A Model of Tournament Incentives with Corruption*

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Abstract

We provide a theory of how growth, corruption, and a low-powered public-sector pay scale coexist in a stable equilibrium in the early stage of China’s development. The regionally decentralized authoritarian regime of China features lower-level government officials competing for promotion to a higher level in the government by generating local economic growth, and calls for high-powered incentives to elicit effort from the officials. However, this is at odds with the generally low-powered public-sector pay scale in China. We propose a principal-agent model, where the principal represents the Chinese people’s desire to pursue economic growth and the agents are the government officials delegated with production tasks and organized in a tournament, to address how a low-powered pay scale can effectively elicit effort in a tournament infested with widespread corruption.

Keywords: Institution; Tournament; Corruption; China.
JEL codes: D73; J45; O43; P26.

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1 Introduction

The political institution that accompanied three decades of rapid economic growth in China is termed in the literature the *regionally decentralized authoritarian* (RDA) regime (Xu, 2011). In its essence, the RDA regime combines the political centralization of cadre management with the regional decentralization of economic management. The local government officials who have a high level of autonomy in terms of economic decision making then engage in a competition that rewards better local economic performance with promotion within the bureaucratic nomenclature. The effectiveness of the RDA regime in incentivizing officials to promote local economic growth has been studied both theoretically (Maskin et al., 2000) and empirically (Li and Zhou, 2005; Jia et al., 2015).

Conceptually, the RDA regime is often referred to as resembling a rank-order tournament labor contract,\(^1\) which can induce an efficient level of effort from workers under a properly designed pay structure (Lazear and Rosen, 1981). The pay structure in such a tournament is shown to be convex in the sense that the pay increase between rungs of career ladders becomes larger as one moves up the rank (Rosen, 1986). While such compensation schemes are found in sports and corporations (Ehrenberg and Bognanno, 1990; Eriksson, 1999), the public-sector pay scale of the Chinese officials almost certainly falls short of it. The “3581” Project implemented in Beijing in 2004 portrays a pay scale that starts from an annual salary of 30,000 *yuan* for township/sub-division level officials, 50,000 *yuan* for county/section level officials, 80,000 *yuan* for prefecture/bureau level officials and 100,000 *yuan* for province/ministry level officials. Leaving the absolute level of the salary aside, the increase in the salary across the bureaucratic spectrum appears hardly high-powered enough to be compatible with a rank-order tournament.

One way out is to recognize that the observed salary is only a “capitulation wage” to be supplemented by, for instance, corruption income (Becker and Stigler, 1974; Besley and McLaren, 1993). Several international rankings of corruption indices confirm the impression that corruption is wide-spread in China (see a review in Svensson (2005)). However, letting the corruption income accrue disproportionately to the top officials is not compatible with the spirit of the RDA design. Under the RDA regime, the personnel management is top-down, which also includes the way the corruption is monitored and penalized.\(^2\) This implies

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\(^1\) Xu (2011) gives the tournament interpretation to a seminal model of the organizational form of the Chinese government (Maskin et al., 2000). More recently, Li et al. (2018) adopt a Tullock contest model embedded in a multi-layer tournament to study target-setting in the Chinese government.

\(^2\) China does not lack anti-corruption laws. The Anti-Unfair Competition Law of the PRC deals mainly with commercial bribery, while the Criminal Law of the PRC prohibits giving and receiving money or property to obtain any undue benefit. Penalties for the offense ranges from fines and confiscation to imprisonment.
effectively zero monitoring at the top. Then to minimize the potential massive abuse of power at the top, the RDA cleverly requires the lower-level governments to control the bulk of the economy, which distances the top from a direct control over economic resources and which limits the scope for corruption at the top.

So what is the grand prix to the winner of the tournament? We provide an explanation of how the rank-order tournament incentive can work in an RDA regime. In its most parsimonious form, there are three logical steps in the argument. First, corruption, interpreted as misuse of power in the allocation of economic resources for personal gains, occurs where economic decisions are made. Under the RDA regime, it can occur anywhere except the top. Second, corruption income cannot be fully consumed contemporaneously, and hence contributes to wealth accumulation that forms the basis of an official’s vested interest in winning the tournament. Third, the fact that the RDA regime can enforce control top-down means that the accumulated wealth is not safe until the official rises to the top. Then, the grand prize at the top is not an opportunity to extract more rents, but the security with which to keep the wealth accumulated over one’s career. This view logically weaves growth, corruption, and all elements of the RDA regime into a coherent story.

We formalize the idea by embedding a rank-order tournament in a principal-agent framework. The principal represents the will of the Chinese people of pursuing economic growth. The agents are the government officials to whom production is delegated. The agents are organized in a bureaucracy and incentivized by a rank-order tournament labor contract, which contains the main features of the RDA regime. The principal faces three hidden action problems in his contracting relationship with the agents. First, the lower-level government officials who undertake production needs to exert costly effort, which is unobservable to the principal. Therefore, the principal cannot contract on the efforts from these officials, but only on the outcomes, i.e. local economic performance. Second, these officials who undertake production faces an opportunity to grow private wealth at an above market rate, which captures the potential for rent-seeking or corruption. The decision to exploit such an opportunity is unobservable to the principal, and hence the principal delegates a monitoring task and the death penalty. The law enforcement is however more subtle. Due to the inseparability of the judicial system and the Party, in practice, it is the Commission for Discipline Inspection of the Communist Party that monitors corruption related to government officials. The top leaders of the country are the members of the Standing Committee of the Politburo. Meanwhile, the highest organ in charge of the anti-corruption regulation is the Central Commission for Discipline Inspection, whose head has an administrative rank no higher than any member from the Standing Committee of Politburo. Disciplinary actions against members of the Standing Committee of the Politburo were unheard of until the recent case of Zhou Yongkang. In general, corruption charges against such high-profile officials are very rare and moreover not institutionalized. A recent paper by Che et al. (2017) studies the implications from granting criminal immunity to top leaders in China.
to the top government official, to be consistent with the centralized personnel management of the RDA regime. Last but not the least, upon disciplinary inspection, the outcome of the investigation is observed only to the inspector or the top official, but not to the principal. This creates potential for collusion between the inspector and the official under investigation, which presents another contractual challenge to the principal (Tirole, 1992). We believe these are very realistic problems the Chinese society at large faces in dealing with politicians in an institution that fundamentally lacks checks and balances.

Within this framework, we solve for the principal’s optimal choice for the level of disciplinary inspection as well as the incentive structure or the wage prizes offered at each stage of the tournament. We show that the only level of disciplinary inspection that can be realistically implemented is a lax one, under which the officials are indifferent between being corrupt or not. This is a consequence of the deep-rooted informational frictions between the principal and the agents. With the epidemic of corruption inherent in the system, the principal then leverages on it by including it as an implicit form of reward such that the grand prize at the end of the tournament is securing the private wealth that the winner has amassed over his career. At an early stage of development, incorporating politician’s personal wealth building as an implicit reward may not only be cost-saving to the principal, but also necessary for a poor principal to elicit an efficient level of effort from the agents.

This paper makes two contributions. It is most closely related to the literature on the Chinese political institution, and in particular that on the RDA regime (Du and Xu, 2007; Landry, 2008; Xu, 2011). We contribute to that literature by clarifying how a seemingly lower-powered public-sector pay scale is able to incentivize the government officials to work hard at promoting economic growth under the RDA regime. More generally, our results suggest that, during the course of development in China, good governance that produced tremendous economic growth may paradoxically require some level of corruption. This is especially true when the principal is budget constrained, so there is a limit to which he can use explicit wages to incentivize agents. Formalizing this particular role of corruption in China’s development is our second contribution, which complements the literature on the role of corruption during economic development (Tirole, 1994; Banerjee, 1997; Laffont and N’Guessan, 1999). As the country grows richer and the government function evolves, the public-sector pay needs to adjust accordingly to elicit the efficient level of effort at all levels of the government.
2 The Model

We examine the RDA regime from the perspective of a principal-agent problem. The principal represents the Chinese people, who have relatively homogenous and single-dimensional preference for economic growth, at least during the first three decades following Deng’s reforms and opening-up policy. The principal delegates two types of production tasks to the agents, interpreted as government officials, who are organized in a hierarchical structure. In addition, since the production entails an opportunity for corruption, the principal also delegates a monitoring task to an agent, or an inspector. The principal and the agents are all risk neutral.

Technology. The production is organized in geographical terms. For simplicity, we consider two levels of geographical division: provinces and prefectures. There are two types of production tasks. The province-level task is to draw provincial development plans and invest in public infrastructure within the province. The prefecture-level task is to draw prefectural development plans and improve local public infrastructure. The province-level task takes into account externalities across prefectures and economies of scale at the level of the province, and therefore has values independent of the tasks in prefectures under the province’s jurisdiction. More formally, let the subscript \( j \) indicate a province. Let the subscript \( ij \) indicate a prefecture \( i \) in the province \( j \). The final output, interpreted as local output growth rate, in each prefecture is denoted \( y_{ij} \), which is jointly determined by the output of the provincial-level task, \( q_j \), and the output of the prefectural-level task, \( q_{ij} \). The parameter \( A \) denotes total factor productivity (TFP):

\[
y_{ij} = A_d^\alpha q_j^\alpha q_{ij}^\beta, \text{ where } \alpha + \beta < 1. \tag{1}
\]

Some justifications are warranted here. The idea that both province-level and prefecture-level tasks contribute to the production of the final output is not new. Li et al. (2018) is a recent example that emphasizes this interdependence. However, our set-up differs from theirs along two dimensions. First, our production function of the final output features decreasing return to scale. This is to ensure, later on, that when we solve for principal’s problem, we have a well-behaved concave objective function optimized over a convex set. Note that we have abstracted away from having an input from the center-level, to be consistent with the decentralized economic decision making under the RDA regime. Second, while in Li et al. (2018), the output at a given task is a one-to-one function of the effort of the agent undertaking that task, we model the moral hazard problem between the principal and the
agent explicitly. To do that, we assume that the output of the province-level (prefecture-level) task is an outcome of both an effort input from the agent who is in charge of that task and a luck component:

\[ \ln q_j = \ln x_j + \ln \eta_j, \]

where \( x_j \) is the effort input and \( \eta_j \) is the luck component that is lognormally distributed: \( \ln \eta_j \sim N(-\frac{1}{2} \alpha \sigma_1^2, \sigma_1^2) \). Denote the distribution of the shock by \( G(\eta_j) \). Likewise, the output of the prefectural-level task, \( q_{ij} \) is:

\[ \ln q_{ij} = \ln x_{ij} + \ln \varepsilon_{ij}, \]

where \( x_{ij} \) is the effort input and \( \varepsilon_{ij} \) is the luck component: \( \ln \varepsilon_{ij} \sim N(-\frac{1}{2} \beta \sigma_2^2, \sigma_2^2) \). Denote the distribution of the shock by \( F(\varepsilon_{ij}) \). The variances of the shocks are set up so that the expected final output in prefecture \( i \) of province \( j \) is

\[ E(y_{ij}) = A E(q_j^\alpha) E(q_{ij}^\beta) = A x_j^\alpha x_{ij}^\beta. \]

Suppose that there are \( n_1 \) province-level tasks and, in each province, there are \( n_2 \) prefecture-level tasks. The expected aggregate final output in this economy is then

\[ E(Y) = \sum_{j=1}^{n_1} \sum_{i=1}^{n_2} E(y_{ij}) = \sum_{j=1}^{n_1} \sum_{i=1}^{n_2} A x_j^\alpha x_{ij}^\beta. \]

**The Informational Structure.** During production, the effort input is private information to whoever undertakes the production. Therefore, for the principal, it is impossible to directly contract with the agents on the effort inputs, instead the rewards are based on realized outputs. As in a typical moral hazard problem, the principal contracts with the agents a compensation scheme where rewards depend on realized output.

In addition, an agent who is in charge of the province-level (prefecture-level) task has an opportunity to grow his personal wealth at some above market rate of return, \( r_1 \) (\( r_2 \)). We interpret the act of growing private wealth at an above market return as a form of corruption. It captures cases such as taking advantage of one’s office to seek benefits for his relatives’ business.\(^3\)

\(^3\)Suppose \( r_1 \geq r_2 \) so that province-level tasks entail more room for corruption than

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\(^3\)In Appendix A, we analyze 119 announcements of corruption charges published on the website of the Central Committee for Discipline Inspection of the CCP. In more than 95% of these announcements, the sacked official is described to “seek personal interests,” “obtain colossal interests,” “help relatives’ business,”
prefecture-level tasks, perhaps due to the fact that a larger part of the economy is affected by province-level decisions.

Since the act of corruption is also private information, the principal delegates a corruption monitoring task to a disciplinary inspector. The inspector investigates the agents in charge of the production probabilistically. Once inspected, whether the agent is corrupt or not is known to the inspector. However, the inspector can collude with the agent and conceal the corruption from the principal in exchange for side payments (similar to Tirole (1992)). We assume that the inspector can conceal a corrupt official’s wrongdoing from the principal, but cannot make corruption charges on a clean official. Therefore, the principal also faces a moral hazard problem in his contracting relationship with the inspector. Since the probability of inspection is not contractable, the principal contracts with the inspector on the outcome of the inspection, namely the percentage of officials caught corrupt.

Throughout the paper, we assume that disciplinary inspection is effortless to the inspector. The argument that a lack of discipline is caused by a high cost of effort in inspection is straightforward but not particularly insightful, hence we shut the channel down. As a consequence, there is no wage to the inspector for the inspection job per se. However, when the principal contracts a certain percentage of convicted officials, he understands the structure of the game, perfectly anticipates the probability of inspection a rational inspector will choose under the contract and expects to see the corresponding percentage of officials being convicted as a result. If the inspector fails to deliver at least that percentage of corrupt officials, he faces a punishment so high that he never finds it optimal to do that. To put it simply, the contract between the principal and the inspector specifies a probability of conviction and a (large) punishment to the inspector if he does not deliver at least the specified number of corrupt officials according to the contract.

The Organizational Structure. The organizational structure describes how the agents are organized and what type of compensation scheme is adopted in the organization. Motivated by the RDA design, we model the organization of officials as a multi-stage tournament. More specifically, imagine a three-layer bureaucracy. There is one official at the top who is the disciplinary inspector. There are $n_1$ officials at the middle level who are in charge of the province-level production tasks and there are $n_1n_2$ officials at the bottom level who are in charge of the prefecture-level production tasks. The middle level officials can be thought of as the provincial governors and the bottom-level officials the prefectural mayors.

"help others to get promoted," or "utilize power to seek interests." As Guo (2013) shows, this conflict-of-interest type of corruption has become the mainstream, while cases of outright embezzlement or bribery are declining, possibly because of better auditing measures in the government.
The promotion and incentive scheme is carried out as follows. At the bottom level, all \( n_2 \) prefectural mayors in the same province compete for the promotion to a provincial governor by exerting effort in the prefecture-level production. At the middle level, all \( n_1 \) provincial governors compete for the promotion to the top by exerting effort in the province-level production. The prizes of the tournament, \( W_0, W_1, \) and \( W_2, \) denote the (life-time) wages offered to the officials whose career ends at the top, the provincial, and the prefectural level respectively. This setup is consistent with the RDA regime, in that economic or production decisions are decentralized to the officials at the two lower levels, but the personnel management is embodied a centralized promotion exercise based on the relative performance of officials at the same rank. In addition, the disciplinary inspection in this setting is done in a top-down fashion, which highlights the fundamental challenge that an authoritarian regime faces, namely there are no effective checks and balances at the top. Figure 1 illustrates the organizational structure with a simplified example of 3 provinces and 2 prefectures per province.

The time line of the tournament is as follows. There are \( n_1n_2 \) ex ante identical prefectural mayors, who enter the tournament, each with private wealth \( K. \) Each chooses the effort, \( x_{ij}, \) and waits for the output, \( q_{ij}, \) to realize. After the realization, the mayor with the highest output is the winner, who will move to the next stage to be a provincial governor, while the losing mayors receive a consolation prize \( W_2. \) The officials then decide whether they will grow their private wealth by \( r_2. \) The losing mayors face a probability of inspection of \( \pi_2, \) when if caught corrupt, will lose all private wealth, either to the inspector during a collusion or to the principal due to a corruption charge. This completes the prefecture-level stage. At the province level, the provincial governors go through a similar process. They exert effort \( x_j, \) after which output, \( q_j, \) realizes. The winner will get to the top in the next stage and the losers get a prize of \( W_1. \) They decide whether to corrupt, understanding that all losing governors are inspected with probability \( \pi_1. \) If caught corrupt, the official will lose all his private wealth. At the last stage, the top leader gets \( W_0 \) and conducts the disciplinary inspection. As an inspector, he may collude with a corrupt lower-level official, in which case he receives transfers from the corrupt official.

We first tackle the contracting problem between the principal and the top official about disciplinary inspection.
2.1 The Contracting Problem between the Principal and the Top Official

Since only the losing officials at each stage are subject to disciplinary inspection, the winner at each stage strictly prefers to be corrupt.\footnote{This assumption is not essential but simplifies the model, otherwise one would need to worry about the rule of replacing a dishonored winner at each stage of the tournament. Another way to think about this assumption is to view the temporary immunity to inspection as part of the reward from winning.} Let’s first consider a losing prefectural mayor’s decision about corruption. He compares the payoff from losing as a corrupt official (i.e. the left hand side of (2)) with the payoff from losing as a clean official (i.e. the right hand side of (3)). He is indifferent between being corrupt and not corrupt if and only if the probability of inspection is $\pi_2^*$ given by:

$$W_2 + (1 - \pi_2^*)r_2K = W_2 + K \tag{2}$$

$$\Rightarrow \pi_2^* = 1 - \frac{1}{r_2}.$$  

Next, a losing provincial governor who was corrupt as a prefectural mayor earlier in his career is indifferent between being corrupt and not corrupt if and only if the probability of inspection is $\pi_1^*$ given by:

$$W_1 + (1 - \pi_1^*)r_1r_2K = W_1 + r_2K \tag{3}$$

$$\Rightarrow \pi_1^* = 1 - \frac{1}{r_1}.$$  

Notice that all losing provincial governors face the same trade-off regarding corruption and so are all losing prefectural mayors. Therefore, for losing officials of the same rank, either all are corrupt or none is corrupt. For simplicity, whenever an official is indifferent between being corrupt and not corrupt, we assume that he is corrupt.

Since the probability of inspection is not observed by the principal, the principal cannot contract directly on it. He can only contract on the outcome of the investigation or the percentage of losing officials convicted. Imagine the principal would like the inspector to inspect the losing officials at probabilities strictly higher than $\pi_1^*$ and $\pi_2^*$ to eliminate corruption. He would realize, however, that there is no contract that can enforce it. The outcome of the investigation that would be consistent with these probabilities is zero conviction. But if the principal contracts with the inspector on an outcome of zero conviction, then the inspector will privately lower the actual probabilities to $\pi_1^*$ and $\pi_2^*$. Since the losing officials react to the probabilities of inspection that are actually implemented, they will all be corrupt and when inspected, they will simply collude with the inspector by transferring wealth to him in
exchange for covering up.

The principal understands that there will be at most a proportion of $\pi_1^*$ among the losing provincial governors and $\pi_2^*$ among the losing prefectural mayors being convicted. Suppose the principal contracts with the inspector a pair of percentages of convicted losing officials, $(\pi_1, \pi_2)$. Then the principal knows that any feasible contract must satisfy $\pi_1 \leq \pi_1^*$ and $\pi_2 \leq \pi_2^*$. Otherwise, the contracted percentages of conviction can never be delivered.

Moreover, he also understands that if the contract specifies $(\pi_1, \pi_2) < (\pi_1^*, \pi_2^*)$, a rational inspector will not inspect at the probabilities $\pi_1$ and $\pi_2$. He will instead inspect the losing officials with probabilities $\pi_1^*$ and $\pi_2^*$, under which all are corrupt, and randomly choose a subset of them, $\pi_1$ and $\pi_2$ of the losing officials, to report to the principal. The principal then confiscates $\pi_1$ of losing provincial governors’ and $\pi_2$ of losing prefectural mayors’ private wealth. In the case of a collusion, the inspector covers up corruption in exchange for side payments from the official. The inspector can ask up to the official’s entire private wealth for the cover-up and we assume this is the case. This way, he can keep to himself a total of $\pi_1^* - \pi_1$ of losing provincial governors’ private wealth and $\pi_2^* - \pi_2$ of losing prefectural mayors’ wealth.

The only contract that does not lead to collusion must have $\pi_1 = \pi_1^*$ and $\pi_2 = \pi_2^*$. Under this contract, the inspector has no choice but to inspect at exactly the aforementioned intensity. If he increased the probabilities marginally, then officials would stop corrupting and he would not be able to deliver the specified number of convictions. If he decreased the probabilities marginally, then he would not have enough convictions to report to the principal.

We summarize these arguments in the following proposition about the intensity of disciplinary inspection.

**Proposition 1.** Let the contract between the principal and the disciplinary inspector which specifies the percentages of convicted losing provincial governors and prefectural mayors be denoted by $(\pi_1, \pi_2)$.

1. A feasible contract must have $(\pi_1, \pi_2) \leq (\pi_1^*, \pi_2^*)$.
2. If $(\pi_1, \pi_2) = (\pi_1^*, \pi_2^*)$, then there is no collusion between the inspector and the officials. The principal confiscates $\pi_1^*$ of losing provincial governors’ and $\pi_2^*$ of losing prefectural mayors’ private wealth.
3. If $(\pi_1, \pi_2) < (\pi_1^*, \pi_2^*)$, then there will be collusion between the inspector and some officials. More specifically, the actual probabilities of inspection implemented by the inspector are $\pi_1^*$ and $\pi_2^*$ and the inspector colludes with $\pi_1^* - \pi_1$ of leaving provincial governors and $\pi_2^* - \pi_2$
of leaving prefectural mayors. The principal (or inspector) receives $\pi_1$ (or $\pi_1^*-\pi_1$) of losing provincial governors’ private wealth and $\pi_2$ (or $\pi_2^*-\pi_2$) of losing prefectural mayors’ private wealth.

From Proposition 1, it is clear that the actual probabilities of inspection that are implemented are always $\pi_1^*$ and $\pi_2^*$, regardless of the contractual arrangement the inspector has with the principal. Under these probabilities, all officials are corrupt. In the case of an inspection, the official always loses all his wealth, either to the principal if his case is reported or to the inspector during a collusion. The result that the principal can never eliminate corruption in this setting is due to the deep-rooted informational friction between the principal and the inspector and the assumption that all agents are selfish (Corollary 1).

By assuming all agents are selfish, we subject the conflict of interest between the Chinese people, where the legitimacy of the government lies, and the government officials, to whom the executive powers are delegated, to the harshest test.

**Corollary 1.** The principal cannot contract with the disciplinary inspector so as to eliminate corruption.

In what follows, we set the probabilities of inspection that the officials face to the ones that the inspector actually implements: $\pi_1 = \pi_1^* = 1 - \frac{1}{r_1}$ and $\pi_2 = \pi_2^* = 1 - \frac{1}{r_2}$. These probabilities need not coincide with the percentage of conviction contracted between the principal and the inspector, for which we maintain the notation of $\overline{\pi}_1$ and $\overline{\pi}_2$. By Proposition 1, $(\overline{\pi}_1, \overline{\pi}_2) \leq (\pi_1^*, \pi_2^*)$.

### 2.2 The Contracting Problem between the Principal and the Lower-Level Officials

We consider a symmetric equilibrium of the multi-stage tournament, where all provincial governors exert effort $x_1$ and all prefectural mayors exert effort $x_2$. Suppose the cost of effort is quadratic, $c(x) = \frac{1}{2}kx^2$.

**The Provincial Governor’s Problem.** Given the contract $(\overline{\pi}_1, \overline{\pi}_2)$, the top leader who is also the inspector expects transfers from inspected officials during a collusion. Denote these transfers as $I(\overline{\pi}_1, \overline{\pi}_2)$. Proposition 1 implies that the transfers are of the magnitude:

$$I(\overline{\pi}_1, \overline{\pi}_2) \equiv (\pi_1^* - \pi_1)(n_1 - 1)r_1r_2K + (\pi_2^* - \pi_2)n_1(n_2 - 1)r_2K.$$

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5In settings where some politicians are of a benevolent type and others of a selfish type, it is possible to at least temporarily eliminate corruption (Besley and Kudamatsu, 2008; Che et al., 2013).
Correspondingly, the wealth confiscated by the principal is \( T(\bar{\pi}_1, \bar{\pi}_2) \equiv \bar{\pi}_1(n_1 - 1)r_1r_2K + \bar{\pi}_2n_1(n_2 - 1)r_2K \).

For the provincial governor of province \( j \), if he wins the promotion, then he becomes the top official, keeps his accumulated wealth \( r_1r_2K \), and receives transfers as an inspector, \( I(\pi_1, \pi_2) \). Otherwise, he gets a prize \( W_1 \), is inspected with probability \( \pi_1^* \) in which case he loses his wealth, and with the complementary probability keeps his private wealth \( r_1r_2K \). The problem he faces can be formulated as:

\[
V_1 = \max_{x_j} P(x_j; x_1) (W_0 + r_1r_2K + I(\pi_1, \pi_2)) + (1 - P(x_j; x_1)) (W_1 + (1 - \pi_1^*)r_1r_2K) - c(x_j)
\]

\[
= \max_{x_j} P(x_j; x_1) (W_0 + r_1r_2K + I(\pi_1, \pi_2)) + (1 - P(x_j; x_1)) (W_1 + r_2K) - c(x_j),
\]

where the probability of winning is:

\[
P(x_j; x_1) = \int_\mathcal{R} G(x_j - x_1 + \eta_j)_{n_1-1}g(\eta_j)d\eta_j,
\]

and \( G(\cdot) (g(\cdot)) \) is the cdf (pdf) of \( \eta_j \). Then,

\[
\frac{\partial P(x_j; x_1)}{\partial x_j} = \int_\mathcal{R} (n_1 - 1)G(x_j - x_1 + \eta_j)_{n_1-2}g(x_j - x_1 + \eta_j)g(\eta_j)d\eta_j.
\]

The first-order necessary condition (FONC) is

\[
\frac{\partial P(x_j; x_1)}{\partial x_j} (W_0 + r_1r_2K + I(\pi_1, \pi_2) - W_1 - r_2K) - c'(x_j) = 0,
\]

which equates the marginal benefit from exerting effort, or the higher expected payoff from increasing the probability of success, with the marginal cost of exerting effort. Evaluated at \( x_1 \) in the equilibrium:

\[
\int_\mathcal{R} (n_1 - 1)G(\eta_j)_{n_1-2}g(\eta_j)^2d\eta_j (W_0 - W_1 + r_1r_2K - r_2K + I(\pi_1, \pi_2)) = \kappa x_1.
\]

Note that the integral, which is the marginal increase in the probability of winning from more effort, is pinned down by the distribution of the shock, \( \eta_j \), and the number of provincial governors, \( n_1 \). To simplify notation, let \( t_1 \equiv \int_\mathcal{R} (n_1 - 1)G(\eta_j)_{n_1-2}g(\eta_j)^2 \) and let \( \Delta W_1 \equiv W_0 - W_1 + (r_1 - 1)r_2K + I(\pi_1, \pi_2) \). \( \Delta W_1 \) combines all the incentives a provincial governor
faces for promotion. The FONC can be written compactly as:

\[ x_1 = \frac{t_1 \Delta W_1}{\kappa}. \]  

(4)

The value function is then

\[ V_1 = \frac{1}{n_1} \Delta W_1 + (W_1 + r_2 K) - \frac{1}{2} \kappa x_1^2. \] 

(5)

**The Prefectural Mayor’s Problem.** For the prefectural mayor \( i \) in province \( j \), if he wins the promotion, he becomes a provincial governor and gets a provincial governor’s value \( V_1 \). Otherwise, he gets a prize \( W_2 \), is inspected with probability \( \pi^*_2 \) in which case he loses his wealth, and with the complementary probability keeps his private wealth \( r_2 K \). The problem he faces can be formulated as:

\[ V_2 = \max_{x_{ij}} P(x_{ij}; x_2) V_1 + (1 - P(x_{ij}; x_2)) (W_2 + (1 - \pi^*_2) r_2 K) - c(x_{ij}) \]

\[ = \max_{x_{ij}} P(x_{ij}; x_2) V_1 + (1 - P(x_{ij}; x_2)) (W_2 + K) - c(x_{ij}), \]

where \( P(x_{ij}; x_2) \) is the probability that he will beat all other mayors in province \( j \):

\[ P(x_{ij}; x_2) = \int_{\mathbb{R}} F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2-1} f(\varepsilon_{ij}) d\varepsilon_{ij}. \]

Here \( F(\cdot) \) \( (f(\cdot)) \) is the cdf (pdf) of \( \varepsilon_{ij} \). Then,

\[ \frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}} = \int_{\mathbb{R}} (n_2 - 1) F(x_{ij} - x_2 + \varepsilon_{ij})^{n_2-2} f(x_{ij} - x_2 + \varepsilon_{ij}) f(\varepsilon_{ij}) d\varepsilon_{ij}. \]

The FONC is

\[ \frac{\partial P(x_{ij}; x_2)}{\partial x_{ij}} [V_1 - W_2 - K] - c'(x_{ij}) = 0, \]

which again equates the marginal benefit and marginal cost of exerting effort. Evaluated at \( x_2 \), it becomes:

\[ \int_{\mathbb{R}} (n_2 - 1) F(\varepsilon_{ij})^{n_2-2} f(\varepsilon_{ij})^2 d\varepsilon_{ij} (V_1 - W_2 - K) = \kappa x_2, \]
where \( V_1 \) is given by (5). Let \( \Delta W_2 \equiv W_1 - W_2 + (r_2 - 1)K \) and define \( t_2 \) analogously as 
\[
 t_2 \equiv \int_{\mathbb{R}}(n_2 - 1)F(\varepsilon_{ij})^{n_2-2}f(\varepsilon_{ij})^2d\varepsilon_{ij},
\]
we have
\[
x_2 = \frac{t_2}{\kappa} \left( \Delta W_2 + \frac{1}{n_1} \Delta W_1 - \frac{1}{2\kappa} t_1^2 \Delta W_1^2 \right). \tag{6}
\]

The second-order sufficient conditions for the provincial governor’s problem and the prefectural mayor’s problem are detailed in Appendix B. Throughout the paper, we assume that the second-order sufficient conditions are satisfied and interior solutions of officials’ efforts are obtained.

**The Principal’s Problem.** The principal chooses the wage scale and the probabilities of conviction to maximize the expected aggregate final output net the cost of incentives:

\[
 \max_{W_0, W_1, W_2, \pi_1, \pi_2} E(n_1n_2Ax_1^{\alpha}x_2^{\beta}) - W_0 - (n_1 - 1)W_1 - (n_2 - 1)n_1W_2 + T(\pi_1, \pi_2),
\]

subject to the incentive compatibility constraints of the provincial governors and prefectural mayors, (4) and (6). We can write the problem equivalently as follows where the principal chooses \( \Delta W_1 \), \( \Delta W_2 \), and \( W_2 \). Note that \( T(\pi_1, \pi_2) \) is canceled out in (7), since one dollar extracted from the convicted officials is one dollar taken from the inspector, which needs to be compensated by increasing the inspector’s wage by one dollar.

\[
 \max_{\Delta W_1, \Delta W_2, W_2} n_1n_2Ax_1^{\alpha}x_2^{\beta} - \Delta W_1 - n_1\Delta W_2 - n_1n_2W_2 + n_1 (r_2(r_1 - 1) + n_2(r_2 - 1)) K. \tag{7}
\]

\[
 \Leftrightarrow \max_{x_1, x_2} n_1n_2Ax_1^{\alpha}x_2^{\beta} - \Delta W_1 - n_1 \left( \frac{\kappa}{t_2} x_2 - \frac{1}{n_1} \Delta W_1 + \frac{1}{2\kappa} x_1^2 \right) + n_1 (r_2(r_1 - 1) + n_2(r_2 - 1)) K \\
 \Leftrightarrow = \max_{x_1, x_2} n_1n_2Ax_1^{\alpha}x_2^{\beta} - n_1 \frac{1}{t_2} \kappa x_2 - n_1 \frac{1}{2\kappa} x_1^2 + n_1 (r_2(r_1 - 1) + n_2(r_2 - 1)) K \tag{8}
\]

Normalize the outside option of a losing prefectural mayor to 0, so \( W_2 \geq 0 \). Hence, the principal optimally sets \( W_2 \) to 0. Then we solve \( \Delta W_1 \) and \( \Delta W_2 \) from (4) and (6) as functions of \( x_1 \) and \( x_2 \) and plug them in principal’s objective, (7), which yields (8). This is a concave objective in two choice variables (Appendix B). The FONCs for \( x_1 \) and \( x_2 \) are both necessary and sufficient:

\[
 \begin{align*}
 n_1n_2A\alpha x_1^{\alpha-1}x_2^\beta &= n_1\kappa x_1 \\
 n_1n_2A\alpha x_1^{\alpha}x_2^{\beta-1} &= n_1\frac{\alpha}{t_2} 
\end{align*}
\]
We can easily solve out the efforts elicited under the optimal vertical tournament:

\[ x_1^* = \left( \alpha^{1-\beta} \beta^{\beta} n_2 A t_2^\beta \kappa^{-1} \right)^{\frac{1}{2-\alpha-2\beta}} \]  
\[ x_2^* = \left( \alpha^\alpha \beta^{2-\alpha} n_2 A^2 t_2^{2-\alpha} \kappa^{-2} \right)^{\frac{1}{2-\alpha-2\beta}}. \]  

(9)  
(10)

The optimal efforts are functions of parameters of the production function (\(\alpha, \beta, \text{ and } A\)), the tournament (\(n_2\) and \(t_2\)), and the cost function (\(\kappa\)). An examination of the transformation of the objective function from (7) to (8) reveals that the cost of incentive to the provincial governors, \(\Delta W_1\), is canceled out from the perspective of the principal. In other words, if the principal structures a bigger incentive to the provincial governors, then this incentive will show up in the continuation value of the prefectural mayors, and hence the principal will only need a smaller incentive to the prefectural mayors to elicit efforts from them. The effect of \(\Delta W_1\) at the provincial and the prefectural level completely cancels out and does not affect the total cost of incentives to the principal.

An examination of (8) shows that there is no distortion in the marginal cost of effort at the provincial level in the sense that the principal’s private cost of \(x_1\) is the same as the sum of the private cost of \(x_1\) to the relevant agents: \(n_1 \frac{1}{2} \kappa x_1^2\). However, the limited liability constraint \(W_2 \geq 0\) distorts the cost of effort at the prefectural level. The cost to the principal of eliciting \(x_2\) then depends not only on cost parameter \(\kappa\), but also how sensitive the probability of promotion is to effort, \(t_2\), which in turn depends on the number of contestants, \(n_2\), and the amount of luck, \(\sigma^2_2\). Another way of saying it is that the two-stage tournament is effectively reduced to a rank-order tournament at the prefectural level only, while at the provincial level the principal acts as if he can contract directly on the efforts.\(^6\) This is the reason why the optimally chosen \(x_1^*\) and \(x_2^*\) only depend on the parameters of the tournament at the prefectural level.

The total incentives given to a provincial governor and a prefectural mayor are then given

\(^6\)More generally, if there are \(m\) stages in the tournament, then the \(m\)-stage sequential-elimination rank-order tournament can be reduced to a single-stage rank-order tournament at the \(m\)th stage, whereas at all further stages the principal effectively contracts on effort. However, if we allow the wage to the lowest level losing official to be negative, which effectively serves as an entry ticket to the tournament, then the principal can elicit the first-best efforts from all officials. Note that since we assume that both the principal and the agents are risk-neutral, we abstract away from the incentive insurance trade-off, and hence without the limited liability constraint, the principal achieves the first best effort levels.
by,

\[
\Delta W_1^* = \frac{\kappa}{t_1} \left( \alpha_1^{1-\beta} \beta^2 n_2 A t_2^2 \kappa^{-1} \right)^{\frac{1}{2-\alpha-2\beta}} \\
\Delta W_2^* = \frac{\kappa}{t_2} \left( \alpha_2^{\alpha} \beta^{1-\alpha} n_2^2 A t_2^2 \kappa^{-2} \right)^{\frac{1}{2-\alpha-2\beta}} - \frac{1}{n_1} \Delta W_1^* + \frac{1}{2\kappa} t_1^2 \Delta W_1^* \cdot
\]

(11)

(12)

The profit, that is the total output net the cost of incentive to the principal is:

\[
\Pi^* = n_1 n_2 A x_1^* x_2^* - \Delta W_1^* - n_1 \Delta W_2^* + n_1 (r_2(r_1 - 1) + n_2(r_2 - 1)) K.
\]

(13)

We summarize the solution to the principal’s problem in Proposition 2.

**Proposition 2.** The optimal wage contract, \{\(W_0, W_1, W_2\)\}, that the principal offers to the government officials satisfies:

\[
\begin{align*}
W_2 &= 0; \\
W_1 &= \Delta W_2^* - (r_2 - 1)K; \\
W_0 &= \Delta W_1^* - W_1 - (r_1 - 1)r_2 K - I(\pi_1, \pi_2),
\end{align*}
\]

where \(\Delta W_1^*\) and \(\Delta W_2^*\) are given by (11) and (12). The efforts elicited from the provincial governors, \(x_1^*\), and from the prefectural mayors, \(x_2^*\), are given by (9) and (10). The profit to the principal is given by (13).

Note that \(\Delta W_1^*\), \(\Delta W_2^*\), \(x_1^*\) and \(x_2^*\) are all pinned down by the parameters that characterizes production technologies, shocks and the organizational structure. As a result, the profit to the principal is independent from his choice of \(\pi_1\) and \(\pi_2\). The easiest way to see this is to realize that the contracted rates of conviction divide the total private wealth from the corrupt officials caught between the inspector and the principal. The lower the contracted conviction rates, \(\pi_1\) and \(\pi_2\), the more inspector can keep the wealth of the inspected corrupt officials to himself (i.e. the larger the \(I(\pi_1, \pi_2)\)). But the total incentive to the provincial governor to become the inspector, \(\Delta W_1^*\), is fixed and given by (11). The principal then optimally lowers \(W_0\) to offset a higher \(I(\pi_1, \pi_2)\). In other words, for every dollar that the principal allows the inspector to keep from colluding with an inspected official, the principal reduces the prize to the inspector by the same dollar. In the end, we have the following corollary.

**Corollary 2.** The principal is indifferent to setting \(\pi_1\) (\(\pi_2\)) to any value between 0 and \(\pi_1^*\) (\(\pi_2^*\)).
It is worth mentioning that this result hinges on the model setup that the officials’ wealth is evaluated 100 cents on the dollar both to the principal and the inspector. If there is loss in the valuation of the assets during its transfer from the official to the principal for instance, then the principal will strictly prefer letting the inspector collude with his subordinates and keep to himself the corruption income seized from his subordinates. The principal will simply reduce the same amount from the inspector’s wage. This creates another component of the top prize which does not need to be paid for explicitly through wage.

In what follows, we assume the principal sets $\pi_1 = \pi_1^*$ and $\pi_2 = \pi^*$, so that the principal eliminates collusion between the inspector and officials: $I(\pi_1, \pi_2) = 0$. By shutting down this channel, we are stacking the cards against us to find a weak powered wage scale. We discuss next how the accumulation of private wealth from past corruption is essential in providing the high powered incentive compatible with a tournament incentive scheme.

2.3 Discussions

Now we are in the position to examine how the wage incentive interacts with the implicit corruption incentives.

2.3.1 Relationship to Tournament Incentives

First of all, to put our results in the perspective of the past literature on tournament labor contracts, we show that when all incentives, explicit and implicit, are considered, our model does imply a convex incentive structure as previously established by (Rosen, 1986). To see this, we prove that a special parametrization of our model, which is consistent with the set-up of Rosen (1986), delivers the key prediction of the Rosen (1986), namely a disproportionately large prize at the top. Recall that the prizes $\Delta W_1$ and $\Delta W_2$ are total incentive that officials face, which include the explicit wages as well as the corruption income from holding the offices.

Corollary 3. Suppose $n_1 = n_2 = 2$, $t_1 = t_2 = t$, and $x_1^* = x_2^* = x^*$. Under these conditions, the equilibrium of our model is equivalent to a symmetric equilibrium of a sequential elimination tournament in pairs considered by Rosen (1986). Then, we have

$$\Delta W_1^* > \Delta W_2^*,$$

as long as the the no-default condition in Rosen (1986) holds.
Suppose \( n_1 = n_2 = 2 \), \( t_1 = t_2 = t \), and \( x_1 = x_2 = x^* \). Then from (4) and (6), we have

\[
\Delta W_1^* = \frac{\kappa}{t} x^* \\
\Delta W_2^* = \frac{\kappa}{2t} x^* + \frac{\kappa}{2} x^{*2}.
\]

Therefore, \( \Delta W_1^* > \Delta W_2^* \) is equivalent to \( t < \frac{1}{x^*} \). Note that in a Tullock style contest as in Rosen’s original paper, the probability of winning is of the functional form, \( Pr(\text{\textit{i wins}}) = \frac{h(x_i)}{h(x_i) + h(x_j)} \), where \( x_i \) is the contestant’s effort, \( x_j \) is the opponent’s effort, and \( h(x) \) is some increasing function. Then, the marginal increase in the probability of winning evaluated at the symmetric equilibrium becomes

\[
\frac{\partial Pr(\text{\textit{i wins}})}{\partial x_i} \bigg|_{x_i=x^*} = \frac{h'(x^*)}{4h(x^*)}.
\]

On the other hand, note that \( t = \frac{c'(x^*)}{2c(x^*)} \). Therefore, the condition for a bigger top incentive \( \Delta W_1^* > \Delta W_2^* \) can be rewritten as

\[
\frac{h'(x^*)}{4h(x^*)} < \frac{c'(x^*)}{2c(x^*)}, \text{ or } \frac{x^* h'(x^*)}{2c(x^*)} < \frac{x^* c'(x^*)}{c(x^*)} < 2,
\]

which is exactly the no-default condition in Rosen (1986).\(^7\)

This example shows that if we keep the degree of competition \( (n_i \text{ and } t_i) \) and the targeted effort level \( (x_i) \) constant across the ranks of the tournament, as long as the no-default condition holds, the optimal prize design must have a big top prize.

In our model, the number of contending officials per promotion, the size of shocks, and the desired level of effort may differ across ranks, which induces differences in required incentives given to the officials to rise through the ranks. Corollary 3 however confirms that our model inherits the essential feature of an elimination tournament type of incentive scheme, that all else equal, it is more costly to elicit effort as the officials get closer to the top. As the explicit wage scale does not appear convex at all, we posit that the difference between the required incentives and the wages is then explained by wealth accumulation from rent-seeking activities. Moreover, such difference increases as the officials rise through the ranks, as the wealth grows exponentially. We turn to this observation next.

### 2.3.2 The Wealth Accumulation Effect

As we have pointed out earlier in Section 2.2, the total incentives needed for an efficient level of efforts from the principal’s point of view are determined by the technologies, the shocks and the organizational structure (see \( \Delta W_1^* \) and \( \Delta W_2^* \) in (11) and (12)). Observe, however, what makes up for these incentives is not only wage payment, but also corruption income.

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\(^7\)The no-default condition rules out the cases in which the elasticity of response of effort, \( x^* h'(x^*)/h(x^*) \), is too large relative to the elasticity of its cost, \( x^* c'(x^*)/c(x^*) \). In those cases, contestants’ efforts to win drive the payoff to each below zero and they do better by defaulting or exerting zero effort.
Rewrite the prizes in Proposition 2 as

\[ \Delta W^*_2 = W_1 - W_2 + (r_2 - 1)K; \]
\[ \Delta W^*_1 = W_0 - W_1 + (r_1 - 1)r_2K. \]

The wage incentives are given by \( W_1 - W_2 \) for prefectural mayors and \( W_0 - W_1 \) for provincial governors. For a given required level of total incentive, \( \Delta W^*_1 \) and \( \Delta W^*_2 \), the bigger the \( K \) the less progressive the wage scale appears:

\[ (W_0 - W_1) - (W_1 - W_2) = \Delta W^*_1 - \Delta W^*_2 - ((r_1 - 1)r_2 - (r_2 - 1))K. \]

Note that \((r_1 - 1)r_2 - (r_2 - 1) > (r_1 - 1) - (r_2 - 1) = r_1 - r_2 \geq 0\). Therefore, the difference in pay differences across two consecutive ranks, or the speed at which pay increase increases, is declining in \( K \). We have the following result.

**Corollary 4.** The bigger the private wealth endowment of the politicians, \( K \), the lower powered the wage scale appears.

Corollary 4 reconciles a seemingly low-powered pay scale in the public sector in China with a tournament-style compensation scheme that calls for a grand prize at the end of the tournament. It is easy to extend this model to tournaments that last more than two stages. The growing private wealth that an official has accumulated over his career implies that there is more to lose as he moves up the rank towards the top. When he gets to the top, he is distanced from the production tasks that are decentralized to the local officials and hence does not have further opportunity to grow his wealth. Though as an inspector he can potentially collude with lower-level officials to extract more income, the principal understands it and contracts with him an intensity of inspection which eliminates collusion. The grand prize at the end of the tournament then is the security with which he can keep his accumulated wealth from various offices held in his career. This multi-stage tournament scheme leverages on the corruption income an official has received during his career and effectively makes it part of the compensation to economize on the cost of incentives.

More broadly, such a incentive scheme may be attractive at an early development stage, when the tax collection on behalf of the principal is weak so that the principal has only a limited budget to fund the government. In this model, if none of the officials are endowed with any capital (\( K = 0 \)) or there is no corruption opportunities (\( r_i = 0, \forall i \)), then the efficient levels of production must be incentivized by \( W_1 = \Delta W^*_2 \) for each provincial governor and \( W_0 = W_1 + \Delta W^*_1 \) for the top leader, which may well exceed the revenue of a poorly funded
government.

Last but not the least, we have focused on the puzzle of the seemingly low wage incentives in one of the most successful tournament style incentive schemes in practice, one that is adopted to incentivize Chinese officials to promote economic growth. We however do not intend to, nor are we able to, comment on the welfare properties of this incentive system. Doing so would require us to model formally the social cost of the rent-seeking behavior, which we leave for future research. In Section 3.3, we will come back to this idea that the rent-seeking behavior interfere with the production side of the economy.

3 Mapping to China

In this section, we calibrate an extension of the model in Section 2 to tie it closer to the Chinese context. In reality, the Chinese government broadly consists of five levels: center, province, prefecture, county, and lastly township and village level. In this simulation exercise, we consider a four-level government from county level up to the central level. It is plausible to view county-level job as an entry job to the country’s bureaucratic ladder. On the data part, county-level output is the most disaggregated macro data that is available.

To extend our model to four levels of government hierarchy, we suppose that, under the central government, there are $n_1$ provinces; each province has $n_2$ prefectures; and each prefecture has $n_3$ counties. The county-level production function is defined analogously to (1):

$$y_{hij} = A q_j^\alpha q_{ij}^\beta q_{hij}^\gamma,$$

with $\alpha + \beta + \gamma < 1$,

where $q_j$ is the output of the provincial task, $q_{ij}$ that of the prefectural task, and $q_{hij}$ the county-level task. Similarly, denote the corresponding levels of efforts by the officials as $x_j$, $x_{ij}$, and $x_{hij}$ and the corresponding levels of the luck component in the production as $\eta_j$, $\varepsilon_{ij}$ and $\zeta_{hij}$.

We start by describing the data that help us pin down some key parameters of the model in Section 3.1. We present the results from the simulation in Section 3.2. In Section 3.3, we introduce another complication to the model, where we allow the rates at which private wealth accumulates to depend on the effort in public good production. By doing so, we introduce another trade-off that the officials face: The provision of public good may come at a cost of rent extraction.

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8The derivation of this extended model with exogenous rates of return is found in Appendix C.2.
3.1 Data

We obtain county-level annual output data from 1997 to 2007 and province-level consumer price indices from 1997 to 2007 to compute county-level real output growth from 1998 to 2007. We exclude from the analysis sample Beijing, Tianjin, Shanghai and Chongqing, the four municipalities directly controlled under the central government, since the internal municipal government structure is somewhat different than those in other provinces. After trimming the outliers of output growth rate to control for data entry errors, we take the average of the annual real growth of a county within the sample period. This gives us 2,613 counties in 274 prefectures and 25 provinces.\(^9\)

Let the output, \(y_{hij}\), denote a county \(h\) in prefecture \(i\) and province \(j\). According to our model,

\[
\ln y_{hij} = \ln A + \alpha \ln q_j + \beta \ln q_{ij} + \gamma \ln q_{hij} = \ln A + \alpha \ln x_j + \beta \ln x_{ij} + \gamma \ln x_{hij} + \alpha \ln \eta_j + \beta \ln \varepsilon_{ij} + \gamma \ln \zeta_{hij}.
\]

This motivates the following series of regressions. First, we regress the county-level growth on a constant and province dummies, \(ProvDummy_j\):

\[
\ln y_{hij} = \text{constant} + \sum_j b_j ProvDummy_j + u_{hij}.
\]

Denote the predicted value of log growth as \(\ln y_j\). Then take the residual and further regress the residuals on a constant and prefecture dummies, \(PrefDummy_{ij}\):

\[
u_{hij} = \text{constant} + \sum_{i,j} b_{ij} PrefDummy_{ij} + v_{hij}.
\]

Denote the predicted value of the above regression as \(\ln y_{ij}\). These two regressions provide a decomposition of the variance of log county growth into a province-, a prefecture- and a county-component:

\[
\text{var}(\ln y_{hij}) = \text{var}(\ln y_j) + \text{var}(\ln y_{ij}) + \text{var}(v_{hij}) = \alpha^2 \text{var}(\ln \eta_j) + \beta^2 \text{var}(\ln \varepsilon_{ij}) + \gamma^2 \text{var}(\ln \zeta_{hij}).
\]

Given a choice of \((\alpha, \beta, \gamma)\), we can back out the variance of the logged shocks, \(\sigma_1^2\), \(\sigma_2^2\) and \(\sigma_3^2\) from the between-province, within-province but between-county and within-county components of the total variance from the data.

---

\(^9\)Detailed description of the data, sample selection, and regression results are in Appendix C.1.

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Having implemented the regressions with our data, we find that the average total variance of county-level annual growth is 0.00255443, of which 11.8% comes from the between-province variance (0.0003011), 20.7% from the within-province yet between-county variance (0.0005289), and 67.5% from the within-county variance (0.0017244).

### 3.2 Simulation

We parametrize the model as in Table 1. In the model with exogenous rates of return on officials’ private wealth, all parameters are given in Panel (a) of Table 1. The number of competitors per position respects the proportions of the number of provinces, prefectures, and counties in the analysis sample: 25 provinces, 274 prefectures (or roughly 10 per province), and 2,613 counties (or roughly 10 per prefecture). The values of the shape parameters in the county production function, $\alpha, \beta$ and $\gamma$, capture the idea that for the county output, inputs at the county and prefecture level may be more relevant than broad schemes drafted at the province level. The variance of the luck component in the competition is obtained from regressing county-level output data as detailed in Section 3.1. The TFP parameter in the county production function, $A$, determines the scale of efforts demanded by the principal, $x_i$’s, which will in turn determine the scale of the output, interpreted as local economic growth. Therefore, $A$ is calibrated so that the average output, $A x_1^\alpha x_2^\beta x_3^\gamma$, is 1.13, or 13% annual growth rate, the average real growth observed in the data. The wealth of a county official that enters the political tournament, $K$, determines the scale of the wages. The cost of effort parameter, $\kappa$, is chosen such that the second order conditions of officials’ problems are satisfied. The exogenous rates of return, $r_1, r_2$ and $r_3$, are such that higher political positions are more lucrative for growing private wealth.

The simulation yields an optimal wage scale of $W_3 = 0, W_2 = 2.2120, W_1 = 6.4512, W_0 = 8.6840$. Examine the wage increments faced by an official who rises from a county post to a province post: $W_2 - W_3 = 2.2120, W_1 - W_2 = 4.2392,$ and $W_0 - W_1 = 2.2328$. These wage increments appear rather low-powered, especially at the top. Coincidentally, that the "3581" Project in Beijing, which we mentioned in the introduction, implies wage increments of 2, 3, 2, which is qualitatively similar to our simulation result.

The corresponding effort levels of the provincial, prefectural, and county leaders in the simulated model are $x_1 = 0.1217, x_2 = 0.2063, x_3 = 0.0407$. The prefectural mayors work the hardest, followed by the provincial governors, while the county heads work the least. This is intuitive. First, since the principal faces a limited liability constraint of offering the county leaders at least 0, and with multiple competitors ($n_3 = 10$) and large noise ($0.0017244/\gamma^2$), the probability of winning at the prefectural stage is the lowest among all stages, the county
leader thus tend to shirk. Second, due to the interdependence of production, low effort at the county level reduces marginal product of effort at higher levels of government, and hence reduces the principal’s demand for effort at higher levels of government. Third, between the provincial governor and the prefectural leader, prefectural output weighs slightly more than provincial output in aggregate production ($\alpha = 0.2$ and $\beta = 0.3$). Therefore, the principal demands more effort from the prefectural mayors than from the provincial governors.

Next we perform a comparative statics exercise in terms of the private wealth endowment of the county leaders, $K$. We vary $K$ from 8 to 11. The results are found in Figure 2. Consistent with Corollary 4, as the private wealth endowment of the politicians increase, the interest from staying in the tournament and accumulating wealth will work as an implicit compensation to the politicians, and hence relieve the principal’s burden of incentivizing efforts by wages. The top prize, or $W_0$, decreases sharply as wealth endowment increases. Wages at lower levels also decrease as one increases $K$. The explicit wage scale looks increasingly low-powered as the implicit compensation becomes more important. The two forms of compensation, the implicit reward from wealth accumulation and the explicit wage payment, exactly cancel out to elicit a same level of effort from the politicians throughout the ranks. The effort inputs and the county average growth rate stay constant.

### 3.3 Endogenous Rates of Return

It is conceivable that there might be conflict of interest as officials exert effort to produce public good, which affects the scope of his rent seeking. For example, when deciding to whom to award a government procurement contract, the official may face the trade-off that the best bid comes from the market and not from a company that has ties to his own family. To capture this trade-off, we modify the above model slightly to make the rate of return on private wealth depend on the effort of producing public good. More specifically, let $r_i(x_i)$ be defined as

$$r_i(x_i) = \bar{r}_i - \frac{1 - e^{-\lambda x_i}}{1 + e^{-\lambda x_i}} (\bar{r}_i - r_i).$$

Note that the rate of return $r_i$ is decreasing in the effort of the official $x_i$, so that when deciding on how much public good to produce, the official not only weighs the cost of effort against winning the tournament, but also factors in cost of reduced rates of private wealth accumulation against winning.

The parameters of the function of the rates of return (14) are found in Panel (b) of Table 1. The highest rates of return at the province, prefecture, and county level are targeted to
match those in the previous model with exogenous rates of return. As the effort increases, the rates can decline to a mere 5% return. The parameter $\lambda$ governs how fast the rates decline in effort. We consider a range of $\lambda$, from 0 (i.e. equivalent to the model with exogenous rates) to 4. The rest of the model is parametrized in the same way as in Panel (a) of the same table. The results are found in Figure 3.\textsuperscript{10}

First, we confirm that when $\lambda$ is 0, the model reverts to the exogenous rate of return case. As $\lambda$ increases, the conflict of interest between private accumulation and public good production becomes more acute. It becomes more and more costly for the principal to induce effort from the officials. Despite paying in general higher wages and in particular promising a higher and higher top prize, the optimal county growth rate declines as the conflict of interest becomes severe. Interestingly, as $\lambda$ increases, the principal relies more on the effort of the provincial governors and less on the effort of the prefectural mayors to produce growth. This is so because, due to the multitude of posts, increasing lower-level officials’ wages increases the total cost of incentive much faster than increasing the top official’s wage. The conflict of interest introduced here tends to reduce efforts across all ranks, the cheapest way to alleviate that is to give the top official a super prize to overcome the provincial governors’ tendency to shirk. However there is a limit to what can be done and the overall consequence is that growth suffers when officials trade growth for private wealth building.

This exercise illustrates that the optimal wage scale must respond to all the trade-offs officials face when making decisions about production. In an environment with severe conflict of interest considered in this section, the optimal wage scale will again need to be high-powered. However, even under a high-powered wage scale, the officials at the lower levels of government tend to shirk and reap private benefit from their positions rather than trying to win promotion, leading to an overall much muted growth performance.

4 Concluding Remarks

In this article, we model the career advancement of Chinese officials under the RDA regime in a principal-agent framework. We explicitly model the informational frictions that challenge the contractual relationship between the principal, representing the Chinese people, and the agents, representing the officials at various ranks in the government. We reconcile the prima facie contradiction between a rank-order tournament incentive scheme and a low-powered public-sector pay scale in the Chinese context. The key insight is to recognize that the inability to eliminate corruption in a bureaucracy without institutionalized checks and

\textsuperscript{10}The derivation of this extended model with endogenous rates of return is found in Appendix C.3.
balance paradoxically provides a form of implicit incentive in an RDA regime. The official’s private wealth that is built by exploiting political rents along their career is only secure when he rises to the top. This defines the grand prize at the end of the tournament. This incentive structure can be especially effective at an early stage of development, when the principal’s taxing power is limited and hence faces a tight budget constraint to offer explicit wage compensations as incentive.

Looking forward, we make several conjectures based on the insight from this model. As the government function evolves from direct participating in resource allocation to safeguarding market practices, the space for rent-seeking may diminish. This would imply changes in the public-sector pay structure. Leaving aside the potential changes on the way production in the public sector takes place, explicit wage payment would have to be increased to take the place of implicit reward from keeping corruption income.

More generally, this article sheds light on the subtle role that corruption plays in tournament schemes, or bureaucracies where law/contract enforcement is also embedded in the same hierarchical structure. For many developing countries, corruption and the lack of the rule of law (in the sense of an independent judiciary system) are two perennial institutional problems, which lead to long-term poverty and political instability. The mechanism in this paper however illustrates how the two can coexist in a stable equilibrium in the Chinese historical context.
References


Table 1: Parameter Values in the Model Simulation

<table>
<thead>
<tr>
<th>Model notation</th>
<th>Interpretation</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Exogenous $r_i, i = 1, 2, 3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n_1, n_2, n_3$</td>
<td>Number of competitors per position at each level</td>
<td>25, 10, 10</td>
</tr>
<tr>
<td>$\alpha, \beta, \gamma$</td>
<td>Shape parameters in the county production function</td>
<td>0.2, 0.3, 0.3</td>
</tr>
<tr>
<td>$A$</td>
<td>TFP parameter in the county production function</td>
<td>7.252</td>
</tr>
<tr>
<td>$\sigma_1^2, \sigma_2^2, \sigma_3^2$</td>
<td>Variance of luck in competition at each level</td>
<td>0.0003011/\alpha^2, 0.0005289/\beta^2, 0.0017244/\gamma^2</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Cost of effort parameter</td>
<td>80</td>
</tr>
<tr>
<td>$K$</td>
<td>Wealth of a county official</td>
<td>10</td>
</tr>
<tr>
<td>$r_1, r_2, r_3$</td>
<td>Rate of return on private wealth at each level</td>
<td>1.6, 1.4, 1.2</td>
</tr>
<tr>
<td>(b) Endogenous $r_i(x_i)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Sensitivity parameter in the rate of return functions</td>
<td>0 to 4 at 0.2 increment</td>
</tr>
<tr>
<td>$\bar{r}_1, \bar{r}_2, \bar{r}_3$</td>
<td>Upper bound on the rate of return at each level</td>
<td>1.6, 1.4, 1.2</td>
</tr>
<tr>
<td>$\underline{r}_1, \underline{r}_2, \underline{r}_3$</td>
<td>Lower bound on the rate of return at each level</td>
<td>1.05, 1.05, 1.05</td>
</tr>
</tbody>
</table>

Notes: This table contains the parameter values of the model in the simulation exercise detailed in Section 3. Panel (a) contains the parameterization of the baseline model with exogenous rates of return from rent-seeking on private wealth (Section 3.2). Panel (b) contains the additional parameters in the function of the rates of return from rent-seeking, when we endogenize those rates (Section 3.3).
Figure 1: The Organizational Structure of Chinese Officials: An Illustration with \( n_1 = 3 \) and \( n_2 = 2 \)

Notes: This figure illustrates the organizational structure of Chinese officials with a simple example. There are three provinces in this example and each province has two prefectures under its jurisdiction. The production is organized in terms of geographical regions. For instance, the growth in Prefecture A uses output from the governor of Province 1 and output from the mayor of Prefecture A as inputs. In terms of promotion, the two prefectural mayors in the same province compete for the position of the governor of that province, while the governors of the three provinces compete for the top position. For a detailed description of the model setup, see Section 2.
Figure 2: Baseline Model: Varying the Wealth Endowment, $K$

(a) Wage Scale $W_i$, Varying $K$

(b) Effort Level $x_i$, Varying $K$

(c) County Growth Rate $y$, Varying $K$

Notes: This figure shows the results from the simulation of the baseline model with four levels of government and exogenous rates of return from rent-seeking, when we vary the county heads’ private wealth endowment, $K$. For a discussion of the results, see Section 2.3.2.
Figure 3: Model with Endogenous Returns: Varying the Sensitivity of Returns to Effort, $\lambda$

(a) Wage Scale $W_i$, Varying $\lambda$

(b) Effort Level $x_i$, Varying $\lambda$

(c) Endogenous Returns $r_i$, Varying $\lambda$

(d) County Growth Rate $y$, Varying $\lambda$

Notes: This figure shows the results from the simulation of the extended model with four levels of government and endogenous rates of return from rent-seeking, when we vary the sensitivity of returns to effort, $\lambda$. For a discussion of the results, see Section 3.3.