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Abstract

The presence of long memory in Realized Volatility (RV) is a widespread stylized fact. The origins of long memory in RV have been attributed to jumps, structural breaks, nonlinearities, or pure long memory. An important development has been the Heterogeneous Autoregressive (HAR) model and its extensions. This paper assesses the separate roles of fractionally integrated long memory models, extended HAR models and time varying parameter HAR models. We find that the presence of the long memory parameter is often important in addition to the HAR models.

JEL Classification: C22; C31

Keywords: Long memory, Restricted $ARFIMA$, Realized volatility, HAR model, Time varying parameters

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1 Introduction

The long memory feature of many time series has long been of interest to statisticians, econometricians and researchers in many of the physical sciences, who have become aware of the very strong persistence in the autocorrelations and other measures of the temporal dependence of some time series. Hurst (1951, 1957) and Mandelbrot and Wallis (1968) noted the phenomena in river flow and hydrological data; and Greene and Fielitz (1977) in financial data. Some of the historical developments are discussed by Baillie (1996). One of the fascinations with long memory processes is their inherent ability to bridge both persistent stationary and non stationary time series. One of the most ubiquitous and also important examples of long memory are to be found in Realized Volatility (RV) time series.

The construction of observable RV series from high frequency financial market data has now become standard practice in empirical finance. One of the attractions with using RV is to reduce emphasis of the formulation and choice of model, with a direct measurement of volatility. It has been found that RV time series are characterized by very strong persistence in their autocorrelations for a wide range of financial assets. An interesting issue has been to provide an explanation for this phenomenon; and to assess whether it could be due to jumps, structural breaks, omitted nonlinearities, contemporaneous aggregation, or to just "pure long memory".

However, a popular way of describing RV has been the Heterogeneous Autoregressive (HAR) model, which was originally due to Corsi (2009). The model is based on an additive cascade of partial volatilities from high frequencies to low frequencies; with each additive cascade having close to an $AR(1)$ structure. This idea of multiple components in the volatility process has been justified in terms of the differences of agents risk profiles, institutional structures, temporal horizons, etc. In general, the HAR model appears attractive as a simplified regression based procedure for approximating the persistence of many RV time series.

This paper examines the relationship between long memory models, the HAR model and the extended versions of the HAR model, which include semivariances, signed jump variations, and "good" and "bad" volatility. We estimate HAR models from simulated fractional white noise processes and find the simulated estimates have certain similarities with the HAR estimates from actual RV data. We also estimate by MLE a restricted long memory model denoted by $RARFIMA$, which includes the long memory feature and also embodies parameter restrictions from the HAR model. The model is theoretically very similar to the basic HAR model with long memory disturbances and is also estimated by a MLE procedure. The overall conclusion is that in many cases both the long memory feature and the HAR structure for short and medium term memory can be important in representing variation within RV series.

Finally, we also consider a time varying parameter, kernel weighted regression approach to estimate HAR models. These estimated models indicate that the relative importance of the partial volatility cascades typically varies throughout the samples. Such a Time Varying Parameter (TVP) model, denoted by $TVP - HAR$, is quite effective in representing some of the long memory characteristics of RV time series. However, model selection information based criteria generally favor the simpler $RARFIMA$ structure with constant long memory and HAR parameters.

The plan of the rest of the paper is as follows: the next section defines some of the theoretical aspects of RV and also includes details of the statistical quantities regularly implemented and arising from RV series. Section 3 briefly describes the RV data and some of their basic characteristics; while Section 4 describes the long memory models and inferential methods and report MLE of both $ARFIMA$ models and also reports semi parametric estimation of the long memory parameter. Section 5 describes the various HAR models and their estimates, including various extensions including jumps and good and bad volatility components. Section 6 is concerned with different methods for attempting to distinguish between HAR and long memory and also for combining these approaches. In particular, we provide simulation evidence on the properties of OLS estimation of HAR models when the true data generating process is a fractional white noise, long memory process. This section also includes results on the MLE of unrestricted $ARFIMA(22, d, 0)$ and $RARFIMA(22, d, 0)$ models, where the restrictions are from the HAR formulation. We also include MLE of extended HAR models which have long memory disturbances. Section 7 describes an alternative approach based on a time varying parameter HAR model which involves kernel weighted regressions with time varying regression coefficients based on the method by Giraitis et al. (2014). Section 8 discusses some of the results concerning comparisons of the models and also provides a brief conclusion.

2 Basics of Realized Volatility

The variable RV is a model free measurement of financial market volatility and was proposed by Andersen et al. (2001, 2003) and Barndorff-Nielsen and Shephard (2002). We define a continuous time diffusion process for the log of price (p_t) as

$$dp(t) = \mu(t)dt + \sigma(t)dW(t), \quad t \geq 0,$$

where $dp(t)$ is the change in the logarithmic price, $\mu(t)$ denotes the drift term which has continuous and locally bounded variations, $\sigma(t)$ is a strictly positive volatility process and $W(t)$ is standard Brownian motion. Assuming a unit for the time length of one day, daily returns can be expressed as

$$r_t = p(t) - p(t-1) = \int_{t-1}^{t} \mu(s)ds + \int_{t-1}^{t} \sigma(s)dW(s).$$

The volatility of an asset's returns is related to the evolution of the spot volatility (σ_t) so that the distribution of returns depends on both the drift and spot volatility components; hence

$$r_t \sim N\left(\int_{t-1}^{t} \mu(s)ds, \int_{t-1}^{t} \sigma^2(s)\right).$$

RV at day t is RV_t and is defined as the sum of high frequency, intraday squared returns. Hence

$$RV_t = \sum_{\tau=1}^{m} r_{t,\tau}^2,$$

where $r_{t,\tau} = p_{t,\tau} - p_{t,\tau-1}$ is the intraday return based on m intraday log-prices of the asset $\{p_{t,\tau}\}_{\tau=1}^{m}$ within day t observed at m fixed time intervals of $\tau = 1, \ldots, m$. Andersen et al. (2003) showed that under suitable conditions, including the absence of serial correlation in the intraday returns, RV_t is a consistent estimator of integrated volatility (IV_t). Hence

$$RV_t = \sum_{\tau=1}^{m} r_{t,\tau}^2 \xrightarrow{p} \int_{t-1}^{t} \sigma_s^2 ds.$$

The basic RV model has been extended to include the effects of jump components. Suppose the log-price process is a Brownian Semi-Martingale with Jumps, then

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t) \quad t \geq 0,$$

where the jump component is $\kappa(t)dq(t)$, with $\kappa(t)$ as the size of the jump and $dq(t)$ as a continuous process with $dq(t) = 1$ if there is a jump at time t and is 0 otherwise. The corresponding discrete-time daily returns are

$$r_t = \int_{t-1}^{t} \mu(s)ds + \int_{t-1}^{t} \sigma^2(s)dW(s) + \sum_{j=N(t-1)+1}^{N(t)} \kappa(s_j),$$

where $N(t)$ counts the number of jumps occurring with possibly time varying intensity and jump size $\kappa(s_j)$. In the presence of jumps, RV_t converges uniformly in probability to

$$RV_t \xrightarrow{p} \int_{t-1}^{t} \sigma^2(s)\,ds + \sum_{j=N(t-1)+1}^{N(t)} \kappa^2(s_j).$$

Hence, RV_t is a consistent estimator of IV_t only in the absence of jumps, while otherwise it converges to a quantity that also accounts for the jump process, $\sum_{j=N(t-1)+1}^{N(t)} \kappa^2(s_j)$. Hence RV provides an ex-post measure of the true total variation including the discontinuous jump part.

To decompose volatility into a component that relates only to positive high-frequency returns and a component that relates only to negative high-frequency returns, we use the realized semivariance quantity proposed by Barndorff-Nielsen and Shephard (2007). The positive (negative) realized semivariance RS_t^+ (RS_t^-) is computed by summing the squared intraday returns associated with an increase (decrease) in the asset price. Hence,

$$\begin{aligned} RS_t^+ &= \sum_{\tau=1}^{m} r_{t,\tau}^2 I\left\{r_{t,\tau} > 0\right\} \xrightarrow{p} \frac{1}{2}\int_{t-1}^{t} \sigma_s^2 ds + \sum_{t-1<s\leq t} \Delta p_s^2 I\left\{\Delta p_s > 0\right\} \\ RS_t^- &= \sum_{\tau=1}^{m} r_{t,\tau}^2 I\left\{r_{t,\tau} < 0\right\} \xrightarrow{p} \frac{1}{2}\int_{t-1}^{t} \sigma_s^2 ds + \sum_{t-1<s\leq t} \Delta p_s^2 I\left\{\Delta p_s < 0\right\}, \end{aligned}$$

where $I(\cdot)$ is an indicator function and $\Delta p_s = p_s - p_{s-}$ captures a jump, if present. Note that $RV_t = RS_t^+ + RS_t^-$.

Following Patton and Sheppard (2015), we compute the signed jump variation as

$$\Delta J_t^2 = RS^+ - RS^- \xrightarrow{p} \sum_{t-1<s\leq t} \Delta p_s^2 I\left\{\Delta p_s > 0\right\} - \sum_{t-1<s\leq t} \Delta p_s^2 I\left\{\Delta p_s < 0\right\}.$$

Note that the continuous part of RV cancels out and only the jump components remain. We analyze whether the impact of jumps depends on the sign of positive and negative jump variation. Hence, following Patton and Sheppard (2015), we further decompose the signed jump variation as

$$\begin{aligned}\Delta J_t^2 &= \Delta J_t^{2+} + \Delta J_t^{2-} \qquad (1)\\ &= \left(RS_t^+ - RS_t^-\right) I\left\{RS_t^+ - RS_t^- > 0\right\} + \left(RS_t^+ - RS_t^-\right) I\left\{RS_t^+ - RS_t^- < 0\right\}.\end{aligned}$$

Consistent estimation of the continuous part of the volatility, or IV, has been achieved by Barndorff-Nielsen and Shephard (2004), who proved that under the regularity condition that jumps have finite activity, the normalized sum of products of the adjacent absolute values of returns, i.e. Bipower Variation (BV), is a consistent estimator of IV even in the presence of jumps. At day t, BV is defined as

$$BV_t^0 = \frac{\pi}{2}\sum_{\tau=2}^{m} |r_{t,\tau}|\,|r_{t,\tau-1}| \underset{p}{\rightarrow} \int_{t-1}^{t} \sigma_s^2 ds \qquad \text{as } m \to \infty.$$

Rather than using BV^0 directly, we use an average of skip-0 through skip-4 BV estimators as in Patton and Sheppard (2015),

$$BV_t = \frac{1}{5}\sum_{q=0}^{4} BV_t^q,$$

where skip-q BV estimator is defined as

$$BV_t^q = \frac{\pi}{2}\sum_{\tau=q+2}^{m} |r_{t,\tau}|\,|r_{t,\tau-1-q}|\,.$$

The skip-q BV estimator corrects small sample bias of the skip-0 BV estimator.

Over the last few years, many techniques have been proposed to estimate, or to at least proxy, asset return volatility from high frequency data. See Meddahi et al. (2011) and Andersen et al. (2006) for details. Some methods have focused on correcting for microstructure noise caused by trade imperfections, market frictions, or informational effects. The most commonly used technique for computing RV is known as *downsampling*, which conventionally uses sampling intervals from 5 to 30 minutes to derive daily RV series. This method does not use all the high frequency data; and other methods have been suggested in the literature to try to deal with the presence of possible micro-structure noise. In particular, Bandi and Russell (2008) have considered the idea of finding the optimal sampling frequency; while Aït-Sahalia et al. (2005)

have used an MLE of a model for RV which assumes additive *i.i.d.* microstructure noise. Zhou (1996) considered corrections for first-order autocorrelation type noise in high frequency data; Barndorff-Nielsen et al. (2008) use a realized kernel to correct for autocorrelation in a more general approach. Zhang et al. (2005) and Zhang (2006) use two-scale and multi-scale estimators which combine subsampled RV computed at lower and higher frequencies.

However, as noted by Liu et al. (2015), there is uncertainty as to the desirability and also choice of the most appropriate method. After considerable amount of initial data analysis and investigation of possible outliers and noise, we decided to use 5-minute data for the computation of RV. This seemed the most appropriate method for calculating RV given the purpose of this study is to compare, contrast and to combine long memory and the HAR modeling approaches.

3 Data

In order to assess the relative merits of HAR and long memory models we use five minute high-frequency, intraday returns data on various assets. We examine five spot exchange rates of the Australian dollar (AUD), the Canadian dollar (CAD), the Euro (EUR), the UK British pound (GBP), and the Japanese yen (JPY) all against the numeraire US dollar (USD); for the period, January 2, 2004 through December 29, 2017. In line with previous studies we exclude the slower trading patterns induced over the weekends by discarding all observations from Friday 21:00 GMT through Sunday 22:00 GMT and measure the rates as the midpoint of the logarithms of the bid and ask rates. This provides a sample size of $T = 3,627$ daily observations from which to compute RV and the semi-variance measures.

For the equity market data, we use the $S\&P500$ index which consists of five minute tick interpolated prices from January 2, 2001 through December 31, 2016. The trading hours span from 9:30 through 16:00 with a total of 78 intraday observations and the total number of the trading days after adjustments is $T = 4,172$ observations.

[FIGURE 1 ABOUT HERE]

[FIGURE 2 ABOUT HERE]

Figure 1 plots the time paths of the various RV series and Figure 2 plots the first 50 lags of the sample autocorrelation function of the RV series. It can be seen that all the autocorrelation functions for the RV series exhibit the strong persistence that is consistent with long memory behavior.

4 Long Memory and RV

Some of the statistical features of the various RV series may be described in terms of the fractionally integrated, or long memory time series process, as defined by

$$(1-L)^d y_t = u_t,\ t = 1, \ldots, T,$$

where L is the lag operator, u_t is a short memory, $I(0)$ process, and the observable time series y_t is defined to be fractionally integrated of order d, or $I(d)$. In this case y_t is generally the RV series. The process generates hyperbolic rates of decay in the autocorrelation function and Impulse Response Function (IRF). The $I(d)$ process is defined as having partial sums that converge weakly to fractional Brownian motion, while d represents the degree of "long memory", or persistence in the series. For $-0.5 < d < 0.5$ the process is stationary and invertible; while for $0.5 \leq d \leq 1$, the process does not have a finite variance, but still has a finite cumulative impulse response function. The IRF, or infinite order moving average representation of this process, is given by

$$y_t = \sum_{k=0}^{\infty} \psi_k \epsilon_{t-k},$$

where $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma^2$, $E(\epsilon_t \epsilon_s) = 0$, $s \neq t$. For large lags k, these coefficients decay at the very slow hyperbolic rates of $\psi_k \sim c_1 k^{d-1}$ and similarly the infinite autoregressive representation coefficients decay at the rate of $c_2 k^{-d-1}$ and autocorrelation coefficients at the rate of $c_3 k^{2d-1}$, where c_1, c_2 and c_3 are constants. The simplest discrete time parameterization is the $ARFIMA$ model, which combines long memory with short run $I(0)$ dynamics and provides a flexible extension of the $ARIMA$ model and was introduced by Granger (1980), Granger and Joyeux (1980), and Hosking (1981). The simplest time domain workhorse model for long memory processes is the $ARFIMA(p, d, q)$ model of the form

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t, \tag{2}$$

where $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator of orders p and q respectively. Maximization of the Gaussian log likelihood is accomplished with respect to the complete vector of parameters $\boldsymbol{\vartheta}' = \left(d, \phi_1, ...\phi_p, \theta_1, ...\theta_q, \sigma^2\right)$. Under these conditions, the asymptotic distribution of the MLE will be

$$T^{1/2}\left(\hat{\boldsymbol{\vartheta}} - \boldsymbol{\vartheta}_0\right) \rightarrow N\{\mathbf{0}, \mathbf{I}(\boldsymbol{\vartheta}_0)^{-1}\},$$

where $\boldsymbol{\vartheta}_0$ denotes the true value of the vector of parameters and $\mathbf{I}(\boldsymbol{\vartheta}_0)$ is the information matrix. The results follow from Fox and Taqqu (1986) and for sake of simplicity a demeaned process is assumed. Then the MLE are $T^{1/2}$ consistent and asymptotically Normal when the unconditional mean is zero or known. The inclusion of an intercept parameter will result in a $T^{1/2-d}$ consistent estimator of the intercept. In some circumstances the assumption of Gaussianity may be inappropriate and can be replaced with the assumption that the innovations in equation (2) merely satisfy some mild mixing conditions. Given the results in Hosoya (1997), the implementation of quasi MLE is then straightforward; and in particular,

$$T^{1/2}\left(\hat{\boldsymbol{\vartheta}} - \boldsymbol{\vartheta}_0\right) \rightarrow N\{\mathbf{0}, \mathbf{A}(\boldsymbol{\vartheta}_0)^{-1}\mathbf{B}(\boldsymbol{\vartheta}_0)\mathbf{A}(\boldsymbol{\vartheta}_0)^{-1}\},$$

where $\mathbf{A}(\cdot)$ is the Hessian and $\mathbf{B}(\cdot)$ is the outer product gradient, both of which are evaluated at the true parameter values $\boldsymbol{\vartheta}_0$; see Baillie and Kapetanios (2013) for further details.

It is worth noting that long memory characteristics can be induced in a time series by many mechanisms. In particular, Granger (1980) showed that the aggregation of contemporaneous stationary $AR(1)$ processes could lead to an aggregate process with fractional integration. Also, occasional break points as in Granger and Hyung (2004); and forms of regime switches, as shown by Diebold and Inoue (2001), can also give rise to the appearance of long memory. In many instances there may not be any obvious explanation as to the occurrence of long memory in time series data. However, such fractional processes can simply be regarded as more general forms of the Wold decomposition than the exponential decay implied by processes with rational spectra, or stationary and invertible $ARMA$ representations. Hence, in some sense, hyperbolic rates of decay do not appear any more in need of justification than the standard exponential rates of decay.

Table 1 reports the MLE of $ARFIMA(p, d, 0)$ models where the order p is selected on the basis of minimizing Schwarz (1978) BIC for $p \in \{0, 1, 2, ..., 10\}$. This is predicated on the assumption that the short memory components are sufficiently well approximated by a finite order $AR(p)$ process. For some of the RV series it was found necessary to have quite high order autoregressive components to deal with fairly substantial short memory $I(0)$ components in addition to the long memory property. The estimated long memory parameters were statistically significantly different from zero for all of the RV series with several cases of borderline non stationarity, which still imply finite cumulative $IRFs$. In all cases the estimates of the short memory parameters are suppressed in the interests of conserving space and all the emphasis is on the estimation of the long memory parameter, d.

A more parsimonious parameterization of the short memory component can theoretically be found from the $ARFIMA(p, d, q)$ model; and estimates of this model are also reported in Table 1[1]. It should be noted that while the $ARFIMA(p, d, q)$ models are expected to provide a more parsimonious parameterization of the short memory components, their use can be complicated due to near cancellation of AR and MA roots. In general there is reasonable consistency across the time domain results with borderline non stationary fractional integration for many of the RV series; and we conclude that the RV series appears to be quite well suited to be represented by the fractionally integrated $ARFIMA$ models.

[TABLE 1 ABOUT HERE]

Following the seminal paper of Geweke and Porter-Hudak (1983), an alternative procedure is to use semi parametric estimation of the long memory parameter, which complements the linear $ARFIMA$ model estimation in Table 1. We report estimates of the long memory parameter from two semi parametric procedures. First, the Local Whittle (LW) estimator, which is obtained by minimizing the objective function,

$$R^{LW}(d) = \ln\left[\frac{1}{m}\sum_{j=1}^{m}\omega_j^{2d}I(\omega_j)\right] - \frac{2d}{m}\sum_{j=1}^{m}\ln(\omega_j),$$

[1]The strategy for model selection of $ARFIMA(p, d, q)$ requires estimation of models of orders of $p \in \{0, 1, 2, ..., P\}$ and $q \in \{0, 1, 2, ..., Q\}$ where P and Q are the maximum orders of the short memory parameters being considered. In this study $P = Q = 8$, so that the implementation of minimizing the BIC required estimation of 81 models.

with respect to d, where $\omega_j = (2\pi j)/T$ for $j = 1, 2, ..., T$ and $I(\omega_j)$ is the periodogram defined as,

$$I(\omega_j) = \frac{1}{2\pi T}\left|\sum_{j=1}^{T} y_t e^{i\omega_j t}\right|^2.$$

The estimator depends on the choice of bandwidth, m, which is generally chosen as $m = \lfloor T^{\delta} \rfloor$ where $0 < \delta < 4/5$; and where $\lfloor \cdot \rfloor$ denotes the integer part. Several important extensions of the LW estimator have been introduced in the literature. In particular, Shimotsu and Phillips (2005) have proposed the Exact Local Whittle (ELW) approach using a "corrected" discrete Fourier transform of the series, where the objective function now becomes,

$$R^{ELW}(d) = \ln\left[\frac{1}{m}\sum_{j=1}^{m} I_{\nabla^d y}(\omega_j)\right] - \frac{2d}{m}\sum_{j=1}^{m}\ln(\omega_j),$$

where $\nabla^d = (1-L)^d$. Given the distinct possibility of non stationary long memory RV series we also use the method of Abadir et al. (2007), who have introduced the Fully Extended Local Whittle ($FELW$) where $d \in (p-1/2, p+1/2]$, for $p = 0, 1, 2, ...$, which has the particular attraction of covering the region of nonstationarity for long memory processes. Then,

$$I^{FELW}(\omega_j) = \left|1 - e^{i\omega_j}\right|^{-2p} I_{\nabla^p y}(\omega_j),$$

where the $FELW$ is obtained by minimizing,

$$R^{FELW}(d) = \ln\left[\frac{1}{m}\sum_{j=1}^{m} j^{2d} I^{FELW}(\omega_j)\right] - \frac{2d}{m}\sum_{j=1}^{m}\ln(j).$$

The LW is known to be a consistent estimator of d in the stationary region of $-1/2 < d < 1/2$ with $m^{1/2}\left(\widehat{d_{LW}} - d_0\right) \to N\{0, (1/4)\}$. While the ELW and $FELW$ estimators are known to be consistent for all values of d.

A particularly important issue concerns the choice of bandwidth, denoted by m, which is generally chosen in the range of $T^{1/2} \leq m \leq T^{4/5}$. The LW and $FELW$ statistics are also reported in Table 1 and similarly to the time domain methods they find very significant long memory features of the RV series. However, both the LW and $FELW$ statistics are very dependent on the choice of bandwidth, m. For this reason the LW and $FELW$ estimators are also reported for a selection of bandwidth choices; including $m = T^{0.5}$, which tends to be the conventional choice, and also $m = T^{0.3}$ and $m = T^{0.7}$. The latter gives considerably more weight to the short frequency components that are apparently of importance as evidenced by the need for relatively large number of short memory parameters selected in the $ARFIMA$ estimation and also the HAR model considered later.

Overall, there is clear evidence of long memory characteristics from the $ARFIMA$ estimation and also the complementary LW and $FELW$ semi parametric results. The overall results suggest that the estimated long memory is either inside, or very close to the region of nonstationarity.

It is possible that the very significant estimates of the long memory parameter are due to nonlinear effects, or due to structural breaks. In particular, Granger and Hyung (2004) show that occasional break points processes are hard to distinguish from a pure fractional, $I(d)$ model. Alternative nonlinear explanations have centered on the possibility of regime switches giving rise to the appearance of long memory in a time series; see Granger and Ding (1996), Granger and Teräsvirta (1999) and particularly Diebold and Inoue (2001) who showed that a Markov Switching regime change model that can generate a long memory time series. Davidson and Sibbertsen (2005) discuss other regime switching and nonlinear models which can generate long memory. For these reasons we also used the tests of Sibbertsen (2004) and Wenger et al. (2018), who have provided a $CUSUM$ test to test for structural breaks in the intercept of long memory process. This change in mean test is used to check the robustness of the long memory hypothesis; and on applying this test to the fractionally filtered series, it is denoted as $CUSUM-\nabla^d$. The test statistic is defined as

$$Q_T = \sup_{r\in(0,1)} \left| \left(\widehat{\sigma^2} T\right)^{-1/2} \sum_{t=1}^{[rT]} \widehat{u}_t^* \right|,$$

where $(1-L)^{\widehat{d}} y_t = y_t^*$, and $\widehat{d}$ is the LW or $FELW$. Furthermore $\widehat{u}_t^* = y_t^* - \overline{y^*}$, where $\overline{y^*} = \frac{1}{T}\sum_{t=1}^{T} y_t^*$ and $\widehat{\sigma^2} = \frac{1}{T}\sum_{j=1}^{m} \widehat{u}_t^{*2}$. Wenger et al. (2018) show that the limiting distribution of Q_T is pivotal with respect to $\widehat{d}$ and that the test statistic follows the conventional distribution as defined by Ploberger and Krämer (1992). The critical values for Q_T at the $0.01, 0.05$ and 0.10 significance levels are 1.63, 1.36 and 1.22 respectively. The $CUSUM$ statistics results indicate that the only RV series with some evidence for structural change in the mean is the Australian dollar.

[TABLE 2 ABOUT HERE]

5 HAR Models

The HAR models require defining h period averages of the observed RV series

$$\overline{RV}_{t,t+h} = \frac{1}{h}\sum_{i=1}^{h} RV_{t+i},$$

where $h = 1, 5,$ and 22 for the one day, one week, and one month cumulative volatilities. This three parameter HAR model is motivated by the additive partial cascade of volatilities model. On further defining $\overline{RV}_t^w = \frac{1}{5}\sum_{j=0}^{4} RV_{t-j}$ as the weekly average, and $\overline{RV}_t^m = \frac{1}{22}\sum_{j=0}^{21} RV_{t-j}$ as the monthly average; then the HAR model reduces to

$$\overline{RV}_{t,t+h} = \phi_0 + \phi_d RV_t + \left(\frac{\phi_w}{5}\right)\sum_{i=0}^{4} RV_{t-i} + \left(\frac{\phi_m}{22}\right)\sum_{i=0}^{21} RV_{t-i} + \varepsilon_{t+h} \quad (3)$$

which is a restricted parameter version of the general $AR(22)$ model and is represented as,

$$\overline{RV}_{t,t+h} = \phi_0 + \phi_d\, RV_t + \phi_w \overline{RV}_t^w + \phi_m \overline{RV}_t^m + \varepsilon_{t+h}. \quad (4)$$

The original model was proposed by Corsi (2009) to explain the persistence of RV series from the heterogeneity of an agent's behavior over distinct time horizons. The HAR model is generally described as an additive volatility cascade, from high frequencies to low frequencies; with each additive cascade having close to an $AR(1)$ structure. The notion of multiple components in the volatility process is justified in terms of differences of agents risk profiles, institutional structures, temporal horizons, etc. The model has been extended by Patton and Sheppard (2015) to include separate effects of volatility due to positive and negative returns and to include good and bad volatility through the signed jump variation. While McAleer and Medeiros (2008) have considered an alternative model formulation which combines smooth transition regimes and long range dependence.

[TABLES 3 AND 4 ABOUT HERE]

Estimates of the HAR model are reported in Table 3 for the six RV series. The OLS estimates of the parameters are largely consistent with those of previous studies with the estimated daily HAR parameter, ϕ_d being statistically significant and in the range of 0.22 to 0.42 for five of the RV series. The value of the estimated $\phi_w$varies substantially across series and is generally statistically significant. While the estimated ϕ_m parameter is in the range of 0.33 to 0.54 and is very significant for all the six RV series. On comparing the estimated $ARFIMA$ models in Table 1 and the estimated HAR models in Table 3, it is clear that the $ARFIMA$ models dominate the HAR models in terms of BIC model selection. Also, in terms of BIC, the $ARFIMA(p,d,0)$ model is preferred to the HAR and extended HAR models with semivariances, for all assets except Canada. While the $ARFIMA(p,d,0)$ is preferred to $EHAR$ with jumps for all but Canada and the Euro; and the $ARFIMA(p,d,q)$ is better than HAR and $EHAR$ with semivariances for all assets; and the $ARFIMA(p,d,q)$ is preferred to $EHAR$ with jumps for all but Canada. In summary, when the HAR, or extended HAR are combined with long memory the estimate of d is significant. However, the $ARFIMA(p,q,0)$ and $ARFIMA(p,d,q)$ models were preferred over HAR or extended HAR in the majority of cases.

Table 4 reports estimates of three versions of the $EHAR$ model which supplement the terms in the basic HAR model to include signed semivariances which distinguish between positive and negative returns in equation (5), signed jumps with BV in equation (6) and separate positive and negative signed jumps as in equation (1) with BV in equation (7). These models are

$$\overline{RV}_{t,t+h} = \phi_0 + \phi_d^+ \ RS_t^+ + \phi_d^- \ RS_t^- + \phi_w \overline{RV}_t^w + \phi_m \overline{RV}_t^m + \varepsilon_{t+h} \tag{5}$$

$$\overline{RV}_{t,t+h} = \phi_0 + \phi_J \Delta J_t^2 + \phi_C BV_t + \phi_w \overline{RV}_t^w + \phi_m \overline{RV}_t^m + \varepsilon_{t+h} \tag{6}$$

and

$$\overline{RV}_{t,t+h} = \phi_0 + \phi_J^+ \Delta J_t^{2+} + \phi_J^- \Delta J_t^{2-} + \phi_C BV_t + \phi_w \overline{RV}_t^w + \phi_m \overline{RV}_t^m + \varepsilon_{t+h}. \tag{7}$$

The last three models were all introduced in Patton and Sheppard (2015). Exactly the same conclusions emerge from a comparison of the estimated $ARFIMA$ models in Table 1 with the extended HAR model in Table 4, where the daily RV component is omitted and replaced with RS^+ and RS^- respectively. The ϕ_C parameter is associated with the "continuous" BV, which is intended to make the continuous part of RV robust to the presence of the jumps. If there are no jumps, then daily RV should be asymptotically identical to BV. Good jumps lead to lower volatility, and bad jumps lead to higher volatility in longer horizons. There is evidence in the second panel of Table 4 that the presence of the signed jump variables are effective in explaining the RV for Japan, but not for the other RV exchange rate series. However, the parameter associated with the negative jump variable is highly and negatively significant for the $S\&P500$ RV series. Similar results have been found by Busch et al. (2011).

However, the Bipower Variation BV_t is highly significant and replaces the need for the daily RV variable associated with the ϕ_d parameter. Interestingly, the estimated ϕ_m parameter is between 0.33 and 0.46 across the various assets RV series. The importance of the essentially $AR(22)$ term seems to indicate the need for higher order dynamics or for long memory. This possibility is pursued in the next section.

6 Distinguishing HAR from Long Memory

So far we have presented favorable evidence for the presence of long memory and also for the validity of the HAR model. While BIC generally favors the long memory models over HAR it is also worth further investigation to try to distinguish between these two theories. One issue with empirical work in this area is to accurately distinguish between long memory and very persistent short memory autoregressive behavior. This problem becomes particularly apparent in the high correlation between the estimated long memory parameter and the estimated short memory $ARMA$ parameters and resulting instability of these parameter estimates in the presence of higher order parameterizations. The same problem is apparent in the frequency domain LW and $FELW$ where the choice of bandwidth is so critical and the noted poor performance of these semi parametric estimators in the presence of very persistent autocorrelation; e.g. see Baillie and Kapetanios (2008) and Nielsen and Frederiksen (2005).

In this section we tackle these problems in several different directions. First, we estimate HAR models from simulated long memory processes and tabulate the properties of the resulting simulated HAR parameter estimates. Second, we estimate both unrestricted $ARFIMA(22, d, 0)$ models and restricted $ARFIMA$ models, (denoted as $RARFIMA$), where the parameter restrictions are implied by the HAR model. Third, we estimate by MLE a similar theoretical model which estimates HAR models with long memory disturbances. The final and fourth approach is to use a $FELW$ estimate of d to filter out the long memory properties of the RV series and to then estimate an $AR(22)$ model to the filtered series. All of these methods provide different pieces of evidence on the issue of distinguishing one model, or property, from another.

6.1 Simulating Estimated HAR Models from a Long Memory Process

The first approach is to generate realizations from $ARFIMA(0, d, 0)$ model with different long memory parameters of $d \in \{0.25, 0.30, 0.35, 0.40, 0.45\}$. Each generated series has $T = 10,000$ observations and we perform $5,000$ replications for each design, to estimate the basic three parameter HAR models. In many respects the simulation results in Table 5 replicate many of the features of HAR estimation in this paper and other literature. The mean of the simulated estimated ϕ_d parameter is 0.20 for a data generating process of $ARFIMA(0, 0.25, 0)$ and increases monotonically as d increases to 0.41 for when the simulated series is from an $ARFIMA(0, 0.45, 0)$ process. The interval 0.37 to 0.44 provides a 95% coverage of the monthly HAR parameter ϕ_d from an $ARFIMA(0, 0.45, 0)$ design and so appears relatively precise. Similar degrees of precision are found for the other simulated parameters. However, ϕ_w and ϕ_m have much less variation with the value of d and lie in the range of 0.23 to 0.29 for all cases.

The above results can be compared with those in Table 3, which have many similar features; although GBP has considerably lower ϕ_d than predicted and rather higher ϕ_m than predicted.

6.2 Restricted ARFIMA Models

The second line of investigation focuses on using MLE to estimate both unrestricted $ARFIMA(22, d, 0)$ models and restricted version of the model, an $RARFIMA(22, d, 0)$. The parameter restrictions on this latter model are those implied in equation (3). Hence the $RARFIMA(22, d, 0)$ model is

$$(1 - L)^d \lambda(L) RV_t = \varepsilon_t, \tag{8}$$

where $\lambda(L) = 1 - \lambda_1 L - \lambda_2 L^2 - \lambda_2 L^3 - \lambda_2 L^4 - \lambda_2 L^5 - \lambda_3 L^6 - \lambda_3 L^7 \cdots - \lambda_3 L^{22}$ with all the roots of $\lambda(L)$ outside the unit circle and ε_t denoting white noise. This model is identical to $\phi(L)(1 - L)^d RV_t = \varepsilon_t$ with 19 restrictions, $\phi_2 = \phi_3 = \phi_4 = \phi_5 \equiv \lambda_2$ and $\phi_6 = \phi_7 = \cdots = \phi_{22} \equiv \lambda_3$.

MLE of the $RARFIMA(22, d, 0)$ model parameters are to be found in Table 6. Results for the unrestricted $ARFIMA(22, d, 0)$ models are not presented due to the high degree of correlation between the long memory parameter and the twenty two unrestricted autoregressive parameters. This was found to cause both unstable parameter estimates and poor quality standard errors of the parameter estimates. However, robust Wald test statistics of the 19 parameter restrictions which reduce the unrestricted $ARFIMA(22, d, 0)$ model to the $RARFIMA(22, d, 0)$ are presented in Table 6. The Wald tests generally reject the restrictions that are consistent with a HAR model. From Table 6 it can be seen that the HAR model restrictions cannot be rejected for Canada, Japan or $S\&P500$; although there is considerable variation among the short memory HAR parameters with several not being significant. While the MLE of the long memory parameter d is around 0.30 for four of the RV series and is not significantly different from zero for the Euro or the $S\&P500$ RV series. In general these results suggest that the HAR model provides a useful representation of some of the low order dynamics of RV, but that long memory also plays an important role to describe higher order dynamics.

[TABLE 5 AND 6 ABOUT HERE]

A related method to the above is to specify the long memory process as a disturbance around the HAR specification and to estimate the model

$$(1-L)^d \left(y_t - x_t'\beta\right) = \varepsilon_t, \qquad t = 1, \ldots, T, \tag{9}$$

where $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma^2$, $E(\epsilon_t \epsilon_s) = 0$, $s \neq t$. While x_t is a k dimensional vector of explanatory HAR type variables at time t, and β is the corresponding vector of parameters.

[TABLE 7 AND 8 ABOUT HERE]

The implementation of MLE to the above model follows as in Section 3. If all the variables are $I(d)$ with $-0.5 < d < 0.5$ then conventional asymptotics are valid and the MLE should be $T^{1/2}$ consistent. However, when HAR variables are included in the regression, there is the possibility of some variables having $d > 0.5$ and hence being non stationary long memory processes; and also the possibility of forms of non standard fractional cointegration occurring.

The long memory property of RV is a feature shared by many other volatility series, which gives rise to the possibility of fractional cointegration between volatility series. This has been considered by Christensen and Nielsen (2006) and Bollerslev et al. (2013), who include the possibility that volatility predicts returns. In fact, several articles consider this in the context of long memory models; see Christensen and Nielsen (2007) and Christensen et al. (2010) who use a $FIEGARCH - M$ model which builds on the $FIGARCH$ model of Baillie et al. (1996) and

$FIEGARCH$ model of Bollerslev and Mikkelsen (1996). The latter paper deals with differences from positive and negative returns and is particularly relevant in the context of realized semi-variances and jump variation in RV. One attraction of high frequency data is that it is model free and does not require the formulation of Stochastic Volatility or $GARCH$ type models. The interest in the HAR approach is that it provides possible alternatives to long memory and fractional cointegration analysis.

The potential borderline non stationarity of long memory of the RV series creates particularly challenging problems and is not pursued in this study. Hence while we believe the estimates are likely consistent, we note that there is some uncertainty associated with the conventional standard error estimates being reported. The resulting estimated models are reported in Tables 7 and 8 with many of the ϕ_d estimates being particularly significant across assets RV series; while the relative importance of the other volatility parameters ϕ_c and ϕ_w being less statistically important. Of particular interest is the magnitude and significance of the long memory parameter d, which is highly statistically significant across all RV series except for the Canadian dollar.

One relevant comparison is the basic HAR estimation in Table 3 with the estimated above model from equation (9) in Table 7. The estimated long memory parameter is significant for five of the six RV series and the BIC prefer the model with long memory to the basic HAR formulation. Interestingly there appears to be less role for the long memory parameter when estimating the extended HAR model with ϕ_c and signed jump variables. Also, the impact of the negative signed jump variable for the $S\&P500$ series still has a negatively signed parameter estimate and is now not significant. This provides an interesting comparison with the results in Table 4 and suggests that the presence of a jump variable maybe picking up discontinuities which are otherwise giving rise to the presence of long memory.

6.3 Two Step Estimation of HAR from Filtered RV Series

The final approach was to use an initial semi parametric estimate of d from the $FELW$, denoted by $\widehat{d}_{FELW}$, from Table 1 and to then apply the filter

$$(1-L)^{\widehat{d}_{FELW}} RV_t = v_t$$

to the RV series to obtain v_t, which was then used to estimate a HAR model. This form of two step estimation with long memory processes has been considered by Baillie and Kapetanios (2008, 2013). In this context the fractional filter appeared too strong a transformation and tended to remove higher frequency data characteristics associated with HAR as well as the long memory features. The results are not reported in the interests of conserving space but are available from our websites.[2]

7 Time Varying Parameter Extended HAR Models

While long memory appears to be an important modeling feature to include with the extended HAR formulation, another possibility worth considering is that the weights attached to the partial cascade volatilities may not be constant over time. If this were the case, the non linearity may well capture some of the long memory aspects of the RV series. If we view the partial cascade volatilities reflecting agent's risk preferences, access to information, geographical location, etc then there are many explanations why the coefficients may have time variation. On modifying equation (4), we can write the $TVP-HAR$ model as

$$\overline{RV}_{t,t+h} = \phi_{0,t} + \phi_{d,t}\, RV_t + \phi_{w,t}\overline{RV}_t^w + \phi_{m,t}\overline{RV}_t^m + \varepsilon_{t+h}$$

where the $\phi_{j,t}$ coefficients are now time varying and are partial volatility parameters that depend on time varying risk premium. This model is implemented as a kernel weighted regression which is facilitated by an extension of the random coefficient approach of Giraitis et al. (2014). Some of the details of the approach are provided in the appendix to this paper.

[FIGURE 3 ABOUT HERE]

Details of the means and standard deviations of the estimated parameters in the $TVP-HAR$ model for all RV assets are presented in Table 9. Similar results for the estimated $TVP-EHAR$ models are available in Table 10. Showing the first two moments of these parameter estimates as they change over time is only part of the story and some idea about their variability can be seen in Figure 3 which plots the time variation of the parameter estimates across the sample. We only report results for the Australian dollar and the Euro vis a vis the US dollar to

[2]Estimation of truncated $AR(p)$ models as in higher order HAR models can have their own distributional issues when the true data generating process is long memory. See Poskitt (2007, 2008) and Baillie and Kapetanios (2013).

conserve space. Full details of the figures for the remaining assets are available from the authors on request. A potentially interesting topic for future research would be to see if the variation in the parameter estimates are related to particular economic episodes.

However, in terms of model comparisons using BIC, the $RARFIMA$ model in equation (8), which includes both HAR parameters and long memory; and the related model in equation (9) outperform the $TVP-HAR$ alternatives in terms of BIC. Hence the $RARFIMA$ based models with constant long memory parameter and constant HAR parameters appear preferable based on model selection criteria.

[**TABLES 9 AND 10 ABOUT HERE**]

8 Conclusions

This paper has investigated the presence of long memory in Realized Volatility (RV) through analysis of the Heterogeneous Autoregressive (HAR) model and fractionally integrated long memory models. We find that the presence of the long memory parameter is often important in addition to the HAR models and that their relative importance seems to vary across the asset process being considered. In several cases the preferred model is a combination of the two approaches. The HAR restricted $ARFIMA$ model, denoted by $RARFIMA$ appears to be a good approximation to the dynamic structure of several RV series.

Time varying parameter versions of the HAR model were also investigated and show the relative importance of different HAR components at different time periods in the sample. In general, the $RARFIMA$ model is preferred to the time varying parameter models on information criteria.

Our results suggest that RV series are quite complex and can involve both HAR components and long memory components. In fact, RV potentially convey a lot of information and are worthy of further research.

9 Appendix: Kernel weighted regression

In order to give HAR models the maximum opportunity to represent the RV process, we implement a non parametric approach for computing the time variation in the regression coefficients that requires minimal theoretical restriction on the functional form. We extend the work of Giraitis et al. (2014) on autoregressive processes to that of a kernel smoothed regression. Giraitis et al. (2014) consider the random coefficient $AR(1)$ process

$$y_t = \phi_{t-1} y_{t-1} + u_t,$$

where u_t is a stationary ergodic martingale difference sequence with respect to some natural filtration, Ω_t, and there is some initialization of the process y_0. Then ϕ_{t-1} is the random coefficient in the above TVP $AR(1)$ process, and $E\left[u_t|\Omega_{t-1}\right] = 0$ and $E\left[\phi_t|\Omega_{t-1}\right] = \phi$. The stability of the model depends on the TVP nature of the AR parameters satisfying various smoothness classes. Giraitis et al. (2014) model ϕ_t as a rescaled random walk, where $\{a_t\}$ is a non stationary process which defines the random drift, and $-1 < \phi < 1$. In this context ϕ_t is a standardized version of a_t so that

$$\phi_t = \phi \frac{a_t}{\max_{0 \leq t \leq T} |a_t|} \ldots.t > 0,$$

where the stochastic process a_t is assumed to be a drift-less random walk, so that $a_t = a_{t-1} + w_t$ and where w_t is a stationary process with zero mean. Also, $\phi \in (0, 1)$ and ϕ_{t-1} is then bounded away from the boundary points of -1 and 1. The above framework can be extended to the time varying $AR(p)$ model

$$y_t = \sum_{i=1}^{p} \phi_{t-1,i} y_{t-i} + u_t$$

and can be used with the boundary conditions

$$\phi_{t,i} = \phi_i \frac{a_{t,i}}{\max_{0 \leq t \leq T} |a_{t,i}|} \ldots.t > 1,$$

where $0 < \phi < 1$ and each $a_{t,i}$ are independent versions of the a_t process defined above. Under these assumptions the maximum absolute eigenvalues of the matrix

$$A_t = \begin{bmatrix} \phi_{t,1} & \phi_{t,2} & \ldots & \ldots & \ldots & \phi_{t,p} \\ 1 & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & 1 & 0 & \ldots & \ldots & 0 \\ \ldots & 0 & 1 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ 0 & \ldots & \ldots & 0 & 1 & 0 \end{bmatrix}$$

are bounded above by unity for all t. Giraitis et al. (2014) show that the coefficient process $\{\phi_t; t = 1, ..., T\}$ converges in distribution as T increases to the limit

$$\{\phi_t; 0 \leq \tau \leq 1\} \rightarrow_D \{\phi \tilde{W}_\tau; 0 \leq \tau \leq 1\}.$$

The approach for estimating the time varying parameter, ϕ_t, is to use the moving window estimator for the $AR(1)$ random coefficient model

$$\widehat{\phi}_t = \frac{\sum_{k=1}^{T} K\left(\frac{t-k}{H}\right) y_k y_{k-1}}{\sum_{k=1}^{T} K\left(\frac{t-k}{H}\right) y_{k-1}^2},$$

where $K\left(\frac{t-k}{H}\right)$ is a kernel and continuously bounded function, such as the Epanechnikov kernel with finite support, or the familiar Gaussian kernel with infinite support. On generalizing a generic regression which can be expressed as

$$y_t = x_t' \beta_t + u_t,$$

with $\beta_t = (\beta_{1,t}, \beta_{2,t}, ...\beta_{k,t})$ and it is assumed that each $\beta_{j,t}$ follows a bounded random walk. x_t' is the vector $(m \times 1)$ containing the time series of the factors. In general the kernel weighted regression estimator for $\beta_{j,t}$ is

$$\hat{\beta}_t = \left(\sum_{k=1} w_{kt} x_k x_k' \right)^{-1} \left(\sum_{k=1} w_{kt} x_k y_k \right),$$

where $w_{kt} = K\left(\frac{t-k}{H}\right)$. From Giraitis et al. (2014), it follows that

$$H^{1/2}(1 - \hat{\beta}_{j,t}^2)^{-1/2}(\hat{\beta}_{j,t} - \beta_{j,t}) \sim N(0, 1).$$

The authors prove that if the bandwidth is $o_p\left(T^h\right)$ with $h = 1/2$, and given homoskedasticity of the error process, then

$$Var\left(\hat{\beta}_t\right) = \hat{\sigma}_u^2 \left(\sum_{k=1} w_{kt} x_k x_k' \right)^{-1} \sum_{k=1} w_{kt}^2 x_k x_k' \left(\sum_{k=1} w_{kt} x_k x_k' \right)^{-1},$$

where

$$\hat{\sigma}_u^2 = \frac{1}{T}\sum_{i=1}^{T}\left(y_t - x_t'\beta_t\right)^2$$

One appealing characteristic of this approach is that they nest rolling window estimates of the regression betas and are equivalent to kernel smoothing estimators using a uniform one-sided kernel instead of a Gaussian two-sided kernel. A key role is played by the decision about the bandwidth and for a given kernel function, $K\left(\frac{t-k}{H}\right)$, the bandwidth, H, represents the degree of smoothness of the estimates. Giraitis et al. (2014) proved that a bandwidth of $H = T^h$, with $h = 0.5$, provides an estimator with desirable properties such as consistency and asymptotic normality and in addition provides valid standard errors.

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Table 1. Estimates of Long Memory Parameter d

	AUD	CAD	EUR	GBP	JPY	S&P500
				$ARFIMA(p,d,0)$		
p	7	4	3	4	0	6
d	0.786	0.625	0.553	0.598	0.396	0.728
	(0.114)	(0.067)	(0.051)	(0.099)	(0.045)	(0.180)
$ln(L)$	–2919.626	14.173	84.690	–714.267	–2106.755	–6086.334
BIC	5921.213	29.028	–120.202	1485.907	4238.098	12247.694
				$ARFIMA(p,d,q)$		
p	7	3	3	3	2	3
q	2	3	3	5	3	1
d	0.725	0.793	0.670	0.685	0.488	0.733
	(0.140)	(0.210)	(0.132)	(0.193)	(0.104)	(0.202)
$ln(L)$	–2875.159	83.275	123.603	–676.273	–2073.290	–6170.745
BIC	5848.672	–92.785	–173.440	1442.704	4212.150	12399.843

Key: The $ARFIMA(p,d,0)$ models are estimated for $p \in \{0,1,\ldots,10\}$ and the model with the smallest *BIC* is chosen. The strategies for model selection of $ARFIMA(p,d,q)$ models involve estimation of $(P+1)(Q+1)$ models where P and Q are the maximum orders of the short memory parameters being considered. They were generally fixed at 8 requiring estimation of 81 models and the model with the smallest *BIC* is chosen. Robust standard errors are in parentheses. $ln(L)$ represents the maximized log-likelihood and *BIC* represents the Bayesian information criterion.

Table 2. Estimates of Long Memory Parameter d using *LW* and *FELW*

		AUD	CAD	EUR	GBP	JPY	S&P500
		LW ($m = \lfloor T^b \rfloor$)					
$b = 0.3$	d	0.316	0.606	0.385	0.381	0.541	0.312
		(0.146)	(0.146)	(0.146)	(0.146)	(0.146)	(0.143)
0.5	d	0.495	0.731	0.756	0.779	0.501	0.473
		(0.270)	(0.270)	(0.270)	(0.270)	(0.270)	(0.268)
0.7	d	0.678	0.598	0.558	0.643	0.380	0.734
		(0.325)	(0.325)	(0.325)	(0.325)	(0.325)	(0.323)
		FELW ($m = \lfloor T^b \rfloor$)					
$b = 0.3$	d	0.274	0.495	0.294	0.344	0.390	0.245
		(0.146)	(0.146)	(0.146)	(0.146)	(0.146)	(0.143)
0.5	d	0.501	0.708	0.743	0.794	0.510	0.474
		(0.270)	(0.270)	(0.270)	(0.270)	(0.270)	(0.268)
0.7	d	0.606	0.564	0.518	0.556	0.336	0.681
		(0.325)	(0.325)	(0.325)	(0.325)	(0.325)	(0.323)
$b = 0.5$	*CUSUM*-$\bigtriangledown^d$	2.020	0.671	0.546	0.674	1.067	1.912

Key: The *LW* and *FELW* estimators are estimated with bandwidths $(m) = \lfloor T^b \rfloor$ with $b \in \{0.3, 0.5, 0.7\}$. Robust standard errors are reported in parentheses. *CUSUM*-$\bigtriangledown^d$ statistic is the usual *CUSUM* statistic applied to the d (estimated with *FELW*, $m = T^{0.5}$) fractionally filtered series.

Table 3. Estimation of the basic *HAR* Model

	$\overline{RV}_{h,t+h}= \phi_0+\phi_d RV_t^{(d)}+\phi_w \overline{RV}_t^{(w)}+\phi_m \overline{RV}_t^{(m)}+\varepsilon_{t+h}$					
	AUD	CAD	EUR	GBP	JPY	S&P500
ϕ_d	0.415	0.270	0.272	0.077	0.223	0.222
	(0.070)	(0.083)	(0.052)	(0.054)	(0.081)	(0.122)
ϕ_w	0.119	0.275	0.244	0.145	0.197	0.330
	(0.087)	(0.096)	(0.069)	(0.069)	(0.065)	(0.144)
ϕ_m	0.343	0.370	0.401	0.542	0.364	0.337
	(0.069)	(0.070)	(0.057)	(0.060)	(0.061)	(0.106)
$ln(L)$	–3001.364	28.371	48.230	–1266.077	–2191.938	–6376.967
BIC	6043.708	–15.762	–55.479	2573.134	4424.857	12795.614

Key: *OLS* estimates of the basic *HAR* model are reported with robust standard errors in parentheses. $ln(L)$ is the maximized log-likelihood.

Table 4. Estimation of the *EHAR* Model

	$\overline{RV}_{h,t+h} = \phi_0 + \phi_d^+ RS_t^+ + \phi_d^- RS_t^- + \phi_w \overline{RV}_t^{(w)} + \phi_m \overline{RV}_t^{(m)} + \varepsilon_{t+h}$					
	AUD	CAD	EUR	GBP	JPY	S&P500
ϕ_d^+	0.601	0.355	0.285	–0.079	0.179	–0.009
	(0.150)	(0.107)	(0.086)	(0.019)	(0.166)	(0.194)
ϕ_d^-	0.157	0.185	0.260	0.550	0.252	0.425
	(0.137)	(0.102)	(0.069)	(0.189)	(0.163)	(0.177)
ϕ_w	0.140	0.278	0.242	0.106	0.202	0.352
	(0.086)	(0.084)	(0.070)	(0.059)	(0.064)	(0.145)
ϕ_m	0.354	0.367	0.401	0.462	0.365	0.333
	(0.068)	(0.070)	(0.057)	(0.072)	(0.060)	(0.105)
$ln(L)$	–2979.832	32.935	48.333	–1133.620	–2191.037	–6344.528
BIC	6008.841	–16.694	–47.489	2316.417	4431.251	12739.072

Key: As in Table 3 with *OLS* estimates of the *EHAR* model with positive and negative semivariances reported.

Table 4 (cont'd). Estimation of the *EHAR* Model

	$\overline{RV}_{h,t+h} = \phi_0 + \phi_J^+ \Delta J_t^{2+} + \phi_J^- \Delta J_t^{2-} + \phi_C BV_t + \phi_w \overline{RV}_t^{(w)} + \phi_m \overline{RV}_t^{(m)} + \varepsilon_{t+h}$					
	AUD	CAD	EUR	GBP	JPY	S&P500
ϕ_J^+	0.362	0.017	–0.060	–0.085	–0.325	0.277
	(0.308)	(0.163)	(0.089)	(0.040)	(0.053)	(0.241)
ϕ_J^-	0.029	0.128	0.031	–0.135	0.312	–0.749
	(0.176)	(0.075)	(0.090)	(0.137)	(0.184)	(0.295)
ϕ_C	0.437	0.443	0.420	0.222	0.590	0.149
	(0.107)	(0.071)	(0.096)	(0.150)	(0.074)	(0.147)
ϕ_w	0.104	0.188	0.165	0.111	0.065	0.327
	(0.098)	(0.087)	(0.075)	(0.066)	(0.047)	(0.140)
ϕ_m	0.355	0.349	0.379	0.481	0.281	0.309
	(0.069)	(0.071)	(0.061)	(0.083)	(0.055)	(0.094)
$ln(L)$	–2939.628	100.703	97.773	–1159.155	–2059.240	–6256.314
BIC	5936.630	–144.033	–138.174	2375.682	4175.853	12570.980

Key: As in Table 3 with *OLS* parameter estimates and robust standard errors of the *EHAR* model with positive and negative signed variation and *BV* reported.

Table 5. Simulated *HAR* Estimations from Fractional White Noise

	$d = 0.25$			$d = 0.30$		
	ϕ_d	ϕ_w	ϕ_m	ϕ_d	ϕ_w	ϕ_m
Mean($\hat{\phi}$)	0.204	0.227	0.228	0.252	0.252	0.235
SD($\hat{\phi}$)	0.017	0.033	0.048	0.017	0.032	0.044
Mean(se($\hat{\phi}$))	0.016	0.031	0.040	0.016	0.030	0.035
	$d = 0.35$			$d = 0.40$		
	ϕ_d	ϕ_w	ϕ_m	ϕ_d	ϕ_w	ϕ_m
Mean($\hat{\phi}$)	0.302	0.270	0.232	0.354	0.281	0.222
SD($\hat{\phi}$)	0.017	0.030	0.040	0.018	0.029	0.036
Mean(se($\hat{\phi}$))	0.017	0.029	0.031	0.016	0.027	0.027
	$d = 0.45$					
	ϕ_d	ϕ_w	ϕ_m			
Mean($\hat{\phi}$)	0.407	0.285	0.206			
SD($\hat{\phi}$)	0.017	0.028	0.031			
Mean(se($\hat{\phi}$))	0.016	0.026	0.024			

Key: For each panel, Mean($\hat{\phi}$) is the average value of each estimated *HAR* parameters across 5,000 iterations. Similarly, SD($\hat{\phi}$) is the standard deviation of those estimates and Mean(se($\hat{\phi}$)) refers to the average standard error of those estimates.

Table 6. Estimation of the *RARFIMA*$(22, d, 0)$ model

The *RARFIMA*$(22, d, 0)$ model is $\lambda(L)(1-L)^d(RV_t-\mu) = \varepsilon_t$ where $\lambda(L) = 1-\lambda_1 L-\lambda_2 L^2-\lambda_2 L^3-\lambda_2 L^4-\lambda_2 L^5-\lambda_3 L^6-\lambda_3 L^7-\lambda_3 L^8-\cdots-\lambda_3 L^{22}$. This model is identical to $\phi(L)(1-L)^d(RV_t-\mu) = \varepsilon_t$ with 19 restrictions, $\phi_2 = \phi_3 = \phi_4 = \phi_5 \equiv \lambda_2$ and $\phi_6 = \phi_7 = \cdots = \phi_{22} \equiv \lambda_3$.

	AUD	CAD	EUR	GBP	JPY	S&P500
λ_1	0.317	0.014	–0.163	–0.006	0.054	0.387
	(0.121)	(0.116)	(0.037)	(0.234)	(0.102)	(1.171)
λ_2	0.045	0.044	–0.060	0.044	–0.008	0.062
	(0.025)	(0.043)	(0.022)	(0.076)	(0.025)	(0.120)
λ_3	0.016	0.031	0.004	0.022	0.015	0.015
	(0.008)	(0.006)	(0.012)	(0.011)	(0.013)	(0.048)
σ^2	0.305	0.058	0.057	0.089	0.187	1.142
	(0.049)	(0.006)	(0.006)	(0.021)	(0.033)	(0.199)
d	0.305	0.310	0.518	0.364	0.362	0.118
	(0.119)	(0.120)	(0.325)	(0.264)	(0.085)	(1.167)
$ln(L)$	–2993.725	10.280	50.416	–756.124	–2105.726	–6196.369
BIC	6028.431	20.421	–51.655	1553.229	4252.434	12434.419
Wald	48.343	35.799	75.575	47.664	25.857	25.814

Key: In the last column, the Wald statistic is computed from the unrestricted *ARFIMA*$(22, d, 0)$ model with these 19 *HAR* restrictions as the null hypothesis. The 1% and 5% critical values for χ^2_{19} distribution are 43.82 and 35.58, respectively. Robust standard errors are in parentheses.

Table 7. Estimation of the *HAR* Model with Long Memory Error Process

	AUD	CAD	EUR	GBP	JPY	S&P500
ϕ_d	0.229	0.144	0.097	0.034	0.065	–0.023
	(0.042)	(0.168)	(0.060)	(0.012)	(0.083)	(0.076)
ϕ_w	0.035	0.170	0.077	0.050	0.071	–0.033
	(0.133)	(0.194)	(0.105)	(0.037)	(0.081)	(0.202)
ϕ_m	0.442	0.518	0.517	0.244	0.238	0.364
	(0.180)	(0.195)	(0.103)	(0.101)	(0.171)	(0.384)
d	0.298	0.146	0.239	0.365	0.295	0.478
	(0.110)	(0.173)	(0.074)	(0.060)	(0.096)	(0.152)
$ln(L)$	–2937.37	35.832	79.35	–732.11	–2091.54	–6204.68
BIC	5923.92	–22.486	–109.52	1513.39	4232.25	12459.37

Key: Approximate *MLE*s of the *HAR* model with *ARFIMA* errors reported. *QMLE* standard errors are in parentheses.

Table 8. Estimation of the *ARFIMA-EHAR* Model

	ARFIMA-EHAR model with positive and negative semivariances					
	AUD	CAD	EUR	GBP	JPY	S&P500
ϕ_d^+	0.374	0.214	0.061	0.034	0.047	–0.211
	(0.134)	(0.222)	(0.099)	(0.031)	(0.079)	(0.147)
ϕ_d^-	0.074	0.111	0.120	0.033	0.077	0.137
	(0.133)	(0.190)	(0.069)	(0.086)	(0.122)	(0.128)
ϕ_w	0.058	0.192	0.075	0.050	0.074	–0.023
	(0.142)	(0.202)	(0.107)	(0.037)	(0.079)	(0.521)
ϕ_m	0.448	0.495	0.516	0.244	0.240	0.370
	(0.175)	(0.221)	(0.105)	(0.103)	(0.176)	(0.376)
d	0.272	0.124	0.244	0.366	0.294	0.487
	(0.114)	(0.192)	(0.076)	(0.058)	(0.097)	(0.254)
$ln(L)$	–2926.69	37.38	80.00	–732.11	–2091.36	–6171.65
BIC	5910.75	–17.38	–102.62	1521.58	4240.09	12401.65

Key: Similar to Table 7 with approximate *MLE*s of the *ARFIMA-EHAR* model with positive and negative semivariances reported.

Table 8 (cont'd). Estimation of the *ARFIMA-EHAR* model

	ARFIMA-EHAR model with positive and negative signed variations and *BV*					
	AUD	CAD	EUR	GBP	JPY	S&P500
ϕ_J^+	0.201	0.067	–0.139	0.030	–0.262	0.104
	(0.317)	(0.218)	(0.095)	(0.025)	(0.077)	(0.197)
ϕ_J^-	0.073	0.107	0.054	–0.087	0.270	–0.469
	(0.183)	(0.088)	(0.078)	(0.192)	(0.132)	(0.304)
ϕ_C	0.305	0.473	0.278	0.048	0.416	–0.053
	(0.079)	(0.113)	(0.167)	(0.020)	(0.121)	(0.090)
ϕ_w	0.037	0.218	0.087	0.040	0.038	0.030
	(0.161)	(0.098)	(0.111)	(0.034)	(0.059)	(0.448)
ϕ_m	0.440	0.300	0.481	0.246	0.283	0.424
	(0.161)	(0.132)	(0.112)	(0.102)	(0.082)	(0.255)
d	0.236	–0.050	0.155	0.361	0.159	0.413
	(0.114)	(0.092)	(0.137)	(0.062)	(0.090)	(0.215)
$ln(L)$	–2897.95	102.06	109.16	–724.77	–2039.08	–6142.23
BIC	5861.47	–138.55	–152.75	1515.12	4143.73	12351.14

Key: Similar to Table 7 with approximate *MLE*s of the *ARFIMA-EHAR* model with positive and negative signed variations and *BV* reported.

Table 9. Estimation of the *TVP-HAR* Model

	AUD	CAD	EUR	GBP	JPY	S&P500
$\phi_{d,t}$	0.287	0.166	0.250	0.189	0.246	0.293
	(0.196)	(0.179)	(0.186)	(0.138)	(0.165)	(0.189)
$\phi_{w,t}$	0.260	0.316	0.180	0.281	0.208	0.308
	(0.206)	(0.155)	(0.142)	(0.182)	(0.191)	(0.186)
$\phi_{m,t}$	0.178	0.256	0.272	0.226	0.166	0.120
	(0.127)	(0.125)	(0.168)	(0.161)	(0.149)	(0.133)
BIC	7791.801	1679.234	1498.431	3241.878	5876.518	14621.189

Key: The mean values of the coefficients of the *TVP-HAR* are reported with standard deviation in parentheses. The Gaussian kernel with a bandwidth of $T^{0.5}$ is used for estimation.

Table 10. Estimation of the *TVP-EHAR* Model

	AUD	CAD	EUR	GBP	JPY	S&P500
	TVP-EHAR model with positive and negative semivariances					
$\phi^{+}_{d,t}$	0.555	0.307	0.290	0.226	0.128	0.024
	(0.507)	(0.328)	(0.309)	(0.222)	(0.375)	(0.238)
$\phi^{-}_{d,t}$	–0.002	0.033	0.221	0.168	0.358	0.553
	(0.273)	(0.185)	(0.238)	(0.232)	(0.377)	(0.261)
$\phi_{w,t}$	0.269	0.311	0.178	0.278	0.223	0.322
	(0.190)	(0.141)	(0.136)	(0.182)	(0.190)	(0.188)
$\phi_{m,t}$	0.187	0.257	0.270	0.225	0.163	0.129
	(0.128)	(0.123)	(0.165)	(0.157)	(0.145)	(0.133)
BIC	8140.775	2078.796	1936.916	3693.370	6276.771	15044.607
	TVP-EHAR model with positive and negative signed variations and *BV*					
$\phi^{+}_{J,t}$	0.281	0.046	–0.004	0.048	–0.201	–0.062
	(0.741)	(0.339)	(0.355)	(0.260)	(0.463)	(0.250)
$\phi^{-}_{J,t}$	0.244	0.200	0.020	–0.035	0.020	–0.445
	(0.310)	(0.329)	(0.225)	(0.424)	(0.617)	(0.412)
$\phi_{C,t}$	0.397	0.333	0.398	0.311	0.445	0.331
	(0.149)	(0.189)	(0.185)	(0.141)	(0.324)	(0.227)
$\phi_{w,t}$	0.213	0.231	0.117	0.230	0.158	0.288
	(0.160)	(0.138)	(0.147)	(0.163)	(0.178)	(0.195)
$\phi_{m,t}$	0.176	0.247	0.257	0.212	0.152	0.134
	(0.131)	(0.129)	(0.155)	(0.147)	(0.154)	(0.127)
BIC	8496.352	2409.951	2345.089	4046.409	6635.650	15317.870

Key: Similar to Table 9 with the mean values of the coefficients of the *TVP-EHAR* model reported with standard deviations in parentheses.

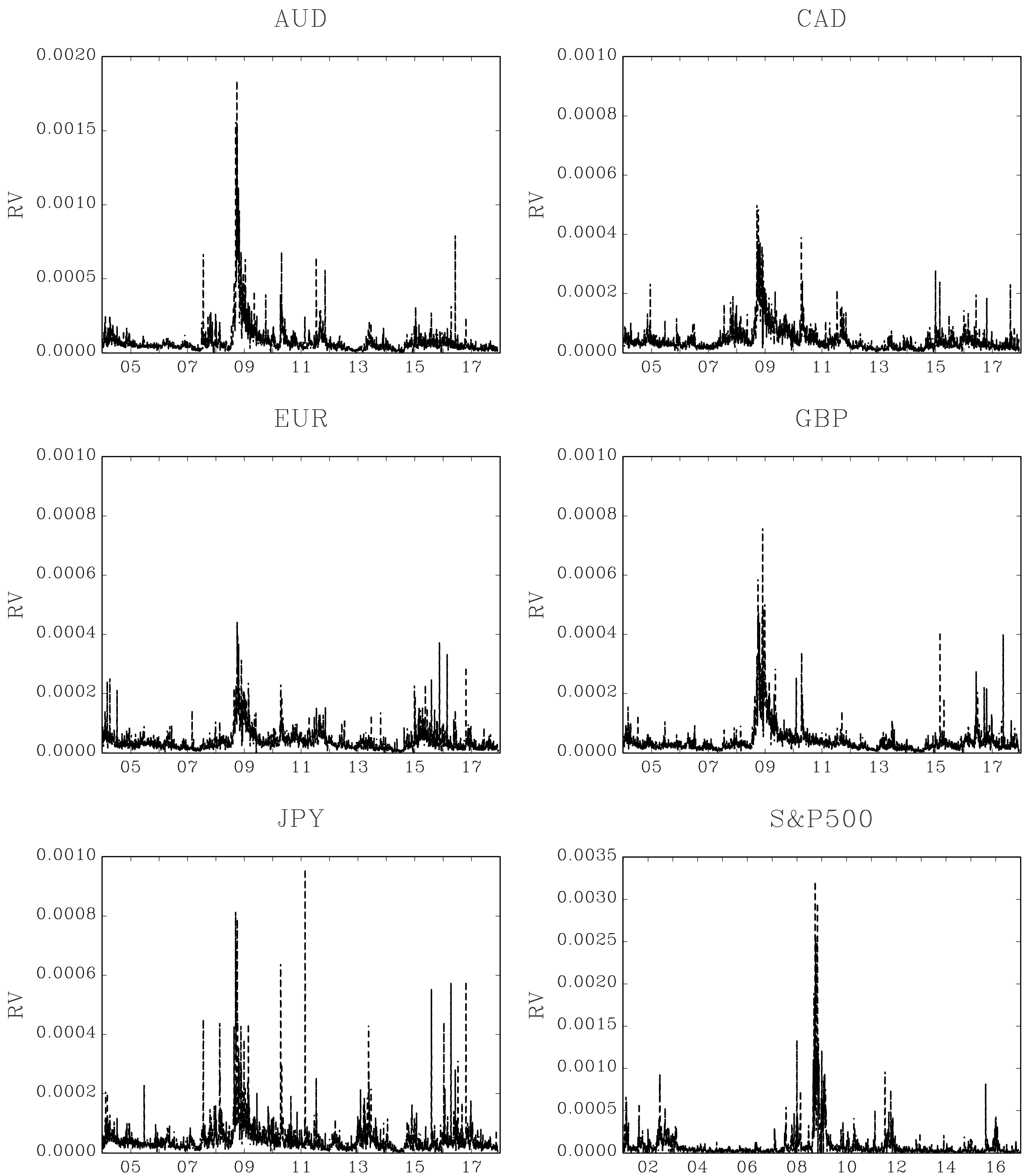


Figure 1. Realized volatility for each financial series

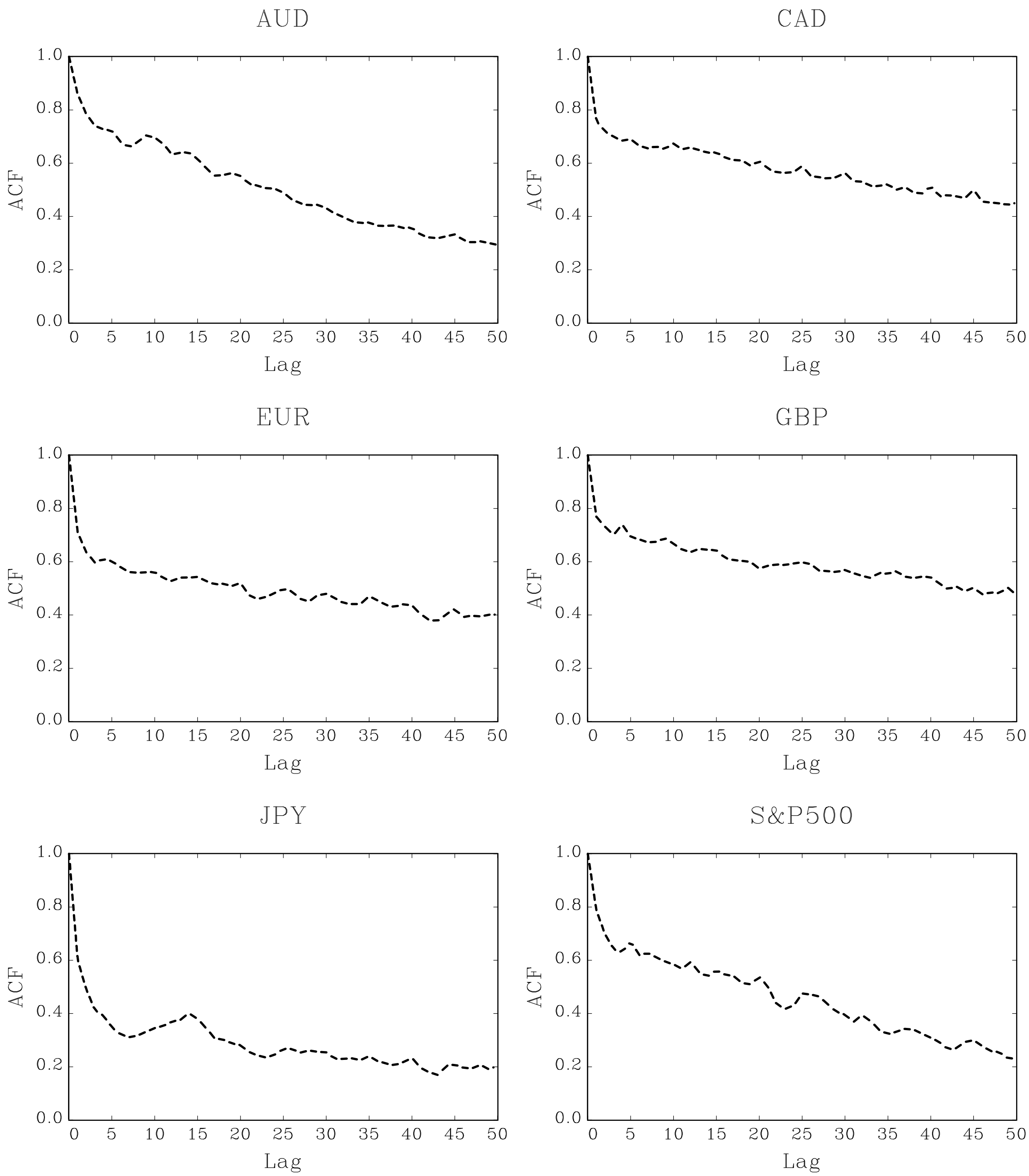
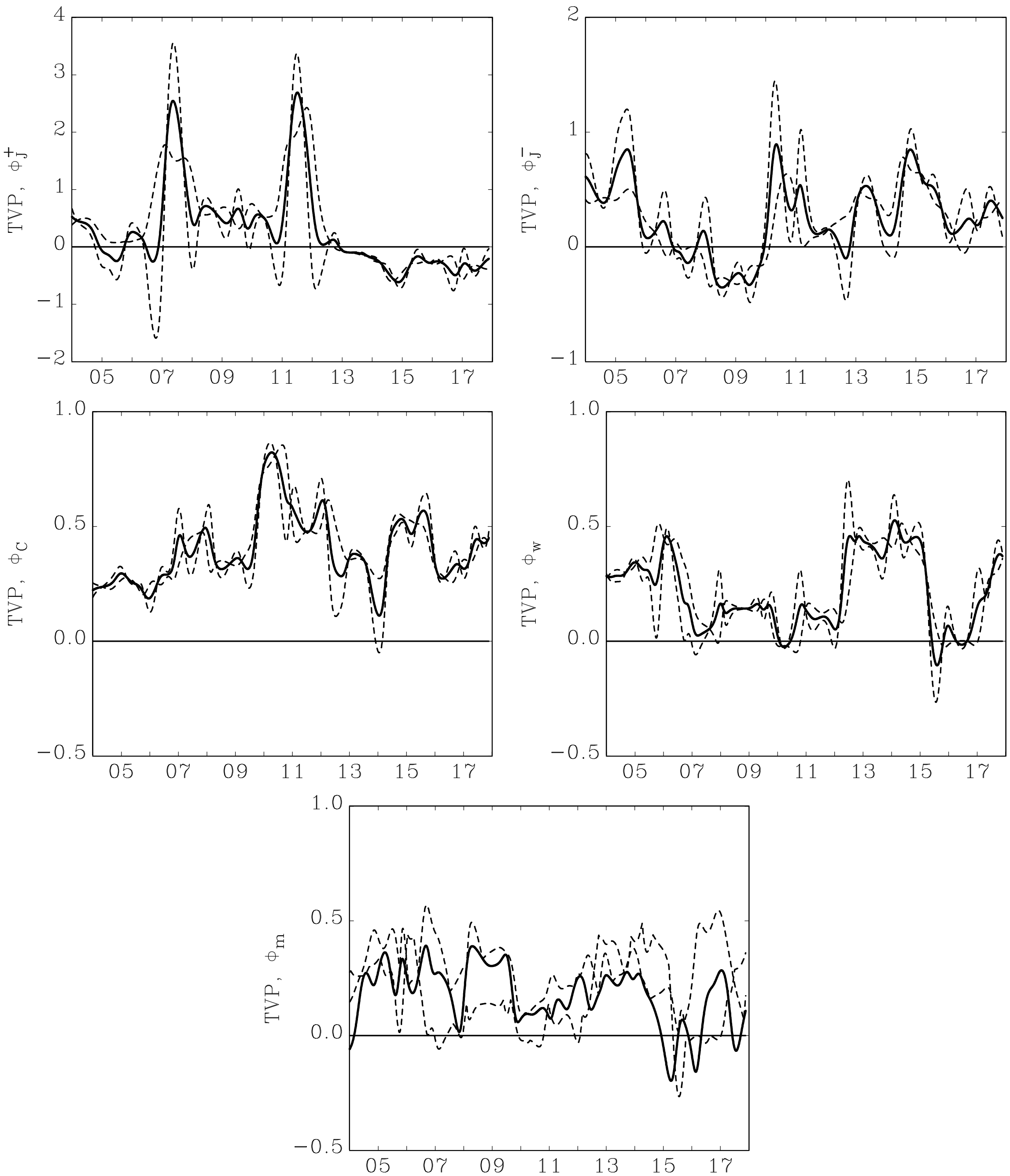
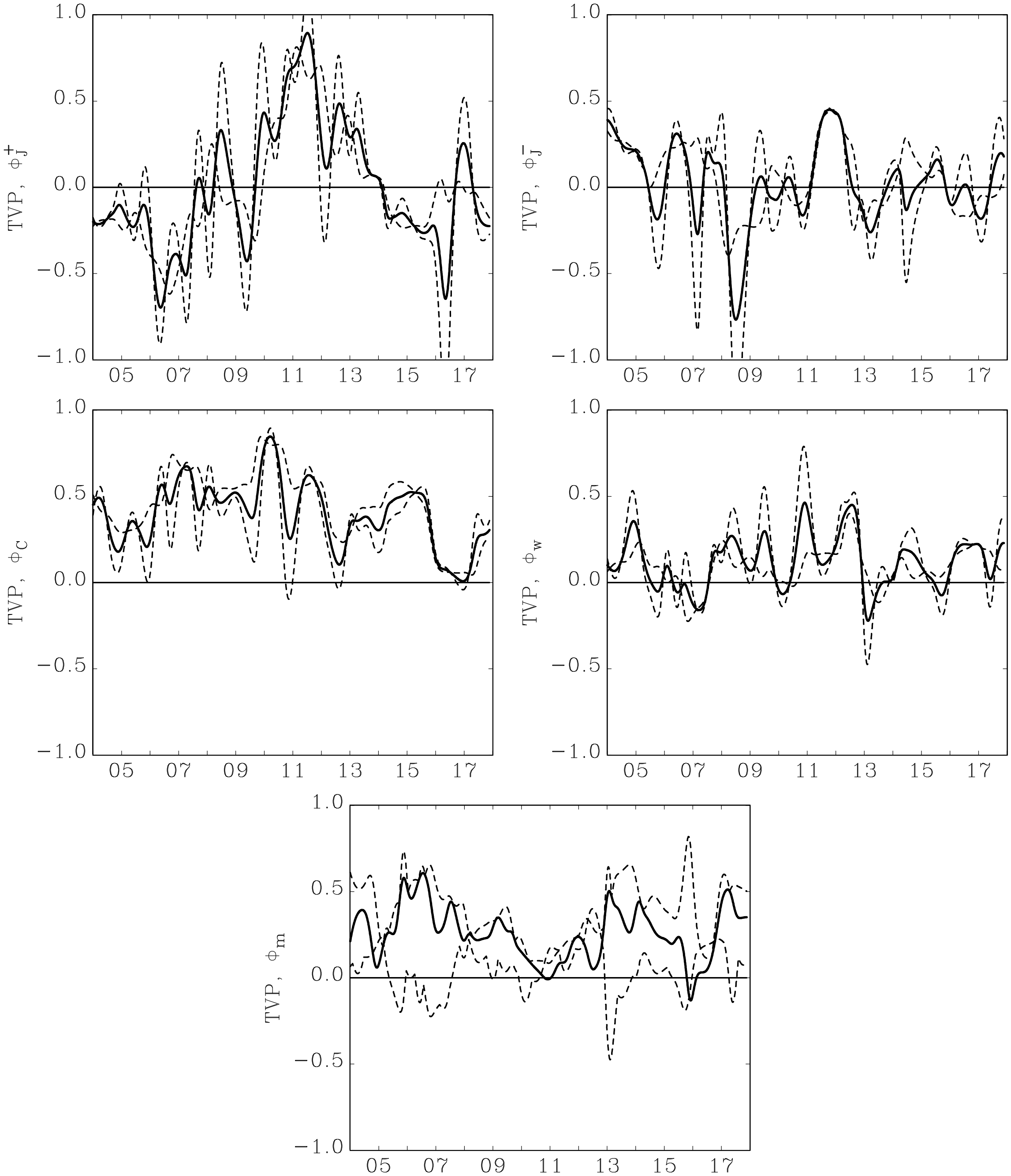


Figure 2. Autocorrelation function for each financial series



(a) AUD

Figure 3. Estimation results from the *TVP-EHAR* model



(b) EUR

Figure 3. Estimation results from the *TVP-EHAR* model (cont'd)

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