \varepsilon_t^{y^*}, \varepsilon_t^{y^*} \sim NID(0, \sigma_{y^*}^2), \tag{7}$$

$$\hat{\pi}_t^* = \rho_{\pi^*}\hat{\pi}_{t-1}^* + \varepsilon_t^{\pi^*}, \varepsilon_t^{\pi^*} \sim NID(0, \sigma_{\pi^*}^2). \tag{8}$$

The model consists of 10 state variables $\hat{X}_t$ which includes includes 2 expectational terms

$$\hat{x}_t = \underbrace{[\hat{y}_t, \hat{\pi}_t, \Delta\hat{e}_t, \hat{R}_t, \Delta\hat{q}_t, \hat{z}_t, \hat{y}_t^*, \hat{\pi}_t^*, \mathbb{E}_t\hat{y}_{t+1}, \mathbb{E}_t\hat{\pi}_{t+1}]'}_{},$$

and 5 exogenous shocks v_t that are independent standard normal variables

$$v_t = \underbrace{[\varepsilon_t^R, \varepsilon_t^z, \varepsilon_t^q, \varepsilon_t^{y^*}, \varepsilon_t^{\pi^*}]'}_{},$$

[2]Under the assumptions of the model, in particular, complete asset markets and perfect risk sharing, the change in terms of trade can be solved for endogenously and provides the following law of motion: $\Delta\hat{q}_t = -\frac{1}{\tau+\lambda}(\Delta\hat{y}_t - \Delta\hat{y}_t^*)$. However, Lubik and Schorfheide (2007) find that the cross-equation restrictions of the fully structural model impair the movement of terms of trade, leading to implausible parameter estimates. Therefore following their direction terms of trade is modelled as an exogenous variable.

[3]Lubik and Schorfheide (2007) emphasis that modeling $\hat{y}_t^*$ and $\hat{\pi}_t^*$ as latent processes relaxes the potentially tight cross-equation restrictions embedded in the model. In particular, $\hat{\pi}_t^*$ incorporates deviations from PPI. However Alstadheim et al (2013) treat the variables as observable and use U.S. GDP and CPI combined with measurement errors as a proxy.

the benchmark DSGE model has 16 parameters Φ

$$\Phi = \underbrace{[\tau, \beta, \alpha, \rho_R, \psi_\pi, \psi_y, \psi_e, \rho_{\Delta q}, \rho_z, \rho_{y^*}, \rho_{\pi^*}, \sigma_q^2, \sigma_z^2, \sigma_{y^*}^2, \sigma_{\pi^*}^2, \sigma_R^2]'}.$$

The model can be rewritten in canonical form as

$$A(\Phi)\hat{x}_t = B(\Phi)\hat{x}_{t-1} + C(\Phi)v_t + D(\Phi)\eta_t. \tag{9}$$

where A, B, C and D are matrices and function of the structural model parameters Φ. Under rational expectations and no regimes shifts, the model can then be solved using a standard rational expectation algorithms such as the *Gensys* solution method proposed in Sims (2002). The solution is expressed in the form of the following first-order VAR

$$\hat{x}_t = \Omega(\Phi)\hat{x}_{t-1} + \Delta(\Phi)v_t,$$

where solution matrices $\Omega(\Phi)$ and $\Delta(\Phi)$ are functions of the structural model parameters Φ. The model can then be estimated using Bayesian methods where the Kalman filter is applied to obtain the likelihood function.

3.2 Markov-switching versions of the model

I allow for time-variation in the parameters that determine the behaviour of areas of the economy prominently put forward to explain movements of inflation persistence. In particular, I consider changes in the monetary policy stance to targeted variables, the level of nominal rigidity and the volatility of structural shocks in the economy.

Four versions of the model are estimated and consider regimes change in 1) Monetary policy; 2) Nominal rigidity; 3) both monetary policy and nominal rigidities; 4) shock volatility. Estimating four alternative MS-DSGE models allows the data to indicate which form of regime change, if any, best describes U.K. data.

In models 1), 2) and 3) the regimes of a selected groups of switching parameters are governed by a latent variable s_t that follows a two-state first-order Markov-process, this enables parameters to switch between two regimes with matrix $\mathbf{p}$ containing the constant transition probabilities

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i),\ i, j = 1, 2;\ \mathbf{p} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}.$$

Model agents know these transitions probabilities and use this information when forming expectations. Therefore, this approach maintains the rational expectations hypothesis. A potential drawback is that the probability of moving across regimes is independent of the state of the economy and how close inflation is to target levels.

It is important to note that all models allow for stochastic volatility of the model shocks, which takes the form of regime change in structural shock variances. The regime change in shock variances is governed by a latent variable S_t which follows an independent Markov-process, with matrix $\mathbf{q}$ containing transition probabilities

$$q_{ij} = \Pr(S_t = j | S_{t-1} = i),\ i, j = 1, 2;\ \mathbf{q} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}.$$

The probability of switching to a new regime is only dependent on the current regime in place.

To identify each regime I impose a normalisation condition whilst computing the likelihood of the switching models which is briefly outlined below alongside additional model specific assumptions.

Monetary policy regimes

The monetary policy rule in equation 4 now allows for two monetary policy regimes and is expressed as

$$\hat{R}_t = \rho_r(\hat{R}_{t-1}) + (1-\rho_r)[\psi^\pi(s_t)(\hat{\pi}_t) + \psi^y(s_t)(\Delta\hat{y}_t + \hat{z}_t) + \psi^e(s_t)e_t + \varepsilon_t^R], \varepsilon_t^R \sim NID(0, \sigma_R^2(S_t)) \tag{10}$$

$$s_t \in \{1, 2\}, S_t \in \{1, 2\}$$

$$\psi_\pi(s_t=1) < \psi_\pi(s_t=2)$$

$$\begin{array}{rl} Accomodative & s_t=1 \\ Inflation\,Targeting & s_t=2 \end{array} \left\{ \begin{array}{c} \psi_1^\pi, \psi_1^y, \psi_1^e \\ \psi_2^\pi, \psi_2^y, \psi_2^e \end{array} \right\},$$

where ψ_π dictates the time-varying response of nominal interest rates to the deviations of inflation from a steady-state value of zero. These two parameters follow the same two-state first-order Markov process governed by the latent variable s_t. The normalisation condition implies, when $s_t = 1$, the central bank adopts a relatively accommodative stance towards inflation, responding less aggressively to deviations away from target, captured by parameter ψ_1^π. It is worth highlighting that ψ_1^π can now be lower than one.[4] I assume that the interest rate smoothness parameter is constant, following sub-sample evidence.

Nominal rigidities

The New-Keynesian Phillips curve in equation 3 now allows for a regime change in the degree of nominal rigidity in the economy and is expressed as

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \alpha\beta\mathbb{E}_t \Delta q_{t+1} - \alpha \Delta q_t + \frac{\kappa(s_t)}{(\tau+\lambda)}\hat{y}_t + \frac{\kappa(s_t)\lambda}{\tau(\tau+\lambda)}\hat{y}_t^* \tag{11}$$

$$s_t \in \{1,2\},$$

$$\kappa(s_t=1) > \kappa(s_t=2)$$

$$\begin{array}{rl} Low\ degree\,of\,NominalRigidity & s_t=1 \\ High\ degree\,of\,NominalRigidity & s_t=2 \end{array} \left\{ \begin{array}{c} \kappa_1 \\ \kappa_2 \end{array} \right\},$$

where $\kappa(s_t)$ the price stickiness parameter is now shifting between κ_1 and κ_2. This model allows for changes in the structural equation governing inflation dynamics in the underlying model and is motivated by the theory that firms adjust prices more frequently during high inflation periods.

Monetary policy and nominal rigidities

This model allows for simultaneous changes in both monetary policy and the degree of price stickiness in the economy. The model is estimated following Chen and MacDonald (2012) who find it best fits U.K. data. A passive monetary policy regime is assumed to occur when prices are more flexible.[5]

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \alpha\beta\mathbb{E}_t \Delta q_{t+1} - \alpha \Delta q_t + \frac{\kappa(s_t)}{(\tau+\lambda)}\hat{y}_t + \frac{\kappa(s_t)\lambda}{\tau(\tau+\lambda)}\hat{y}_t^* \tag{12}$$

$$\hat{R}_t = \rho_r(\hat{R}_{t-1}) + (1-\rho_r)[\psi_\pi(s_t)(\hat{\pi}_t) + \psi_y(s_t)(\Delta\hat{y}_t + \hat{z}_t) + \psi_e(s_t)e_t + \varepsilon_t^R], \varepsilon_t^R \sim NID(0, \sigma_R^2(S_t))$$

$$s_t \in \{1,2\},$$

$$\kappa(s_t=1) > \kappa(s_t=2)$$

$$\psi_\pi(s_t=1) < \psi_\pi(s_t=2)$$

$$\begin{array}{rl} Low\ degree\,Nominal\,Rigidities\,\&\,Accomodative\,Monetary\,Policy & s_t=1 \\ High\ degree\,Nominal\,Rigidities\,\&\,Inflation\,Targeting\,Monetary\,Policy & s_t=2 \end{array} \left\{ \begin{array}{c} \kappa_1, \psi_1^\pi, \psi_1^y, \psi_1^e \\ \kappa_2, \psi_2^\pi, \psi_2^y, \psi_2^e \end{array} \right\}.$$

This normalisation condition implies that periods of relatively accommodative monetary policy are coupled with periods of low price stickiness coincide. This condition is motivated by the theory that firms adjust prices more frequently during high inflation periods in addition to the assumption of high inflation periods being linked to accommodative monetary policy.

[4]This feature is conditional on the relative differences of the inflation response ψ_1^π and ψ_2^π across regimes and also the duration of the regime with an accommodative stance. These conditions are known as the generalised Taylor principle following Davig and Leeper (2007).

[5]Chen and MacDonald (2012) model comparison exercise involves two alternative switching models that allow for changes in monetary policy with constant shock volatility and consistent monetary policy with changes in shock volatility.

Switching volatility of shocks

It is important to highlight that all models allow for changes in the variances of structural shocks as a change in economic volatility is one of the key arguments to explain the great moderation. The latent variable s_t governs the regime of shock volatilities in place and follows an independent Markov process

$$\sigma_1^X(s_t = 1) > \sigma_2^X(s_t = 2),\ X \in \{R, z, q, y^*, \pi^*\}$$

$$\begin{array}{ll} \textit{High volatility} & S_t = 1 \\ \textit{Low volatility} & S_t = 2 \end{array} \left\{ \begin{array}{l} \sigma_1^R, \sigma_1^z, \sigma_1^q, \sigma_1^{y^*}, \sigma_1^{\pi^*} \\ \sigma_2^R, \sigma_2^z, \sigma_2^q, \sigma_2^{y^*}, \sigma_2^{\pi^*} \end{array} \right\}.$$

In principle, each shock could change regime according to their own independent Markov chains. However, this creates identification issues for each regime which relate the length of time in a regime and the interpretation of individual shock regimes. Therefore, I focus on models with synchronised switching in volatilities following Davig and Doh (2014). Allowing for two regimes for the variance of shock should suffice to describe the overall volatility of the economy as when looking in the models that allow for changes in monetary policy or nominal rigidities there will be four different covariance matrices to effectively pin down the variance of the data.

In this model it is useful to note that solution methods for the constant parameter model are applied as shocks do not have real effects on the decisions made by economic agents in the model.

4 Solving and estimating a Markov-switching DSGE model

Maintaining the rational expectations hypothesis at the centre of DSGE models while allowing areas of the economy to change, introduces complexities in solution and estimation. This paper applies the solution algorithm of Farmer, Waggoner and Zha (2011). This section gives a brief explanation of the chosen solution algorithm and discusses alternative methods.

4.1 Solution method

Agents in the MSDSGE model know the probability of moving across regimes, and they use this information when forming expectations. Standard solution methods no longer apply, as the solution of the model now not only depends on the structural parameters but additionally, the regime in place and the probability of moving across regimes.

Obtaining the optimal decision rules of agents in MS-DSGE models involves solving a quadratic polynomial that in turn gives rise to multiple solutions that can be unbounded. A literature has developed around identifying the full and concentrating on a subset of solutions and conditions to determine there uniqueness and stability. Davig and Leeper (2007) and Farmer, Waggoner and Zha (2008) describe a class of minimal state variable (MSV) solutions following McCallum (1983) and rewrite the non-linear model to have a constant parameter representation which can then be solved by the standard algorithms used on constant parameter models such as the *Gensys* algorithm of Sims (2002). Both studies define conditions that provide a unique solution. However, Famer, Waggoner and Zha (2011), demonstrate that the determinacy conditions of Davig and Leeper (2007) do not hold when considering the non-linear model. Farmer, Waggoner and Zha (2009a) prove that any MSV solution is also a solution to the original non-linear model. In turn, Farmer, Waggoner and Zha (2011) show that the algorithm in Farmer, Waggoner and Zha (2008) is not always able to find solutions even though they exist.[6] Farmer et al. (2011) also show that solution methods dealing with the non-linear system directly such as the functional iteration algorithm of Svensson and Williams (2009) also do not always find solutions when they exist. Cho (2016) applies a functional iteration approach and introduces forward conditions that deduce stability from solving the model ahead. Farmer et al. (2010) propose an algorithm that employs Newton methods and can identify a full set of MSV equilibria. This algorithm is the one considered in this chapter. Although the authors provide some instruction on how to choose between different solutions I only consider parameter estimates that yield a unique solution.[7]

[6]The Davig and Leeper (2007) algorithm rewrites the model to form a linear representation by introducing regime dependent state-variables and stacking the model conditional on each regime in place and Sims (2001) algorithm Gensys solves the now linear model. This algorithm was used in an earlier version of this paper and is discussed in the appendix as it provides a convenient introduction to solving MS-DSGE models.

[7]Farmer et al. (2009b) suggest the data fit should be used to select from multiple solutions.

The above methods focus on models in which Markov-switching parameters do not affect the steady-state. Dealing with regime-dependent steady-state parameters and the effect of regime change on uncertainty requires the use of perturbation methods which have only recently been extended to the case of Markov-switching by Maih (2015), Barthèlemy and Marx (2017) and Forester et al. (2016) and is discussed in Zakipour-Saber (2019).

The Markov-switching models presented in section 3 can be expressed in the following form:

$$\overset{A}{\begin{bmatrix} \underset{(n-l)\times n}{a_1(s_t)} \\ \underset{l\times n}{a_2(s_t)} \end{bmatrix}} \underset{n\times 1}{\hat{x}_t} = \overset{B}{\begin{bmatrix} \underset{(n-l)\times n}{b_1(s_t)} \\ \underset{l\times n}{b_2(s_t)} \end{bmatrix}} \underset{n\times 1}{\hat{x}_{t-1}} + \overset{C}{\begin{bmatrix} \underset{(n-l)\times k}{c_1(s_t)} \\ \underset{l\times k}{c_2(s_t)} \end{bmatrix}} \underset{k\times 1}{\varepsilon_t} + \overset{D}{\begin{bmatrix} \underset{(n-l)\times l}{d_1(s_t)} \\ \underset{l\times l}{d_2(s_t)} \end{bmatrix}} \underset{k\times 1}{\eta_t}, \tag{13}$$

where x_t is a $n \times 1$ vector of model state variables, $a_1, a_2, b_1, b_2, c_1, c_2, d_1$ and d_2 are conformable parameter matrices that are regime-dependent, ε_t is a $k \times 1$ vector of i.i.d. stationary exogenous shocks, and η_t is an $l \times 1$ vector of expectational errors. The variable s_t is an exogenous stochastic process that determines the monetary policy regime or degree of nominal rigidity in place and follows a two-regime Markov chain. Therefore, I concentrate on the case of two regimes when describing the solution algorithm of Farmer, Waggoner and Zha (2011). It is important to note that I deal with the first-order approximation of the underlying non-linear system and in this case, the presence of switching volatilities of structural shocks does not affect the model solution due to the presence of certainty equivalence.

The state variables $x_t =[y_t', \, z_t', \, \mathbb{E}_t y_{t+1}']$ is partitioned to separate forward looking variables, the first pair $[y_t', \, z_t']$ is of dimension $n-l$. Specifically, x_t consists of an endogenous component y_t and z_t represents the predetermined components consisting of lagged and exogenous variables. The second block of x_t is of the form $y_t = \mathbb{E}_{t-1} y_t + \eta_t$.

The algorithm of Farmer, Waggoner and Zha (2011) concentrates on MSV solutions of equation 13, the first requirement of this solution class is that it be fundamental and cannot contain a sunspot. The MSV solution takes the following form:

$$\hat{x}_t = V_{s_t} F_{1,s_t} \hat{x}_{t-1} + V_{s_t} G_{1,s_t} \varepsilon_t \tag{14}$$

$$\eta_t = -(F_{2,s_t} \hat{x}_{t-1} + G_{2,s_t} \varepsilon_t) \tag{15}$$

where the matrix $[A(i) V_i \, D]$ is invertible and

$$[A(i) V_i \, D] \begin{bmatrix} F_{1,i} \\ F_{2,i} \end{bmatrix} = B(i) \tag{16}$$

$$[A(i) V_i \, D] \begin{bmatrix} G_{1,i} \\ G_{2,i} \end{bmatrix} = C(i) \tag{17}$$

$$\left(\sum_{i=1}^{2} p_{i,j} F_{2,i} \right) V_j = \mathbf{0_{l,n-l}}. \tag{18}$$

The dimension of V_i is $n \times (n-l)$, $F_{1,i}$ is $(n-l) \times n$, $F_{2,i}$ is $l \times n$, $G_{1,i}$ is $(n-l) \times k$, and $G_{2,i}$ is $l \times k$.

Equations 14 and 15 define the process $\{x_t, \eta_t\}_{t=1}^{\infty}$, equations 16 and 17 ensure that the process satisfies the Markov-switching model presented in equation 13. Equation 18 ensures that $\mathbb{E}_{t-1}[\eta_t] = 0$. To find an MSV equilibrium, the key is to find V_i. When V_i is known, equations 16 and 17 can be used to find $F_{1,i}, F_{2,i}, G_{1,i}, G_{2,i}$. If V_i and $F_{2,i}$ satisfy equation 18, then this is a candidate MSV equilibrium. It still must be verified that the solution is stationary, Farmer et al. (2011) verify stationarity using the mean-square-stable definition of Costa et al. (2004). As shown in Costa et al. (2004), the candidate MSV solution is stationary if and only if the eigenvalues of

$$(P \otimes I_{n^2}) diag(V_1 F_{1,1} \otimes V_1 F_{1,1}, V_2 F_{1,2} \otimes V_2 F_{1,2})$$

are all inside the unit circle.

Since $D = [0_{l,n-l} I_l]'$, the matrix $[A(i) V_i \, D]$ is invertible if and only if, the upper $(n-l) \times (n-l)$ block of $A(i) V_i$ is invertible. Without loss of generality, equation 18 can be written as

$$\sum_{i=1}^{2} p_{i,j} [X_i \, I_l] B(i) A(j)^{-1} \begin{bmatrix} I_{n-l} \\ -X_j \end{bmatrix} = 0_{l,n-l}.$$

for some $l \times (n-l)$ matrix X_j. The method of Farmer, Waggoner and Zha (2011) reduces the task of finding an MSV solution to that of computing the roots of a quadratic polynomial in several variables. Farmer, Waggoner and Zha (2011) apply Newton's method to calculate these roots due to savings in computational time. The authors suggest that by choosing a large enough grid of initial conditions, all possible MSV solutions can be found. Maih (2015) extends this algorithm to improve the efficiency of the newton procedure to deal with larger models. However, for the model considered I find that the Farmer, Waggoner and Zha (2011) algorithm converges quickly.

Let $X = (X_1, X_2)$ reflect the case of two regimes, define f_j to be the function from $\mathbb{R}^{2l(n-l)}$ to $\mathbb{R}^{l(n-l)}$ given by

$$f_j(X) = \sum_{i=1}^{2} p_{i,j}[X_i\, I_l]B(i)A(j)^{-1} \begin{bmatrix} I_{n-l} \\ -X_j \end{bmatrix},$$

and f be a function from $\mathbb{R}^{2l(n-l)}$ to $\mathbb{R}^{2l(n-l)}$ given by

$$f(X) = (f_1(X), f_2(X)).$$

The quadratic polynomial equations, $f(X) = 0$, are the same as the constraints represented in equation 18. This implies that finding an MSV equilibrium is equivalent to finding the roots of $f(X)$ and the authors construct the following constructive algorithm for finding MSV solutions.

The Farmer, Waggoner and Zha (2011) algorithm starts with an initial guess $X^{(1)} = (X_1^{(1)}, X_2^{(1)})$. If the kth iteration is $X^{(k)} = (X_1^{(k)}, X_2^{(k)})$, then the $(k+1)^{th}$ iteration is given by

$$vec(X^{(k+1)}) = vec(X^{(k)}) - f'(X^{(k)})^{-1} vec(f(X^{(k)}))$$

where

$$f'(X) = \begin{bmatrix} \frac{\delta f_1}{\delta X_1}(X) & \frac{\delta f_1}{\delta X_2}(X) \\ \frac{\delta f_2}{\delta X_1}(X) & \frac{\delta f_2}{\delta X_2}(X) \end{bmatrix}.$$

The sequence $X^{(k)}$ converges to a root of $f(X)$.

To obtain the initial guess $X^{(1)} = (X_1^{(1)}, X_2^{(1)})$, I follow the procedure described in Farmer Waggoner and Zha (2011).

To be consistent with the rest of the thesis the MSV solution shown in equation 14 is relabeled as

$$\hat{x}_t = \mathbf{\Omega}(s_t)\hat{x}_{t-1} + \mathbf{\Delta}(s_t)v_t,\ v_t \sim N(0, \Sigma_v)$$

where $\mathbf{\Omega}(s_t)$ and $\mathbf{\Delta}(s_t)$ are the solution matrices that are a function of the model parameters.[8]

4.2 Estimation

The model solution in equation 14 is combined with a measurement equation to form the following state-space representation

$$y_t = \mathbf{H}\hat{x}_t$$
$$\hat{x}_t = \mathbf{\Omega}(s_t)\hat{x}_{t-1} + \mathbf{\Delta}(s_t)v_t,\ v_t \sim N(0, \Sigma_v)$$

where y_t represents the data used for estimation and $\hat{x}_t$ describes the model state variables. Where $\mathbf{\Omega}(s_t)$ and $\mathbf{\Delta}(s_t)$ are the solution matrices that are a function of the model parameters.

The filtering algorithm proposed by Kim (1994) is used to approximate the likelihood of the MS-DSGE model. The presence of different regimes causes the predictions of the Kalman filter to be conditional on the entire history of regimes in place throughout the sample leaving the standard filter intractable. To address the problem of predictions dependent on the history of regimes, Kim (1994) suggests to focus on a limited number of predictions of model variables, which are carried forward from the Kalman filter iterations of each period, and these are then collapsed at the end of each iteration. Therefore, the algorithm proposed by Kim (1994) approximates the likelihood function. Following Kim and Nelson (1999) and Liu and Mumtaz (2011), I track forecasts that depend on the regime in place in period t, $t-1$ and $t-2$ and accounts for eight possible paths of the of model variables. The probabilities assigned to

[8] The solution from section 4 $\hat{x}_t = V_{s_t}F_{1,s_t}\hat{x}_{t-1} + V_{S_t}G_{1,S_t}\varepsilon_t$ is rewritten $\hat{x}_t = \mathbf{\Omega}(s_t)\hat{x}_{t-1} + \mathbf{\Delta}(s_t)v_t$ to simplify the definition of the inflation persistence measures in the following section 6.

each path are used as weights to perform a weighted average and are obtained using the filter proposed by Hamilton (1989). The algorithm is described in more detail in the appendix.

Table 1: Prior distributions of MS-DSGE models

	Mean	Variance	Domain	Density
α	0.2	0.05	[0,1)	Beta
τ	1.5	0.4	$\mathbb{R}^+$	Gamma
α	0.2	0.05	[0,1)	Beta
ψ_π	1.5	0.5	$\mathbb{R}^+$	Gamma
ψ_y	0.25	0.25	$\mathbb{R}^+$	Gamma
ψ_e	0.25	0.25	$\mathbb{R}^+$	Gamma
κ	0.5	0.2	$\mathbb{R}^+$	Gamma
τ	1.5	0.4	$\mathbb{R}^+$	Gamma
α	0.2	0.05	[0,1)	Beta
ρ_z	0.2	0.1	[0,1)	Beta
ρ_{y^*}	0.2	0.1	[0,1)	Beta
ρ_{y^*}	0.2	0.1	[0,1)	Beta
$\rho_{\triangle q}$	0.2	0.1	[0,1)	Beta
σ_R^2	0.5	5	$\mathbb{R}^+$	Inverse Gamma
σ_z^2	0.85	5	$\mathbb{R}^+$	Inverse Gamma
$\sigma_{y^*}^2$	1.5	5	$\mathbb{R}^+$	Inverse Gamma
$\sigma_{\pi^*}^2$	2.55	5	$\mathbb{R}^+$	Inverse Gamma
σ_q^2	1.2	5	$\mathbb{R}^+$	Inverse Gamma
q_{11}	18	1	[0,1)	Dirichlet
q_{22}	18	1	[0,1)	Dirichlet

Notes: This table displays the prior distribution of the model parameters. The first and second columns represent the means and standard deviations for Gamma, Normal, and Beta distributions; s and ν for the Inverse-Gamma Type-I (IG-1) distribution with density $p(\sigma) \propto \sigma^{-v-1}\exp(\frac{vs^2}{2\sigma^2})$. The third column represents the range of the support for the parameters. The final column displays the prior distribution. The Markov transition probabilities are assumed to follow a Dirichelet distribution with α1 = 18 and α2 = 1. This gives the probability of staying in the same regime to be 0.95.

Priors

The Bayesian approach involves incorporating prior information in the form of priors when estimating model parameters and can exclude implausible estimates of parameters. The priors includes information, which is not contained in the data and augments the likelihood. As the likelihood of DSGE models may often not be globally concave, prior information may resolve this problem as described in Lindley (1971). However, caution should be taken as even in a Bayesian framework the existing nonlinearities in DSGE models induce difficulties in identification as discussed in Canova and Sala (2009) and Koop, Pesaran, and Smith (2013).

The prior distribution for the model parameters is set broadly following Lubik and Schorfheide (2007). The priors for the transition probabilities follow a Dirichlet prior distribution as in Liu and Mumtaz (2011).[9] Table 1 displays the prior distributions of all model parameters.

The discount factor is calibrated at 0.998 as this parameter is widely agreed upon by the literature. The degree of openness α follows a beta distribution that is set around a mean of 0.2 reflecting the sample average of imports and exports for the United Kingdom following Liu and Mumtaz (2011). The intertemporal elasticity of substitution τ follows a Gamma distribution of mean 1.5 and has a relatively large standard deviation of 0.5 to reflect a wide range of estimates. The parameters of the policy rule ρ_r,ψ_π,ψ_e,ψ_y are comparatively loose compared to Lubik and Schorfheide (2007) to reflect a wide range of estimates reported in MS-DSGE models. In addition, the prior distribution for Phillips curve slope coefficient κ, is chosen to follow a gamma distribution with a mean of 0.5 and variance of 0.2 to be consistent with the range of values typically found in the New Keynesian DSGE literature. The prior distribution for the exogenous shocks and the processes describing rest-of-the-world output y_t^*, rest-of-the-world inflation π_t^* and terms of trade $\triangle q_t$ are consistent with Lubik and Shorfheide (2007). The priors of model parameters are the same for each model and do not depend on the regimes in place, this assumption allows the data decide to on the magnitude of regime changes. The transition probabilities are assumed to have a Dirichlet distribution with $\alpha_1 = 18$ and $\alpha_2 = 1$ following Liu and Mumtaz (2011),

[9] The parameters for the Dirichlet prior are assumed to be $\alpha_1 = 18$ and $\alpha_1 = 1$, implying a probability of staying in the same regime to be 0.95. Let $\alpha_0 = \alpha_1 + \alpha_2$, the mean of the Dirichlet distribution is $E(x_i) = \frac{\alpha_i}{\alpha_0}$

implying the probability of staying in the same regime is 0.95 and an expected duration of each regime to be 20 quarters.

MCMC algorithm

The prior distributions and the likelihood are combined to approximate the posterior distribution of model parameters. A combination of numerical optimisers is used to find the mode of the posterior to initiate the MCMC algorithm. The simplex algorithm is first applied to refine starting values that are used as input for Chris Sims' optimisation routine CSMINWEL, this follows Mumtaz and Liu (2011). The simplex algorithm refines values by using a non-derivative based and the CSMINWEL algorithm updates the derivative approach of Broyden, Fletcher, Goldfrab and Shannon method to deal which extreme changes in likelihood during the parameter search.[10]

The posterior mode is used to initiate the Metropolis-Hastings algorithm. I run 100,000 replications, burning the first 50,000 and then save every 10th draw to leave 5000 draws that form the approximate posterior. The convergence of the algorithm is assessed by computing the recursive mean of parameter draws of the best-fitting model as in Mumtaz and Liu (2011) which are plotted in the appendix. The sequence of draws for most parameters fluctuate around a relatively stable mean providing some evidence of convergence to the posterior distribution.

4.3 Data

Quarterly observations of U.K. data from 1965Q1-2009Q1 are used to estimate the models. Specifically, real GDP, CPI, export and import price deflators, the nominal effective exchange rate and the annualised nominal interest rate are used and obtained from the National Office of Statistics and Global Financial Data.[11] The change in terms of trade $\triangle q_t$ is computed by taking the first log difference of the relative price of exports in terms of imports, calculated using the respective price deflators. The nominal interest rate R_t is converted to quarterly rates. CPI inflation π_t and Exchange rate depreciation $\triangle e_t$ are quarter on quarter log differences of CPI and the nominal effective exchange rate. The first difference of Real GDP provides the growth rate used as an observable. All series are then demeaned and scaled by 100 following Lubik and Schorfheide (2007).

The data is related to the state variables via the measurement equation

$$y_t = H\hat{x}_t$$

$$\begin{pmatrix} Real\,GDP\,Growth \\ CPI\,Inflation_t \\ Exchange\,Rate\,Depreciation_t \\ Interest\,Rate_t \\ Change\,in\,Terms\,of\,trade_t \end{pmatrix} = 100 \begin{pmatrix} \hat{y}_t - \hat{y}_{t-1} + z_t \\ \hat{\pi}_t \\ \triangle\hat{e}_t \\ \hat{R}_t/4 \\ \triangle q_t \end{pmatrix}.$$

The growth rate of real GDP is related to the stationary model variable aggregate domestic output growth $\hat{y}_t - \hat{y}_{t-1}$ in addition to the quarterly growth rate of technology z_t to take into the stochastic trend in output induced by the non-stationary world technology process. The transformed data Y_t is displayed in Figure 1.

4.4 Model comparison

Before examining movements in inflation persistence, it is essential to determine which areas of the economy were subject to regime change, if any. To assess the relative model fit, I compute the marginal likelihood using the harmonic mean method of Gelfand and Dey (1994) for each version of the estimated DSGE model following Bianchi (2009), Davig and Doh (2008) and Liu and Mumtaz (2011).

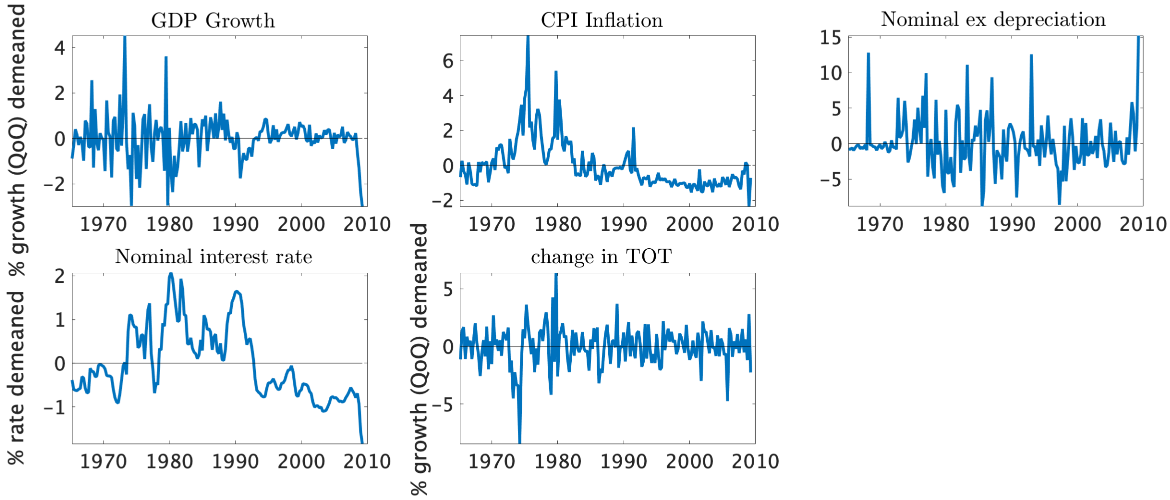
The marginal likelihood, is obtained by integrating the approximate posterior density of the entire parameter space in each model and is expressed as

$$\Pr(Y_t) = \int_\theta f(Y_t|\theta)\Pr(\theta)$$

[10] Dejong and Dave (2011) provide a insightful description of optimaisation procedures.

[11] The dataset closely follows Liu and Mumtaz (2011) and I am grateful to the authors for making their data publicly available.

Figure 1: Demeaned U.K. data from 1970-2009



Note: This figure displays the transformed data used for estimation.

where θ denotes the model parameters, $f(Y_t|\theta)$ is the likelihood while $\Pr(\theta)$ represents the prior distributions. The marginal likelihood can be approximated using the modified harmonic mean (MHM) method of Gelfand and Dey (1994), which employs the following theorem

$$\frac{1}{\Pr(Y_t)} = \int_{\Theta} \frac{h(\theta)}{f(Y_t|\theta)\Pr(\theta)} f(\theta|Y_t)d\theta$$

where $h(\theta)$ denotes a weighting function, i.e. a probability density function whose support is in Θ. Numerically the integral above can be evaluated as

$$\frac{1}{\Pr(Y_t)} = \sum_{i=1}^{N} \frac{h(\theta^i)}{f(Y_t|\theta^i)\Pr(\theta^i)}$$

where $i = 1...N$ indexes the draws from the MCMC samples. Following Geweke (1999) a normal density is used as the weighting function. Although Sims, Waggoner and Zha (2008) suggest an elliptical density is more appropriate for comparing time-varying DSGE model. However, in practice Davig and Doh (2014) and Mumtaz and Liu (2011) find that the resulting estimate of the marginal likelihood using a elliptical density is unstable and highly sensitive to draws away from the posterior.

Table 2 presents the estimated value of each model considered, where a higher marginal likelihood indicates a preference for the model by the data. It is clear from the comparison that the data rejects the time-invariant model. Within, the models that allow for regime change, imposing that the degree of nominal rigidity changes with monetary policy generates the lowest marginal likelihood and hence, the poorest data-fit, therefore, is not used to examine inflation persistence, the parameter estimates and filter probabilities of this model are presented in the appendix.[12] The models that allow for changes in the Phillips curve slope and only changes in the variance of structural shocks provide only small improvements in marginal likelihood. The finding of small improvements to allowing for changes in the price stickiness is in line with Lhuissier and Zabelina (2015). The preferred model allows for independent regimes in monetary policy and the volatility of shocks. This finding suggests that for the U.K. economy, a change in the policy rule as well as a change in shock variances is a crucial feature of the data and supports the findings of Liu and Mumtaz (2011).

Figure 2 shows that the preferred model with independent monetary and volatility regimes has a reasonably accurate in-sample fit for the inflation series. The solid blue line plots demeaned inflation, and the red-dashed line represents the one-step-ahead Kim-filter prediction using posterior median estimates.

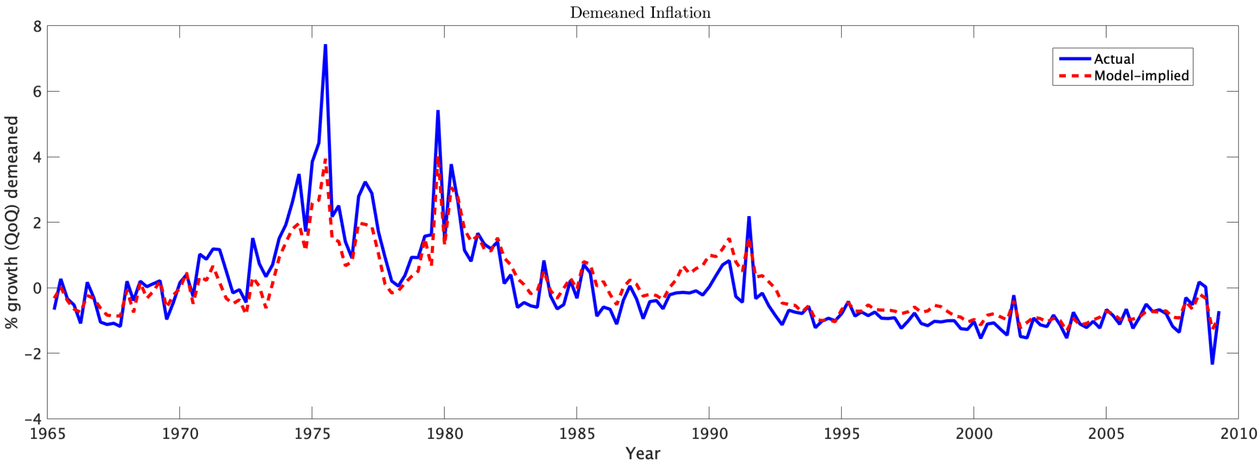
[12]This finding contrasts with Chen and MacDonald (2012), who find the model which allows of simultaneous changes in monetary policy and nominal rigidity, has the highest marginal likelihood. However, I consider a broader range of competing models and a more extended sample period.

Table 2: Log marginal likelihood of estimated models

	Marginal likelihood
Time-invariant	-1414.13
Shock variances	-1323
Monetary policy	**-1273**
Nominal rigidities	-1318
Monetary policy and nominal rigidities	-1333

Note: This table presents the marginal likelihood in order to find the best model that describes the UK data. The model preferred by this measure has the largest marginal likelihood.

Figure 2: Actual versus predicted inflation from best-fitting MS-DSGE model



Note: The solid blue line plots demeaned inflation, and the red-dashed line represents the one-step-ahead Kim-filter prediction using posterior median estimates. The best fitting MS-DSGE model allows for independent changes in the monetary policy rule and volatility of all structural shocks.

5 Parameter estimates

Table 3 presents the median of the posterior distribution of parameter estimates across the models of the preferred switching model alongside the time-invariant model and sub-sample estimation of the model, with the 90% Bayesian credible intervals in square brackets.

5.1 Time-invariant model

The results of the constant parameter model are broadly consistent with studies considering similar models and sample periods, such as Lubik and Schorfheide (2007) and Liu and Mumtaz (2011).

The inter-temporal elasticity of substitution τ is estimated at 0.21, which is lower than the estimates of Lubik and Schorfheide (2007) however, the credible intervals are similar to the time-invariant model estimates of Chen and MacDonald (2012). The estimated degree of openness in the U.K. economy α which is 0.11 is larger than the value of Lubik and Schorfheide (2007) and closer to the calibrated value of 0.185 in Liu and Mumtaz (2011).[13] The estimated interest rate smoothing parameter ρ_r is 0.81, indicating the Bank of England's preference for gradual policy implementation. The Taylor rule inflation parameter ψ_π is estimated at 1.48, implying a relatively aggressive monetary policy stance towards inflation movements. The Taylor rule exchange rate depreciation parameter ψ_e is estimated at 0.126. This indicates that U.K. monetary policy directly responds to movements in exchange rates and is line with Lubik and Schorfheide (2007). The finding of a significant monetary policy to exchange rates is a reasonable finding as the sample consists of explicit currency pegs such as the European Exchange Rate Mechanism in 1990-1992 and the implicit peg to the dollar implied by Bretton Woods which ended in 1971 when the pound became free-floating. The estimated monetary policy response to a percentage increase in the level of output gap in isolation implies a 0.5% increase in nominal interest rate. This is substantially larger than the constant parameter estimates of Lubik and Schorfheide (2007) and Liu and Mumtaz but consist with those of Chen and MacDonald (2012). The estimated posterior mean of the Phillips curve slope parameter κ is 0.34 and lies between those reported in Lubik and Schorfheide (2007) and Chen and MacDonald (2012). The persistence of structural shocks $\rho_z, \rho_{y^*}, \rho_{\pi^*}, \rho_q$ and their variances are consistent with Lubik and Schorfheide (2007), the global output shock has the greatest degree of persistence an is close to following a unit root process. The global inflation shock is estimated as the most volatile with a variance $\sigma^2_{\pi^*}$ of 3.63 and the monetary policy shock has the lowest volatility with an estimated variance σ^2_r of 0.38.

5.2 Sub-sample analysis

The subsamples represent periods before and after the adoption of inflation-targeting monetary policy 1965Q1-1992Q4 and 1993Q1-2009Q1, respectively. Comparing estimates of the two sub-samples raises doubt on the time-invariant modeling of the parameters describing the Taylor rule, household preferences and shocks volatilities. However, as previously sub-sample estimation does not allow agents to incorporate the possibility of regime change in their expectations. The Taylor rule inflation parameter ψ_π increases from 1.27 to 2.28 in the post-1992 subsample. In the pre-1992 subsample the lower bound of the credible interval of ψ_π is 1.01. A value of 1.01 is the lowest value of this parameter can take in time-invariant models that guarantees a unique and stable solution.[14]

The mean policy responses to the rate of output growth and interest rate smoothing decrease in the post-1992 subsample, however, the credible sets overlap considerably suggesting that this decrease is not significant. In contrast, the monetary policy response to exchange rates decreases from 0.16 to 0.04. The parameter values of κ substantially decreases during the second subsample which support the theory that prices are less rigid during high-inflation periods. The inter-temporal elasticity of substitution also increases considerably from 0.24 to 0.58. The mean estimates of the degree of openness increase from 0.14 to 0.21. Volatilities of all structural shocks decreases in the post-1992 subsample, with monetary policy and technology shocks being subject to the largest decreases, relative to their pre-1992 estimates. The decrease in shock volatiles implies that monetary policy was not the only area of the economy to change after 1992 and highlights the importance of isolating these changes when trying to examine sources of inflation persistence.

[13] Liu and Mumtaz calibrate α to 0.185 reflecting the sample average of imports and exports for the United Kingdom during there sample between 1970Q1 to 2009Q1.

[14] This proposition has been termed as the Taylor principal. Davig and Leeper (2007) show that in Markov-switching models the Taylor principal does not have to hold in one monetary policy regime allowing for possible passive responses to inflation, this is subject to conditions relating to the time spent in this regime and the active that the authors label as the generalised Taylor principal.

Table 3: Parameter estimates from time-invariant models versus best fitting MS-DSGE model

	Time-invariant	Subsample		Monetary Policy	
		1965Q1-1992Q4	1993Q1-2009Q1	0.94 [0.87,1.13]	0.63 [0.46,1.04]
α	0.11 [0.08,0.14]	0.14 [0.09,0.20]	0.22 [0.16,0.28]	0.13 [0.11,0.16]	
τ	0.21 [0.14,0.29]	0.24 [0.16,0.34]	0.59 [0.46,0.71]	0.16 [0.13,0.18]	
κ	0.34 [0.09,0.59]	1.84 [1.18,2.54]	0.44 [0.11,0.97]	0.8 [0.46,1.04]	
ρ_r	0.81 [0.76,0.86]	0.78 [0.70,0.85]	0.73 [0.63,0.80]	0.80 [0.77,0.83]	
ψ_π	1.48 [1.08,1.99]	1.27 [1.01,.1.71]	2.28 [1.68,2.93]	0.91 [0.76,.1.16]	1.97 [1.59,2.23]
ψ_y	0.51 [0.27,0.80]	0.48 [0.21,0.98]	0.56 [0.30,0.90]	0.54 [0.42,0.68]	0.6 [0.36,0.8]
ψ_e	0.13 [0.07,0.20]	0.16 [0.09,0.27]	0.04 [0.02,0.09]	0.31 [0.21,0.46]	0.06 [0.03,0.10]
ρ_z	0.49 [0.32,0.63]	0.40 [0.20,0.59]	0.56 [0.47,0.65]	0.44 [0.35,0.55]	
ρ_{y^*}	0.93 [0.90,0.97]	0.91 [0.86,0.97]	0.94 [0.85,0.97]	0.96 [0.95,0.98]	
ρ_{π^*}	0.30 [0.21,0.38]	0.29 [0.18,0.41]	0.62 [0.46,0.77]	0.21 [0.17,0.26]	
ρ_q	0.11 [0.04,0.19]	0.13 [0.06,0.23]	0.13 [0.04,0.24]	0.1 [0.05,0.16]	
σ_R^2	0.38 [0.33,0.44]	0.47 [0.39,0.58]	0.29 [0.24,0.36]	0.43 [0.38,0.48]	0.22 [0.2,0.25]
σ_z^2	0.82 [0.68,0.98]	1.03 [0.81,1.27]	0.45 [0.38,0.53]	1.1 [0.99,1.25]	0.28 [0.25,0.33]
$\sigma_{y^*}^2$	1.12 [0.78,1.66]	1.29 [0.82,2.29]	1.27 [0.85,1.79]	0.54 [0.43,0.69]	0.2 [0.17,0.25]
$\sigma_{\pi^*}^2$	3.63 [3.29,3.94]	3.84 [3.47,4.25]	3.19 [2.72,3.71]	4.39 [4.01,4.83]	2.22 [2.00,2.44]
σ_q^2	1.72 [1.58,1.88]	1.87 [1.68,2.06]	1.37 [1.2,1.58]	1.93 [1.79,2.08]	0.89 [0.57,1.24]
p_{11}	0.98 [0.97,0.99]	-		0.98 [0.97,0.99]	
p_{22}	0.95 [0.92,0.98]	-		0.95 [0.92,0.98]	
q_{11}	0.93 [0.87,0.96]	-		0.93 [0.87,0.96]	
q_{22}	0.95 [0.91,0.97]	-		0.95 [0.91,0.97]	

Note: The first column presents the baseline time-invariant rational expectation model, the second column presents estimates of the same time-invariant model across subsample split pre and post inflation-targeting, and the third column presents the results of the switching. The posterior median parameter estimates are displayed alongside 90% credible interval in square brackets. This comparison is made to highlight the impact of only allowing a particular subset of parameters to switch freely over time as opposed to sub-sample estimation that allows for a one-off change in potentially all parameters. The best fitting MS-DSGE model allow for independent changes in the monetary policy rule and volatility of all structural shocks.

5.3 Monetary policy regimes and switching shock volatility

The model preferred by the data identifies two monetary policy regimes and two independent volatility regimes. The first monetary regime characterises a weak reaction to inflation with the posterior mean of ψ_π estimated at 0.91. This value is worth highlighting as it implies when considering an increase in inflation in isolation, the Bank of England would raise interest rates less than one for one and therefore, suggests an accommodative stance towards inflation. The probability of remaining in this regime p_{11} is estimated at 0.95 implying an expected duration of 20 quarters.[15] The second regime responds relatively aggressively towards inflation with a posterior mean of ψ_π estimated at 1.97. This regime is relatively persistent with p_{22} estimated at 0.98 implying an expected duration of 50 quarters.[16] I label the second monetary policy regime as inflation-targeting due to the greater emphasis placed on inflation reflected in the significantly larger values of ψ_π. The point estimates and credible sets of the monetary policy reaction to changes in the output gap ψ_y do not suggest any significant change across regimes, this again supports the finding of Chen and MacDonald (2012). Whereas, the policy reaction to exchange rates is statistically significant in the first regime (henceforth, accommodative) and the posterior mean estimate of ψ_e being 0.33 and contrasts with the estimate in the inflation-targeting regime of 0.06. The policy weight given to exchange rates almost disappears in the inflation-targeting regime. High and low volatility regimes are significantly identified and broadly resemble subsample estimates. The constant parameters are consistent with the estimates of the time-invariant model.

The top left panel of Figure 3 plots the posterior mean of the filter probabilities of the inflation-targeting regime being in place and provides a historically consistent version of events. From the beginning of the sample until to the mid-1970s the probability leans more towards the accommodative monetary policy regime is in place. From 1975 until the end of the 1980s the accommodative regime is identified to be in place, after the breakdown of Bretton Woods, the pound was freely floating, however, polices during the time were more focused on influencing unemployment with price controls being in place.

The mean probability indicates that the inflation-targeting regime is in place throughout the 1990s until the end of the sample. It is worth highlighting that before the mid-1970s and the European Exchange rate Mechanism 1990-1992, the error bands increase and suggests the inflation-targeting and accommodative monetary policy regimes allowed for are not well identified in period of fixed-exchange rates.[17]

The top right panel plots the probability of the low volatility regime during the sample. The high volatility state appears to be in place during the 1970s and the mid-1980s. The mid-1980s until 1992 can be described by many sudden switches to low volatility, however, the error bands are large around these spikes in probability. After 1992 the low volatility regime is predominantly in place except in 2002 and the end of 2008, which can be explained by the dot-com bubble and the beginning of the global financial crisis. Table 4 presents the median of the posterior distribution of parameter estimates across the other alternative MSDSE models that allow for regime-switching, with the 90% Bayesian credible intervals in square brackets.

5.4 Nominal rigidity regimes and switching shock volatility

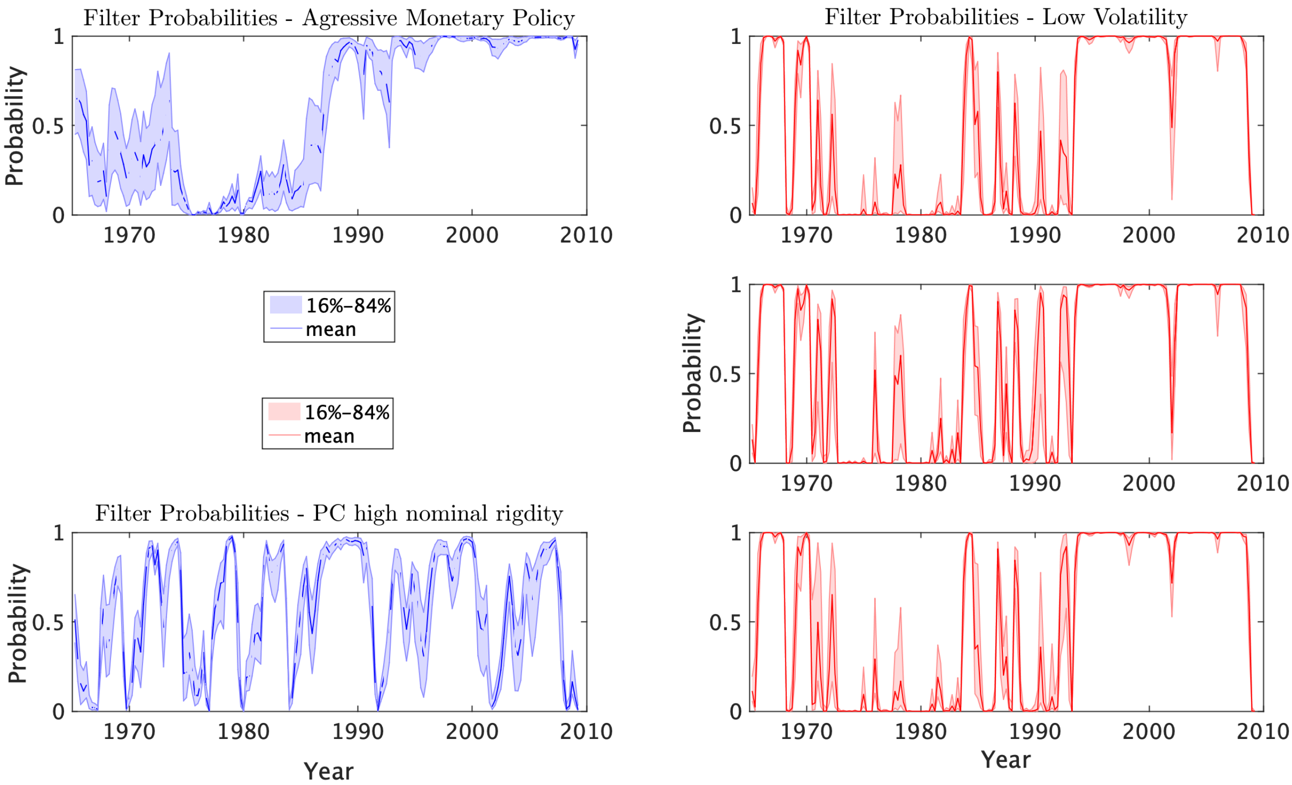
This model also allows for regime shifts along two dimensions. First, as in all switching models, the variance of the structural shocks is regime dependent. Secondly, the Phillips curve slope parameter κ is allowed to switch and can be interpreted as changes in the degree of price stickiness. The estimated parameters that do not switch are consistent with the estimates of the fixed parameter model. There is a significant shift in this parameter, identifying a low and high degree of nominal rigidity with posterior estimates of κ centered around 0.94 and 0.02, respectively. The bottom left panel of Figure 3 displays the filter probabilities of the high nominal rigidity regime. The degree of nominal rigidity is subject to frequent regime changes with the transition probability of the low nominal rigidity regime p_{11} estimated at 0.89, which implies an expected duration of 10 quarters and the estimate for the high nominal rigidity

[15]The expected duration of a regime is the inverse of $\frac{1}{1-p_{ij}}$.

[16]The estimates of ψ_π^1 and ψ_π^2 are in line with Chen and MacDonald (2012).

[17]The model considered in the paper does not take into account changes in rest of the world monetary policy. An alternative could be to use an open-economy model where the monetary policy of the rest of the world has a domestic effect such as Chin et al. (2018). In their model, U.S. data is used to proxy the rest of the world and has its own Taylor rule. However, U.S. monetary policy also changed during this sample period and distinguishing between changes in domestic and foreign monetary policy would require additional Markov changes that can increase computational burden of the model.

Figure 3: Filter probabilities from three MS-DSGE models



Note: Each row represents the filter probabilities of a specific model. The solid lines are the posterior mean filter probabilities and the shaded area represents the 68% credible sets. In the first row blue represents the probability of the aggressive monetary policy regime or the high nominal rigidity. In the third row blue represents the probability of the regime with relatively high nominal rigidity. In all rows red represents the probability of the low volatility regime.

Table 4: Parameter estimates of alternative MS-DSGE models

	Nominal rigidities		Shock volatilities	
α	0.1 [0.086,0.12]		0.13 [0.11,0.17]	
τ	0.15 [0.1,0.17]		0.19 [0.15,0.23]	
κ	0.94 [0.87,1.13]	0.02 [0.01,0.07]	0.64 [0.39,1.03]	
ρ_r	0.82 [0.78,0.86]		0.81 [0.78,0.84]	
ψ_π	1.24 [1.04,1.67]		1.50 [1,24,.1.74]	
ψ_y	0.41 [0.24,0.72]		0.44 [0.31,0.57]	
ψ_e	0.04 [0.03,0.07]		0.07 [0.05,0.11]	
ρ_z	0.37 [0.29,0.45]		0.42 [0.34,0.53]	
ρ_{y^*}	0.95 [0.93,0.97]		0.96 [0.94,0.98]	
ρ_{π^*}	0.27 [0.21,0.3]		0.23 [0.19,0.28]	
ρ_q	0.13 [0.09,0.16]		0.12 [0.07,0.16]	
σ^2_R	0.4 [0.37,0.46]	0.15 [0.14,0.17]	0.46 [0.42,0.51]	0.19 [0.17,0.22]
σ^2_z	1.09 [0.96,1.26]	0.36 [0.30,0.45]	1.00 [0.87,1.12]	0.28 [0.25,0.33]
$\sigma^2_{y^*}$	0.54 [0.43,0.67]	0.36 [0.30,0.45]	0.61 [0.48,0.79]	0.2 [0.17,0.25]
$\sigma^2_{\pi^*}$	4.66 [4.20,.5.40]	2.19 [2.00,2.45]	4.42 [4.08,4.80]	2.22 [2.00,2.44]
σ^2_q	1.97 [1.8,2.20]	1.2 [1.14,1.28]	1.99 [1.83,2.14]	1.2 [1.12,1.30]
p_{11}	0.89 [0.81,0.94]		-	
p_{22}	0.91 [0.89,0.94]		-	
q_{11}	0.93 [0.87,0.96]		0.92 [0.87,0.95]	
q_{22}	0.90 [0.87,0.94]		0.93 [0.87,0.97]	

Note: The first column of this table presents the estimates of MS-DSGE model which allow for changes in the degree of nominal rigidities and independent shock volatility regimes, the second column presents estimates of the MS-DSGE model that allows solely for regime changes in the volatility of structural shocks. The posterior median parameter estimates are displayed alongside 90% credible interval in square brackets.

regime p_{22} at 0.91. The probability of the high nominal rigidity regime is low during the mid-1970s, 1992 and then finally during the recent recession in 2008. This finding supports Liu and Mumtaz (2012) who associate a regime with low price stickiness with periods of low GDP growth and high inflation. The magnitudes and filter probabilities of the volatility regimes are identical to the previous model.

5.5 Switching shock volatilities

This model solely allows for the volatility of shocks to switch across two regimes. The time-invariant parameter estimates are similar to those shocks obtained in the fixed parameter specification. The parameter estimates of switching volatilities are identical to more flexible versions of the model which allow for additional sources of time-variation. Both high and low volatility regimes are persistent with transition probabilities p_{11} and p_{22} estimated at 0.92 and 0.93, respectively. The middle right panel presents the filter probabilities of the low volatility regime, this regime is in place after 1992 and coincides with the period of inflation-targeting.

6 Changes in Inflation persistence

This section examines how changes in monetary policy and volatility regimes affect inflation persistence. Taking into account disagreements in measuring persistence, I consider three measures applied in recent literature.[18]

6.1 Inflation predictability - Cogley, Sargent and Premceri (2009)

This measure defines persistence as the fraction of the total variation of future inflation $\hat{\pi}_{t+j}$ that is due to shocks inherited from the past, relative to those that will occur in future. This is equivalent to the ratio of the conditional and unconditional variance; since future shocks account for the forecast error. It is possible to isolate the effect of each regime on inflation persistence, as the measure is calculated using the solution matrices of the MSDGE model $\Omega(s_t, P)$ and $\triangle(s_t, P)$ and the covariance matrix of shocks $\varSigma(S_t)$ that dependent on both the coefficient and the volatility regime in place. Which makes it possible to isolate the effect of each regime on inflation persistence

$$\hat{x}_t = \Omega(s_t, P)\hat{x}_{t-1} + \triangle(s_t, P)v_t, v_t \sim N(0, \varSigma(S_t))$$

$$R^2_{j(s_t,S_t)} \approx 1 - \frac{\mathbf{e}_\pi[\sum_{h=0}^{j-1}((\Omega(s_t)^h)\triangle(s_t)\varSigma(S_t)\triangle(s_t)'(\Omega(s_t)^h)']\mathbf{e}'_\pi}{\mathbf{e}_\pi[\sum_{h=0}^{\infty}((\Omega(s_t)^h)\triangle(s_t)\varSigma(S_t)\triangle(s_t)'(\Omega(s_t)^h)']\mathbf{e}'_\pi},$$

where $R^2_{j(s_t,S_t)}$ captures inflation persistence and e_π is a selection vector. This measure is analogous to the R^2 statistic for j-step ahead forecasts. This fraction must be between zero and one, with large values implying that past shocks die out slowly, making the inflation gap more persistent and hence more predictable. The measure converges to zero as the forecast horizon j lengthens. I consider the measure one period ahead as this is when past shocks have their greatest effect. Within this model when the horizon j increases to more than 10 periods ahead the measure quickly converges to zero. It is also worth noting that as the model inflation is in terms of deviations from steady-state level the estimates should be interpreted as persistence of the inflation gap as in Cogely, Premiceri and Sargent (2010).[19]

Table 5 reports the posterior estimates of the model implied $R^2_{j(s_t,S_t)}$ persistence statistic for each switching model with the 16% and 84% credible intervals. The preferred model estimates are in the first column and are very similar regardless of the regime in place. When the inflation-targeting regime is in place estimates of persistence are slightly lower, moving from 0.59 to around 0.55. However, the credible sets are very similar indicating no significant change of inflation persistence. This finding suggests the persistence of the inflation gap has not changed over the sample and neither the regimes identified in monetary policy and volatility have an effect on inflation persistence. The models with switching nominal rigidities implies significantly higher inflation persistence in the regime with low price stickiness. Similar to the other switching models, there is no impact of changes in the volatility of shocks on persistence. These results are not consistent with the reduced-form subsample estimates of Benati (2009, 2011) that find a significant reduction in inflation persistence after 1992. To highlight this finding Figure 4 plots the model implied persistence before and after 1992. The scatter graph measures the model-implied persistence of a subset of 1000 posterior draws weighted by their filter probability for each quarter of the sample and then average before, and after 1992, the solid black line represents no change during the periods. For all models, the draws are centered around the line equality.

6.2 Normalised spectrum

Following Cogely and Sargent (2005) and Mumtaz and Surico (2011) the normalised spectrum is used to measure inflation persistence. Specifically, the spectrum allows for a decomposition of the variance of inflation $\hat{\pi}_t$ by frequency. These are calculated for each regime as

$$\frac{S_\pi(\omega, s, S)}{\int_\omega S_\pi(\omega, s, S)}$$

[18] Pivetta and Reis (2009) suggest that inflation persistence can be sensitive to the measure used.

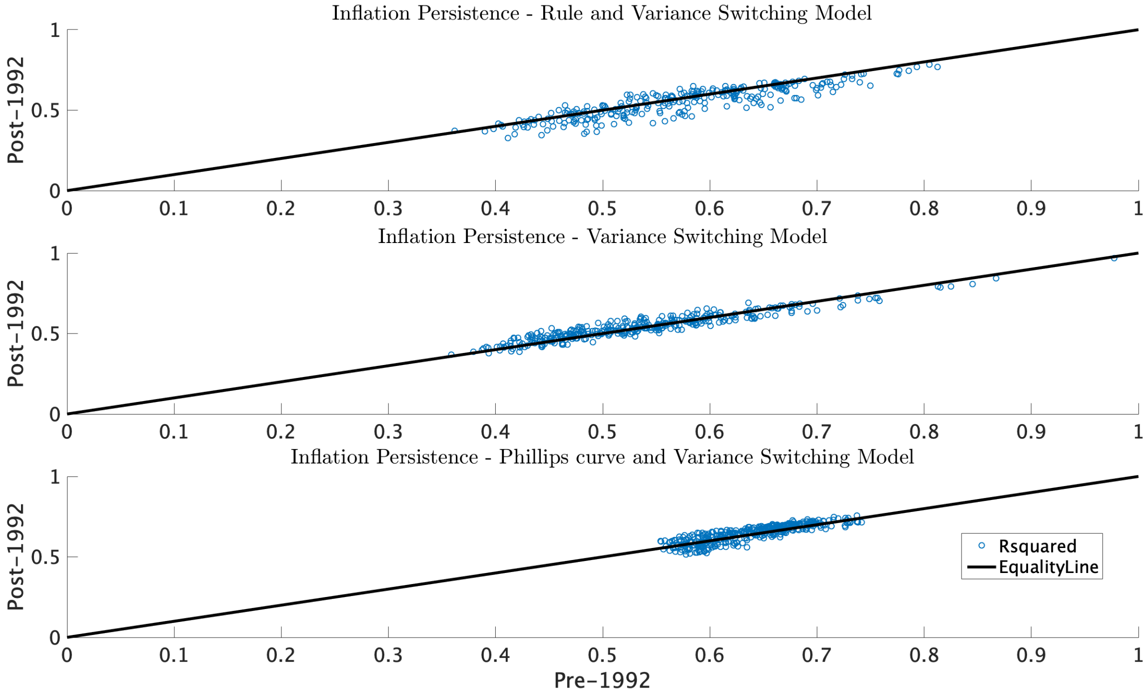
[19] However, unlike Cogely, Premiceri and Sargent (2010) target inflation or equivalently steady state inflation is assumed to be constant. Davig and Doh (2014) find adding a continuously time-varying target does not significantly affect model fit or persistence.

Table 5: Inflation predictability for selected MS-DSGE models

Regime	Monetary Policy	Nominal rigidities	Shock Volatilities
$R^2_{1(s_t=1,S_t=1)}$	0.59 [0.48,0.70]	0.79 [0.73,0.84]	0.53 [0.47,0.61]
$R^2_{1(s_t=1,S_t=2)}$	0.59 [0.49,0.69]	0.80 [0.76,0.85]	0.52 [0.44,0.61]
$R^2_{1(s_t=2,S_t=1)}$	0.56 [0.45,0.63]	0.46 [0.41,0.52]	
$R^2_{1(s_t=2,S_t=2)}$	0.55 [0.44,0.66]	0.46 [0.41,0.52}	

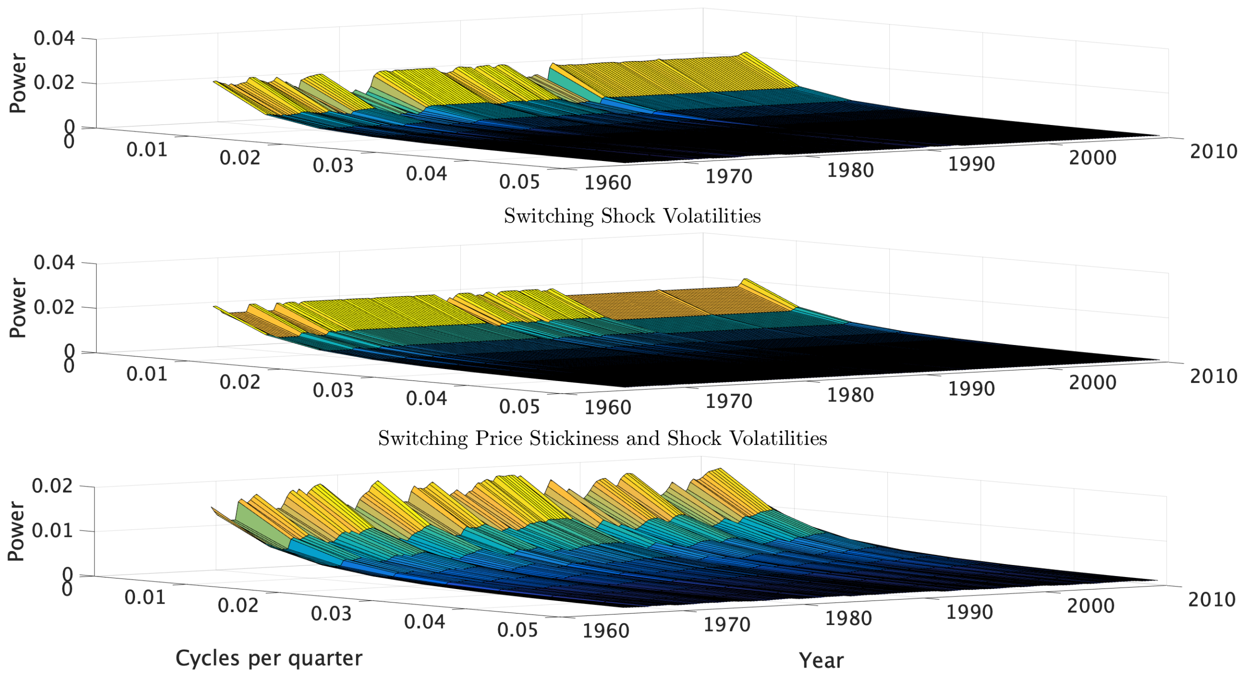
Note: Where $R^2_{1(s_t=2,S_t=2)}$ measures the median predictability of inflation one horizon ahead during the coefficient regime 2 (less aggressive monetary policy or high nominal rigidity) and volatility regime 2 (low volatility). The 68% credible sets are presented in square brackets.

Figure 4: Joint distribution of one-horizon ahead inflation predictability (R^2_1) for best-fitting MS-DSGE models



Note: The scatter graph measures model-implied inflation persistence of a subset of 1000 posterior draws from three MS-DSGE models. Each posterior parameter draw is used to calculate inflation persistence under the alternative regimes which is then weighted the by their filter probabilities for each quarter of the sample and then average before, and after 1992, the solid black line represents no change across the two periods. For all models, the draws are centered around the line equality. Persistence for each period weighted by its filter probabilities takes the following expression. $\Pr(s_t = 1, S_t = 1)R^2_{1(s_t=1,S_t=1)} + \Pr(s_t = 1, S_t = 2)R^2_{1(s_t=1,S_t=2)} + \Pr(s_t = 2, S_t = 1)R^2_{1(s_t=2,S_t=1)} + \Pr(s_t = 2, S_t = 2)R^2_{1(s_t=1,S_t=2)}$

Figure 5: Median posterior spectrum of inflation implied by selected MS-DSGE models



Note: This figure plots the posterior median normalised spectrum of inflation concentrating on low frequencies and weighted by the respective filter probabilities to provide an evolution of the measure over time. The y-axis represent the power of the spectrum, x-axis measures frequency regarding cycles per quarter and the z-axis represents the year. The first panel represents the model with monetary policy regimes. The filter probability weighted spectrum for each period is calculated as follows $\Pr(s_t = 1, S_t = 1)S_\pi(\omega, 1, 1) + \Pr(s_t = 1, S_t = 2)S_\pi(\omega, 1, 2) + \Pr(s_t = 2, S_t = 1)S_\pi(\omega, 2, 1) + \Pr(s_t = 2, S_t = 2)S_\pi(\omega, 2, 2)$.

where

$$S_\pi(\omega, s, S) = \frac{1}{2\pi}(I - \Delta_s e^{-i\omega})^{-1}\Sigma_S(I - \Delta_s^{'} e^{-i\omega})^{-1}$$

and ω denotes the frequency, I is a conformable identity matrix, Δ_s is the solution matrix for coefficient regime s and Σ_S is the the covariance matrix for volatility regime S. The power of the entire spectrum is normalised following Cogely and Sargent (2005), to adjust for changes in variance, in order to measure autocorrelation rather than auto-covariance. The regime-dependent solution matrices are used to compute the inflation persistence measured by the normalised spectrum across a range of frequencies measured in units of cycles of per quarter. Figure 5 provides an evolution of the measure over time of inflation persistence implied by the normalised spectrum and weighted by the posterior mean estimates of the filter probabilities. The magnitude of the frequency power indicates the amount of variance is explained at this frequency.

The first panel indicates the spectrum implied by the preferred model with monetary policy and volatility regimes. This spectrum indicates a sharp, brief reduction of inflation gap persistence during the mid-1970s and mid-1980s and a small more permanent decrease in inflation after 1992. The spectrum implied by the model with only switching shock volatility implies a similar reduction in persistence after 1992 with an increase in persistence after the crisis. The last panel shows that the model with a switching Phillips curves, provides an alternative dynamic pattern of persistence that changes more frequently in periods of relatively higher persistence in the early 1970s and late 1980s and toward the end of the crisis. Comparing to the measure of Cogely, Premiceri and Sargent (2009) the changes in point estimates also do not imply significant changes in persistence for all models.

6.3 Population moment of autocorrelation - Davig and Doh (2014)

An alternative measure of persistence is the population moment for the autocorrelation of inflation proposed by Davig and Doh (2014). This measure is a weighted average of parameters $\rho_{\triangle q}, \rho_z, \rho_{y^*}, \rho_{\pi^*}$

Table 6: Posterior spectrum at zero frequency of inflation implied by selected MS-DSGE models

Regime	Monetary Policy	Nominal rigidities	Shock Volatilities
$Spectrum_{0(s_t=1,S_t=1)}$	0.019 [0.014,0.023]	0.027 [0.02,0.03]	0.01 [0.007,0.013]
$Spectrum_{0(s_t=1,S_t=2)}$	0.02 [0.014,0.022]	0.028 [0.021,0.03]	0.0146 [0.011,0.019]
$Spectrum_{0(s_t=2,S_t=1)}$	0.017 [0.014,0.021]	0.015 [0.012,0.018]	
$Spectrum_{0(s_t=2,S_t=2)}$	0.017 [0.014,0.022]	0.015 [0.012,0.0175]	

Note: $Spectrum_{0(s_t=2,S_t=2)}$ measures the median normalised spectrum at frequency during the coefficient regime 2 (less aggressive monetary policy or high nominal rigidity) and volatility regime 2 (low volatility). Posterior median estimates are highlighted alongside the 68% credible sets are presented in square brackets.

Table 7: Population moment of autocorrelation for selected MS-DSGE models

Regime	Monetary Policy	Nominal rigidities	Shock Volatilities
$AR(\hat{\pi}_t \vert s_t = 1, S_t = 1)$	0.84 [0.78,0.82]	0.88 [0.86,0.89]	0.70 [0.59,0.78]
$AR(\hat{\pi}_t \vert s_t = 1, S_t = 2)$	0.83 [0.81,0.86]	0.9 [0.89,090]	0.84 [0.75,0.87]
$AR(\hat{\pi}_t \vert s_t = 2, S_t = 1)$	0.78 [0.69,0.82]	0.86 [0.84,0.87]	
$AR(\hat{\pi}_t \vert s_t = 2, S_t = 2)$	0.77 [0.71,0.82]	0.86 [0.84,0.87}	

Note: Where $AR(\hat{\pi}_t|s_t = 2, S_t = 2)$ measures the median population moment of autocorrelation during the coefficient regime 2 (less aggressive monetary policy or high nominal rigidity) and volatility regime 2 (low volatility). The 68% credible sets are presented in square brackets.

describing the persistence of structural shocks. Inflation inherits the persistence of structural shocks which are determined exogenously and are not subject to regime change. The weights placed on these parameters are determined by the variance of shocks $\sigma_1^R, \sigma_1^z, \sigma_1^q, \sigma_1^{y^*}, \sigma_1^{\pi^*}$ and the effect of past shocks captured by the regime-dependent solution matrix $\Omega(s_t)$. These weights provides a channel for changes in monetary policy and shock volatility to affect inflation persistence. The statistic measuring inflation persistence conditioning on a given regime is:

$$AR(\hat{\pi}_t|S_t = i, s_t = j) = w_{\triangle q}(i,j)\rho_{\triangle q} + w_{\triangle z}(i,j)\rho_{\triangle z} + w_{y^*}(i,j)\rho_{y^*} + w_{\pi^*}(i,j)\rho_{\pi^*},$$

where the weights are for $k \in \{\triangle q, z, y^*, \pi^*\}$

$$w_k(s_t = i, S_t = j) = \Omega\pi k_i W(i,j)\left(\frac{\sigma_{k_j}^2}{1-\rho_k}\right),$$

and

$$W(i,j) = \Omega\pi\triangle q_i\left(\frac{\sigma_{\triangle q_j}^2}{1-\rho_{\triangle q}}\right) + \Omega\pi z_i\left(\frac{\sigma_{z_j}^2}{1-\rho_z}\right) + \Omega\pi {y^*}_i\left(\frac{\sigma_{y_j^*}^2}{1-\rho_z}\right) + \Omega\pi {\pi^*}_i\left(\frac{\sigma_{\pi_j^*}^2}{1-\rho_z}\right) + \Omega\pi R_i\left(\sigma_{R_j}^2\right)$$

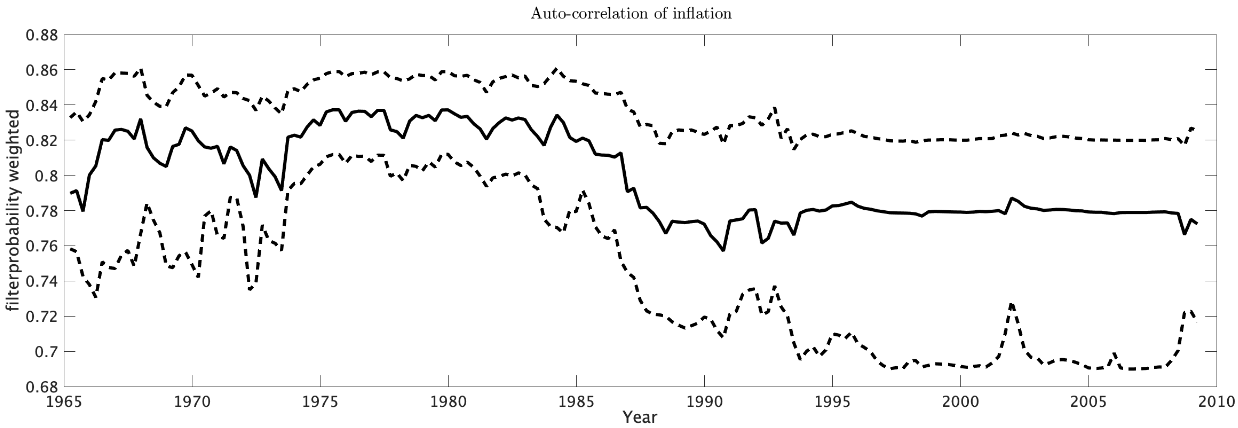
where $\Omega\pi\triangle q_i$ indicates the row of matrix Ω corresponding the behavior of inflation and the column indexing the impact of $\triangle q_i$.[20] The weights provides a channel for changes in monetary policy and shock volatility to affect inflation persistence.

This measure is used by Davig and Doh (2014) as it offers a structural interpretation of movements in persistence; the weights offer a direct link to regime shifts in the economic structure and volatility.

Table 7 presents the inflation persistence of each regime of the Markov-switching models considered. Focusing on the model preferred by the data, the inflation-targeting regime provides a reduction in mean persistence however, credible sets suggest these changes are not significant. These findings are broadly consistent with estimates of the other measures of persistence considered in this chapter.

Figure 6 displays the dynamics of this measure of persistence over time implied by the preferred model. The dynamics of the mean estimate indicate a reduction in inflation persistence during the late

Figure 6: Model-Implied inflation persistence of best fitting MS-DSGE model



Note: The solid line and the dashed lines represent the posterior median and the 68% credible sets, respectively, weighted by the respective filter probabilities to provide an evolution of the measure over time.

Table 8: Univariate model - Subsample analysis vs Markov-switching

CPI Inflation AR1				CPI Inflation MS-AR1			
Subsample	μ	ρ	σ_{ε}	Markov-switching	μ	ρ	σ_{ε}
1965-1991	0.2 $[-0.02,0.42]$	0.68 $[0.54,0.83]$	1.13	High volatility	0 $[-0.635,0.635]$	0.76 $[0.45,1.07]$	3.54
1992- 2009	-0.9 $[-1.14,-0.65]$	0.06 $[-0.18,0.3]$	0.14	Low volatility	-0.13 $[-0.14,-0.12]$	0.73 $[0.72,0.73]$	0.21

Note: Posterior median estimates are highlighted alongside the 90% credible sets are presented in square brackets.

1980s and resemble those implied by the normalised spectrum. However, the wide error bands around this decrease suggest no evidence of systematic change in inflation persistence during the sample.

6.4 Reduced-form univariate Markov-switching model

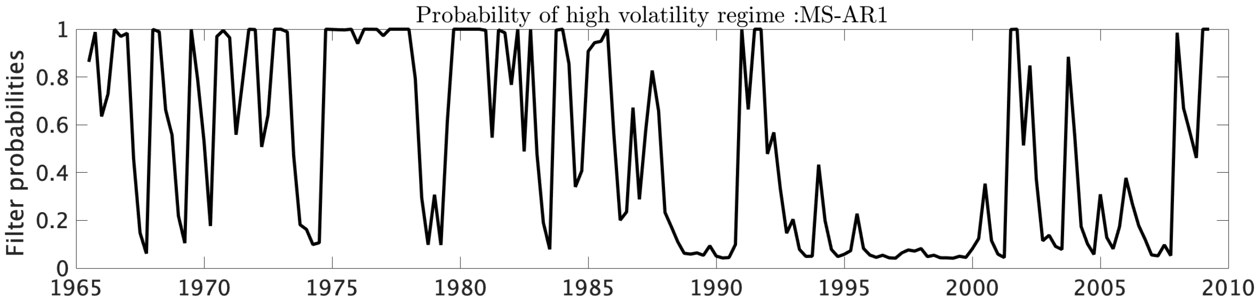
The best-fitting MS-DSGE model suggests no statistically significant changes in inflation persistence in the U.K. during 1965-2009. To compare this to inflation persistence from a reduced-form model I follow Davig and Doh (2014) by estimating a simple univariate Markov-switching model using Bayesian methods. The following univariate model is considered

$$\hat{\pi}_t = \mu_{S_t} + \rho_{St}\hat{\pi}_{t-1} + \varepsilon_t,\ \varepsilon_t \sim NIID(0, \sigma^2_{\varepsilon_{St}}). \tag{19}$$

Within the switching model in equation 19, all parameters are allowed to change simultaneously with S_t, which follows a first-order two-state Markov chain. The autoregressive parameter ρ measures the persistence of inflation based on this model. The results of this experiment presented in Table 8 are interesting, sub-sample estimates suggest that the pre-1992 period is characterised by significantly higher mean inflation, persistence and volatility relative to the subsequent post-1992 subsample. In contrast, the regimes identified by Markov-switching univariate model indicate only significant changes in the standard deviation of residual and only a small insignificant change in inflation persistence during the high volatility regime. The filter probabilities for the high volatility regime of the univariate model are displayed in figure 7. The Markov-switching regression supports the finding of the MS-DSGE model preferred by the data of no significant changes in inflation persistence across the the sample. The high volatility regime appears to be in place during the 1970s however the regime change is very frequent in this model compared to those implied by the MS-DSGEs.

[20]Since the monetary policy shock ε_t^R is modeled as IID, it has no persistence. However, the impact of monetary regime changes is captured by changing weights in the solution matrix $\Omega(s_t)$ used to weight the of measures of persistence.

Figure 7: Posterior median filter probabilities of high volatility regime in a simple MS AR1 model



Note: This figure presents the posterior mean filter probabilities of the high volatility regime implied the MS-AR(1).

7 What effect did monetary policy regime change have on inflation and the economy?

Although, the preferred model identifies two monetary policy regimes that differ substantially on their stance to inflation, the analysis presented in the previous section implies that inflation persistence was unchanged during the sample. This section looks at the effect of this change in monetary policy on inflation and the economy by examining impulse responses and counterfactual analysis.

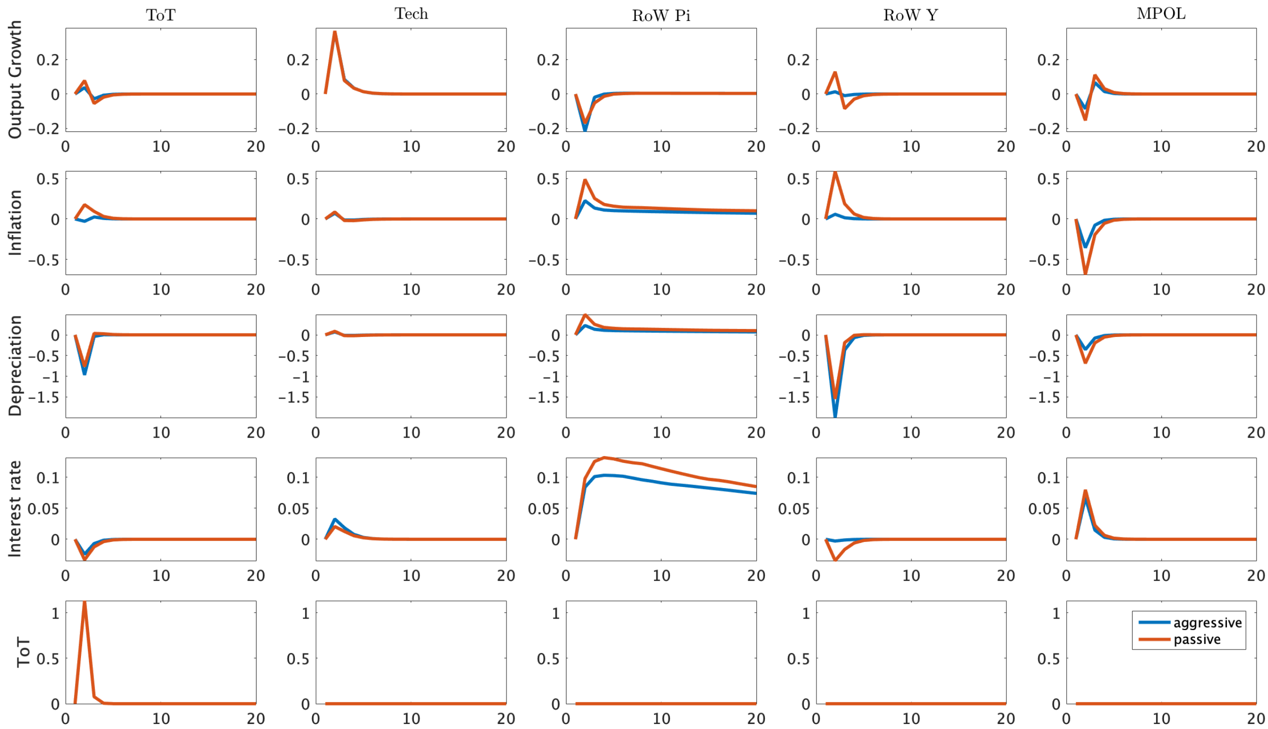
7.1 Regime-dependent impulse responses

Figure 8 presents the impulse responses of the observable variables implied by the preferred model the to five structural shocks. Impulse response functions are computed conditional on one regime being in place over the entire horizon in order to highlight the difference between each regime. The shock is of a magnitude of one standard-deviation under the low volatility regime. The red line displays the responses of the accommodative inflation regime, and the blue line represents the responses of the inflation-targeting regime. The impulse responses for all variables to the shocks to world output and world inflation are amplified when the accommodative monetary policy regime is in place. The finding is consistent with Davig and Leeper (2007), who find that a Taylor rule inflation coefficient less than unity results in more substantial fluctuations in the economy. The presence of the inflation-targeting regime dampens the response of inflation and exchange rate depreciation to a monetary policy shock and is consistent with the finding of Liu and Mumtaz (2011). It is worth noting that the time taken for shocks to inflation to completely dissipate is longer when the accommodative regime is in place and is an indicator of higher inflation gap persistence during this regime.

7.2 Counterfactual analysis

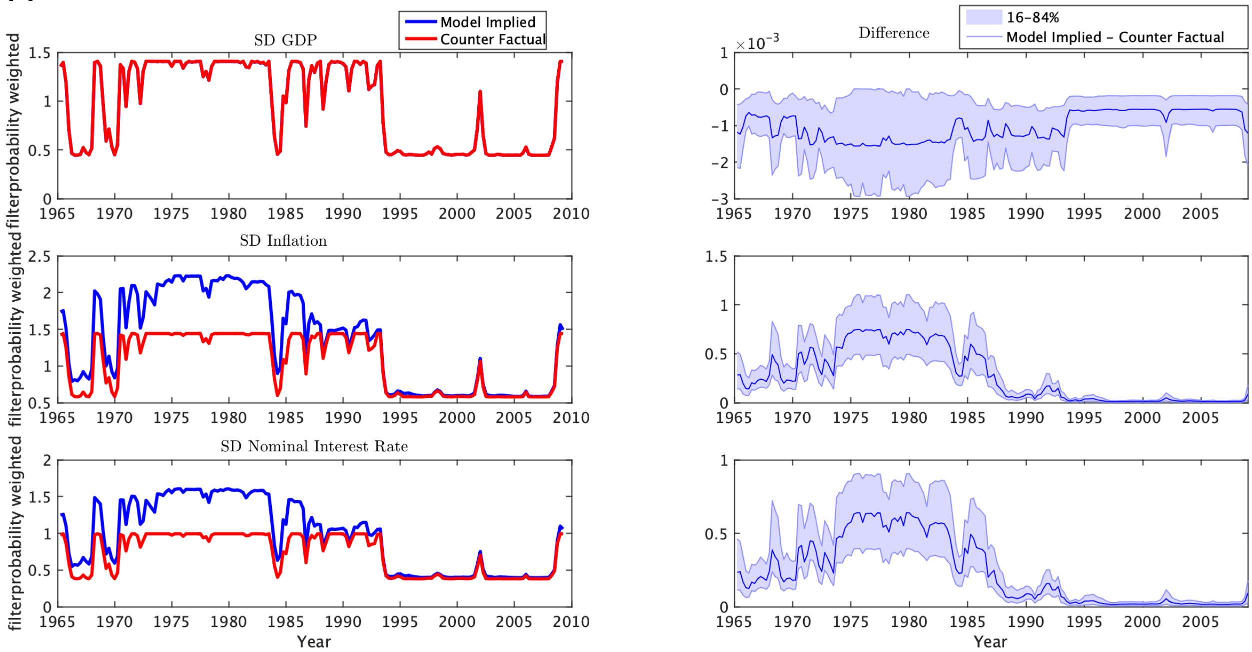
To focus on the role of monetary policy, I follow Liu and Mumtaz (2011) and use counterfactual experiments to consider how the estimated change in the policy rule from the preferred model, contributed to changes in the volatility and level of inflation and other key macroeconomic variables. The reader is referred to Pesaran and Smith (2016, 2018), for a detailed discussion on conducting counterfactual experiments. Specifically, the counterfactual scenario considered, assumes the inflation-targeting monetary policy stance is in place for the entire sample. The experiment assumes the evolution of shock volatilities of the preferred model are left unchanged. This counterfactual experiment is not subject to the Lucas critique as model parameters are pinned down by microeconomic foundations. To impose the counterfactual scenario, the preferred model is resolved for every draw assuming the inflation targeting regime is in place. The unconditional standard deviations of the headline macroeconomic indicators, GDP level, inflation and the interest rate are computed using model estimates and the counterfactual scenario. They are weighted by the filter probabilities of the stochastic volatility regime. Figure 9 presents the results of the experiment. The blue line represents the model implied unconditional standard deviation over the sample period. The volatility of output, inflation and interest rates share a similar pattern, with peaks in the mid-1970s, mid-1980s, early 1990s and 2008. As in Liu and Mumtaz (2011), the standard

Figure 8: Posterior median regime-dependent impulse response functions for best-fitting MS-DSGE model



Note: The red and blue lines represent the median impulse response function of the less aggressive and aggressive monetary policy regimes, respectively. The columns represent responses to specific shocks, whilst rows represent specific variables. For example the response of output growth to a one-standard deviation monetary policy shock can be found in the fifth column of the first row.

Figure 9: Model and counterfactual estimates of unconditional volatility from best-fitting MS-DSGE



Note: The blue lines in the first column show the model implied unconditional standard deviation while the red lines show the outcome under the counterfactual scenario of the inflation-targeting regime over the entire sample period. The second column present the difference between the model implied estimates and the counterfactual experiment, where the shaded area represents the 68% error band.

deviation of inflation reached its peak during periods where the accommodative monetary policy regime was identified to be in place. Also, the fall in volatility coincides with the move from a passive policy rule to a more active one. This result suggests that a change in policy rule was partly responsible for the reduction in inflation volatility during this period. The red line represents the counterfactual, with the inflation-targeting regime in place for the entire sample and suggests that the volatility of inflation would be significantly lower during the mid-1970s until the mid-1980s. The same is true for the nominal interest rate and output. However, the reduction in output is relatively small. The right-hand panels display the differences between the model implied and counterfactual unconditional volatility, where the shaded intervals represent one-standard deviation error bands .

8 Conclusion

This paper examines the persistence of price inflation in the United Kingdom using a structural open economy models. The models are non-linear and allow for regime shifts in areas of the economy most commonly related to changes in persistence – monetary policy, the volatility of structural shocks and the degree of nominal rigidity present in price-setting. Applying three measures of persistence proposed in the recent literature that isolate the effect of each change in the economies structure, I find that U.K. inflation persistence has not significantly changed between 1965-2009. This finding is somewhat in contrast with the current literature that finds a sharp decline in persistence after the adoption of inflation-targeting in 1992. The second contribution lies in identifying a disconnect between this finding and counterfactual analysis of the model. Assuming the inflation-targeting regime is in place for the entire sample period results in a substantial decrease in the unconditional standard deviation of the inflation. This analysis point towards adopting new tools for measuring inflation persistence as the current literature provides considerably different historical accounts of both the level and changes of inflation persistence.

References

[1] Alogoskoufis, G. and Smith, R.P. (1991). The Phillips Curve, The Persistence of Inflation, and the Lucas Critique: Evidence from Exchange-Rate Regimes. The American Economic Review, Vol. 81, No. 5, pages 1254-1275.

[2] Alstadheim, R., Bjørnland, H. C. and Maih, J. (2013). Do Central Banks Respond to Exchange Rate Movements? A Markow-Switching Structural Investigation. Norges Bank Research Working paper, 24.

[3] Barnett, A. Groen, J. and Mumtaz, H. (2010). Time-varying inflation expectations and economic fluctuations in the United Kingdom: a structural VAR analysis, Bank of England working papers 392, Bank of England.

[4] Barthélemy, J. and Marx, M. (2012). Generalizing the Taylor Principle: New Comment. working papers 403. Banque de France.

[5] Barthélemy, J. and Marx, M. (2017). Solving endogenous regime switching models. Journal of Economic Dynamics and Control, 77, pages1-25.

[6] Benati, L. (2008a). Investigating Inflation Persistence Across Monetary Regimes. The Quarterly Journal of Economics, Oxford University Press, vol. 123(3), pages 1005-1060.

[7] Benati, L. (2008b). The great moderation in the United Kingdom. Journal of Money, Credit and Banking.

[8] Bianchi, F. (2012). Evolving monetary/fiscal policy mix in the united states. American Economic Review. 102, pages167–172.

[9] Bianchi, F. (2013). Regime switches, agents' beliefs, and post-world war ii u.s. macroeconomic dynamics. Review of Economic Studies 80 (2), pages 463–490.

[10] Blake, A.P. and Zampolli, F. (2011). Optimal policy in Markov-switching rational expectations models. Journal of Economic Dynamics and Control 35, pages 1626–1651.

[11] Blake. A and Mumtaz. H. (2012). Applied Bayesian econometrics for central bankers, Technical Books, Centre for Central Banking Studies, Bank of England, edition 1, number 4.

[12] Blake. A and Mumtaz. H. (2017). Applied Bayesian econometrics for central bankers.

[13] Caggiano, G., Castelnuovo, E. and Figueres, J.M. (2017). "Economic policy uncertainty and unemployment in the United States: A nonlinear approach," Economics Letters, Elsevier, vol. 151(C), pages 31-34.

[14] Caggiano, G., Castelnuovo, E. and Groshenny, N. (2014). "Uncertainty shocks and unemployment dynamics in U.S. recessions," Journal of Monetary Economics, Elsevier, vol. 67(C), pages 78-92.

[15] Caldara, D. and Herbst, E. (2018). Monetary Policy, Real Activity, and Credit Spreads: Evidence from Bayesian Proxy SVARs, American Economic Journal: Macroeconomics, forthcoming.

[16] Calvo, G. A., 1983. Staggered prices in a utility-maximizing framework, Journal of Monetary Economics, Elsevier, vol. 12(3), pages 383-398, September.

[17] Canova, F. and Sala, L., 2009. Back to square one: Identification issues in DSGE models, Journal of Monetary Economics, Elsevier, vol. 56(4), pages 431-449, May.

[18] Chen, X. and Macdonald, R. (2012). Realized and Optimal Monetary Policy Rules in an Estimated Markov-Switching DSGE Model of the United Kingdom, Journal of Money, Credit and Banking, Blackwell Publishing, vol. 44(6), pages 1091-1116, September.

[19] Chin, M., De Graeve,F., Filippeli, T. and Theodoridis, K. (2018). "Understanding International Long-Term Interest Rate Comovement," Cardiff Economics Working Papers E2018/19, Cardiff University, Cardiff Business School, Economics Section.

[20] Cho, S. (2015). Sufficient conditions for determinacy in a class of Markov-switching rational expectations models. Review of Economic Dynamics.

[21] Clarida, R., Galí, J. and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: evidence and some theory. Quarterly Journal of Economics. 115, pages 147–180.

[22] Costa, O., Fragoso, M. and Marques, R. (2005). Discrete-Time Markov Jump Linear Systems. Springer.

[23] Cogley, T. and Sargent, T. J. (2005). Drift and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S.Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 8(2), pages 262-302, April.

[24] Cogley, T., Primiceri, G.and Sargent, T. J. (2010). Inflation-Gap Persistence in the U.S.. American Economic Journal: Macroeconomics, American Economic Association, vol. 2(1), pages 43-69, January.

[25] Davig, T. and Doh, T. (2014). Monetary Policy Regime Shifts and Inflation Persistence. Review of Economics and Statistics, 96(5): pages 862–875.

[26] Davig, T. and Leeper, E.M. (2007). Generalizing the Taylor principle. American Economic Review. 97, pages 607–635.

[27] DeJong, D. and Dave, C. (2011). Structural Macroeconometrics Second Edition, Economics Books, Princeton University Press, edition 1, number 9622.

[28] Del Negro, M. and Schorfheide, F. (2009). Inflation Dynamics in a Small Open Economy Model under inflation-targeting: Some Evidence from Chile. Central Banking, Analysis, and Economic Policies Book Series,in: Klaus Schmidt-Hebbel and Carl E. Walsh and Norman Loayza (Series Editor) and Klaus Schmidt-Hebbel (Series (ed.), Monetary Policy under Uncertainty and Learning, edition 1, volume 13, paper13, pages 511-562 Central Bank of Chile.

[29] Eo, Y. (2008). Bayesian Analysis of DSGE Models with Regime Switching, MPRA paper13910, University Library of Munich, Germany, revised 11 Feb 2009.

[30] Farmer, R.E.A., Waggoner, D. F. and Zha, T., 2008. Minimal state variable solutions to Markov-switching rational expectations models, FRB Atlanta Working Paper 2008-23, Federal Reserve Bank of Atlanta.

[31] Farmer, R. E. A., Waggoner, D. F. and Zha, T. (2009a). Indeterminacy in a forward-looking regime switching model. International Journal of Economics. Theory 5, pages 69–84.

[32] Farmer, R.E.A., Waggoner, D. F. and Zha, T. (2009b). Understanding Markov-switching rational expectations models. Journal Econometric Theory 144, pages 1849–1867.

[33] Farmer, R. E.A., Waggoner, D. F. and Zha, T. (2011). Minimal state variable solutions to Markov-switching rational expectations models. Journal of Economics Dynamics and Control 35 (12), pages 2150–2166.

[34] Foerster, A., Rubio-Ramirez, J. and Waggoner, D., Zha, T. (2016). Perturbation methods for Markov-switching DSGE models. Quant. Econ. 7, pages 637–669.

[35] Fuhrer, J.C. (2010). Inflation persistence. Handbook of Monetary Economics in: Friedman and Woodford, edition 1, vol.3. paper9, pages 423-486 Elsevier.

[36] Galí, J. and Monacelli, T. (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Review of Economic Studies, Oxford University Press, vol. 72(3), pages 707-734.

[37] Gelfand, A.E., Dey, D.K. (1994). Bayesian model choice: Asymptotic's and exact calculations. Journal of the Royal Statistical Society (SeriesB) vol.56, pages 501–514.

[38] Geweke, J. (1999). Using simulation methods for bayesian econometric models: inference, development, and communication, Econometric Reviews, Taylor and Francis Journals, vol. 18(1), pages 1-73.

[39] Hamilton, J.D. (1986). "State-space models," Handbook of Econometrics,in: R. F. Engle & D. McFadden (ed.), Handbook of Econometrics, edition 1, volume 4, paper50, pages 3039-3080 Elsevier.

[40] Hamilton, J.D. (1989). A new approach to the economic analysis of non-stationary time series and the business cycle, Econometrica 57, pages 357–384.

[41] Kim C.-J. and Nelson C. R. (1999). State-space models with regime switching: Classical and Gibbs-sampling approaches with applications, volume 2. MIT Press, Cambridge, MA.

[42] Judd, K. (1992). Projection methods for solving aggregate growth models. Journal of Economic Theory. 58, pages 410–452.

[43] Koop, G., Pesaran, H.M. and Smith, R.P. 2013. On Identification of Bayesian DSGE Models. Journal of Business & Economic Statistics, Taylor & Francis Journals, vol. 31(3), pages 300-314, July.

[44] Leeper, E.M. and Zha, T. (2003). Modest policy interventions. Journal of Monetary Economics. Vol. 50, pages 1673–1700.

[45] Levin, A and Piger, J. M. (2004). Is inflation persistence intrinsic in industrial economies?, working paper Series 334, European Central Bank.

[46] Lindley,D. (1971). Bayesian Statistics: A review. Society for Industrial and Applied Mathematics

[47] Liu, P. and Mumtaz, H. (2011). Evolving macroeconomic dynamics in a small open economy: an estimated Markov-switching DSGE model for the United Kingdom, Journal of Money, Credit and Banking. 43(7), pages 1444-1474.

[48] Liu, Z., Waggoner, D.F. and Zha, T. (2010). Sources Of Macroeconomic Fluctuations: A Regime-Switching DSGE Approach. Quantitative Economics, Econometric Society, vol.2(2), pages 251-301, July.

[49] Lhuissier, S. and Tripier, (2016). Do Uncertainty Shocks Always Matter for Business Cycles?, working papers 2016-19, CEPII research center.

[50] Lhuissier, S. and Zabelina, M, 2015. On the stability of Calvo-style price-setting behavior, Journal of Economic Dynamics and Control, Elsevier, vol. 57(C), pages 77-95.

[51] Lubik, T.A. and Schorfheide, F. (2004). Testing for indeterminacy: an application to u.s. monetary policy. American Economic Review. 94, pages 190–217.

[52] Maih, J. (2015). Efficient Perturbation Methods for Solving Regime-Switching DSGE Models. Norges Bank Working paper.

[53] Meng, X., and Wong, W. (1996). Simulating Ratios of Normalising Constants via a Simple Identity: A Theoretical Exploration. Statistica Sinica, 6(4), pages 831-860.

[54] Mumtaz, H. and Surico, P. (2009). The international Transmission of International Shocks: A factor-augment VAR. Journal of Money, Credit an Banking, 41:pages47-100.

[55] Pivetta, F. and Reis, R. (2007). The persistence of inflation in the United States. Journal of Economic Dynamics and Control, 31, 1326-1358.

[56] Pesaran, H. M. and Smith, R.P. (2016). Counterfactual analysis in macroeconometrics: An empirical investigation into the effects of quantitative easing, Research in Economics, Elsevier, vol. 70(2), pages 262-280.

[57] Pesaran, H. M. and Smith, R.P., (2018). Tests of Policy Interventions in DSGE Models, Oxford Bulletin of Economics and Statistics, Department of Economics, University of Oxford, vol. 80(3), pages 457-484, June.

[58] Roberts, G., Gelman, A. and Gilks, W. (1997). Weak convergence and optimal scaling of random walk metropolis algorithms. The Annals of Applied Probability 7(1): 110-120.

[59] Sims, C.A. (1982). Policy analysis with econometric models. Brooking chapters Econ. Act. 13, 107–164.

[60] Sims, C. A., Waggoner, D.F., and Zha, T, (2008). Methods for inference in large multiple-equation Markov-switching models. Journal of Econometrics, Elsevier, vol. 146(2), pages 255-274, October.

[61] Sims, C.A. and Zha, T. (2006). Were there regime switches in U.S. monetary policy? American Economic Review. 96, 54–81.

[62] Svensson, L., Williams, N. (2009). Optimal monetary policy under uncertainty in DSGE models: a Markov jump-linear-quadratic approach. In: Schmidt-Hebbel, K., Walsh, C.E., Loayza, N. (Eds.), Central Banking, Analysis, and Economic Policies Book Series, Monetary Policy Under Uncertainty, 13, pages77–114.

[63] Taylor, J.B., (1993). Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy, Elsevier, vol. 39(1), pages 195-214, December.

[64] Woodford, M. (2003). Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

[65] Zakipour-Saber, S., (2019). State-dependent Monetary Policy Regimes. Research Technical Papers 4/RT/19, Central Bank of Ireland.

9 Appendix

9.1 Underlying model

This section describes the agents' behaviour and optimisation problems underlying the log-linearised model presented in Lubik and Schorfhiede (2006) and closely follows the appendix of Del Negro and Schorfheide (2008). This small-open economy model is within the class of models defined by Gali and Monacelli (2005). The domestic economy is one among a continuum of small economies making up the world economy. Since each economy is of measure zero, its domestic policy decisions do not have any impact on the rest of the world. All economies are assumed to share identical preferences, technology and market structure. The model describes the behaviour of a single economy known as the domestic economy in this case the U.K., and its interaction with the complete continuum of small economies which represents the rest of the world.

Time t decisions are made after observing all current shocks and asset markets are assumed to be complete.

Households

Domestic households solve the following decision problem

$$\max_{\{C_{h_t},N_{h_t},D_{h_t},D^*_{h_t}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t/Z_t)^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$s.t.\ P_{H,t}C_t + Q_t D_{t+1} + e_t Q_t^* D_{t+1}^* \leq W_t N_t + D_t + e_t D_t^* + \int \Omega_t(i)di,$$

where C_t is consumption of a composite good, N_t is hours worked, $P_{H,t}$ is the nominal price level of the composite good. D_{t+1} (D^*_{t+1}) is holdings of security that pays 1 unit of the domestic currency (foreign currency) and Q_t its current price in pounds. e_t is the nominal exchange rate (domestic currency/foreign currency), T_t are nominal transfers, and Ω_t are nominal dividends earned from a domestic firm i. Z_t is a world technology process, which is assumed to follow a random walk with drift.

The first order-conditions can be written as

$$N_t^{\varphi} = c_t^{-\sigma} w_t \tag{20}$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left[R_t c_{t+1}^{-\sigma} (z_{t+1}\pi_{t+1})^{-1} \right] \tag{21}$$

$$0 = \mathbb{E} \left[(R_t - R_t^* \triangle e_{t+1}) \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (z_{t+1}\pi_{t+1})^{-1} \right] \tag{22}$$

where consumption and wages are detrended and denoted as $c_t = C_t/Z_t$ and $w_t = W_t/PZ_t$. $z_t = Z_t/Z_{t-1}$, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, and $\triangle e_t = e_t/e_{t-1}$ is the gross depreciation rate.

Terms of trade and real exchange rate

Let P_t and $P_{F,t}$ be the domestic price and home and foreign produced goods, respectively.

The terms of trade:

$$q_t = P_{H,t}/P^*_{F,t}.$$

The law of one price for foreign goods holds:

$$P_{F,t} = e_t P^*_{F,t}.$$

Here $P^*_{F,t}$ is the price of the foreign produced good in foreign country, measured in foreign currency and P_t represents domestic CPI. Domestically produced goods are assumed to have a negligible weight in foreign consumption. Specifically, let ϑ represent the relative size of the domestic economy, $\alpha^* = \vartheta\alpha$ and $\vartheta \to 0$. Hence, $P^*_{F,t}$ will be approximately equal to foreign CPI P^*_t and I can express the terms of trade as

$$Q_t = P_t/(e_t P_t^*).$$

An exchange rate depreciation and foreign inflation would reduce the terms of trade, making imports more expensive.

The real exchange rate is defined as

$$S_t = e_t P_t^*/P_t. \tag{23}$$

The relative price is expressed as

$$P_{H,t}/P_t = Q_t S_t. \tag{24}$$

Composite Goods

There are firms that buy quantities $C_{H,t}$ and $C_{F,t}$ of domestic and foreign produced goods and package them into a composite good that is used for consumption by the households. These firms maximise profits in a perfectly competitive environment:

$$\max_{C_t, C_{H,,t}, C_{F,t}} P_t C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t}$$

$$s.t.\ C_t = \left[(1-\alpha)^{1/\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{1/\eta} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

The first-order conditions and a zero profit condition that

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t,\ C_{F,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t,$$

$$P_t = \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

Dividing by P_t and rearranging terms lead to the following relationship between the real exchange rate and the terms of trade:

$$S_t = \left[(1-\alpha) Q_t^{1-\eta} + \alpha\right]^{\frac{1}{1-\eta}}. \tag{25}$$

The domestically produced good, supplied in overall quantity Y_t, is itself a composite made of a continuum of domestic intermediate goods $Y_t(i)$:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}.$$

Firms producing composite goods are perfectly competitive, they buy the domestic intermediate goods, package them, and resell the composite good to the firms that aggregate $C_{H,t}$ and $C_{F,t}$ as well as abroad. These firm solve the following problem:

$$\max_{Y_t, Y_{,t}(i)} P_{H,t} Y_t - \int_0^1 P_{H,t} Y_{,t}(i) di$$

$$s.t.\ Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}.$$

The first-order conditions and a zero-profit condition lead to

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} Y_t,\ P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

Domestic Intermediate Goods

The producers of the domestic intermediate goods $Y_t(i)$ are monopolistically competitive. Firms can re-optimise prices in each period with probability $1-\theta$. Firms that are unable to re-optimise their price, $P_{H,t}(i)$ will increase according to the steady-state inflation rate $\pi_{H,*}$. The firms use today's prices of state-contingent securities to discount future nominal profits. The firms' production function is linear with respect to labour and takes the following form

$$Y_t(i) = Z_t N_t(i).$$

Where productivity Z_t is not firm specific and its growth rate $z_t = Z_t/Z_{t-1}$ follows an AR(1):

$$(\ln z_t - \gamma) = \rho_z(\ln z_{t-1} - \gamma) + \varepsilon_t^z,$$

and γ is the steady state growth rate of productivity. The firms' problems is given by:

$$\max_{\tilde{P}_{H\,t},\{Y_{t+\tau}(i)\}_{\tau=0}^{\infty}} \mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\theta^{\tau}Q_{t+\tau}Y_{t+\tau}(i)\left(\tilde{P}_{H,t}(i)\pi_{H,*}^{\tau} - MC_{t+\tau}^{n}\right)\right]$$

$$s.t.\; Y_{t+\tau}(i) \leq \left(\frac{\tilde{P}_{H,t}(i)\pi_{H,*}^{\tau}}{P_{H,t+\tau}}\right)^{-\epsilon} Y_{t+\tau},$$

where $MC_{t+\tau}^{n} = W_{t+\tau}/Z_{t+\tau}$ is the nominal marginal cost and $Q_{t+\tau}$ is the time t price of a security that pays 1 unit of domestic currency in period $t+\tau$.

In symmetric equilibrium in which all firms solve the same problem, the firms' first-order condition can then be written as:

$$\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\theta^{\tau}Q_{t+\tau}\left(\frac{\tilde{P}_{H,t}(i)\pi_{H,*}^{\tau}}{P_{H,t+\tau}}\right)Y_{t+\tau}\left[(\epsilon-1)\tilde{P}_{H,t}(i)\pi_{H,*}^{\tau} - \epsilon MC_{t+\tau}^{n}\right]\right] = 0.$$

The fraction of firms that are allowed to re-optimise their price is $1-\theta$, while the remaining firms update their price by the steady state inflation rate.

$$P_{H,t} = \left[\theta\tilde{P}_{H,t}^{1-\epsilon} + (1-\theta)(\pi_{H,*}P_{H,t-1})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

Both the nominal marginal costs and the price chosen by firms are able to re-optimise in terms of the price of the domestic good:

$$mc_t = \frac{MC_t^n}{P_t^H} = \frac{W_t}{Z_t P_t}\frac{P_t}{P_t^H} = w_t Q_t^{-1} S_t \; and \; \tilde{p}_{H,t} = \frac{\tilde{P}_{H,t}}{P_{H,t}}. \tag{26}$$

In equilibrium

$$Q_{t+\tau} = \beta^{\tau}\frac{\lambda_{t+\tau}}{\lambda_t} = \beta^{\tau}\frac{C_{t+\tau}^{-\sigma}P_t Z_t}{C_t^{-\sigma}P_{t+\tau}Z_{t+\tau}}.$$

The optimal pricing rule can be re-stated as

$$\tilde{p}_{H,t} = \frac{\epsilon}{\epsilon-1}\frac{\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\theta^{\tau})\frac{c_{t+\tau}^{-\sigma}}{c_t^{-\sigma}}\left(\frac{\pi_H^{\tau},*}{\prod_{s=1}^{\tau}\pi_{H,t+s}}\right)^{-\epsilon}S_{t+\tau}Q_{t+\tau}y_{t+\tau}mc_{t+\tau}\prod_{s=1}^{\tau}\pi_{H,t+s}\right]}{\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\theta^{\tau})\frac{c_{t+\tau}^{-\sigma}}{c_t^{-\sigma}}\left(\frac{\pi_H^{\tau},*}{\prod_{s=1}^{\tau}\pi_{H,t+s}}\right)^{-\epsilon}S_{t+\tau}Q_{t+\tau}y_{t+\tau}\pi_H^{\tau},*\right]}. \tag{27}$$

$$1 = \left[\theta\tilde{p}_{H,t}^{1-\epsilon} + (1-\theta)(\pi_{H,*}/\pi_{H,t-1})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{28}$$

Market Clearing and Aggregate Production

The market for domestically produced goods clears if the following condition is satisfied

$$y_t = c_{H,t} + c^*_{H,t}.$$

Then substituting the first-order conditions for the composite goods one obtains

$$y_t = c_{H,t} + c^*_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} c_t + \alpha\vartheta\left(\frac{P_{H,t}/e_t}{P^*_t}\right)^{-\eta} c^*_t$$

$$= (1-\alpha)(S_t Q_t)^{-\eta} c_t + \alpha\vartheta Q_t^{-\eta} c^*_t.$$

The aggregate production function for the domestic economy is given by

$$y_t = N_t\left[\int\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di\right]^{-1}. \tag{29}$$

The households receive the profits generated by the monopolistically competitive domestic intermediate goods producing firms. Firm i generates the following profit:

$$\Omega_t(i) = Y_t(i) P_{H,t}(i) - N_t(i) W_t.$$

The demand function can be written

$$\Omega_t(i) = P_{H,t}^{1-\epsilon}\frac{Y_t}{P^{\epsilon}_{H,t}} - N_t(i) W_t.$$

Integrating both sides using the expression for the price of the composite good, I obtain

$$\int \Omega_t(i) di = P_{H,t} Y_t - W_t N_t.$$

Finally, the following condition is deduced from the firms budget constraint

$$P_t C_t - P_{H,t} Y_t = D_t + e_t D^*_t - (Q_{t+1} D_{t+1} + e_t Q^*_{t+1} D^*_{t+1}.$$

The rest of the world

The perfect-risk-sharing assumption obtains a relationship between domestic and foreign consumption

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma}\pi_{t+1} = \left(\frac{c^*_{t+1}}{c^*_t}\right)^{\sigma}\pi^*_{t+1} e_{t+1}.$$

The equation can be rewritten as:

$$\left(\frac{c_{t+1}}{c^*_{t+1}}\right)^{\sigma}\frac{P_{t+1}}{e_{t+1} P^*_{t+1}} = \left(\frac{c_t}{c^*_t}\right)^{\sigma}\frac{P_t}{e_t P^*_t}.$$

This equation links the consumption growth of the domestic country and the rest of the world. To obtain implications about the level of consumption in the two countries, assume that in period $t = 0, S_0 = 1$ and that $\vartheta = C_0/C^*_0$. This assumption implies

$$c_t = \vartheta c^*_t S_t^{1/\sigma}. \tag{30}$$

The market clearing conditions for the domestically produced goods can be rewritten

$$y_t = \vartheta c^*_t Q_t^{-\eta}\left[(1-\alpha) S_t^{1/\sigma-\eta} + \alpha\right]. \tag{31}$$

Equilibrium conditions

The equilibrium conditions are given by equations 32-43. I refer the reader to Del Negro and Schorfheide (2009) for details on the steady-state and the log-linearization of the model.

$$N_t^{\varphi} = c_t^{-\sigma} w_t \tag{32}$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left[R_t c_{t+1}^{-\sigma} (z_{t+1}\pi_{t+1})^{-1} \right] \tag{33}$$

$$0 = \mathbb{E} \left[(R_t - R_t^* \triangle e_{t+1}) \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (z_{t+1}\pi_{t+1})^{-1} \right] \tag{34}$$

$$S_t = e_t P_t^* / P_t \tag{35}$$

$$P_{H,t}/P_t = Q_t S_t \tag{36}$$

$$S_t = \left[(1-\alpha) Q_t^{1-\eta} + \alpha \right]^{\frac{1}{1-\eta}}. \tag{37}$$

$$mc_t = \frac{MC_t^n}{P_t^H} = \frac{W_t}{Z_t P_t} \frac{P_t}{P_t^H} = w_t Q_t^{-1} S_t \; and \; \tilde{p}_{H,t} = \frac{\tilde{P}_{H,t}}{P_{H,t}}. \tag{38}$$

$$\tilde{p}_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\theta^{\tau}) \frac{c_{t+\tau}^{-\sigma}}{c_t^{-\sigma}} \left(\frac{\pi_H^{\tau},*}{\prod_{s=1}^{\tau} \pi_{H,t+s}} \right)^{-\epsilon} S_{t+\tau} Q_{t+\tau} y_{t+\tau} mc_{t+\tau} \prod_{s=1}^{\tau} \pi_{H,t+s} \right]}{\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\theta^{\tau}) \frac{c_{t+\tau}^{-\sigma}}{c_t^{-\sigma}} \left(\frac{\pi_H^{\tau},*}{\prod_{s=1}^{\tau} \pi_{H,t+s}} \right)^{-\epsilon} S_{t+\tau} Q_{t+\tau} y_{t+\tau} \pi_H^{\tau}, * \right]}. \tag{39}$$

$$1 = \left[\theta \tilde{p}_{H,t}^{1-\epsilon} + (1-\theta)(\pi_{H,*}/\pi_{H,t-1})^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{40}$$

$$y_t = N_t \left[\int \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} di \right]^{-1} \tag{41}$$

$$c_t = \vartheta c_t^* S_t^{1/\sigma}. \tag{42}$$

$$y_t = \vartheta c_t^* Q_t^{-\eta} \left[(1-\alpha) S_t^{1/\sigma - \eta} + \alpha \right]. \tag{43}$$

9.2 Approximating the likelihood of the MSDGE model

The model solution in equation 14 is combined with a measurement equation to form the following state-space representation

$$y_t = H\hat{x}$$
$$\hat{x}_t = \Omega(s_t)\hat{x}_{t-1} + \Delta(s_t)v_t, \; v_t \sim N(0, \Sigma_v)$$

where y_t represents the data used for estimation and $\hat{x}_t$ describes the model state variables.

The presence of different regimes causes the predictions of the Kalman filter to be conditional on the entire history of regimes in place throughout the sample leaving the standard filter intractable. Therefore, the algorithm proposed by Kim (1994) approximates the likelihood function. To address the problem of predictions dependent on the history of regimes, a limited number of predictions of model variables are carried forward from the Kalman filter iterations each period, and these are then collapsed at the end of each iteration. Following Kim and Nelson (1999) and Liu and Mumtaz (2011), I track forecasts that depend on the regime in place in period t, $t-1$ and $t-2$ and accounts for for the model with switching deep parameters and stochastic volatility this implies $4^3 = 64$ possible paths of the model variables.[21]

The algorithm begins with a Kalman filtering step to predict the state variables $\hat{x}_t$

[21] For the model that only allows for stochastic volatility regime this implies $2^3 = 8$ possible paths.

$$\hat{x}_{t|t-1}^{(h,i,j)} = \Omega^j \hat{x}_{t|t-1}^{(i,j)}$$

$$P_{t|t-1}^{(h,i,j)} = \Omega^j \hat{x}_{t|t-1}^{(i,j)} \Omega^{j'} + \Delta^j \Sigma_v \Delta^{j'}$$

$$\eta_{t|t-1}^{(h,i,j)} = \mathbf{y_t} - H\hat{x}_{t|t-1}^{(h,i,j)}$$

$$f_{t|t-1}^{(h,i,j)} = HP_{t|t-1}^{(h,i,j)} H'$$

$$\hat{x}_{t|t}^{(h,i,j)} = \hat{x}_{t|t-1}^{(h,i,j)} + P_{t|t-1}^{(h,i,j)} H(f_{t|t-1}^{(h,i,j)})^{-1} \eta_{t|t-1}^{(h,i,j)}$$

$$P_{t|t}^{(h,i,j)} = (I - P_{t|t-1}^{(h,i,j)} H'(f_{t|t-1}^{(h,i,j)})^{-1}) P_{t|t-1}^{(h,i,j)}$$

where $\hat{x}_{t|t-1}^{(h,i,j)}$ represent the predictions the model state variables that are conditioned on past information $t-1$ and the regime in place at time t, in the previous period $t-1$ and in $t-2$. Similarly $P_{t|t-1}^{(h,i,j)}$ is the covariance of this predictions, $\eta_{t|t-1}^{(h,i,j)}$ denotes the forecast error and $f_{t|t-1}^{(h,i,j)}$ represents covariance of the forecast error. $\hat{x}_{t|t-1}^{(i,j)}$ represents the collapsed states and the predictions of the model that are conditioned on past information $t-1$, the regime in place at time t and the regime of the previous period $t-1$. The collapsing stage reduces the 64 predictions of the states to 16 and is conducted using weights based on the joint and marginal probabilities of the regimes in place.

$$\hat{x}_{t|t}^{(i,j)} = \frac{\Pr(s_{t-2} = h, s_{t-1} = i, s_t = j)}{\Pr(s_{t-1} = i, s_t = j)} \hat{x}_{t|t}^{(h,i,j)}$$

To obtain the probability terms, the Hamilton filter is applied

$$\Pr(s_{t-2} = h, s_{t-1} = i, s_t = j|\varphi_{t-1}) = \Pr(s_t = j|s_t = i, \hat{z}_{t-1}) \times \sum_{g=1}^{4} \Pr(s_{t-3} = g, s_{t-2} = h, s_{t-1} = i|\varphi_{t-1})$$

where $|\varphi_{t-1}$ represents conditioning on information in time period $t-1$, this term updated with φ_t

$$\Pr(s_{t-2} = h, s_{t-1} = i, s_t = j|\varphi_t) = \frac{f(y_t|s_{t-2} = h, s_{t-1} = i, s_t = j, \varphi_{t-1}) \times \Pr(s_{t-2} = h, s_{t-1} = i, s_t = j|\varphi_{t-1})}{\sum_{h=1}^{2} \sum_{i=1}^{2} \sum_{j=1}^{2} f(y_t|s_{t-2} = h, s_{t-1} = i, s_t = j, \varphi_{t-1})}$$

where the conditional densities are defined as

$$f(y_t|s_{t-2} = h, s_{t-1} = i, s_t = j, \varphi_{t-1}) = 2\pi^{-n/2} \left| f_{t|t-1}^{(h,i,j)} \right|^{-1/2} \exp\left(-\frac{1}{2} \eta_{t|t-1}^{(h,i,j)} \left(f_{t|t-1}^{(h,i,j)'} \right)^{-1} \eta_{t|t-1}^{(h,i,j)'} \right)$$

and the filter probability of regime j being in place at time is given by

$$\Pr(s_t = j|\varphi_t) = \sum_{h=1}^{4} \sum_{i=1}^{4} \Pr(s_{t-2} = h, s_{t-1} = i, s_t = j|\varphi_t).$$

The by-products of the filter can be used to the calculate marginal density of y_t

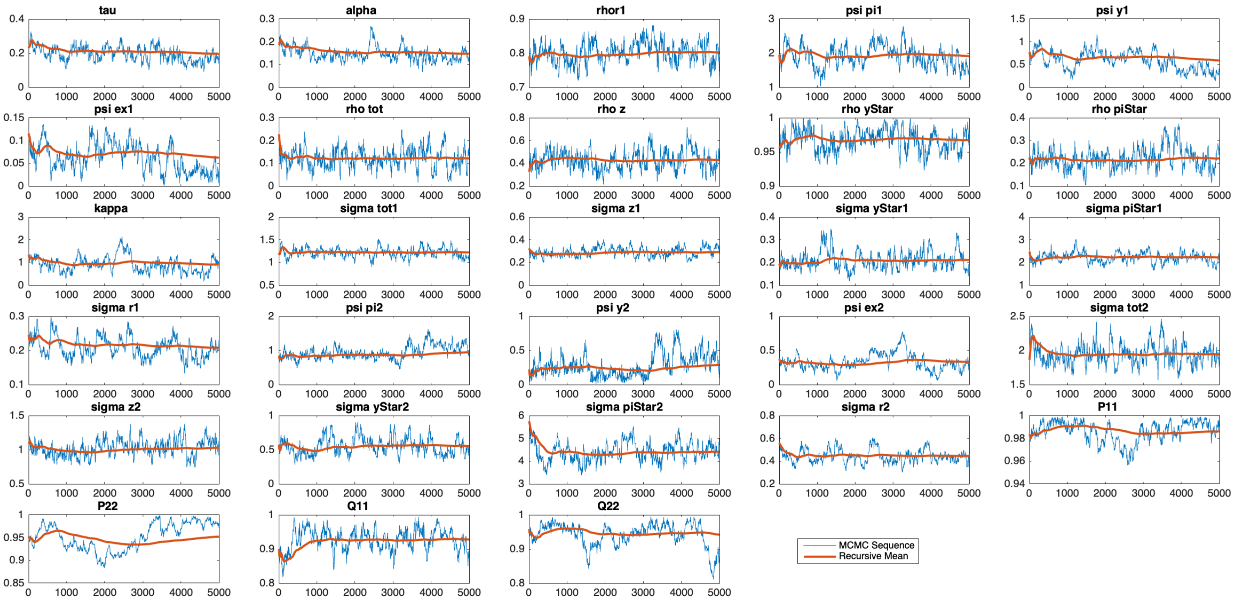
$$f(y_t|\varphi_{t-1}) = f(y_t, s_{t-2} = h, s_{t-1} = i, s_t = j|\varphi_{t-1})$$

the approximate log likelihood is then given by

$$LL = \sum_{t=1}^{T} log(f(y_t|\varphi_{t-1})).$$

Any set of parameter that do not satisfy the normailisation conditions described in section 3 that identify the regimes are discarded by receiving a likelihood close to zero.

Figure 10: Sequence and recursive means of Metropolis Hastings draws used to approximate the posterior distribution of parameters



Note: Each subplot of Figure 10 represents draws of a specific parameters of the best-fitting model with monetary policy and independent volatility regimes. The solid blue line represents the sequence of draws used to approximate the posterior distribution while the red line represents the recursive mean of the respective sequence of the parameter draws.

9.3 Convergence of the best-fitting model - Monetary policy regimes and independent volatility regimes

Figure 10 presents the sequence and recursive means of the Metropolis Hastings draws used to approximate the posterior of parameters of the best fitting model that allows for monetary policy regimes and independent volatility regimes. The sequence of draws of most parameters appears to fluctuate around a relatively stable mean providing some evidence of convergence to the posterior distribution.

9.4 Parameter estimates and filter probabilities of model with switching Phillips curve and monetary policy

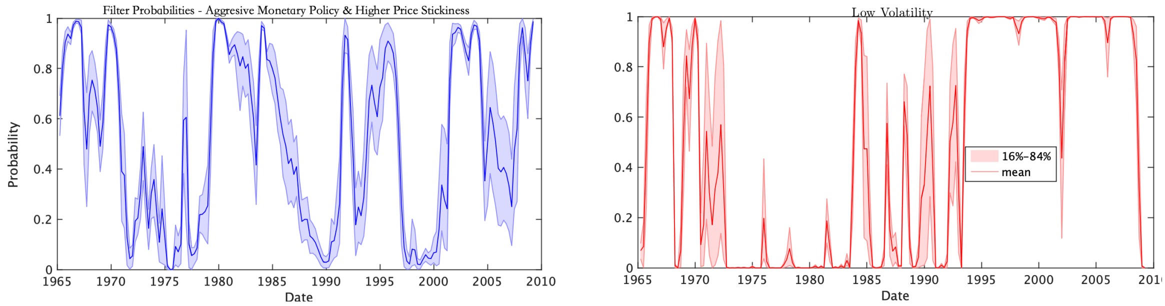
This model also allows for regime shifts along two dimensions. First, as in all switching models, the variance of the structural shocks is regime dependent. Secondly, the Phillips curve slope parameter κ and the parameters of the monetary policy rule ψ_π, ψ_y, ψ_e are allowed to switch simultaneously following Chen and MacDonald (2012). The estimated parameters that do not change are consistent the estimates of the fixed parameter model presented in the section. Similar to the model that allows the degree of nominal rigidity to switch in isolation, there is a significant shift in this slope of the Phillips curve parameter identifying a low and high degree of nominal rigidity with posterior estimates of κ centered around 0.94 and 0.03. There are also two monetary policy regimes identified, which differ considerably on the stance taken to deviations from inflation. Figure 11 displays the filter probabilities of the aggressive monetary policy and high nominal rigidity regime. Allowing the degree of nominal rigidity to change alongside the monetary policy regime induces frequent regime changes with the transition probability of the high nominal rigidity regime at p_{11} at 0.93 implying an expected duration of 14 quarters and the estimate for the low nominal rigidity regime p_{22} at 0.95. The probability of the regime characterized by a higher degree of nominal rigidity and aggressive monetary policy is low during the mid-1970s, 1990, 1999, and then finally, during the recent recession in 2008. The magnitudes and filter probabilities of the volatility regimes are identical to the other regime switching models.

Table 9: Parameter estimates of alternative MS-DSGE model - Monetary policy and nominal rigidities switch simultaneously

	Monetary policy and nominal rigidities	
α	0.1 [0.086,0.12]	
τ	0.13 [0.11,0.17]	
κ	0.94 [0.87,1.13]	0.03 [0.01,0.04]
ρ_r	0.77 [0.74,0.81]	
ψ_π	0.97 [0.86,.1.06]	1.97 [1.59,2.23]
ψ_y	0.33 [0.27,0.42]	0.37 [0.22,0.59]
ψ_e	0.08 [0.04,0.12]	0.13 [0.09,0.17]
ρ_z	0.43 [0.37,0.54]	
ρ_{y^*}	0.98 [0.97,0.99]	
ρ_{π^*}	0.25 [0.22,0.28]	
ρ_q	0.1 [0.07,0.14]	
σ^2_R	0.37 [0.35,0.41]	0.19 [0.16,0.22]
σ^2_z	1.01 [0.88,1.16]	0.32 [0.28,0.41]
$\sigma^2_{y^*}$	0.60 [0.47,0.79]	0.19 [0.16,0.21]
$\sigma^2_{\pi^*}$	4.31 [4.00,4.60]	2.18 [2.01,2.48]
σ^2_q	1.99 [1.87,2.15]	1.2 [1.08,1.34]
p_{11}	0.93 [0.92,0.93]	
p_{22}	0.93 [0.90,0.98]	
q_{11}	0.93 [0.90,0.96]	
q_{22}	0.95 [0.93,0.97]	

Note: The posterior median parameter estimates are displayed alongside 90% credible interval in square brackets

Figure 11: Filter probabilities for all MSDSE model - Monetary policy and nominal rigidities switch simultaneously



Note: The figure presents the filter probabilities of the model which allow the Phillips curve slope parameter and monetary policy reaction function to change simultaneously. The solid lines are the posterior mean filter probabilities and the shaded area represents the 68% credible sets. In the first panel the blue lines represent the probability of the aggressive monetary policy regime and the high nominal rigidity regime. In the second panel the red lines represent the probability of the low volatility regime.

9.5 Alternative solution methods - Davig and Leeper (2007)

Although this paper applies the solution method of Farmer, Waggoner and Zha (2011) to solve MS-DSGE models an early draft of this paper uses the alternative solution method of Davig and Leeper (2007). As mentioned in the subsection 4.1, Farmer, Waggoner and Zha (2011) demonstrate that the determinacy conditions proposed in Davig and Leeper (2007) do no hold when considering the non-linear model. However, the parameter estimates, model comparison and implied inflation persistence using the solution method of Davig and Leeper (2007) are consistent with the results presented in sections 5 and 6. Therefore, this section briefly outlines this algorithm as it provides a convenient introduction to solving MS-DSGE models.[22]

It is assumed that agents do not know the current regime; however, transition probabilities are know and used by agents to make inferences about the current regime and these inferences are updated using Bayes rule and following Eo (2009). The agents incorporate the probability of regime change when forming expectations of model variables in periods ahead. Therefore, expectational terms $\mathbb{E}_t\widehat{y}_{t+1}$ and $\mathbb{E}_t\hat{\pi}_{t+1}$ are redefined:

$$\mathbb{E}_t[\hat{y}_{t+1}|S_t = i] = p_{i1}\mathbb{E}_t\hat{y}_{1t+1} + p_{i2}\mathbb{E}_t\hat{y}_{2t+1} \tag{44}$$

$$\mathbb{E}_t[\hat{\pi}_{t+1}|S_t = i] = p_{i1}\mathbb{E}_t\hat{\pi}_{1t+1} + p_{i2}\mathbb{E}_t\hat{\pi}_{2t+1} \tag{45}$$

where $\hat{y}_{1t+1}$ ($\hat{y}_{2t+1}$) represent values of output in time $t+1$ when regime 1 (regime 2) is place. The presence of transition probabilities in agents expectations incorporates the possibility of monetary policy regime in the following period. This is in contrast to subsample estimation of a constant parameter model, where agents expect to remain in the same policy regime forever. This difference in the formation of expectations alters the dynamics of the forward-looking Markov-switching model; Davig and Leeper (2007) refer to this difference as 'expectation formation effects'. These effects disappear when the economy is faced with an absorbing regime. [23]

The model equations 1-8 are rewritten to capture the effects of expected regime changes on the dynamics of state variables. The state variables are now regime-dependent and the expectational terms are replaced using equations 44-45. The probabilistic inferences on the regimes in place requires the lagged state variables to also be replaced by weighted averages under alternative regime for example $\hat{R}_{t-1} = p_{1i}\hat{R}_{1t-1} + p_{2i}\hat{R}_{2t-1}$.

The equations of the best-fitting Markov-switching model, that allows for independent monetary policy and shock volatility regimes, are rewritten to present the solution method of Davig and Leeper (2007) and presented below.

$$\hat{y}_{it} = p_{i1}\mathbb{E}_t y_{1t+1} + p_{i2}\mathbb{E}_t y_{2t+1} - (\tau+\lambda)(\hat{R}_{it} - p_{i1}\mathbb{E}_t\pi_{1t+1} + -p_{i2}\mathbb{E}_t\pi_{2t+1} - \rho_z\hat{z}_{it})...$$

$$+\alpha(\tau+\lambda)(\rho_{\Delta q}\Delta q_{it+1}) + \frac{\lambda}{\tau}\rho_{y^*}(\hat{y}^*_{it} - p_{1i}\hat{y}^*_{1t-1} - p_{2i}\hat{y}^*_{2t-1}) \tag{46}$$

$$\hat{\pi}_{it} = \beta(p_{i1}\mathbb{E}_t\pi_{1t+1} + p_{i2}\mathbb{E}_t\pi_{2t+1}) + \alpha\beta\mathbb{E}_t\rho_{\Delta q}\Delta q_{it+1} - \alpha\Delta q_{it} + \frac{\kappa}{(\tau+\lambda)}\hat{y}_{it} + \frac{\kappa\lambda}{\tau(\tau+\lambda)}\hat{y}^*_{it} \tag{47}$$

$$\hat{R}_{it} = \rho_r(p_{1i}\hat{R}_{1t-1}+p_{2i}\hat{R}_{2t-1})+(1-\rho_r)[\psi^{\pi}(s_t)(\hat{\pi}_{it})+\psi^{y}(s_t)(\hat{y}_{it}-p_{1i}\hat{y}_{1t-1}-p_{2i}\hat{y}_{2t-1}+\hat{z}_{it})+\psi^{e}(s_t)e_{it}+\varepsilon^R_{it}]...$$

$$\varepsilon^R_{it} \sim NID(0,\sigma^2_R(S_t)) \tag{48}$$

$$\Delta\hat{e}_{it} = \hat{\pi}_{it} - (1-\alpha)\Delta\hat{q}_{it} - \hat{\pi}^*_{it} \tag{49}$$

$$\hat{z}_{it} = \rho_z(p_{1i}\hat{z}_{1t-1} + p_{2i}\hat{z}_{2t-1}) + \varepsilon^z_{it}, \varepsilon^z_{it} \sim NID(0,\sigma^2_z(S_t)) \tag{50}$$

$$\Delta\hat{q}_{it} = \rho_{\Delta q}(p_{1i}\Delta q_{1t-1} + p_{2i}\Delta q_{2t-1}) + \varepsilon^q_{it}, \varepsilon^q_{it} \sim NID(0,\sigma^2_q(S_t)) \tag{51}$$

[22] Davig and Doh (2014) apply this solution method Davig and Leeper (2007) to solve a small scale MS-DSGE model.

[23] For example if the transition probability $p_{11} = 1$ if the economy enters regime 1 it will remain there for all time periods.

$$\hat{y}^*_{it} = \rho_{y^*}(p_{1i}\hat{y}^*_{1t-1} + p_{2i}\hat{y}^*_{2t-1}) + \varepsilon^{y^*}_{it}, \varepsilon^{y^*}_{it} \sim NID(0, \sigma^2_{y^*}(S_t)) \tag{52}$$

$$\hat{\pi}^*_{it} = \rho_{\pi^*}(p_{1i}\hat{\pi}^*_{1t-1} + p_{2i}\hat{\pi}^*_{2t-1}) + \varepsilon^{\pi^*}_{it}, \varepsilon^{\pi^*}_{it} \sim NID(0, \sigma^2_{\pi^*}(S_t)) \tag{53}$$

$$s_t = i,\ i = 1, 2\, and\, S_t = k,\ k = 1, 2$$

As described in subsection 4.1, the first approximation of the underlying non-linear system implies that independent switching in the volatilities of structural shocks does not affect the model solution due to the presence of certainty equivalence.

The linearised described in equations 46-53 is written assuming the coefficient regime for $s_t = 1$ is in place and then repeated for when $s_t = 2$, stacking the models implied by each regime provides a linear representation

$$A\hat{x}_t = B\hat{x}_{t-1} + Cv_t + D\eta_t. \tag{54}$$

where A, B, C and D are matrices and function of the structural model parameters Φ. Under rational expectations and no regimes shifts, the model can then be solved using a standard rational expectation algorithms such as the *Gensys* solution method proposed in Sims (2002).

$$A(\Phi)\underbrace{\left[\begin{array}{c} \hat{x}_{1t}\left\{\begin{array}{c} \hat{y}_{1t} \\ \hat{\pi}_{1t} \\ \Delta\hat{e}_{1t} \\ \hat{R}_{1t} \\ \Delta\hat{q}_{1t} \\ \hat{z}_{1t} \\ \hat{y}^*_{1t} \\ \hat{\pi}^*_{1t} \\ \mathbb{E}_t y_{1t+1} \\ \mathbb{E}_t \pi_{1t+1} \end{array}\right\} \\ \hline \hat{x}_{2t}\left\{\begin{array}{c} \hat{y}_{2t} \\ \hat{\pi}_{2t} \\ \Delta\hat{e}_{2t} \\ \hat{R}_{2t} \\ \Delta\hat{q}_{2t} \\ \hat{z}_{2t} \\ \hat{y}^*_{2t} \\ \hat{\pi}^*_{2t} \\ \mathbb{E}_t y_{2t+1} \\ \mathbb{E}_t \pi_{2t+1} \end{array}\right\} \end{array}\right]}_{\hat{x}_t} = B(\Phi)\underbrace{\left[\begin{array}{c} \hat{x}_{1t}\left\{\begin{array}{c} \hat{y}_{1t-1} \\ \hat{\pi}_{1t-1} \\ \Delta\hat{e}_{1t-1} \\ \hat{R}_{1t-1} \\ \Delta\hat{q}_{1t-1} \\ \hat{z}_{1t-1} \\ \hat{y}^*_{1t-1} \\ \hat{\pi}^*_{1t-1} \\ \mathbb{E}_{t-1} y_{1t} \\ \mathbb{E}_{t-1} \pi_{t} \end{array}\right\} \\ \hline \hat{x}_{2t}\left\{\begin{array}{c} \hat{y}_{2t-1} \\ \hat{\pi}_{2t-1} \\ \Delta\hat{e}_{2t-1} \\ \hat{R}_{2t-1} \\ \Delta\hat{q}_{2t-1} \\ \hat{z}_{2t-1} \\ \hat{y}^*_{2t-1} \\ \hat{\pi}^*_{2t-1} \\ \mathbb{E}_{t-1} y_{2t} \\ \mathbb{E}_{t-1} \pi_{2t} \end{array}\right\} \end{array}\right]}_{\hat{x}_{t-1}} + C(\Phi)\underbrace{\left[\begin{array}{c} v_{1t}\left\{\begin{array}{c} \varepsilon^R_{1t} \\ \varepsilon^z_{1t} \\ \varepsilon^q_{1t} \\ \varepsilon^{y^*}_{1t} \\ \varepsilon^{\pi^*}_{1t} \end{array}\right\} \\ \hline v_{2t}\left\{\begin{array}{c} \varepsilon^R_{2t} \\ \varepsilon^z_{2t} \\ \varepsilon^q_{2t} \\ \varepsilon^{y^*}_{2t} \\ \varepsilon^{\pi^*}_{2t} \end{array}\right\} \end{array}\right]}_{v_t} + D(\Phi)\underbrace{\left[\begin{array}{c} \eta^y_{1t} \\ \eta^\pi_{1t} \\ \hline \eta^y_{1t} \\ \eta^\pi_{1t} \end{array}\right]}_{\eta_t}$$

The solution is rearranged to be expressed in the following form

$$\hat{x}_{1t} = \Omega(s_t = 1)\hat{x}_{1t-1} + \Delta(s_t = 1)v_{1t},$$
$$\hat{x}_{2t} = \Omega(s_t = 2)\hat{x}_{2t-1} + \Delta(s_t = 2)v_{2t},$$

where solution matrices $\Omega(\Phi)$ and $\Delta(\Phi)$ are functions of the structural model parameters. The model is then estimated using Bayesian methods as described in subsection 4.2, with the Kim filter described in section 9.2 applied to obtain the likelihood function. The parameter estimates and filter probabilities of the best fitting model that allows for monetary policy regimes and independent shifts in the volatility are presented in table 10 and figure 12. The parameter estimates are consistent with the results presented in section 5 using the solution algorithm of Farmer et al. (2011). However, the filter probabilities differ as the probability of being in the aggressive regime is close to zero during early 1990s.

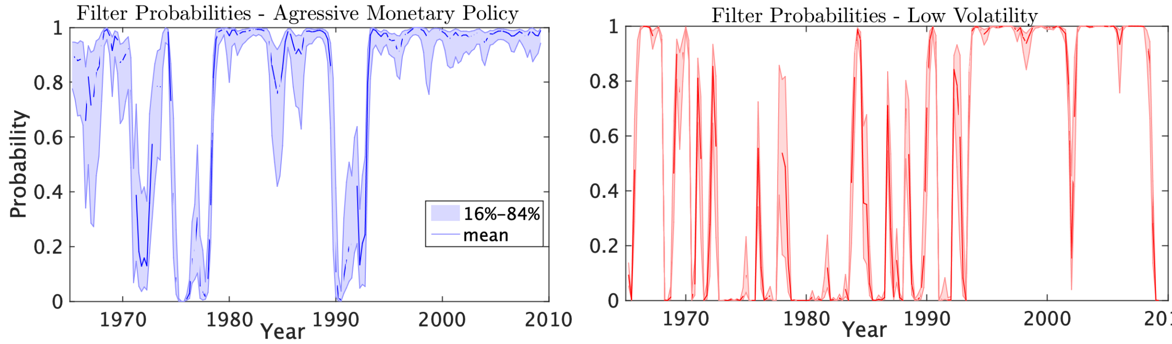
Table 11 reports the posterior estimates of the inflation persistence measure applied in Davig of Doh (2014) for the best fitting MS-DSGE model with the 16% and 84% credible intervals. This measure is described in subsection 6.3. The preferred model estimates are in the first column and are very similar regardless of the regime in place. When the inflation-targeting regime is in place estimates of persistence are slightly lower, moving from 0.88 to around 0.85. However, the credible sets are very similar indicating no significant change of inflation persistence. This finding suggests the persistence of the inflation gap has not changed over the sample and neither the regimes identified in monetary policy and volatility have an effect on inflation persistence.

Table 10: Parameter estimates of best-fitting MS-DSGE model using Davig and Leeper (2007) solution method - Monetary policy and independent shock volatility regimes

	Monetary policy	
α	0.09 [0.07,0.13]	
τ	0.27 [0.16,0.35]	
κ	0.26 [0.08,0.44]	
ρ_r	0.83 [0.78,0.86]	
ψ_π	1.25 [0.93,.1.59]	2.26 [1.84,2.67]
ψ_y	0.45 [0.26,0.61]	0.26 [0.12,0.49]
ψ_e	0.17 [0.06,0.26]	0.09 [0.02,0.15]
ρ_z	0.41 [0.26,0.64]	
ρ_{y^*}	0.98 [0.97,0.99]	
ρ_{π^*}	0.24 [0.17,0.31]	
ρ_q	0.1 [0.03,0.21]	
σ_R^2	0.48 [0.42,0.54]	0.20 [0.17,0.23]
σ_z^2	1.25 [1.01,1.53]	0.43 [0.34,0.51]
$\sigma_{y^*}^2$	1.69 [0.75,1.14]	0.94 [0.75,1.14]
$\sigma_{\pi^*}^2$	2.31 [2.04,4.60]	5.02 [4.19,5.89]
σ_q^2	2.25 [1.92,2.61]	1.28 [1.18,1.42]
p_{11}	0.92 [0.90,0.95]	
p_{22}	0.95 [0.91,0.97]	
q_{11}	0.84 [0.93,0.97]	
q_{22}	0.91 [0.84,0.96]	

Note: This table presents the estimates of the best-fitting MS-DSGE model estimated that allows for monetary policy regime change and independent switching in shocks volatilities with solution method of Davig and Leeper (2007). The posterior median parameter estimates are displayed alongside 90% credible interval in square brackets.

Figure 12: Filter probabilities for best-fitting MS-DSGE model using Davig and Leeper (2007) solution method - Monetary policy and independent shock volatility regimes regimes



Note: The figure presents the filter probabilities of the model which allow the monetary policy reaction function to change estimated with the solution method of Davig and Leeper (2007). The solid lines are the posterior mean filter probabilities and the shaded area represents the 68% credible sets. In the first panel the blue lines represent the probability of the aggressive monetary policy regime and the relatively aggressive regime. In the second panel the red lines represent the probability of the low volatility regime.

Table 11: Population moment of autocorrelation for best-fitting MS-DSGE model using Davig and Leeper (2007) solution method - Monetary policy and independent shock volatility regimes regime

Regime	Monetary Policy
$AR(\hat{\pi}_t \mid s_t = 1, S_t = 1)$	0.88 [0.8,0.98]
$AR(\hat{\pi}_t \mid s_t = 1, S_t = 2)$	0.87 [0.82,0.88]
$AR(\hat{\pi}_t \mid s_t = 2, S_t = 1)$	0.85 [0.82,0.91]
$AR(\hat{\pi}_t \mid s_t = 2, S_t = 2)$	0.84 [0.77,0.9]

Note: Where $AR(\hat{\pi}_t|s_t = 2, S_t = 2)$ measures the median population moment of autocorrelation during the coefficient regime and volatility regime 2 (less aggressive monetary policy and low volatility) . The 68% credible sets are presented in square brackets.

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