State-dependent Monetary Policy Regimes

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Abstract

Are monetary policy regimes state-dependent? To answer the question this paper estimates New Keynesian general equilibrium models that allow the state of the economy to influence the monetary authority's stance on inflation. I take advantage of recent developments in solving rational expectations models with state-dependent parameter drift to estimate three models on U.S. data between 1965-2009. In these models, the probability of remaining in a monetary policy regime that is relatively accommodative towards inflation, varies over time and depends on endogenous model variables; in particular, either deviations of inflation or output from their respective targets or a monetary policy shock. The main contribution of this paper is that it finds evidence of state-dependent monetary policy regimes. The model that allows inflation to influence the monetary policy regime in place, fits the data better than an alternative model with regime changes that are not state-dependent. This finding points towards reconsidering how changes in monetary policy are modeled.

Key words: Markov-Switching DSGE, State-dependence, Bayesian Estimation

JEL Codes: C13, C32, E42, E43

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1 Introduction

How to model changes in monetary policy is an important and relevant question; as in the last thirty years most advanced economies have experienced at least two major changes in the way their central bank conducts monetary policy. The first and the one this paper examines, is the adoption of inflation-targeting, which explicitly began in the early 1990s where beforehand, other nominal variables such as exchange rates or the monetary base were targeted. The second is the more recent move towards unconventional monetary policy instruments after the nominal interest rate hit its effective lower-bound, prompted in most economies by the global financial crisis of 2008. Despite these shifts in monetary policy, estimated structural models that are used for policy analysis often assume the behaviour of the central bank remains constant across the entire estimation sample. The primary example is New Keynesian general equilibrium models, that are widely used by central banks and typically estimated on at least two decades of quarterly data. In this framework, the monetary policy instrument is usually the nominal interest rate, and it is set following a systematic rule that depends on how far target variables such as price inflation and output are away from target levels.

Recent advances in the literature propose empirical approaches to modeling changes in monetary policy within this class of models that allow for time-variation in parameters of the systematic rule. The time-varying parameters are typically those that determine either the elasticity of the nominal interest rate with respect to deviations of inflation away from target level or the inflation target. However, parameter change that is consistent with the rational expectations hypothesis complicates the solution of these models, as additional non-linearities are introduced to the underlying micro-founded model. Therefore to keep additional complexities to a minimum, the time-varying parameters are assumed to follow an exogenous process. This assumption implies that endogenous model variables are not allowed to influence changes in monetary policy regime directly. Therefore, when agents form expectations of future variables needed to construct their optimal decision rules, the probability they assign to future policy changes is not directly effected by the recent state of the economy.

The assumption of a completely exogenous source of time-variation although simplifying, seems unrealistic as ultimately the state of the economy determines how central banks choose their targets and the strength with which these targets are pursued. Also, central banks, as public institutions are judged on their performance, i.e. how well they deliver on their objectives; therefore, it seems natural to assume that central banks will be under pressure to change their policy approach if targets are considerably or persistently missed.

Therefore, this paper relaxes the assumption of a completely exogenous source of time-variation by estimating New Keynesian general equilibrium models that allow for changes in the monetary authority's stance on inflation to depend on the performance of the economy and finds supportive evidence of this mechanism. The mechanism takes the form of a logit function that explicitly links the probability of remaining in a monetary policy regime to endogenous model variables. Model dynamics are affected through the formation of agents expectations as the probability of future regime change can now depend on variables that gauge the performance of the economy relative to target levels such as inflation and output gaps. This paper estimates three models with state-dependence on U.S. data, in which, the inflation gap, output gap or a monetary policy shock that captures unsystematic changes in monetary policy, can influence the probability of remaining in a monetary policy regime that is relatively accommodative towards inflation.

To estimate the effects of allowing for state-dependent monetary policy regimes, I focus on the well-studied case of the adoption of inflation targeting by the U.S. Federal Reserve, spearheaded by the appointment of chairman Paul Volcker in 1979. During the regime prior to Volcker, the central bank is believed to have been more accommodative of inflation and instead pursued other nominal targets such as the monetary base and exchange rates, in addition to placing more emphasis on influencing unemployment. The increased focus on inflation policy is put forward as one of the leading causes of the remarkable stabilisation of the U.S. economy from the mid-1980s until the recent global financial crisis in 2008, often referred to as the great moderation period. However, as the existing literature does not allow for the type of state-dependence considered in this paper, this particular regime change is an ideal candidate for evaluating the mechanism.

The best fitting model identifies the period between 1973 and 1985 as being relatively accommodative towards inflation. In this model, the probability of remaining in this regime has an inverse relationship to the amount of inflation in the previous period is above the target level. This form of state-dependence reduces the expected duration of the accommodative regime from eight quarters in 1973 to four quarters in 1978, when inflation reached peak rates. The time-varying probabilities show some evidence of dampening responses to cost-push shocks relative to a model estimated with no state-dependence. Parameter estimates are also affected, most importantly, the estimated inflation target in the accommodative regime is lower when state-dependence is enabled.

This rest of this paper is organised as follows. Section 2 presents a brief review of how this paper fits in with the existing literature. Section 3 illustrates the underlying New Keynesian model and introduces the concept of state-dependent monetary policy regimes. Section 4 discusses the estimation strategy and presents simulation results. In Section 5, I consider which of the estimated models provides the best fit with the data. Section 6 presents impulse responses from the selected model, as this model is state-dependent results are compared to estimated models with no state-dependence. Finally, Section 7 concludes.

2 Existing literature

This paper extends the work of Barthélemy and Marx (2017) to estimate general equilibrium models with state-dependent monetary policy regimes.

Barthélemy and Marx (2017) develop an algorithm to solve general equilibrium models with regime-switching parameters in which the transition probabilities are time-varying and smooth functions of endogenous model variables. They then apply their method to solve a calibrated model with two monetary policy regimes. The regime that is relatively accommodative towards inflation, with a higher inflation target and a relatively weaker response to deviations away from this target, is state-dependent. Specifically, the probability of leaving this regime is a logit function of the distance past inflation is away from the inflation target of this regime. As the degree of state-dependence is unknown, Barthélemy and Marx (2017) experiment with a broad range of values to show that the mechanism alters model-implied dynamics for macroeconomic variables and their responses to shocks.

To extend the calibration exercise of Barthélemy and Marx (2017) to estimation, I adopt a Bayesian approach and take this model to U.S. data. Thus providing an estimate of the degree of state-dependence of the relatively inflation accommodating regime believed to be in place during the 1970s by the seminal studies of Clarida, Gali and Gertler (2000) and Lubik and Schorfheide (2004). Additionally, I also consider two alternative forms of state-dependence, in which, the output gap and a monetary policy shock that captures unsystematic changes in monetary policy, can influence the probability of remaining in a monetary policy regime that is relatively accommodative towards inflation.

From the models considered, the model preferred by the data is the one considered in Barthélemy and Marx (2017) and allows for state-dependence through that deviations of inflations from the relatively higher target level of the accommodative regime. This form of state-dependence in general equilibrium models was first considered in Davig and Leeper (2008), who solve a model with two similar monetary policy regimes that differ with respect to their stance on inflation. However, in the case of Davig and Leeper (2008), only the monetary policy instrument's inflation elasticity can discretely change, with the switch being triggered by the previous periods inflation gap from a target level crossing a certain threshold. Their threshold-switching approach relies on global approximation methods that can have large computational costs, which are undesirable when following a Bayesian approach to estimation.[1 2]

Solutions algorithms that allow for feasible estimation of general equilibrium models with state-dependence, such as those proposed by Barthélemy and Marx (2017), Maih (2015) and Forester et al. (2018) have only recently become available. Therefore, to my knowledge only a handful of studies have conducted empirical analysis with this class of models.

The closest current research to this paper is Chang, Tan and Wei (2018), who

[1] Applying Bayesian techniques to model estimation is now the conventional approach to estimating New Keynesian general equilibrium models; benefits are described in detail in An and Schorfheide (2003) and Herbst and Schorfheide (2015).

[2] Markov Chain Monte Carlo methods are used to approximate the posterior distribution of parameters and object of interest which require solving the general equilibrium model over a hundred thousand times, therefore a solution method with a low computational method is highly desirable.

also adopt the solution method of Barthélemy and Marx (2017) to estimate a regime-switching model with state-dependent monetary policy regimes. Chang, Tan and Wei (2018) consider a similar underlying model and also study the Federal Reserve's adoption of a more aggressive stance towards inflation during 1980s. The main difference between the work of Chang, Tan and Wei (2018) with this paper is the form of state-dependence considered. Chang, Tan and Wei (2018)assume the level of a latent factor determines the regime in place, and state-dependence is allowed for by correlation between the latent factor and the monetary policy shock. As the latent factor follows an autoregressive process, their mechanism allows for an endogenous feedback channel in which, the recent history of policy shocks lead to a cumulative effect on the regime factor that eventually trigger a regime shift. The economic intuition behind this mechanism is that assuming a monetary policy regime that accommodates inflation is in place a series of contractionary shocks agents would lead to agents expecting an aggressive monetary policy regime would come in its place. Whereas, I follow Barthélemy and Marx (2017) by enabling a logit function to link the probability of switching regimes to endogenous model variables, which in the case of output and inflation are observed in the data. Chang, Maih and Tan (2018) conduct a similar exercise using the latent factor to induce state-dependence but apply a solution algorithm along the lines of Maih (2015). An additional difference in this paper is that I allow the elasticity of the monetary policy instrument with respect to inflation and the inflation target to change across monetary policy regime. I present evidence of significant change in both the inflation target and sensitivity of the monetary policy instrument to changes in inflation across regimes.

Forester et al. (2018) also estimate a state-dependent general equilibrium model by allowing the probability of a crisis regime in Mexico to depend on the country's leverage to output ratio. They assume state-dependence by directly linking endogenous model variables to time-varying transition probabilities; they do so via a logistic function. Maih, Linde and Wouters (2018) estimate a larger scale general equilibrium model with monetary regimes on U.S. data, to consider how to model periods where nominal interests rate reached the effective lower bound. The model estimated with regimes that are not state-dependent compares favourably with the other approaches the authors consider. However, they suggest introducing state-dependence through time-varying transition probabilities that are a logistic function of the shadow rate, would allow for changes in the propagation mechanism during the period of the effective lower bound regime.

Although the estimation of New Keynesian general equilibrium models with state-dependent regimes is new, empirical work on regime-switching models is a developed strand of literature since the seminal work of Hamilton (1989). Markov-switching reduced-form models with state-dependence in the form of time-varying transition probabilities were first considered in economic applications by Filardo (1994) and Filardo and Gordon (1998). Kim et al. (2003) extend this method of introducing state-dependence to Markov-switching multivariate models. The theoretical work of Leeper and Zha (2003), emphasises that even small but persistent changes in monetary policy alter the way agents form expectations and therefore affects their decision rules. Further theoretical work by Davig and Leeper (2007) shows that allowing for monetary regimes through discrete switches in the monetary policy instrument's inflation elasticity response to inflation within a general equilibrium framework increases the parameter space that yields determinate solutions.

Davig and Doh (2014) and Bianchi (2013) estimate New Keynesian general equilibrium models with switching monetary policy rules and both concentrate on the adoption of an aggressive stance to inflation in the U.S. around the 1980s, as in this paper. Davig and Doh (2014) estimate a small-scale model along the lines of this paper and find a significant reduction in inflation persistence after the 1980s is primarily explained by the change in monetary policy towards aggressively pursuing inflation. Whereas, Bianchi (2013) estimates a richer model and constructs counterfactuals that suggest the possibility of moving to a monetary policy regime with an even more aggressive inflation stance than estimated would considerably mitigate the recessions in the 1970s through expectations effects.

Both Davig and Doh (2014) and Bianchi (2013) find that the best-fitting models allow regime-switching in monetary policy in addition to independent switching in the variance of structural shocks. However, despite their findings, this paper does not allow for heteroskedasticity due to the additional computational time added to solve the model and leaves this for future work that is ongoing.

The literature has yet to distinguish whether the monetary policy instrument's inflation elasticity response or inflation target or both have changed in the U.S. during the 1980s. Therefore I follow Barthélemy and Marx (2017) in allowing for two monetary policy regimes, where the relative inflation accommodating regime has a high inflation target and a relatively low Taylor rule response to deviations away from this target. Liu et al. (2011) model monetary policy regimes in the U.S. as changes in the inflation target rather than changes in the Taylor rule response to inflation. Although within this model the difference between the inflation targets of the two regimes is substantial, they find that the data prefer alternative models allowing solely for stochastic volatility. Forester (2017) conducts a simulation exercise to examine the implications of modelling monetary policy regimes as changes in the inflation target or changes in the Taylor rule response to inflation. The main findings indicate a switching target alters the level of model-implied variables, whilst a switching inflation response alters the parameter space.

Outside from the U.S., regime-switching general equilibrium models have been applied to examine the economic effects of adopting inflation targeting monetary policy regimes in small open economies as in Mumtaz and Liu (2011) and Alstaheim, Bjørnland and Maih (2013).

3 Modeling monetary policy regimes within a general equilibrium framework

To explicitly allow for changing monetary policy, some form of parameter drift must be introduced into the reaction function the central bank uses to set its policy instrument. This paper estimates a class of models where the type of parameter drift is discrete and the state of the economy can influence the monetary policy regime in place.

In this section, I briefly describe the underlying economic model and then spend more time introducing the concept of state-dependent monetary policy regimes.

3.1 Benchmark model

For the empirical analysis presented in this paper, I consider a standard small-scale New Keynesian monetary DSGE model. This model is admittedly elementary compared to those currently estimated by central banks but is selected as this is one of the first passes at estimating a general equilibrium model with state-dependent parameter changes. The model is identical to the setup of Barthélemy and Marx (2017). Davig and Doh (2014) and Chang, Tan and Wei (2018) also estimate similar models where the parameters of the monetary policy rule are subject to regime shifts.

Households

The representative household chooses $\{C_t, N_t, B_t\}_{t=0}^{\infty}$ to maximise liftime utility,

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\tau}}{1-\tau} - L_t \right),$$

where C_t denotes consumption of a composite good, L_t is hours worked, $\beta \in (0,1)$ is the discount factor and τ measures the inverse of the intertemporal elasticity of substitution. Utility maximisation is subject to the intertemporal budget constraint

$$P_t C_t + Q_t B_t = B_{t-1} + W_t L_t + P_t D_t - P_t T_t,$$

where B_t are nominal bond holdings, the variable P_t denotes the price level, Q_t is the price of a zero-coupon bond at time t yielding 1 in period $t+1$, W_t is the nominal wage per hour, D_t are the real profits from ownership of firms, and T_t are lump-sum taxes. The preference shock ξ_t represents a shock affecting the discount factor.

Firms

A continuum of intermediate goods-producing firms, denoted by the subscript $j \in [0,1]$, firm j produces output according to

$$Y_{jt} = L_{jt},$$

where L_{jt} is the labour input hired by firm j and is the only input. The labour market is perfectly competitive.

Nominal rigidities are introduced *à la* Rotemberg (1982). The monopolistic intermediate goods-producing firms pay a cost of adjusting their price, given by

$$AC = \frac{\phi}{2}\left(\frac{P_{jt}}{\Pi^*_{st} P_{jt-1}} - 1\right)^2 Y_t,$$

where $\phi \geq 0$ determines the magnitude of the price adjustment cost, Π^*_{st} denotes the regime-dependent steady-state that coincides with the central bank's inflation target in regime s_t and P_{jt} denotes the nominal price set by firm j at time t. The price adjustment cost is in terms of the final good Y_t. Each intermediate goods-producing firm maximises the expected present value of profits,

$$\sum_{s=0}^{\infty} \beta^s \lambda_{t+s} \frac{D_{jt+s}}{P_{t+s}},$$

where λ_{t+s} is the representative household's stochastic discount factor and D_{jt} are nominal profits of firm j at time t. Real profits are

$$\frac{D_{jt}}{P_t} = \frac{P_{jt} Y_{jt}}{P_t} - \frac{W_t}{P_t} Y_{jt} - \frac{\phi}{2}\left(\frac{P_{jt}}{\Pi^*_{st} P_{jt-1}} - 1\right)^2 Y_{jt}.$$

The profit-maximisation problem for the final-goods producing firm implies that the demand for each intermediate good is give by

$$y_{jt} = \left(\frac{P_{jt}}{P_{jt-1}}\right)^{-\theta_t} Y_t,$$

where $\theta_t > 0$ is the time-varying elasticity of substitution between goods.

The steady-state elasticity of substitution is θ, and the steady-state markup is given up $u = \frac{\theta}{\theta - 1}$.

Monetary Policy

The monetary authority set the short-term nominal rate using the following rule

$$\frac{R_t}{R^*(s_t)} = \left[\frac{R_{t-1}}{R * (s_{t-1})}\right]^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi^*(s_t)}\right)^{\alpha(s_t)} \left(\frac{Yt}{\bar{Y}}\right)^{\gamma} \varepsilon_t\right]^{1-\rho_r}$$

where R_t is the gross nominal interest rate, $\Pi_t = \frac{P_t}{P_{t-1}}$, $\Pi_t = 1 + \pi_t$ and R^*, Π^* and $\bar{Y}$ represent target values of the nominal interest rate, inflation and output the monetary authority pursues.

The above rule suggests the central bank adjusts the nominal interest rate in response to deviations of inflation and output from their respective targets. The shock ε_t captures unanticipated deviations from the policy rule and its standard deviation is denoted by σ_r. The parameter ρ is the degree of interest rate smoothing, and α and γ are the reaction coefficients to inflation and output gaps respectively. The monetary policy shock ε_t captures unanticipated deviations from the policy rule. This rule suggests that the targets a central bank sets and the strength it pursues deviations away from these targets are constant and do not change.

Shocks

The steady-state markup is given up $u = \frac{\theta}{\theta-1}$, while the time t markup shock follows an autoregressive process with drift

$$\ln u_t = (1-\rho_u)\ln u + \rho_u \ln u_{t-1} + \sigma_\xi \varepsilon_\xi.$$

The preference shock follows a autoregressive process

$$\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \sigma_\xi \varepsilon_\xi.$$

First-order conditions

The first-order conditions and equilibrium conditions lead to the following non-linear system

$$\theta u_t C_t^{\tau} - \phi(\theta u_t - \theta + 1)\frac{\Pi}{\Pi^*(s_t)}\left[\frac{\Pi}{\Pi^*(s_t)} - 1\right] + \beta\phi(\theta u_t - -\theta + 1)\mathbb{E}_t\left[\frac{Y_{t+1}}{Y_t}\frac{C_{t+1}^{-\tau}}{C_t^{-\tau}}\left(\frac{\Pi_{t+1}}{\Pi^*(s_{t+1})} - 1\right)\right] = \frac{\theta}{\theta-1} \tag{1}$$

$$Y_t = C_t + \frac{\phi}{2}\left[\frac{\Pi_t}{\Pi(s_t)} - 1\right]^2 Y_t \tag{2}$$

$$\frac{R_t}{R^*(s_t)} = \left[\frac{R_{t-1}}{R*(s_{t-1})}\right]^{\rho_r}\left[\left(\frac{\Pi_t}{\Pi^*(s_t)}\right)^{\alpha(s_t)}\left(\frac{Yt}{\bar{Y}}\right)^{\gamma}\varepsilon_t\right]^{1-\rho_r} \tag{3}$$

$$\mathbb{E}_t\left[\frac{\beta R_t}{\Pi_{t+1}}\left(\frac{C_t}{C_{t+1}}\right)^{\tau}\left(\frac{\xi_{t+1}}{\xi_t}\right)\right] = 1 \tag{4}$$

$$z_t = \underbrace{C_t, Y_t, R_t, \Pi_t, \xi_t, u_t}$$
$$v_t = \underbrace{\varepsilon_r, \varepsilon_\xi, \varepsilon_u}$$
$$\theta = \underbrace{\beta, \tau, \kappa, \alpha, \gamma, \sigma_r, \sigma_\xi, \sigma_u}$$

The benchmark model has one constant monetary policy regime in place and is the non-linear system of equations 1-4 which can be solved by applying perturbation methods as in Judd (1999). An equivalent method is to log-linearise the system of equations around steady-state values and then solve the linearised system using the algorithm proposed by Sims (2002).

3.2 Monetary policy regimes with no state-dependence

The assumption of rational expectations implies that within a constant parameter model, agents know that monetary policy will not change regardless of how well the central bank delivers on its targets. And to ensure equations to 1-4 have a unique and stable solution the central bank must increase the real interest rate in response to inflation, meaning $\alpha > 1$. The constant policy rule can be used to examine regime changes if estimated across subsamples. However, this approach ignores the effect of regime changes on agents expectations and therefore is not consistent with the rational expectations hypothesis.

Therefore, explicitly modelling policy regimes requires introducing time-variation in the behaviour of the monetary authority. I allow for two monetary policy regimes in the following rule

$$\frac{R_t}{R^*(s_{t-1})} = \left[\frac{R_{t-1}}{R^*(s_t)}\right]^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi^*(s_t)}\right)^{\alpha(s_t)} \left(\frac{Yt}{\bar{Y}}\right)^{\gamma} \varepsilon_t\right]^{1-\rho_r} \tag{5}$$

$$\begin{gathered} s_t \in \{1,2\}, \Pi^* \in \{\Pi_L^*, \Pi_H^*\}, \alpha \in \{\alpha_L, \alpha_H\} \\ \Pi_L^* < \Pi_H^*, \alpha_L < \alpha_H \\ \begin{array}{lc} Accomodative & s_t = 1 \\ InflationTargeting & s_t = 2 \end{array} \left\{ \begin{array}{c} \Pi_H^*, \alpha_L \\ \Pi_L^*, \alpha_H \end{array} \right\} \end{gathered} \tag{6}$$

where $\alpha(s_t)$ dictates the regime-dependent response of nominal interest rates to the deviations of inflation from a regime-dependent target $\Pi^*(s_t)$. These two parameters follow the same two-state first-order Markov process governed by the latent variable s_t. Equations 5 and 6 suggest when $s_t = 1$, the central bank adopts a relatively accommodative stance towards inflation by setting a high inflation target Π_H^* and responding relatively weaker to deviations away from this target captured by parameter α_L. It is worth highlighting that α_L can now be lower than 1 as shown in Davig and Leeper (2007). Alternatively, when $s_t = 2$, the central bank adopts a regime that targets inflation relatively aggressively by setting a low inflation target Π_L^* and responds with increased sensitivity to deviations away from this target captured by α_H. In this case, regimes are not state-dependent, therefore, the probability of moving across regimes depends solely on the regime that is currently in place. Implying constant transition probabilities, which are represented by

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i),\ i,j = 1,2;\ \mathbf{p} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}.$$

Agents in the model know the transitions probabilities and use this information when forming expectations and therefore approach maintains the rational expectations hypothesis. A drawback is that the probability of moving across regimes is independent of the state of the economy and how close the of macro variables are to central bank targets.

3.3 Monetary policy regimes with state-dependence

The probability of switching away from the regime with a relatively accommodative stance towards inflation is now assumed to also depend on key endogenous variables $\tilde{z}_{t-1}$.

$$p_{1j_t} = \Pr(s_t = j | s_{t-1} = 1,\ \tilde{z}_{t-1});\ i, j = 1, 2;\ \tilde{z}_{t-1} = \left\{ \hat{\Pi}_{t-1}\ or\ \hat{Y}_{t-1}\ or\ \varepsilon_t \right\}$$

$$p_{11_t} = \frac{e^{V_{11}(\tilde{z}_{t-1})}}{1 + e^{V_{11}(\tilde{z}_{t-1})}}\ with\ V_{11}(\Pi_{t-1}) = \tilde{p}_{11} + \lambda_{11}(\tilde{z}_{t-1})$$

The transition probability p_{11} is now assumed to be a logit function that depends either on the distance that output or inflation are away from their respective targets or the level of an exogenous monetary policy shock. Therefore, I consider three types of state-dependence where $\tilde{p}_{11}$ will determine the steady-state level of the probability of remaining in the accommodative regime (regime 1) and λ_{11} measures the degree of state-dependence. If $\lambda_{11} = 0$, monetary policy regimes are independent of the economy and transition probabilities are constant. I assume that the aggressive inflation targeting regime is not state-dependent. This assumption reduces the number of parameters estimated and is consistent with the view that after the adoption of inflation targeting the approach remained in place until the end of the sample considered.

Past inflation

$$State\ Dependence\ 1 : p_{11_t} = \frac{e^{V_{11}(\hat{\Pi}_{t-1})}}{1 + e^{V_{11}(\hat{\Pi}_{t-1})}}\ with\ V_{11}(\Pi_{t-1}) = \tilde{p}_{11} + \lambda_{11}(\Pi_{t-1} - \Pi_H^*)$$

The first case of state-dependence allows the transition probability p_{11_t} to depend on the level of past inflation. It would seem plausible to assume that λ_{11} is negative, this implies a high rate of inflation above target level leads to a greater probability of switching to the regime that aggressively targets inflation. As the economy is in the high inflation regime the inflation target is assumed to take the larger target value Π_H^*, the one of this regime. The economic rationale for this form of state-dependence is that if the central bank delivers high inflation, they will be opposed by political or public pressures to adopt a new regime.

Output Gap

$$State\ Dependence\ 2 : p_{11_t} = \frac{e^{V_{11}(\hat{Y}_{t-1})}}{1 + e^{V_{11}(\hat{Y}_{t-1})}}\ with\ V_{11}(\hat{Y}_{t-1}) = \tilde{p}_{11} + \lambda_{11}(Y_{t-1} - \bar{Y})$$

The next case allows the monetary policy stance on inflation to be driven by the size of the output gap. The rationale for this form of state-dependence, comes through the output gap being a central variable in determining the state of the economy. Empirical literature on state-dependence in structural VARs usually links the output gap or an alternative measure of economic slack with determining wether the economy is in a recession or in a period of expansion state. Following this line of thought, I would expect λ_{11} to be negative; implying that agents would expect the central bank to be more accommodative to inflation during periods where the output gap is negative.

Monetary policy shock

$$\textit{State Dependence } 3: p_{11_t} = \frac{e^{V_{11}(\varepsilon_t)}}{1 + e^{V_{11}(\varepsilon_t)}} \textit{ with } V_{11}(\varepsilon_t) = \tilde{p}_{11} + \lambda_{11}(\varepsilon_t)$$

The last form of state-dependence allows the monetary policy shock to influence the probability of remaining in the accommodative regime. This rule is included to proxy the form of state-dependence introduced by Chang, Tan and Wei (2018). Within their research, an exogenous latent factor determines the monetary policy regime in place and state-dependence is introduced by allowing for correlation with monetary policy shocks. As the latent factor follows an autoregressive process, their mechanism allows for an endogenous feedback channel in which, the recent history of policy shocks lead to a cumulative effect on the regime factor that eventually trigger a regime shift. The economic intuition behind this mechanism is that assuming a monetary policy regime that accommodates inflation is in place a series of contractionary shocks agents would lead to agents expecting an aggressive monetary policy regime would come in its place.

4 Solving and estimating a general equilibrium model with state-dependent regimes

Allowing the state of the economy to influence parameter changes within a general equilibrium framework introduces additional challenges in both the solution and estimation of non-linear DSGE models.

To solve the model, I apply the solution algorithm introduced by Barthélemy and Marx (2017) and employ Bayesian methods for estimation.

4.1 Solution

To the extent of my knowledge, the only methods that are capable of solving general equilibrium models with state-dependent parameter changes are those proposed by Davig and Leeper (2008), Maih (2015), Barthélemy and Marx (2017) and Forester el al. (2018). These papers focus on approximate local solutions and follow a perturbation approach. I choose the algorithm of Barthélemy and Marx (2017) as it provides determinacy conditions that ensure a unique and stable equilibrium. I offer a brief description of the algorithm below but refer the reader to Barthélemy and Marx (2017) for a detailed exposition.

The problem of solving the non-linear general equilibrium model with state-dependent regimes described in the previous section can take the following representation

$$\mathbb{E}_t[f_{s_t}(z_{t+1}, z_t, z_{t-1}, \sigma v_t)] = 0 \tag{7}$$

$$\Pr(s_t = j|s_t = i) = p_{ij}(z_{t-1}, \sigma v_t) \tag{8}$$

where z_t is a vector of state variables, v_t, is a vector of exogenous shocks, and σ is a perturbation parameter. s_t represents the current regime. For any regime i, $f_i()$ is a smooth function and $\mathbb{E}_t$ is the expectation operator given information available at the time period t. Equation 8 reaffirms that regimes are state-dependent, that is, s_t follows a Markov process with time-varying transition probabilities that are a function of lagged endogenous variables or contemporaneous exogenous shocks. The only restriction on the transition probabilities is that state-dependence cannot be introduced by contemporaneous endogenous variables.[3]

The central proposition of the solution algorithm of Barthélemy and Marx (2017) is that the determinacy of the solution of the regime-switching model with state-dependence above can be deduced from a simpler version of the model that is quasi-linear with no state dependence. Therefore the first step of the algorithm requires checking if the linearised model assuming no state-dependence admits a stationary equilibrium. Dynare is used to take first derivates of the model represents in equation 7 around a regime-specific steady-state assuming transition probabilities are constant. The linearised model considering no state-dependence is

$$\mathrm{E_t}\left[\mathbf{A}(\mathrm{s_t}, \mathrm{s_{t+1}})\hat{\mathrm{z}}_{\mathrm{t+1}}\right] + \mathbf{B}(\mathrm{s_t})\hat{\mathrm{z}}_{\mathrm{t}} + \mathbf{C}(\mathrm{s_t})\hat{\mathrm{z}}_{\mathrm{t-1}} + \sigma\mathbf{D}(\mathrm{s_t})\mathrm{v_t} \tag{9}$$

where matrices $a(s_t, s_{t+1}), b(s_t), c(s_t), d(s_t)$ are regime-dependent matrices constructed using the first derivates of equation 7 around a regime specific steady-state assuming transition probabilities are constant.

The existence of a unique and stationary equilibrium in the simpler model of equation 9 is then assessed against the determinacy conditions proposed in Barthélemy and Marx (2012) that focus on models that are stable when solved forward and are similar to the forward conditions of Cho (2015).

The final step of the solution derives the Taylor expansion of the true solution for small perturbations around regime-dependent steady-states. The first-order approximation of the model solution is

$$\hat{z}_t = \mathbf{\Omega}(s_t)\hat{z}_{t-1} + \mathbf{\Delta}(s_t)v_t \tag{10}$$

where $\Omega(s_t), \Delta(s_t)$ are solution matrices that are a non-linear function of the model parameters and transition probabilities. To enable feasible estimation of the model in equation 7, I only consider the first-order approximation

[3] This is not the case for the solution method of Maih (2015), where contemporaneous model variables can introduce state-dependence, however, these variables must have a unique steady-state.

and avoid additional computational burdens associated with higher-order approximations.[4]

Nevertheless, the complications introduced by allowing for state-dependent regimes impose a relatively higher computational burden when compared to a constant parameter model. Resulting in a considerably longer solution time, where the main proportion of time is spent on checking for the determinacy of the solution.

4.2 Estimation

I adopt a full information Bayesian approach to model estimation. This involves combining the approximate likelihood function with prior distributions and then employing a Markov chain Monte Carlo (MCMC) algorithm to approximate the posterior distribution of the model parameters. Adjustments must be made to the procedure used to approximate the likelihood function to allow for time-varying transition probabilities. However, the main problem I faced when estimating these models was the estimation time; primarily, due to the considerably longer time it takes to solve the model given a set of parameters. This computational burden is the primary reason why estimating state-dependent models with independent changes in the volatility of structural shocks is left as future work.

Likelihood

The model solution in equation 10 is combined with a measurement equation to form the following state-space representation

$$
\begin{aligned}
y_t &= z^*(s_t) + \mathbf{H}\hat{z}_t \\
\hat{z}_t &= \mathbf{\Omega}(s_t)\hat{z}_{t-1} + \mathbf{\Delta}(s_t)v_t,\ v_t \sim N(0, \Sigma_v)
\end{aligned}
$$

where y_t represents the data used for estimation and $z^*(s_t)$ represents the steady state of model variables that depend on the data; specifically, inflation, output and the nominal interest rate.

The presence of different regimes causes the predictions of the Kalman filter to be conditional on the entire history of the regimes in place throughout the sample leaving the standard Kalman filter intractable. Therefore, an approximation of the likelihood function is formed following the algorithm proposed by Kim (1994). The key feature of this approximation is that a limited number of predictions of model variables are carried forward from the Kalman filter iterations each period, and these are then collapsed at the end of each iteration. Following Kim and Nelson (1999) and Mumtaz and Liu (2011), I track forecasts that depend on the regime in place in period t, $t-1$ and $t-2$ and accounts for eight possible paths of the of model variables. However, their approach must

[4] Forester et al. (2018) estimate state-dependent regime-switching general equilibrium models with second-order approximations. However, their solution method does not allow for regime-dependent steady-states which are used in the application of this paper.

be amended when considering state-dependent regimes. I do this by replacing constant transition probabilities with their now time-varying counterparts. The equations governing state-dependence are used to obtain the transition probabilities that are then passed directly to the filtering algorithm in the cases where state-dependence is linked to lagged observations of the data such as output and inflation. However, the contemporaneous monetary policy shock is unobserved and as the filter proposes multiple predictions of this state variable an average is used. Chang, Tan and Wei (2018b), take an alternative alternative approach by augmenting their state-space with the latent factor determining the regime in place.

The algorithm begins with a Kalman filtering step to predict the state variables $\hat{z}_t$

$$\hat{z}^{(h,i,j)}_{t|t-1} = \mathbf{\Omega^j} \hat{z}^{(i,j)}_{t|t-1}$$

$$P^{(h,i,j)}_{t|t-1} = \Omega^j \hat{z}^{(i,j)}_{t|t-1} \Omega^{j'} + \Delta^j \Sigma_v \Delta^{j'}$$

$$\eta^{(h,i,j)}_{t|t-1} = \mathbf{y_t} - z^{*^j} - H\hat{z}^{(h,i,j)}_{t|t-1}$$

$$f^{(h,i,j)}_{t|t-1} = \mathbf{H} P^{(h,i,j)}_{t|t-1} \mathbf{H'}$$

$$\hat{z}^{(h,i,j)}_{t|t} = \hat{z}^{(h,i,j)}_{t|t-1} + \mathbf{P^{(h,i,j)}_{t|t-1}} \mathbf{H'} (f^{(h,i,j)}_{t|t-1})^{-1} \eta^{\mathbf{(h,i,j)}}_{\mathbf{t|t-1}}$$

$$P^{(h,i,j)}_{t|t} = (I - P^{(h,i,j)}_{t|t-1} H' (f^{(h,i,j)}_{t|t-1})^{-1}) \mathbf{H} P^{(h,i,j)}_{t|t-1}$$

where $\hat{z}^{(h,i,j)}_{t|t-1}$ represent the predictions the model state variables that are conditioned on past information $t-1$ and the regime in place at time t, in the previous period $t-1$ and in $t-2$. Similarly $P^{(h,i,j)}_{t|t-1}$ is the covariance of this predictions, $\eta^{(h,i,j)}_{t|t-1}$ denotes the forecast error and $f^{(h,i,j)}_{t|t-1}$ represents covariance of the forecast error. $\hat{z}^{(i,j)}_{t|t-1}$ represents the collapsed states and represents the predictions of the model that are conditioned on past information $t-1$, the regime in place at time t and the regime of the previous period $t-1$. The collapsing stage reduces the 8 predictions of the states to 4 and is conducted using weights based on the joint and marginal probabilities of the regimes in place.

$$\hat{z}^{(i,j)}_{t|t} = \frac{\Pr(s_{t-2} = h, s_{t-1} = i, s_t = j)}{\Pr(s_{t-1} = i, s_t = j)} \hat{z}^{(h,i,j)}_{t|t}$$

To obtain the probability terms, the Hamilton filter is applied, this step has been modified to allow for time-varying transition probabilities.

$$\Pr(s_{t-2}=h,s_{t-1}=i,s_t=j|\varphi_{t-1})=\Pr(s_t=j|s_t=i,\hat{z}_{t-1})\times\sum_{g=1}^{2}\Pr(s_{t-3}=g,s_{t-2}=h,s_{t-1}=i|\varphi_{t-1})$$

where $|\varphi_{t-1}$ represents conditioning on information in time period $t-1$, this term updated with φ_t [5]

$$\Pr(s_{t-2}=h,s_{t-1}=i,s_t=j|\varphi_t)=$$

$$\frac{f(y_t|s_{t-2}=h,s_{t-1}=i,s_t=j,\varphi_{t-1})\times\Pr(s_{t-2}=h,s_{t-1}=i,s_t=j|\varphi_{t-1})}{\sum_{h=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{2}f(y_t|s_{t-2}=h,s_{t-1}=i,s_t=j,\varphi_{t-1})}$$

where the conditional densities are defined as

$$f(y_t|s_{t-2}=h,s_{t-1}=i,s_t=j,\varphi_{t-1})=2\pi^{-n/2}\left|f_{t|t-1}^{(h,i,j)}\right|^{-1/2}\exp\left(-\frac{1}{2}\eta_{t|t-1}^{(h,i,j)}\left(f_{t|t-1}^{(h,i,j)'}\right)^{-1}\eta_{t|t-1}^{(h,i,j)'}\right)$$

and the filter probability of a regime j being in place at time is given by

$$\Pr(s_t=j|\varphi_t)=\sum_{h=1}^{2}\sum_{i=1}^{2}\Pr(s_{t-2}=h,s_{t-1}=i,s_t=j|\varphi_t)$$

The by-products of the filter can be used to the calculate marginal density of y_t

$$f(y_t|\varphi_{t-1})=f(y_t,s_{t-2}=h,s_{t-1}=i,s_t=j|\varphi_{t-1})$$

the approximate log likelihood is then given by

$$LL=\sum_{t=1}^{T}log(f(y_t|\varphi_{t-1})).$$

Finally, the likelihood of parameter sets that do not meet the following normalisation condition are set to extremely low values that ensure the parameter draw is discarded.

$$\begin{array}{lc} Accomodative & s_t=1 \\ InflationTargeting & s_t=2 \end{array}\left\{\begin{array}{c}\Pi_H^*,\alpha_L\\ \Pi_L^*,\alpha_H\end{array}\right\}$$

where $\Pi_L^*<\Pi_H^*$ and $\alpha_L<\alpha_H$. This condition is imposed to ensures identification and avoid the problem of label-switching

[5] Information sets include observations of the data and lagged levels of the states.

Table 1: Priors distributions for parameters of State-Dependent MS-DSGE models

	Mean	Standard Deviation	Domain	Density
α	1.5	0.25	$\mathbb{R}^+$	Gamma
$\pi^*(annualised)$	2	0.1	$\mathbb{R}^+$	Gamma
$\bar{p}_{11}$	0.95	0.15	[0,1)	Beta
p_{22}	0.95	0.15	[0,1)	Beta
λ_{11}	0	**150**	$\mathbb{R}$	Normal
ρ_r	0.75	0.4	[0,1)	Beta
γ	0.15	0.4	$\mathbb{R}^+$	Gamma
κ	0.5	0.2	$\mathbb{R}^+$	Gamma
τ	1.5	0.4	$\mathbb{R}^+$	Gamma
β	0.998	Fixed	[0,1)	Beta
θ	10	4	$\mathbb{R}^+$	Gamma
ρ_ξ	0.5	0.4	[0,1)	Beta
ρ_u	0.5	0.4	[0,1)	Beta
$100\sigma_\varepsilon$	0.21	0.16	$\mathbb{R}^+$	Inv. Gamma
$100\sigma_\xi$	1	0.52	$\mathbb{R}^+$	Inv. Gamma
$100\sigma_u$	0.38	0.38	$\mathbb{R}^+$	Inv. Gamma

Priors

Incorporating prior information when estimating model parameters provides additional curvature for the posterior density and excludes implausible estimates of parameters. The prior distribution for the model parameters is set following Lubik and Schorfheide (2004) and Davig and Doh (2014) and are presented in Table 1. The prior distributions of switching parameters, i.e., annualised inflation target π^* and monetary policy sensitivity to inflation α, are the same in each regime. The prior distribution of the transition probabilities are centered around values of 0.95; this implies an expected duration of each regime to be 20 quarters. For the state-dependent regime, prior beliefs are expressed on the steady-state of the probability of remaining in this regime $\bar{p}_{11}$. The degree of state-dependence λ_{11} follows a normal distribution centered around zero, though the prior variance allows for a wide range of possible values. This prior distribution is set to reflect an agnostic belief of the extent of state-dependence.

MCMC algorithm

The prior distributions and the approximate likelihood are combined to approximate the posterior distribution of model parameters. A combination of numerical optimisers is used to find the mode of the posterior to initiate the MCMC procedure. The simplex algorithm is first applied to refine starting values that are used as input into for Chris Sims' optimisation routine CSMINWEL.

The posterior mode is used to initiate the Metropolis-Hastings algorithm. I run 100,000 replications, burning the first 50,000 and then save every 10th draw to leave 5000 draws that form the approximate posterior.

4.3 Data

Estimation is based on U.S. data consisting of output, inflation and nominal interest rates from 1965Q1-2009Q1. The sample dates are chosen to reflect the two monetary policy regimes accounted for. For example, after the global financial crisis, the Federal Reserve hit the effective lower bound of nominal interest rates and turned to alternative instruments to conduct monetary policy. Therefore, this regime does not fit either the accommodative or inflation targeting regimes I allow for.

Output is log real GDP per capita HP detrended following Lubik and Schorfheide (2004). Inflation is annualised percentage change in CPI. Nominal interest rate is the effective federal fund rate in percent. All data is obtained from the FRED database.

The data is related to the state variables via the measurement equation

$$y_t = z^*(s_t) + \mathbf{H}\hat{z}_t$$

$$\begin{pmatrix} GDP\,per\,Captia\,Detrended_t \\ Inflation_t \\ Interest\,Rate_t \end{pmatrix} = \begin{pmatrix} \\ \Pi^*(A) \\ \Pi^*(A) + r^*(A) \end{pmatrix} + 100 \begin{pmatrix} \hat{y}_t \\ 4\hat{\pi}_t \\ \hat{R} \end{pmatrix}$$

4.4 Simulation Evidence

The validity of the estimation procedure is tested on data generated from a general equilibrium model with state-dependent monetary policy regimes. The data generating process is the solution of a version of the model presented in section 3 simplified by assuming a constant inflation target of zero. Therefore, a monetary policy regime is now defined as a change in the Taylor rule response to inflation. The model is state-dependent as the probability of remaining in the relatively accommodative regime is inversely related to the level of inflation in the previous period.

$$\begin{array}{lc} Accomodative & s_t = 1 \\ InflationTargeting & s_t = 2 \end{array} \left\{ \begin{array}{c} \alpha_L \\ \alpha_H \end{array} \right\}, \alpha_L < \alpha_H$$

$$State\ Dependence : p_{11_t} = \frac{e^{V_{11}(\Pi_{t-1})}}{1 + e^{V_{11}(\Pi_{t-1})}}\ with\ V_{11}(\Pi_{t-1}) = \bar{p}_{11} + \lambda_{11}(\Pi_{t-1})$$

The model parameters are set to values assumed in the calibration exercise of Barthélemy and Marx (2017) . The economy is assumed to begin in steady state and is then simulated for 1000 periods. The data used for estimation discards the first 100 generated observations and is plotted Figure 1below alongside the regimes in place at each period of the sample. Initial values for estimation are

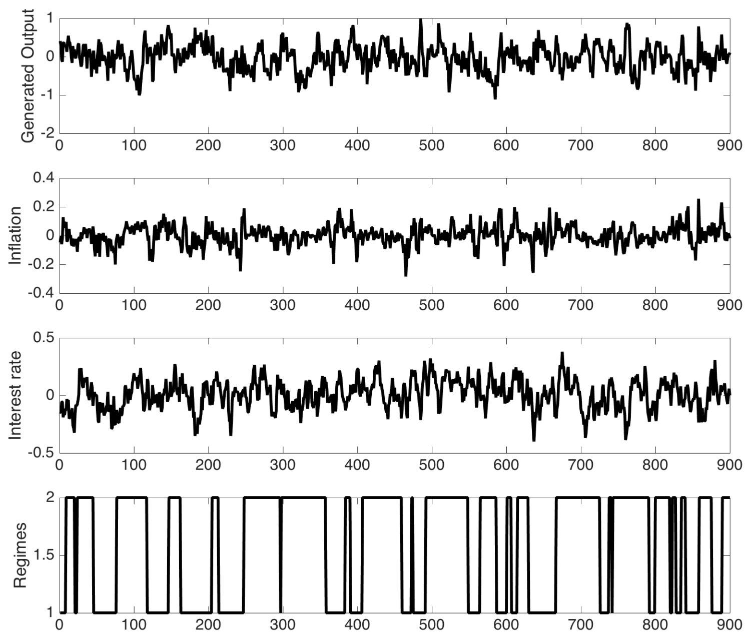

Figure 1: Generated data from MS-DSGE model
Note: The first three panels display simulated data generated from the model economy with state-dependent monetary policy regimes, while, the last panel displays the history of regimes.

prior values except for the degree of state-dependence that is set to -200. The estimation procedure is the same as the one described in the previous subsection and the results of the MCMC algorithm in Table 2. Overall, the posterior mean estimates are close to the parameters values used to generate the data.

5 Results

This paper estimates three models that allow state variables to influence the probability of remaining in a monetary policy regime that is relatively accommodative towards inflation. To evaluate the empirical significance of the non-linearities considered I also estimate a model with switching regimes that are not state-dependent and a constant parameter model.

5.1 Model comparison

To assess which form of state-dependence, if any, is accepted by the data, I calculate two model comparison measures. The first, the marginal likelihood, is

Table 2: Parameters comparison of simulation exercise to evaluate s

	Actual		Estimated	
α	1.1	4.3	1.22 [1.0,1.61]	4.46 [3.76,5.2]
p_{11}	0.90		0.93 [0.93,0.94]	
p_{22}	0.95		0.98 [0.98,0.99]	
λ_{11}	-400		-375 [-375,-374]	
ρ_r	0.7		0.74 [0.72,0.76]	
γ	0.2		0.25 [0.2,0.27]	
κ	0.17		0.24 [0.21,0.27]	
τ	1		0.95 [0.70,1.16]	
β	0.998 $(Calibrated)$		0.998 $(Calibrated)$	
ρ_ξ	0.8		0.67 [0.62,0.72]	
ρ_u	0.8		0.8 [0.78,0.83]	
$100\sigma_\varepsilon$	0.1		0.1 [0.09,0.1]	
$100\sigma_\xi$	0.1		0.09 [0.065,0.1]	
$100\sigma_u$	0.2		0.17 [0.16,0.18]	

obtained by integrating the approximate posterior density of the entire parameter space in each model and is expressed as

$$\Pr(Y_t) = \int_{\theta} f(Y_t|\theta)\Pr(\theta)$$

where θ denotes the model parameters, $f(Y_t|\theta)$ is the likelihood while $\Pr(\theta)$ represents the prior distributions. The marginal likelihood can be approximated using the modified harmonic mean (MHM) method which employs the following theorem

$$\frac{1}{\Pr(Y_t)} = \int_{\Theta} \frac{h(\theta)}{f(Y_t|\theta)\Pr(\theta)} f(\theta|Y_t)d\theta$$

where $h(\theta)$ denotes a weighting function, i.e. a probability density function whose support is in Θ. Numerically the integral above can be evaluated as

$$\frac{1}{\Pr(Y_t)} = \sum_{i=1}^{N} \frac{h(\theta^i)}{f(Y_t|\theta^i)\Pr(\theta^i)}$$

where $i = 1...N$ indexes the draws from the MCMC samples. Following Geweke (1999) a normal density is used as the weighting function. Although Sims, Waggoner and Zha (2008) suggest an elliptical density is more appropriate for comparing time-varying DSGE model. However, in practice Davig and Doh (2014) and Mumtaz and Liu (2011) find that the resulting estimate of the marginal likelihood using a elliptical density is unstable and highly sensitive to draws away from the posterior.

Taking into account the difficulty in the accurate computation of the marginal likelihood for the class of models estimated, the deviance information criterion (DIC) introduced by Spiegelhalter et al. (2002), is also used for model comparison. The DIC is a generalisation of the Akaike information criterion; penalising model complexity and emphasising model fit to the data. The DIC is defined as

$$DIC = \bar{D} + p_D$$

where $\bar{D}$ measures model fit and is referred to as the deviance, it is the average of the log likelihood multiplied by -2 after the evaluated for each MCMC draw and is given as

$$\bar{D} = \frac{1}{N}\sum_{i=1}^{N}(-2\ln(Y_t|\theta^i))$$

$$p_D = \bar{D} - (-2\ln(Y_t|\bar{\theta}))$$

p_D is the effective number of parameters and is defined as the deviance subtracted by the log-likelihood evaluated at the posterior mean multiplied by -2.

Table 3: Log marginal likelihood and Deviance Information Criterion for each estimated model

	Marginal Likelihood	DIC
Time-Invariant	-1538	2893
Switching Rule: Without state-dependence	-1327	2673
State-Dependent: lag Inflation	$\mathbf{-1278}$	**2640**
State-Dependent: lag Output	-1320	2690
State-Dependent: Monetary Policy shock	-1325	2678

.

Table 3 below reports the values of the log marginal likelihood and DIC for each model. The preferred model achieves the maximum marginal likelihood and the minimum DIC value.

Both measures indicate that the preferred model allows for state-dependence in the form of deviations of inflation in the previous period from its target levels and firmly reject the time-invariant model. However there are differences between the rankings implied by each measure. The marginal likelihood indicates that models allowing for state-dependence fit better than the switching model without state-dependence. Whereas the DIC shows a preference to the model without state-dependence relative to those that allow state-dependence to be linked to the output in the previous period or the contemporaneous monetary policy shock.

Reflecting the model comparison exercise, the remainder of the paper will focus on comparing the results of the preferred model that allows for state-dependence linked to inflation and the model with switching monetary policy but no state-dependence.

5.2 Parameter estimates

The estimation results are presented in the Table 4. Parameter estimates are the median of the approximate posterior with the 90% interval in brackets. The first column displays the parameter estimates of the time-invariant model; which are broadly consistent with those of Lubik and Schorfheide (2004) and Davig and Doh (2014). The second column displays the estimates of the switching model without state-dependence, and the last column presents the estimates of the switching model where the accommodative regime is state-dependent and linked to the distance of inflation away from its target in the previous period.

There are two main results worth highlighting. Firstly, regardless of state-dependence, both the accommodative and the inflation targeting regime are well identified. There is no overlap in the 90% intervals of the annulised inflation target π^* and the policy response to inflation α across the regimes.

The filtered probabilities of the state-dependent model suggests that the

Table 4: Parameter estimates of alternative approaches to modelling monetary policy regime shifts

	Constant	Without state-dependence		State-dependence π_{t-1}	
α	1.48 [1.13,2]	1.05 [.92,1.26]	2.86 [1.92,3.2]	1.02 [0.94,1.34]	2.4 [2.14,2.75]
$\pi^*(annualised)$	2.72 [1.45,3.96]	4.10 [3.18,4.76]	1.56 [1.04,2.14]	3.23 [2.78,3.65]	1.65 [1.17,2.04]
p_{11}	-	0.90 [0.82,95]		0.91 [0.86,94]	
p_{22}	-	0.98 [0.96,0.99]		0.98 [0.97,0.99]	
λ_{11}	-	-		-33 $[-52,-8]$	
ρ_r	0.87 [0.71,0.95]	0.71 [.53,.96]		0.76 [0.62,0.83]	
γ	0.43 [0.15,0.63]	0.41 [0.23,0.65]		0.47 [0.34,0.64]	
κ	0.64 [0.13,1.53]	0.43 [0.19,0.73]		0.32 [0.16,0.53]	
τ	1.12 [0.78,1.63]	1.27 [0.56,1.83]		1.44 [1.13,1.75]	
β	0.998 $(Calibrated)$	0.998 $(Calibrated)$		0.998 $(Calibrated)$	
θ	8.42 [4.2,13.63]	10.25 [5.66,14.83]		11.25 [6.13,14.75]	
ρ_ξ	0.75 [0.44,0.94]	0.65 [0.46,0.74]		0.59 [0.55,0.67]	
ρ_u	0.77 [0.6,0.84]	0.65 [0.52,0.74]		0.59 [0.48,0.67]	
$100\sigma_\varepsilon$	0.45 [0.21,0.72]	0.29 [0.15,0.42]		0.32 [0.18,0.47]	
$100\sigma_\xi$	1.01 [0.55,1.32]	0.89 [0.35,1.42]		0.74 [0.52,0.92]	
$100\sigma_u$	0.53 [0.18,0.83]	0.63 [0.27,0.92]		0.72 [0.42,0.95]	

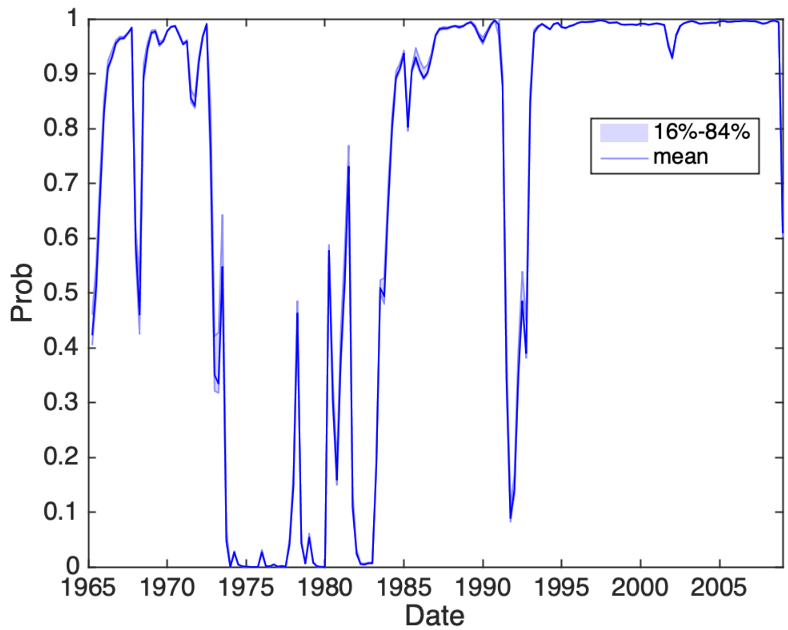

Figure 2: Filter probabilities from MS-DSGE model with state-dependent monetary policy regimes
Note: The solid blue lines are the posterior mean filter probabilities and the shaded area represents the 68% credible sets.

accommodative regime was in place from 1973-1985 and is consistent with Davig and Doh (2014) and Bianchi (2013). However, the model also identifies a recession period during 1991-1992 as period of accommodative monetary policy and is contrary to the narrative provided by Davig and Doh (2014) and Bianchi (2013). This inconsistency may be due to not allowing for stochastic volatility in the structural shocks, as the available monetary policy switch detects large sources of variations which is further highlighted by decrease in filter probability at the end of the sample.

The second result and the main finding of this paper is that the degree of state-dependence is significant. λ_{11} is negative as expected, taking a value of -35 and the 90% interval is wide ranging between -52 and -8 but does not include zero which implies no state-dependence. This finding indicates that the mechanism is accepted by the data and reinforces the result of the model comparison exercise. The posterior mean of λ_{11} is -33 and is much lower than the values of -400 and -800 that Barthélemy and Marx (2017) consider in their calibration exercise. Although, it is worth mentioning that as the time-varying transition probabilities are a logisitic function of endogenous, a value of -33 still implies a large degree of state-dependence.

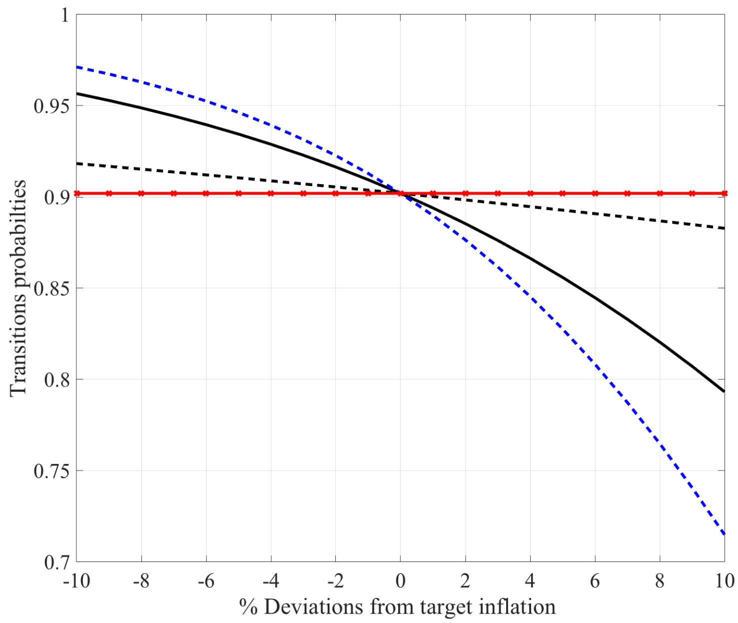

Figure 3: Time-varying transition probabilities

Note: The black solid, black dashed and blue dashed line represents values of λ_{11} at the posterior median, 10% and 90% percentile respectively. The red line with crosses represents the constant probability of the switching model with no state-dependence.

The economic interpretation of the estimated magnitude of λ_{11} is best described by observing the dynamics of the time-varying transition probabilities. The logit function is plotted in Figure 3 for values of inflation $\pm 10\%$ away from a target of 3.23% and the steady-state transition probability is fixed to the posterior mean estimate of 0.91. The black solid, black dashed and blue dashed line represents values of λ_{11} at the posterior median, 10% and 90% percentile respectively. The red line with crosses represents the constant probability of the switching model with no state-dependence. As inflation rises above the target level the probability of remaining in the accommodative regimes decreases, implying a reduction in the expected duration of the regime given by $\frac{1}{1-p_{11_t}}$. For example, looking at the period identified when the accommodative regime was in place, inflation in 1973 was around 5% and reached a peak level of close to 15% percent in 1979. Concentrating on the posterior mean estimates would result in transitions probabilities moving from 0.88 to 0.77 and decreasing the expected duration of the regime from eight quarters to four.

6 Impulse responses

Figure 4 presents the impulse responses conditional on the accommodative regime being in place at each period of the considered horizon of ten quarters. All shocks are inflationary and are of the magnitude of 1-standard deviation.[6]

To evaluate the effect of state-dependence on the propagation of shocks, three sets of responses are considered. The solid black line represents the impulse response implied by the state-dependent model; the solid red line represents the responses implied by the model estimated imposing no state-dependence and lastly the dashed black line represents the responses of the state-dependent model setting the degree of state-dependence to zero.

Overall, the propagation of the shocks is theoretically consistent, as expected from structural models. The responses of inflation are the main difference between the models estimated with and without state-dependence. The median response of inflation is considerably lower for each inflationary shock in the model with state-dependence.

This result could come from two possible channels; the difference between the set of parameters estimated and the expectations effects generated by the reduction in transition probability of remaining in the accommodative regime. A way to isolate the expectations effects is the comparison of the black solid and dashed line. However, these are almost identical suggesting the non-existence of the expectations channel. Therefore, the lower inflation responses implied by the state-dependent model are primarily caused by the correspondingly lower inflation target relative to the model with no-state dependence.

As a robustness check, I also follow Barthélemy and Marx (2017) in considering the difference between the expected responses of a simulated economy that is in the accommodative regime and hit by a shock at time t with the same economy in the absence of this shock. The benefit of this definition of the impulse response is that it considers multiple histories of regimes and provides a way of observing whether the findings of Barthélemy and Marx (2017) hold when their model is taken data. The impulse responses are presented in Figure 5 and can be defined mathematically as $\mathbb{E}[y_{t+k}|\varepsilon_t = \sigma^{\varepsilon}, s_t = 1] - \mathbb{E}[y_{t+k}|\varepsilon_t = 0, s_t = 1]$. All shocks are again inflationary with a magnitude of 1 standard deviation. I label these responses as unconditional for ease of comparison.

The signs of these responses are comparable with the conditional impulse responses. It is also worth noting the effect of the lower inflation target in the state-dependent model is now removed as these are differences from simulated paths. The responses of inflation are similar across the model with and without state-dependence. The expectations channel now emerges but only appears to dampen the response of inflation to a markup shock.

[6] Inflationary shocks are defined as an positive preference shock , a positive markup shock, and an expansionary monetary policy shock.

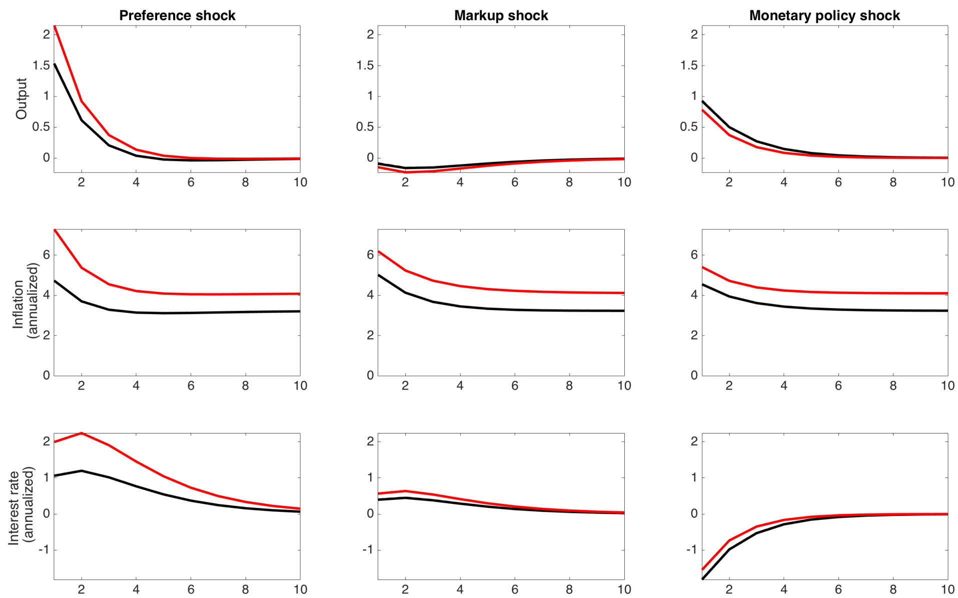

Figure 4: Impulse responses conditional on being in the less aggressive monetary policy regime

Note: The solid black line represents the impulse response implied by the state-dependent model; the solid red line represents the responses implied by the model estimated imposing no state-dependence and lastly the dashed black line represents the responses of the state-dependent model setting the degree of state-dependence to zero.

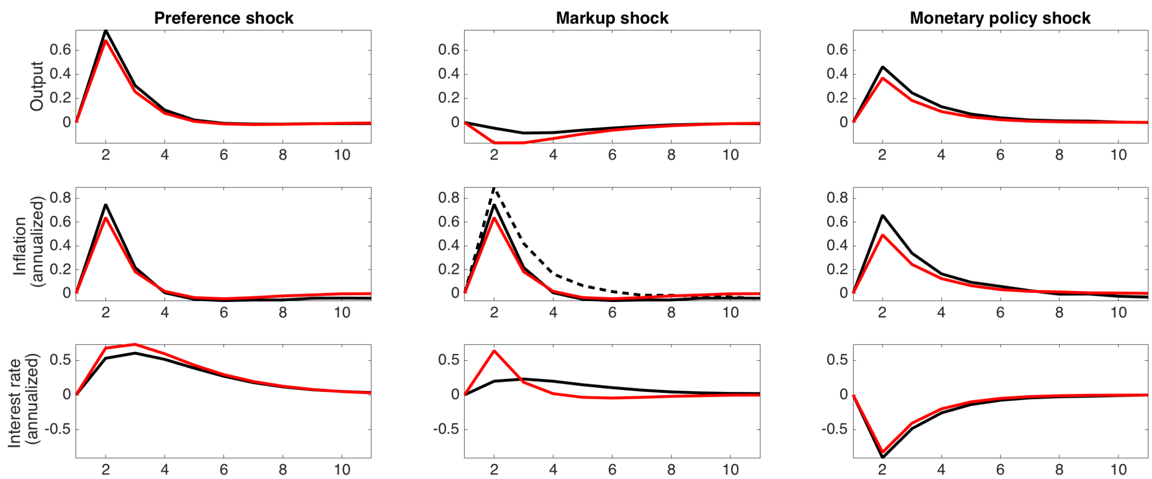

Figure 5: Unconditional impulse response functions

Note: The solid black line represents the impulse response implied by the state-dependent model; the solid red line represents the responses implied by the model estimated imposing no state-dependence and lastly the dashed black line represents the responses of the state-dependent model setting the degree of state-dependence to zero. Following Barthélemy and Marx (2017), the economy is initially ($t = 0$) at the steady state, and then simulated until a approximation of the ergodic distribution is formed and then impulse responses are then computed (t=100, following Barthélemy and Marx (2017). Simulation is across 200000 trajectories of regimes and shocks.

7 Conclusion

This paper investigates the possibility of monetary policy regime changes being influenced by the state of the economy in a small scale New Keynesian model estimated on U.S. data. I estimate versions of the model that allow the probability of remaining in a monetary policy regime that is relatively accommodative towards inflation to vary over time and depend on endogenous model variables; in particular, either deviation of inflation or output from their respective targets or a monetary policy shock. The model comparison exercise indicates that U.S. data is best described by the model that allows for state-dependence linked to inflation and the model with switching monetary policy but no state-dependence. Estimates from this model suggest that the period between 1973-1985 was associated with a regime characterised by a relatively accommodative stance towards inflation. The degree of state-dependence is significant and implies that while in an accommodative regime, higher levels of inflation increase the probability of switching to a regime that aggressively targets inflation. I present mixed evidence that state-dependence alters agents expectations through impulse response analysis suggesting a dampening of inflation responses only to cost-push shocks relative when the mechanism is shutdown. The estimated inflation target in the accommodative regime is lower when state-dependence is enabled.

For future work I would like to examine the effect of state-dependent monetary policy regimes in richer models and also allow for stochastic volatility of structural shocks. I would also like to encourage work on the micro-foundations of state-dependence in New Keynesian models.

8 Appendix

For the empirical analysis presented in this paper, I consider a standard small-scale New Keynesian monetary DSGE model. This model is admittedly elementary compared to those currently estimated by central banks but is selected as this is one of the first passes at estimating a general equilibrium model with state-dependent parameter changes. After log-linearization, the model without regime changes can be summarised by the following three equations:

$$\hat{Y}_t = \beta \mathbb{E}_t \hat{Y}_{t+1} - \tau^{-1}(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - (\xi_t - \mathbb{E}_t \xi_{t+1})) \tag{11}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa(\hat{Y}_t + \frac{u_t}{\tau}) \tag{12}$$

$$\hat{R}_t = \rho_r(\hat{R}_{t-1}) + (1-\rho_r)[\alpha(\hat{\pi}_t) + \gamma(\hat{Y}_t) + \varepsilon_t] \tag{13}$$

$$\hat{z}_t = \underbrace{\hat{Y}_t, \hat{R}_t, \hat{\pi}_t, \xi_t, u_t, \mathbb{E}_t \hat{Y}_{t+1}, \mathbb{E}_t \hat{\pi}_{t+1}}$$

$$
\begin{aligned}
v_t &= \underbrace{\varepsilon_r, \varepsilon_\xi, \varepsilon_u,} \\
\theta &= \underbrace{\beta, \tau, \kappa, \alpha, \gamma, \sigma_r, \sigma_\xi, \sigma_u}
\end{aligned}
$$

where the index t denotes time belong to integers, Y is output, π is inflation, and R the nominal interest. The variables with hats i.e $\hat{Y}, \hat{\pi}$ and $\hat{R}$ express log deviations from their respective steady-states.

Equation (1) is an intertemporal Euler equation obtained from the households' optimal choice of consumption and bond holdings. The current output gap $\hat{Y}_t$ depends on it's expected future value $\mathbb{E}_t\hat{Y}_{t+1}$, the real rate interest $\hat{R}_t - \mathbb{E}_t\hat{\pi}_{t+1}$ and a preference shock ξ_t. The parameter β is the households' discount factor and, τ denotes the elasticity of intertemporal substitution.

The new Keynesian Phillips curve in (2) suggests that inflation is related to expected future inflation $\mathbb{E}_t\hat{\pi}_{t+1}$ via discount factor β, the output gap through the slope term κ and a markup level u_t.

The third equation describes the behavior of the monetary authority. The central bank adjusts the nominal interest rate in response to deviations of inflation and output from their respective targets. The shock ε_t captures unanticipated deviations from the policy rule and its standard deviation is denoted by σ_r. The parameter ρ is the degree of interest rate smoothing, and α and γ are the reaction coefficients to inflation and output gaps respectively.

The preference shock ξ_t and markup $\hat{u}_t$, evolve according to univariate AR(1) processes with coefficients ρ_ξ and ρ_u, their innovations have standard deviations by σ_ξ and σ_u.

After collecting the state variables, shock innovations and model parameters in vectors, z_t, v_t and θ. This constant parameter model can be written in the canonical form

$$
\mathbf{A}\mathbb{E}_{\mathrm{t}}\left[\hat{\mathrm{z}}_{\mathrm{t+1}}\right] + \mathbf{B}\hat{\mathrm{z}}_{\mathrm{t}} + \mathbf{C}\hat{\mathrm{z}}_{\mathrm{t-1}} + \mathbf{D}\mathrm{v}_{\mathrm{t}}. \tag{14}
$$

The model is then solved using the methods of Sims (2002) and estimated using Bayesian methods. The Kalman filter is applied to obtain the likelihood function.

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