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Abstract

This paper uses a panel Threshold VAR model to estimate the regime-dependent impact of oil shocks on stock prices. We find that an adverse oil supply shock has a negative effect on stock prices when oil inflation is low. In contrast, this impact is negligible in the regime characterised by higher oil price inflation. Using a simple DSGE model, we suggest that the explanation for this result may be tied to the behaviour of credit spreads. When oil inflation is low, lower policy rates encourage firms to get highly leveraged. A negative oil shock in this scenario leads to a substantial increase in spreads, reducing profits and equity prices. In contrast, at higher rates of inflation, spreads are less affected by the oil shock, ameliorating the impact on the stock market.

Key words: Threshold VAR, Hierarchical Prior, DSGE model, Oil shocks.

1 Introduction

What is the impact of oil shocks on the stock market? Kilian and Park (2009) shows that the answer to this question depends on the source of the shock. For example, a rise in oil prices is associated with a fall in stock prices only when the oil price increase is driven by oil-market specific demand shocks. In contrast, supply shock based oil price increases are estimated to have a negligible impact on stock returns. The analysis in

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Kilian and Park (2009), thus offers the key insight that not all oil price increases are the same as far as their impact on the stock market is concerned.

In the current paper, we extend this analysis and investigate if the timing, magnitude and sign of oil price changes matters as well. The motivation for pursuing this is two fold. First, a large literature strongly indicates that oil shocks may have a non-linear impact on macroeconomic variables such as GDP. In a classic paper, Mork (1989) argues that oil price increases may be more important than decreases. The subsequent investigations in Hamilton (1996) and Hamilton (2003) also suggest that the magnitude of the increase in current oil prices relative to past increases may matter when determining the impact of oil price shocks on output.[1] Second, if the relationship between oil shocks and stock prices is non-linear, then restricting analyses to linear specifications may understimate the importance of these shocks. Therefore, our investigation is potentially important both from the perspective of investors and policy makers.

To examine the question at hand, we use a non-linear Bayesian VAR model for seventeen OECD countries. In order to account for the cross-sectional dimension of our data set, we allow for the possibility of pooling via a hierarchical structure for the prior distributions. This set up enables us to estimate the posterior distribution of the *average* VAR parameters across countries and provides more precise estimates of dynamic responses than those based on a single country data set.

The empirical analysis in our paper is related to Sim and Zhou (2015) who show that there is a relationship between the quantiles of the US stock returns and quantiles of oil price shocks. Similarly, Chen (2010) shows that the probability of a bear market is affected positively by oil price increases. In a recent contribution, Jiménez-Rodríguez (2015) employs the methodology used in Hamilton (2003) to test for a non-linear relationship between oil price changes and stock returns for Canada, Germany, the UK and the US and finds some evidence against linearity.[2] Our empirical model generalises these analyses in two important ways. First, as we use a structural VAR, we identify oil shocks and estimate their impact simultaneously. Most of the existing literature examines non-linearity in a single-equation framework using off-model measures of oil shocks. Second, our large panel dimension enables us to derive average effects for the OECD while allowing for heterogeneity across countries. This feature also distinguishes our work from Holm-Hadulla and Hubrich

[1] For a recent critical survey of this large literature, see Hamilton (2011).
[2] See Kang *et al.* (2015) for a detailed survey of this literature.

(2017). who estimate a Markov Switching VAR using aggregate Euro-Area data to examine the response to oil price shocks. While Holm-Hadulla and Hubrich (2017) consider the response to composite oil price shocks, we investigate shocks that originate from the demand or supply side. Another key contribution of our paper relative to existing work on this topic is the fact that we also examine the non-linear transmission of the oil shocks from a theoretical perspective.

Our results suggest the following conclusion for the average OECD country in our panel: Oil supply shocks are associated with a fall in equity prices that is substantially larger during the regime characterised by low oil price inflation. In contrast, when oil price inflation is high, these shocks have a negligible impact on stock prices. There is weaker evidence that a similar result holds for speculative demand shocks to oil. Our results suggest, therefore, that not only the source of the oil shock is important for stock prices, but the timing may matter as well. As a consequence, while innovations such as oil supply shocks appear to be unimportant for stock prices in a linear VAR model (see Kilian and Park (2009)), our analysis shows that the assumption of linearity may mask the important effect of such shocks in specific regimes.

To investigate a possible mechanism for these results, we extend the DSGE model of Blanchard and Gali (2010) to include a working capital friction that responds to liquidity conditions in a non-linear way. In particular, to capture the dynamics of the firms' liquidity and risk premia, the cost of finance is assumed to increase as liquidity in the economy declines. When the economy is in a low oil price and low aggregate inflation environment, policy interest rates are low. The firms increase their borrowing and become over leveraged. An adverse oil shock such as a decrease in oil supply pushes up inflation and induces the monetary authority to raise interest rates. This pushes up credit spreads substantially because of an increase in default probability. As a consequence, firms' profits decline and stock prices fall. This channel is muted in a high oil price/aggregate inflation environment as increases in the policy rate have a smaller impact on credit spreads.

The paper is organised as follows: The empirical model and results are presented in Section 2. We consider an explanation for the empirical results in Section 3. Section 4 concludes.

2 Empirical Analysis

2.1 Empirical model

For each country in our panel, we estimate the following threshold VAR (TVAR) model:

$$\begin{aligned} Z_{it} &= \left(c_{1i} + \sum_{j=1}^{P} b_{1i,j} Z_{it-j} + u_{it} \right) S_{it} + \\ & \left(c_{2i} + \sum_{j=1}^{P} b_{2i,j} Z_{it-j} + u_{it} \right) (1 - S_{it}) \end{aligned} \tag{1}$$

where $i = 1, 2, ...17$ denotes the 17 OECD countries included in the study (see data description in section 2.1.3). The matrix of endogenous variables is denoted by Z_{it}, which in the benchmark case includes the following eight variables sampled at the monthly frequency: (1) oil production (P_t), (2) oil inventories (I_t), (3) a measure of global real activity (Y_t), (4) nominal oil price (o_t) and (5) Industrial production for country i (ip_t), (6) CPI inflation for country i (π_t), (7) the spread between the 10 year government bond yield and a short-term interest rate (sp_t) and (8) the stock market index for country i (s_t). With the exception of Y_t which is stationary by construction, I_t which is defined in differences, sp_t which is differenced to induce stationarity, the remaining variables enter in log differences and the lag length P is fixed to 13.

The covariance matrix of the residuals is also regime dependent and defined as:

$$var\left(u_{it}\right) = \Sigma_{it} = S_{it} \odot \Sigma_{1i} + (1 - S_{it}) \odot \Sigma_{1i} \tag{2}$$

The regime switches in the model are governed by the variable S_{it}:

$$S_{it} = 1 \Longleftrightarrow \tilde{o}_{t-d_i} \leq o_i^* \tag{3}$$

The threshold variable $\tilde{o}_t$ is the 12 month moving sum of monthly oil price inflation, an approximation to the annual growth in o_t. This choice is partly motivated by earlier studies that emphasise the importance of increases in the oil price relative to previous highs (see for e.g. Hamilton (1996)). The structure in equation 3 implies that the dynamic relationship between the oil market and the economy is allowed to change if the

difference in o_t relative to its past exceeds an unknown threshold value o_i^*. The importance of the magnitude of changes in the oil price is also implied by the DSGE model describe in Section 3 below. Note also that the lag or delay in the threshold variable d_i is treated as an unknown parameter with and is allowed to take on values $d_i = 1, 2, ..., 12$.

In order to account for possibility that the dynamic relationships amongst the endogenous variables may have similarities across OECD countries, we introduce a hierarchical prior for a number of key parameters in the model. Defining the VAR coefficients as $\beta_{1,i} = vec\left([b_{1i,1}, .., b_{1i,P}]\right)$ and $\beta_{2,i} = vec\left([b_{2i,1}, .., b_{2i,P}]\right)$, we follow Ruisi (2018) and Mumtaz and Sunder-Plassmann (2017) and assume a priori that:

$$p\left(\beta_{1,i}|\bar{\beta}_1, \lambda_1\right) \tilde{} N\left(\bar{\beta}_1, \lambda_1 \Lambda_i\right) \tag{4}$$

$$p\left(\beta_{2,i}|\bar{\beta}_2, \lambda_2\right) \tilde{} N\left(\bar{\beta}_2, \lambda_2 \Lambda_i\right)$$

where $\bar{\beta}_1$ and $\bar{\beta}_2$ denote the weighted cross-sectional averages of the VAR coefficients, Λ_i is a matrix with diagonal elements reflecting the scale of the coefficients and the variances λ_1 and λ_2 control the degree of pooling. As $\lambda_1 \to 0$, for example, the prior places a strong weight on $\beta_{1,i}$ being close to the average coefficients $\bar{\beta}_1$ while larger values for λ_1 imply heterogenous dynamics across countries.

The regime dependent residual covariance matrices are factored as:

$$\begin{aligned} \Sigma_{1i} &= A_{1i}^{-1} H_{1i} A_{1i}^{-1\prime} \\ \Sigma_{2i} &= A_{2i}^{-1} H_{2i} A_{2i}^{-1\prime} \end{aligned} \tag{5}$$

where H_{1i} and H_{2i} are diagonal matrices with the variances of orthogonalised shocks on the main diagonal. A_{1i} and A_{2i} are lower triangular. As in Mumtaz and Sunder-Plassmann (2017) we assume the following prior for the non-zero and non-one elements of A_{1i} and A_{2i} (denoted by $a_{1,i}$ and $a_{2,i}$):

$$p\left(a_{1,i}|\bar{a}_1, \delta_1\right) \tilde{} N\left(\bar{a}_1, \delta_1 \Xi_i\right) \tag{6}$$

$$p\left(a_{2,i}|\bar{a}_2, \delta_2\right) \tilde{} N\left(\bar{a}_2, \delta_2 \Xi_i\right)$$

where $\bar{a}_1$ and $\bar{a}_2$ represent the cross-sectional weighted average, Ξ_i are diagonal matrices to account for scale differences in the elements of $a_{1,i}$ and $a_{2,i}$ while the degree of pooling across countries is controlled by δ_1 and δ_2 in the two regimes.

Finally, the prior on the threshold o_i^* also has a hierarchical structure:

$$p\left(o_i^* | \bar{o}, \varpi\right) \tilde{} N\left(\bar{o}, \varpi \Psi_i\right) \tag{7}$$

where $\bar{o}$ is the average value of the threshold across countries. As before, values of ϖ close to zero imply that the threshold for each country takes on similar values.

Note that the model allows the constants c_{1i}, c_{2i} and the error variances H_{1i}, H_{2i} to be different across countries thus accounting for heterogeneity in initial conditions and the magnitude of shocks hitting the economies.

This structure for the prior distributions offers two key advantages. First, as we describe below, by using the posterior distribution for the average VAR parameters $\left(\bar{\beta}_1, \bar{\beta}_2, \bar{a}_1, \bar{a}_2, \bar{o}\right)$ we can estimate the regime-dependent impulse responses for the average country in our sample. The fact that this calculation exploits the cross-sectional dimension implies that the estimated responses are likely to more precisely estimated than those from a purely time-series model where the estimates would be based on the sub-sample implied by each regime. Second, while the model allows for heterogeneity across countries, country-specific posterior distributions are based on priors centered on cross-country averages. This additional information may improve the precision of country-specific estimates when compared to set ups where cross-sectional information is not incorporated in the prior.

2.1.1 Estimation and impulse responses

Details on prior distributions and the estimation algorithm are presented in the technical appendix. However, it is useful to highlight some important features. The prior for the variances controlling the degree of pooling $\lambda_1, \lambda_1, \delta_1, \delta_2, \varpi$ is assumed to be an inverse Gamma distribution $IG\left(s, v\right)$. As discussed in Gelman (2006) and Jarocinski (2010) the usual 'agnostic prior' with small positive values for s and v can be quite informative in some circumstances. We follow the suggestion in Gelman (2006) and use $v = -1$ and $s = 0$ which implies

a uniform prior for the standard deviations. The marginal posterior distributions are approximated using a Metropolis within Gibbs algorithm. As described in the technical appendix, the Metropolis step is used to sample from the conditional posterior distribution of the threshold o_i^* while the remaining conditional posteriors are standard. The technical appendix presents a small Monte-Carlo experiment which indicates that the proposed algorithm performs fairly well.

As the model is non-linear we use the methods described in Koop *et al.* (1996) to estimate the impulse responses. Our interest mainly centers on the average responses across countries. In this case, the contemporaneous impact matrices A_1 and A_2 are based on the average parameters $\bar{a}_1$ and $\bar{a}_2$. As described below, our benchmark model identifies three oil market shocks using sign restrictions. Given these contemporaneous impact matrices, the impulse responses are defined as:

$$IRF = E\left(Z_h|\varepsilon\right) - E\left(Z_h\right) \tag{8}$$

where $h = 0, 1, .., 60$ is the horizon and ε represents the oil shock of interest. The expectations in equation 8 are estimated using Monte-Carlo integration using the posterior distribution of the average VAR parameters (see Koop *et al.* (1996)).[3] This procedure takes into account the dynamic impact of the shock on the probability of regime switches over the impulse response horizon. We compute these impulse responses using the sequence of 'histories' or lagged values of the average state data Z_t as initial conditions and report the average.

2.1.2 Identification of oil shocks

We identify three oil market shocks: (1) oil supply shock, (2) oil demand shock and (3) speculative demand shock. In the benchmark case, the identification scheme follows Kilian and Murphy (2014) and is based on the contemporaneous sign restrictions listed in Table 1 (see also Peersman and Robays (2012)).

The oil supply shock is defined as the innovation that leads to an increase in the price of oil but reduces oil production and world real activity. In contrast, an increase in oil demand driven by world real activity leads to rise in oil price and production and is accompanied by a rise in the measure of global activity. Oil

[3] Note that $E\left(Z_h\right)$ represents the expected future value of the endogenous variables in an average or typical state.

	Supply	Demand	Speculative demand
Oil production	≤ 0	≥ 0	≥ 0
Real activity	≤ 0	≥ 0	≤ 0
Oil price	≥ 0	≥ 0	≥ 0
Inventories			≥ 0

Table 1: Sign restrictions for the benchmark model

specific, or speculative demand shocks are also associated with a rise in oil price and production but this innovation leads to a fall in real activity and a rise in inventories. As discussed in Kilian and Murphy (2014), speculative demand shocks represent the expected future demand for oil in excess of supply. Such a shortfall may be anticipated due to political uncertainty in oil producing countries, with these type of demand shocks pushing up inventories and pushing down real activity as a result of a fall in oil consumption. Following Kilian and Murphy (2014), we also impose bounds on the price elasticity of oil supply and demand on impact. In particular, we impose the condition that the price elasticity of supply is less than 0.025 on impact, while the impact elasticity of oil demand is constrained to remain above -0.8.

In a robustness analysis presented below, we show that if a recursive identification scheme is used instead, the key findings regarding non-linearity of impulse responses are not over turned.

2.1.3 Data

As mentioned above, the TVAR model is estimated for 17 OECD countries that include: Austria, Belgium, Germany, Denmark, Spain, Finland, France, Canada, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Sweden, United Kingdom and the United States. The oil market variables are common across countries. Data for global oil production measured in millions of barrels is obtained from the Energy Information Administration (EIA). Following Kilian and Murphy (2014), inventories are measured by the EIA data for US crude oil inventories scaled by the ratio of OECD petroleum stocks to US petroleum stocks. The measure of global real activity is taken from Kilian (2009) and is based on the dry cargo shipping freight rates. We use the West Texas intermediate measure of the spot crude oil price with the data obtained from the Federal Reserve bank of St Louis (FRED) data base. The remaining country-specific variables are obtained from the OECD data base.

The sample runs from 1973M1 to 2016M12 for all countries except Portugal where the sample starts from

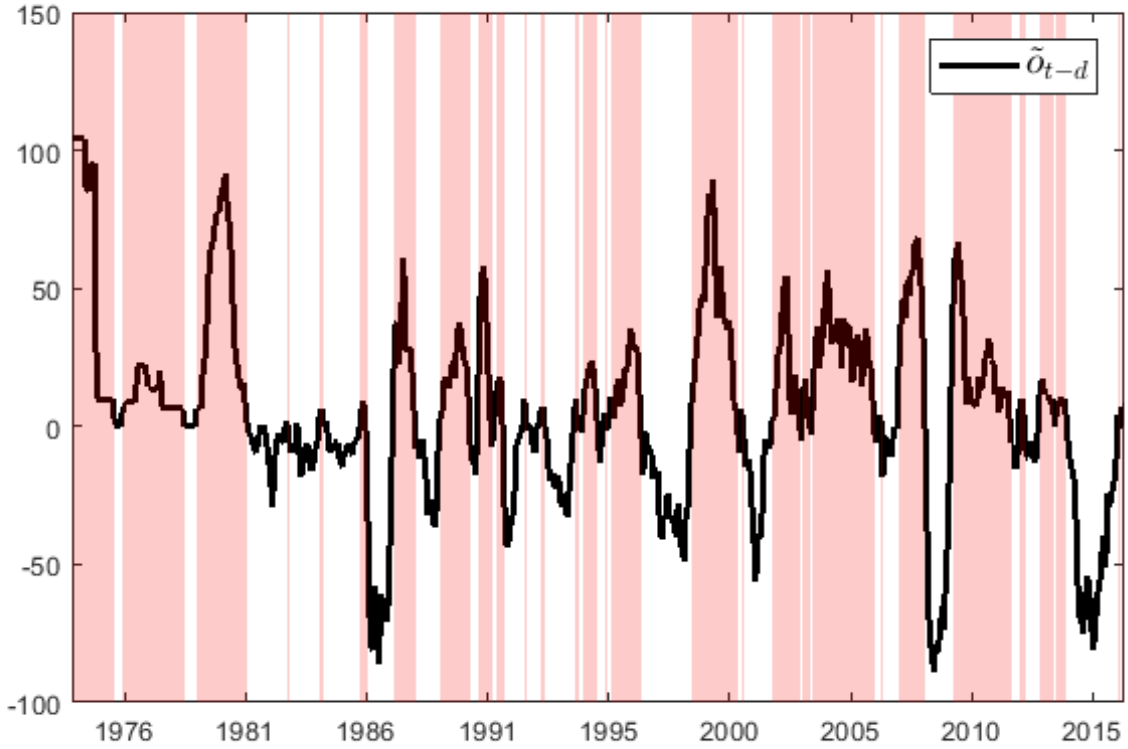

Figure 1: The shaded area indicates periods where $\tilde{o}_{t-d} > \bar{o}$ where the posterior median for the delay d is used.

1985M8.

2.2 Empirical results

To investigate the fit of the TVAR model relative to a linear VAR, we calculate the deviance information criterion (DIC) based on the cross-sectional average of the data (see Spiegelhalter *et al.* (2002)). The DIC rewards model fit while penalising model complexity with smaller values of the statistic preferred. The estimated DIC for the linear model is 10353.27. The estimated DIC for the TVAR is lower at 9900.71 suggesting that an improvement in model fit.

2.2.1 Impulse Responses

We consider impulse responses in the two regimes identified by the empirical model. The estimated posterior median of $\bar{o}$, the average value of the threshold across countries, is 2 percent, while, on average, the delay

is estimated to be 1. Based on these estimates, the regime classification is presented in Figure 1 with the shaded area depicting the periods belonging to the second regime characterised by positive and high annual oil price growth. This regime was predominant during the 1970s, the early and mid-1990s and returned in the form of several persistent episodes during the last two decades of the sample.

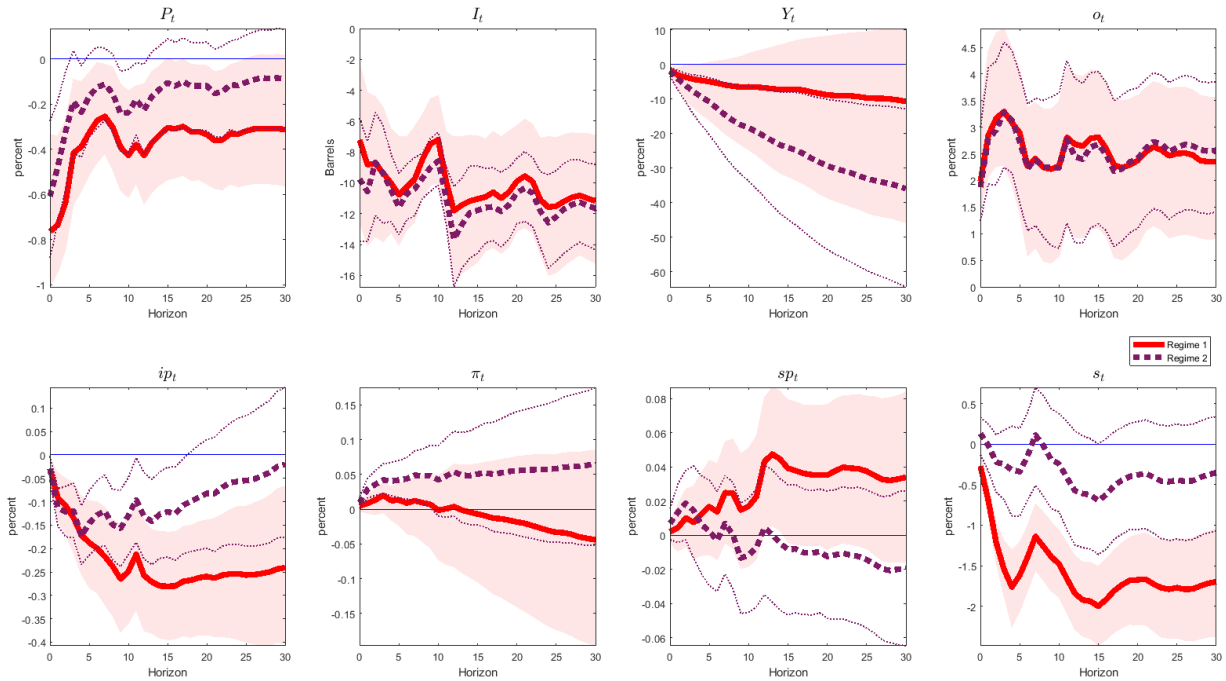

Figure 2: Cumulated impulse response to 1 SD adverse oil supply shock in each regime. Regime 1 denotes periods when $\tilde{o}_{t-d} \leq \bar{o}$ and Regime 2 denotes periods when $\tilde{o}_{t-d} > \bar{o}$. The shaded area and the dotted line represent 68 percent error bands.

Figure 2 shows the response of endogenous variables in the two regimes. As described above, these impulse responses are based on average parameters and can be interpreted as the response in a typical OECD country. The supply shock is associated with an increase in the oil price by an amount that is almost identical across regimes. The decline in oil production and inventories is also similar in the two states. On the other hand, global real activity declines by a substantially larger amount when the process is in the second regime. While the error bands are large, the median response of industrial production shows the opposite pattern – ip_t declines by a larger amount during the regime characterised by low and negative oil inflation. This result is mirrored in the response of stock prices, the variable of interest in our study. In regime 2, the null hypothesis of a zero response of s_t cannot be rejected. This result supports the conclusions reached by Kilian and Park (2009) who shows that oil supply shocks have a negligible effect on stock prices in the context of a linear VAR model. However, the response of stock prices in regime 1 is statistically different from zero. In fact, stock prices decline by about 1.5 percent in response to the shock in this regime indicating that responses from linear models may mask a more complex picture. The estimated error bands suggest that the response in this regime is systematically different from the response when oil inflation is in the high state. Thus, the results indicate that adverse oil supply shocks have a stronger negative impact on stock prices in the average OECD country during periods of low oil inflation.

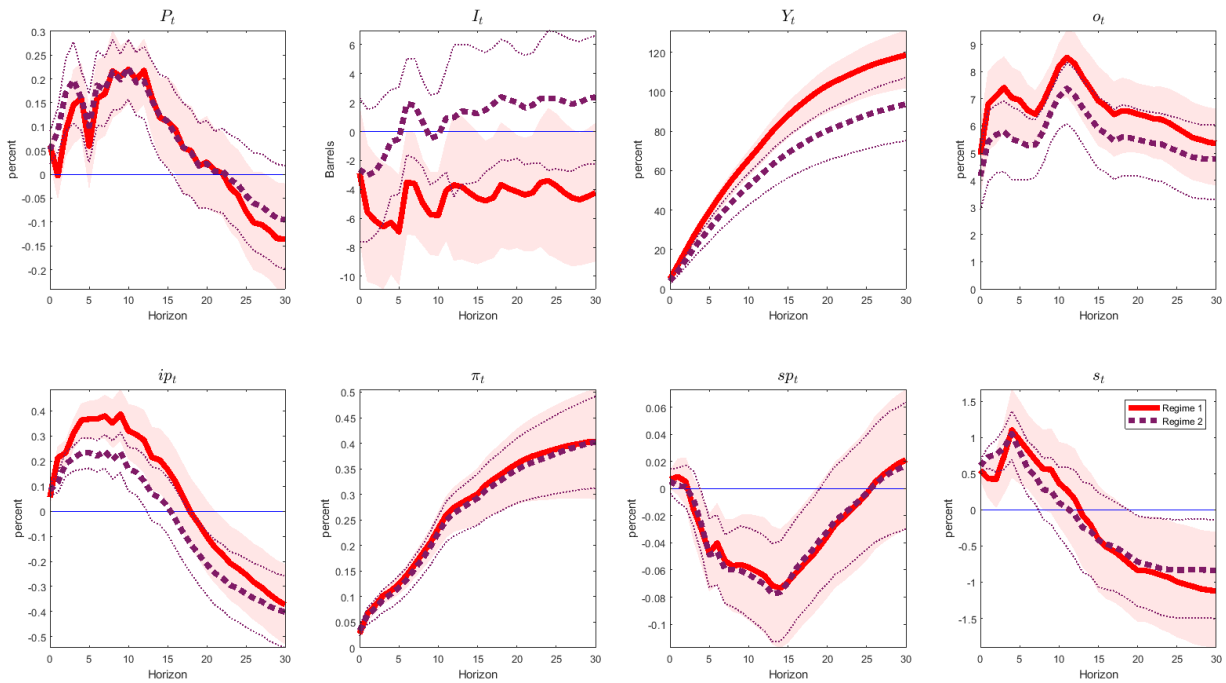

Figure 3: Cumulated impulse response to 1 SD positive oil demand shock in each regime. See notes to Figure 2.

In contrast to the response to oil supply shocks, there appears to be limited evidence of non-linear stock price dynamics in response of oil demand shocks driven by world activity (see Figure 3). The shock raises oil prices, oil production and world real activity where the regime 1 response of Y_t and o_t is estimated to be larger at long horizons. While there is weak evidence that ip_t increases by a larger amount in regime 1, the remaining impulse responses indicate that responses are almost identical across regimes.

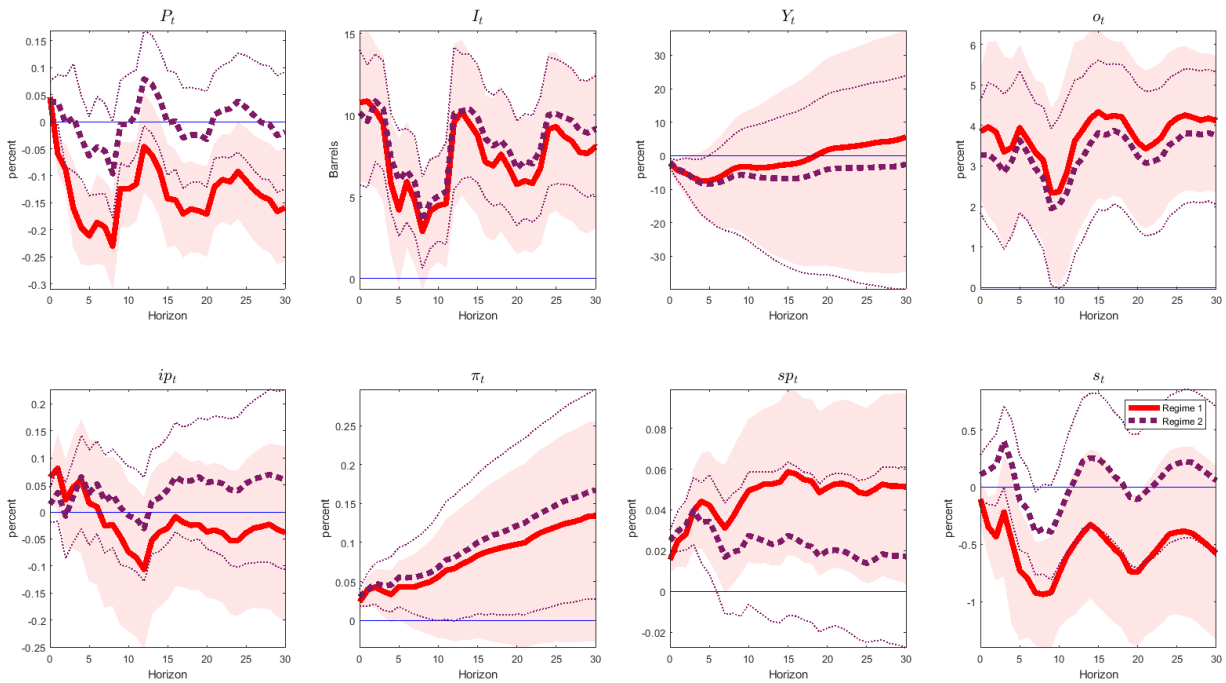

Figure 4: Cumulated impulse response to 1 SD adverse speculative oil demand shock in each regime. See notes to Figure 2.

Figure 4 displays the estimated response to a 1 standard deviation adverse speculative oil demand shock – the shock pushes up oil inventories and oil prices but reduces world activity and oil production. There is some evidence that the shock depresses stock prices by a larger amount in regime 1, with s_t showing a decline of about one percent. These results are close to the linear impulse responses presented in Kilian and Park (2009). In regime 2, however, the shock has a negligible impact on s_t. Thus, as in the case of oil supply shocks, the results suggest that stock prices react by a larger amount during the regime characterised by low oil inflation. However, the difference in the response of stock prices across regimes is smaller than in the case of oil supply shocks with some overlap in the estimated error bands.

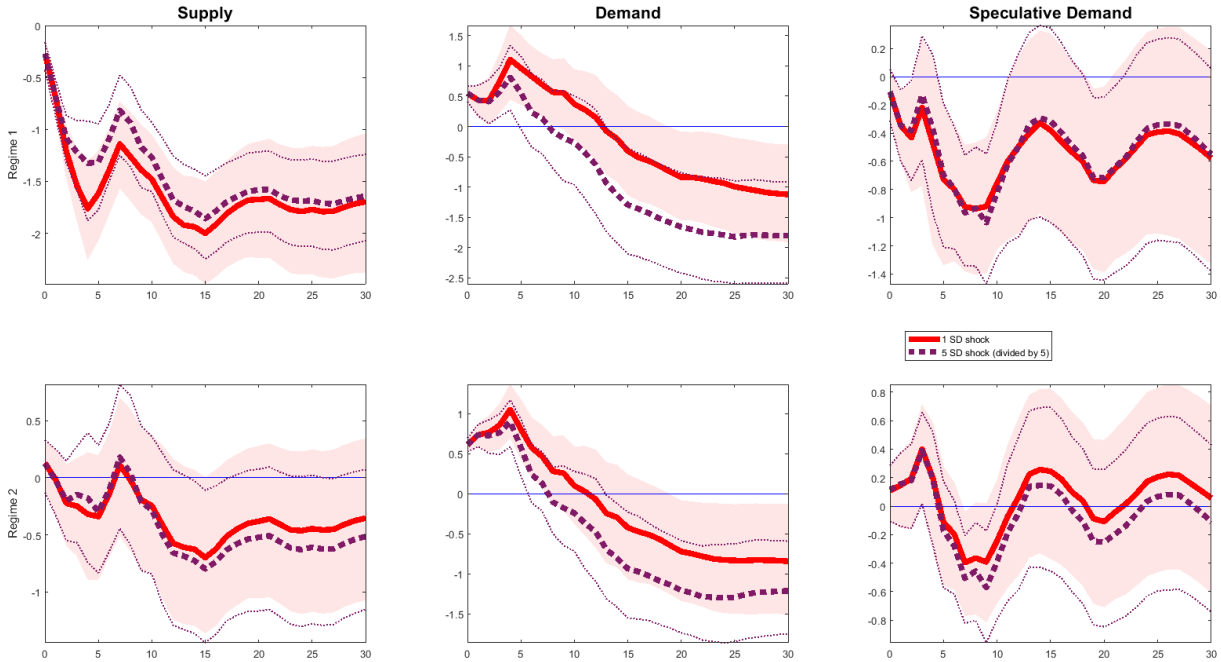

Figure 5: Cumulative response of stock prices to 1 SD and 5 SD shocks. The responses to 5 SD shocks are divided by 5 and then cumulated.

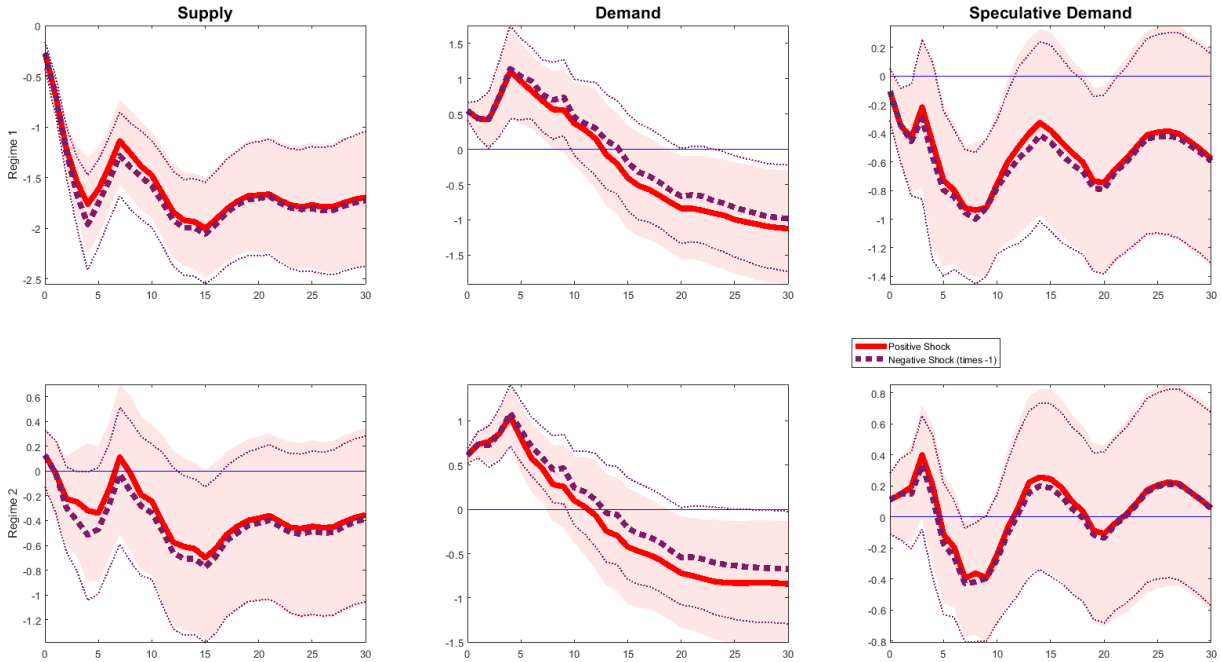

Figure 6: Cumulative response of stock prices to 1 SD and -1 SD shocks. The responses to negative shocks are multiplied by -1.

Figure 5 shows that evidence for the presence of non-linearity associated with the size of the shock is limited. The figure presents the cumulated response of stock prices to one and five standard deviation shocks. The responses to the larger shock are divided by 5 prior to cumulation for comparison. When considering supply shocks, there is weak evidence that the larger shock has a slightly smaller effect, proportionally, in regime 1 at short horizons. In the same regime, the positive effect of demand shocks is less persistent when the size of the shock is large and the decline in s_t begins earlier. There appears to be no evidence for size non-linearity when considering the response to speculative demand shocks. In Figure 6 we investigate if responses to positive and negative one standard deviation shocks display significant differences. It is immediately clear from the figure that this is not the case – the response of stock prices to positive and negative shocks is virtually identical.

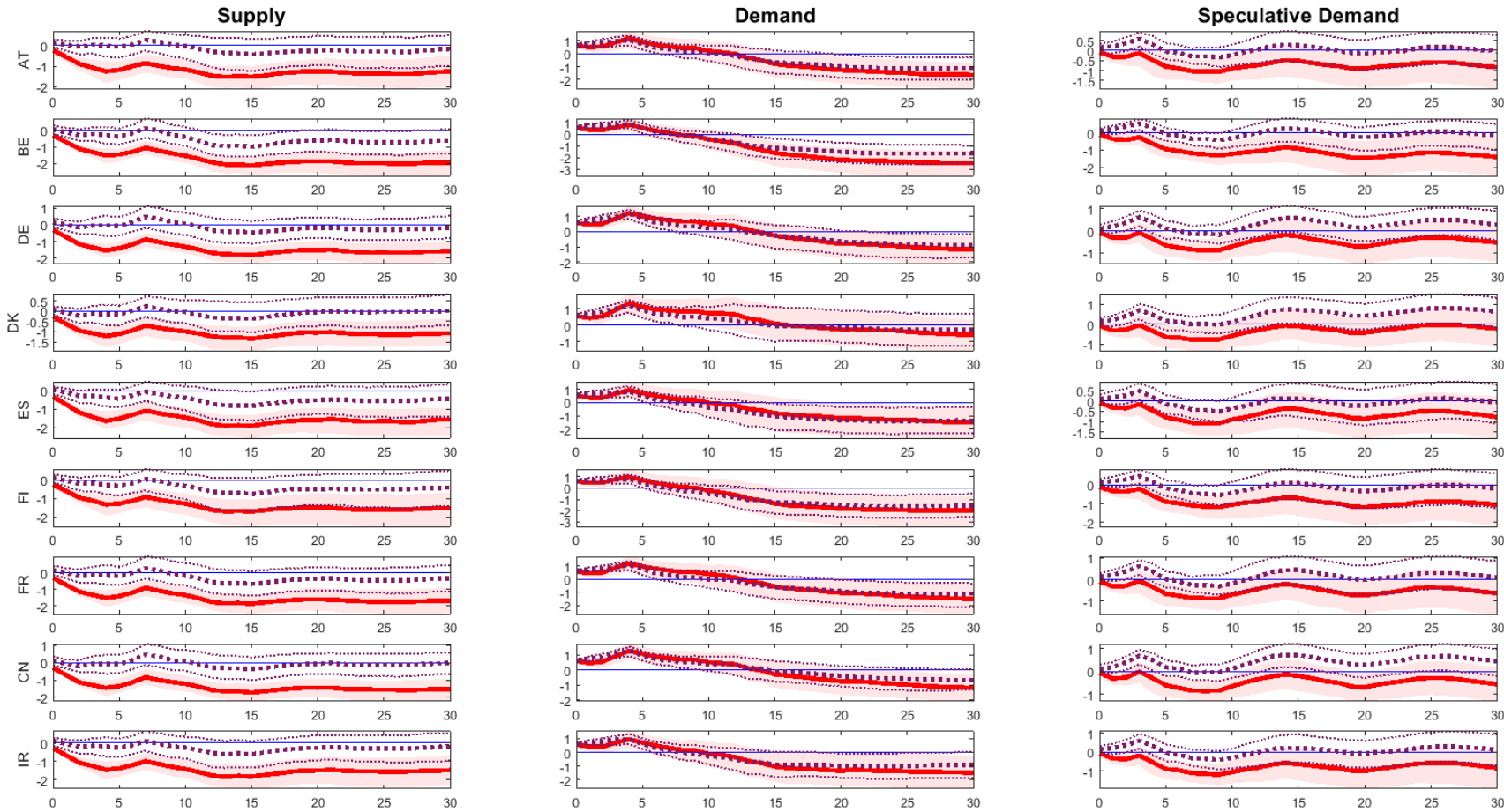

Figure 7: Country-specific responses of stock prices to oil shocks in regime 1 (red solid line and shaded area) and regime 2 (dotted lines)

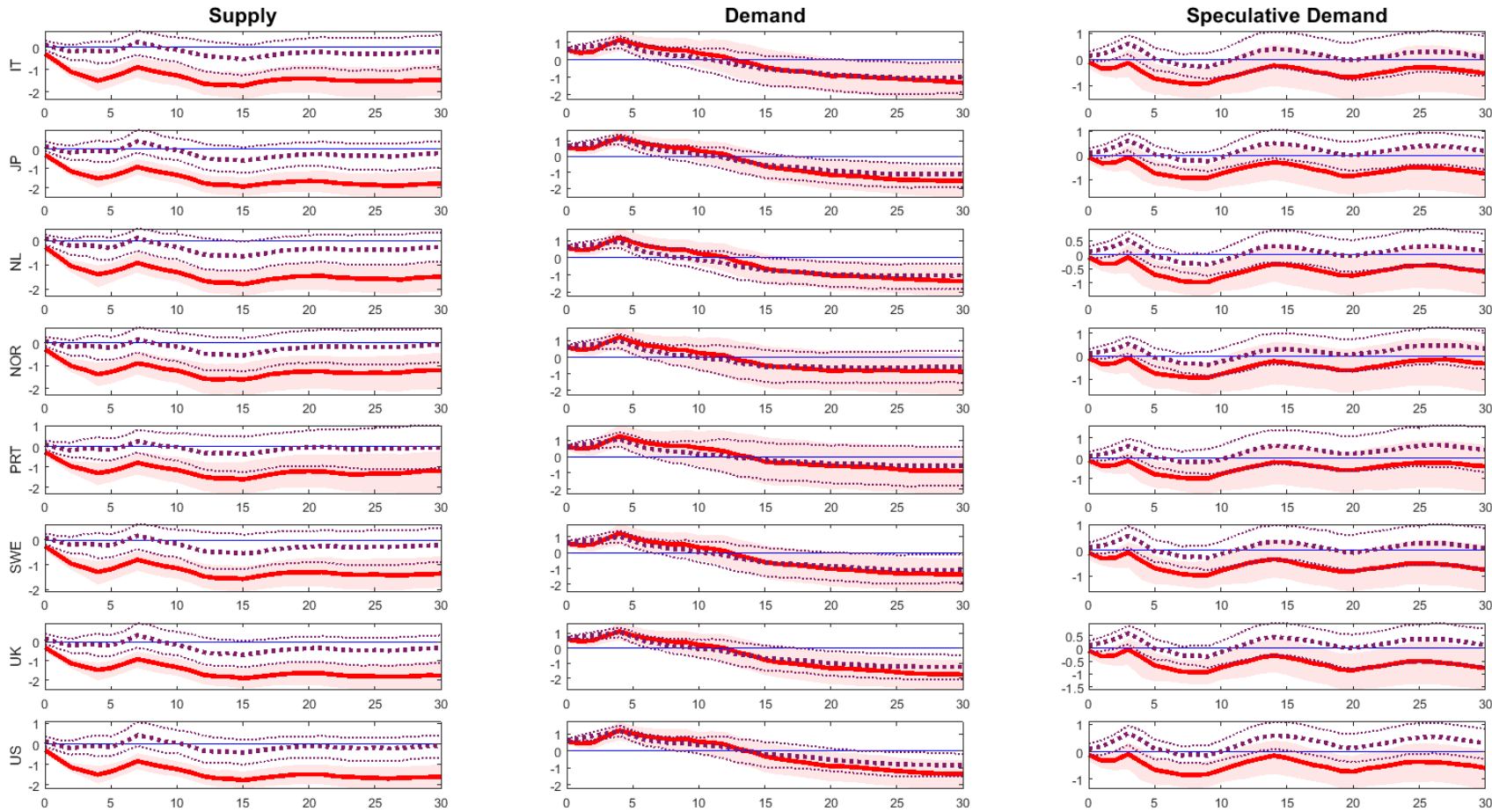

Figure 8: Country-specific responses of stock prices to oil shocks in regime 1 (red solid line and shaded area) and regime 2 (dotted lines)

Robustness The results presented above are based on the (cross-section) average model parameters. In Figures 7 and 8, we consider the regime-specific response of stock prices to oil shocks for each country in the panel. It is evident from the figures that the key results discussed above are present for all countries. First, the response of stock prices to oil supply shocks is negative and different from zero in regime 1 while the regime 2 response is close to zero or mildly positive. Second, the regime non-linearity appears to be important in the case of supply shocks. Differences in responses across regimes are less pronounced in the case of speculative demand shocks and almost non-existent in the case of demand shocks. There is a small amount of heterogeneity across countries in terms of the degree of non-linearity in response to oil supply shocks.[4] Canada, Japan and the US are examples of countries where the divergence between the responses across regimes is the largest. This divergence is estimated to be slightly smaller for European countries such as Austria, Norway, Denmark and Sweden. However, variation across countries does not appear to be an important feature of the results.

We carry out further robustness checks to test if the main results are sensitive to model specification choices. Results for these exercises are presented in the technical appendix. First, we truncate the end of the sample for each country to 2007 M12. The aim is to check if the recent financial crisis plays an important role in driving the main results regarding the differences in the response to oil shocks across regimes. Second, we follow Kilian and Park (2009) and identify the oil shocks using via a Cholesky decomposition with the variables ordered as P_t, Y_t, o_t, s_t.[5] As shown in the technical appendix, the response to oil supply shocks in these alternative models display the same feature as in the benchmark case – The regime 1 response of stock prices is negative while stock prices are close to zero or positive after this shock in regime 2.

3 Explaining the non-linear response to oil supply shocks

The empirical analysis above suggests one key result: *The response of stock prices to oil supply shocks is larger during the regime characterised by low oil inflation.* In this section, we use a DSGE model to discuss a possible explanation for this result. Note that our aim is to show that the empirical results do have a plausible economic explanation. Given the stylised nature of the model, it is, of course, not possible to

[4] We estimate the degree of non-linearity as the difference in the response across regimes at the 2 year horizon.
[5] This alternative model is estimated using these four variables only.

rule out alternative mechanisms that are also consistent with the results. However, the key features of the model proposed below (e.g. financial frictions) are now widely acknowledged as being crucial for an accurate description of the transmission mechanism.

The model employed here is very similar to the one developed by Blanchard and Gali (2010). This is a New Keynesian model where agents consume and supply labour. However, in this economy, consumption is an aggregate of non durable and oil consumption. Furthermore, oil is used by intermediate good producers in the production process. This implies that the CPI inflation is not only affected directly via oil prices but also indirectly; via the domestically generated inflation as oil prices enter the producers' marginal cost. Intermediate good producers have monopoly power over setting their prices. A fraction of those firms receives a random signal and set prices optimally, while the remaining fraction set prices based on a backward indexation rule. Monetary authorities set the policy rate based on a Taylor type rule.

In addition to all these features we add to the model a working capital financial friction that responds non-linearly to liquidity conditions. To be precise, the friction consists of two parts. The first part captures the standard working capital friction (as in Christiano *et al.* (2005) and Christiano *et al.* (2015)), where firms borrow a fraction of the wage and oil bill that needs to be paid in advance (before the production takes place). In addition to this, we allow this cost to increase as liquidity in the economy dries out or when the interest rate increases. This additional cost is at its minimum and constant when there is excess liquidity in the economy (i.e. when the interest is below the effective zero lower bound). The friction aims to capture the situation where firms try to get an advantage of cheap funding (low interest rates) and they get over leveraged. As the availability of financial resources decreases this has a nonlinear effect on the cost of additional finance as firms become more constrained.

The motivation behind introducing this friction is that equity prices not only reflect expectations about future profits but also the firms' financial and risk conditions. In other words, this reduced-form financial friction aims to reflect risk and liquidity premium dynamics at different levels of the policy rate (or liquidity). Bernanke and Kuttner (2005) show that monetary policy has an effect on equity prices primarily through its effect on excess returns. They find that excess returns decrease in response to an expansionary monetary policy shock. In contrast, excess returns increase in response to a contractionary policy. Drechsler *et al.*

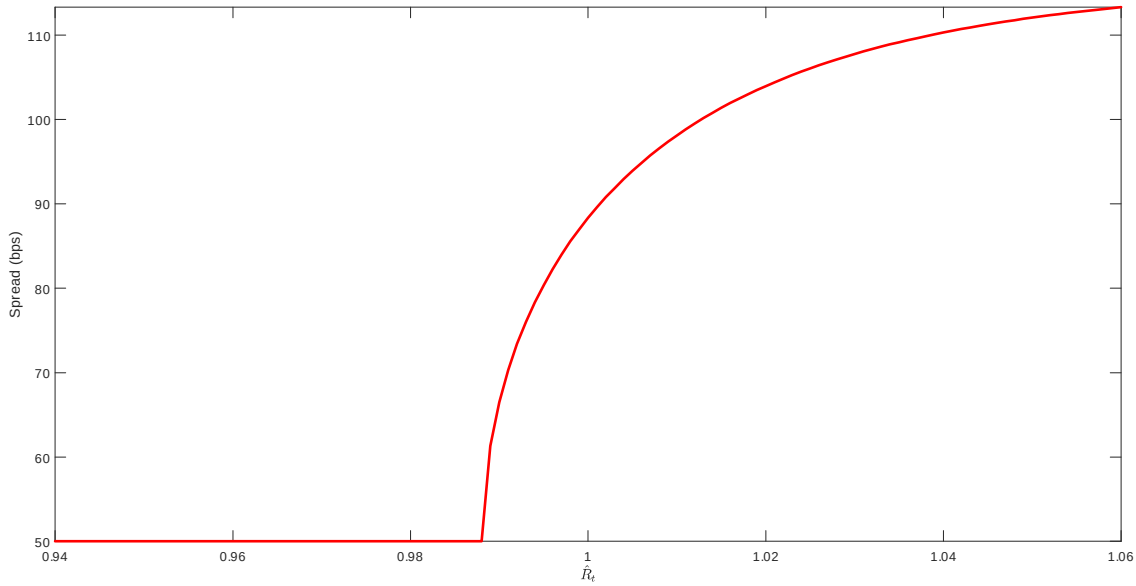

Figure 9: The figure plots quarterly credit spread against deviation of policy rate from its steady-state value (i.e. $\hat{R}_t$). Policy rate equals its steady-state value at $\hat{R}_t = 1$. The credit spread reaches its minimum value (i.e. 50 basis points) when the policy rate hits the zero lower bound (i.e. $\hat{R}_t = 0.99$).

(2018) develop an asset pricing model to show the same. Expansionary monetary policy decreases risk premium.

Several researchers have also provided evidence for the systematic risk-taking channel of monetary policy (Borio and Zhu (2012), Jimenez *et al.* (2014), Dell'Ariccia *et al.* (2017), Colletaza *et al.* (2018)). Others have also pointed to the protracted period of low interest rates, between 2002 to 2006, as a reason for buildup of risk in the financial system (Acharya and Richardson (2009), Diamond and Rajan (2009)). The increase in risk tolerance by financial institutions in periods of low policy rates, therefore, decreases risk premium.

Another strand of literature finds that an increase in policy rate has significant impact on default rates. Jacobson *et al.* (2013) show this for Swedish firms. They further find that the effect is stronger in sectors with highly leveraged firms. Gonzalez-Aguado and Suarez (2015) use a dynamic model to explain why an increase in policy rate may increase the default probability.

We capture these dynamics by assuming that the credit spread is an increasing and concave function in the policy rate. Figure 9 plots the credit spread function (see equation 16) for calibrated values in section 3.4. When the policy rate is low, credit spreads are low as well. An environment of low interest rate and low credit

spread encourages private sector to accumulate more debt. Subsequently, when policy rate increases, credit spreads increase substantially due to an increase in default probability. Credit spreads may also increase as financial institutions move away from investing in risky assets towards investing in safe assets.

However, worsening balance sheet conditions, following an increase in policy rate, also cause private sector to take corrective measures such as portfolio readjustment, de leveraging etc. The assumption of concavity in credit spreads is intended to capture this behaviour. The marginal increase in credit spread is higher when interest rates are low than when interest rates are high.

In rest of this section, we first explain the behavior of firms in the model followed by the behavior of households. We take third-order taylor approximation of the model solution to allow non-linearity in credit spread to affect model dynamics in response to an oil price shock.

3.1 Non-Oil goods producing firms

There is a continuum of firms $f \in [0, 1]$. Each firm produces a single differentiated good in an imperfectly competitive market. Firm-specific differentiated goods, $Y_{f,t}$, are then combined to produce a final good, Y_t. The production function for Y_t is given by:

$$Y_t = \left[\int_0^1 (Y_{f,t})^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}} \tag{9}$$

where θ is the elasticity of substitution. Unlike differentiated goods producing firms, final good producing firms are perfectly competitive. Profit maximisation by final goods producing firms gives the following demand function for each differentiated good:

$$Y_{f,t} = \left(\frac{P_{f,t}}{P_t^n} \right)^{-\theta} Y_t \tag{10}$$

where Y_t can also be interpreted as aggregate demand for differentiated goods and P_t^n is the aggregate price of non-oil goods. Under perfect competition, P_t^n is given by:

$$P_t^n = \left[\int_0^1 (P_{f,t})^{1-\theta} df \right]^{\frac{1}{1-\theta}} \tag{11}$$

The differentiated goods producing firms use labor and oil in the production process. We assume that oil is imported from the international market. We assume a Cobb-Douglas production function of the form:

$$Y_{f,t} = \left(A_t N_{f,t}\right)^{1-\mu} \left(O_{f,t}\right)^{\mu} - \Phi \tag{12}$$

where A_t is stationary labor-augmenting technology shock, $N_{f,t}$ is labor input used by firm f and $O_{f,t}$ is imported oil used by firm f. μ and Φ are oil-output elasticity and fixed cost, respectively.

Each firm minimises the cost subject to equation (12):

$$\min \bar{W}_t N_{f,t} + \bar{P}_t^o O_{f,t} \tag{13}$$

where $\bar{W}_t$ and $\bar{P}_t^o$ are effective nominal wage and effective nominal oil price, respectively. We follow Christiano *et al.* (2015) in assuming that firms finance a fraction of their input cost through borrowing. Effective input prices (i.e. $\bar{W}_t$ and $\bar{P}_t^o$) reflect these borrowing costs and take the following form:

$$\bar{W}_t = \left\{1 - \psi^n + \psi^n [R_t + \phi(\hat{R}_t)]\right\} W_t \tag{14}$$

$$\bar{P}_t^o = \left\{1 - \psi^o + \psi^o [R_t + \phi(\hat{R}_t)]\right\} P_t^o \tag{15}$$

where W_t and P_t^o are market prices for labor and oil, respectively. $\hat{R}_t$ is the deviation of interest rate from its steady-state (i.e. $\frac{R_t}{\bar{R}}$). ψ^j, for $j = n, o$, is a fraction of the cost of input j financed through borrowing.

When ψ^j equals zero, we recover the baseline model in Blanchard and Gali (2010).

The working capital channel in this paper differs from Christiano *et al.* (2015) in the argument $\phi(.)$. $\phi(.)$ captures credit spread which firms pay on top of risk-free rate, R_t. We assume the following functional form for credit spread:

$$\phi(.) = \kappa + \sqrt{\frac{\hat{R}_t^{1-\epsilon} - \gamma}{1000 * (1-\epsilon)}} \tag{16}$$

where κ puts a minimum limit on the value of $\phi(.)$. For our purpose, $\kappa \geq 1$. This ensures that the credit spread does not take a negative value. γ determines the lowest value of the interest rate, relative to its steady-state, at which the economy enters the state of lowest credit spreads. ϵ affects how quickly diminishing marginal spread sets in. Figure 9 plots equation 16 as a function of $\hat{R}_t$.

Firms take effective input prices as given and choose the quantity of inputs which minimise costs subject to the production function. The cost minimisation problem then gives the following expression for marginal costs:

$$MC_t = \left(\frac{1}{1-\mu}\right)^{1-\mu} \left(\frac{1}{\mu}\right)^{\mu} \left(\frac{\bar{W}_t}{A_t}\right)^{1-\mu} \left(\bar{P}_t^o\right)^{\mu} \tag{17}$$

where MC_t is marginal costs and is independent of f. Differentiated goods producing firms then set a price which maximises her discounted profits.

We assume that firms set their prices according to the Calvo mechanism as explained in Calvo (1983). Specifically, in a given period, only a fraction of firms can set their price to the desired level. We further assume that firms which are not able to adjust their price index their price by last period's inflation. Firms' profit maximisation problem gives the following non-linear Phillips Curve:

$$g_{1,t} = \lambda_t \frac{MC_t}{P_t^n} Y_t^d + \beta\zeta E_t \left[\frac{\pi_{n,t}^{\iota}}{\pi_{n,t+1}}\right]^{-\theta} g_{1,t+1} \tag{18}$$

$$g_{2,t} = \lambda_t \Pi^*_{n,t} Y^d_t + \beta\zeta E_t \left[\frac{\pi^{\iota}_{n,t}}{\pi_{n,t+1}}\right]^{1-\theta} \frac{\Pi^*_{n,t}}{\Pi^*_{n,t+1}} g_{2,t+1} \tag{19}$$

$$\theta g_{1,t} = (\theta - 1) g_{2,t} \tag{20}$$

where β is discount factor, ζ is the fraction of firms which cannot change their price in a given period and ι is the degree of indexation in firms pricing decision. $\Pi^*_{n,t}$ is the reset price and $\pi_{n,t}$ is aggregate inflation in the non-oil sector.

Oil prices are determined in the international market and are exogenous to the domestic economy. The empirical exercise in section 2 focuses on the effect of innovations to nominal oil prices. To be consistent, we assume that the shock affects nominal oil price inflation rather than real oil price. The expression for real oil price is:

$$p^o_t = \frac{p^o_{t-1}}{\pi_t} \pi_{o,t} \tag{21}$$

where $\pi_{o,t}$ is oil price inflation and follows an AR(1) process of the form:

$$\pi_{o,t} = \rho_o \pi_{o,t-1} + \epsilon_{o,t} \tag{22}$$

where $\epsilon_{o,t}$ is an i.i.d. shock $\sim N(0, \sigma_o)$.

3.2 Households

There is also a continuum of households $h \in [0, 1]$. Households choose the path for consumption and labor supply which maximises their lifetime utility. We assume a CRRA utility function which is seperable in consumption and hours worked:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\varphi}}{1+\varphi}\right] \tag{23}$$

where σ and φ are the inverse of elasticity of substitution and inverse Frisch labor supply elasticity, respectively. C_t is the final consumption good and N_t is hours worked. C_t includes both oil and non-oil goods and takes the following form:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} {C_{n,t}}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} {C_{o,t}}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{24}$$

where $C_{n,t}$ and $C_{o,t}$ represents consumption of non-oil and oil, respectively. As in the case of firms, oil used in households consumption is imported as well. α is the weight on oil in households consumption basket. η is the elasticity of substitution between oil and non-oil goods. Demand for oil and non-oil goods is given by:

$$C_{n,t} = (1-\alpha)\left(\frac{P_{n,t}}{P_t}\right)^{-\eta} C_t \tag{25}$$

$$C_{o,t} = \alpha\left(\frac{P_{o,t}}{P_t}\right)^{-\eta} C_t \tag{26}$$

The price index associated with equation (24) is:

$$P_t = \left[(1-\alpha){P_{n,t}}^{1-\eta} + \alpha {P_{o,t}}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{27}$$

Households maximise (23) subject to the following budget constraints:

$$P_t C_t + P_t^{eq} S_t + B_t \leq W_t N_t + R_{t-1} B_{t-1} + (P_t^{eq} + \Pi_t^{eq}) S_{t-1} - T_t \tag{28}$$

where B_t is bonds, S_t is equity shares and T_t is lump-sum taxes. Each equity share has a price of P_t^{eq} and also pays a dividend of Π_t^{eq}. We assume that firms are owned by households and, therefore, all of firms' profit

is transferred to households in the form of dividend. The solution to households maximisation problem with respect to S_t gives the following expression for equity prices:

$$p_t^{eq} = \beta M_{t+1}(p_{t+1}^{eq} + \pi_{t+1}^{eq}) \tag{29}$$

where p_t^{eq} is the real price of equity share and π_t^{eq} is dividend in real terms. M_{t+1} is the stochastic discount factor (i.e. λ_{t+1}/λ_t) between period t and $t+1$. λ_t is the marginal utility of wealth.

The complete set of model equations are given in the technical appendix to the paper.

3.3 Monetary Policy

The central bank conducts policy according to a Taylor-type rule. We also assume that the central bank targets core inflation, $\pi_{n,t}$, instead of aggregate inflation.[6] Clark and Terry (2010) show that the Fed's responsiveness to oil price shocks has decreased since 1985. Furthermore, Blinder and Reis (2005) and Mehra and Sawhney (2010) show that a Taylor-type rule with core inflation targeting tracks the observed policy rate better than with inflation targeting.

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_r} \left[\left(\frac{\pi_{n,t}}{\bar{\pi}}\right)^{r_\pi} \left(\frac{Y_t}{Y^{ss}}\right)^{r_y}\right]^{1-\rho_r} \tag{30}$$

where $\bar{R}$ and $\bar{\pi}$ are steady-state interest rate and inflation, respectively. Y^{ss} is steady-state output. ρ_r, r_π and r_y are constant parameters.

3.4 Model Calibration

We calibrate model parameters to study the affect of oil price shock on equity prices. Calibrated values of model parameters are provided in Table 2. We first discuss parameters governing the working capital channel. ψ^j equals 0.7. This is similar to the value assumed in Christiano *et al.* (2011). The calibrated value of κ at 0.005 ensures that annual credit spread does not go below 100 basis points. ϵ and γ determine

[6] Our results are not sensitive to assuming core inflation targeting.

Parameters	Values	Parameters	Values
β	0.99	θ	11
ζ	0.60	ι	0.25
σ	2	σ_l	2
ψ	8.5	η	0.5
ψ^o	0.7	ψ^n	0.7
μ	0.02	α	0.02
κ	0.005	γ	5
ϵ	35	r_π	1.5
r_y	0.7	ρ_r	0.7
ρ_o	0.5	σ_o	0.03

Note: *This table provides calibrated values of structural parameters in the model.*

Table 2: **Model Parameters**

curvature of the spread function and also the policy rate at which credit spread reaches its minimum. We calibrate the two parameters such that annual credit spread at the steady-state is close to 4%. Moreover, credit spread reaches its minimum at the effective zero lower bound (i.e. $\hat{R}_t = 0.99$).

In line with literature, we assume complementarity between oil and non-oil consumption goods. Elasticity of substitution between oil and non-oil goods (i.e. η) equals 0.5. The weight on oil in households consumption (i.e. α) and output-oil elasticity in firms production function (i.e. μ) is calibrated to equal 0.02. Finally, the persistance parameter of oil price inflation shock (i.e. ρ_o) takes a value of 0.5. We calibrate σ_o to match the empirical response of a 3% increase in oil price inflation at impact. Therefore, σ_o equals 0.03. Calibrated values for remaining parameters are in line with the New Keynesian literature.

3.5 Results

We use third-order taylor approximation of model solution to compute impulse responses to an oil price inflation shock. We compute two sets of IRFs at different points on the credit spread function: one when the policy rate is 0% (i.e. $\hat{R}_t = 0.99$) and another when the policy rate is 6% (i.e. $\hat{R}_t = 1.02$). These reflect two different inflationary regimes. A low inflationary regime (with low oil price inflation) is consistent with a lower policy rate and vice versa.

Figure 10 plots IRFs at different points on the credit spread function. An increase in oil price inflation increases both aggregate and core inflation. Since monetary authorities want to stabilise inflation, policy rate increases. The contractionary effect of an oil price shock decreases firms' profit. Since equity prices

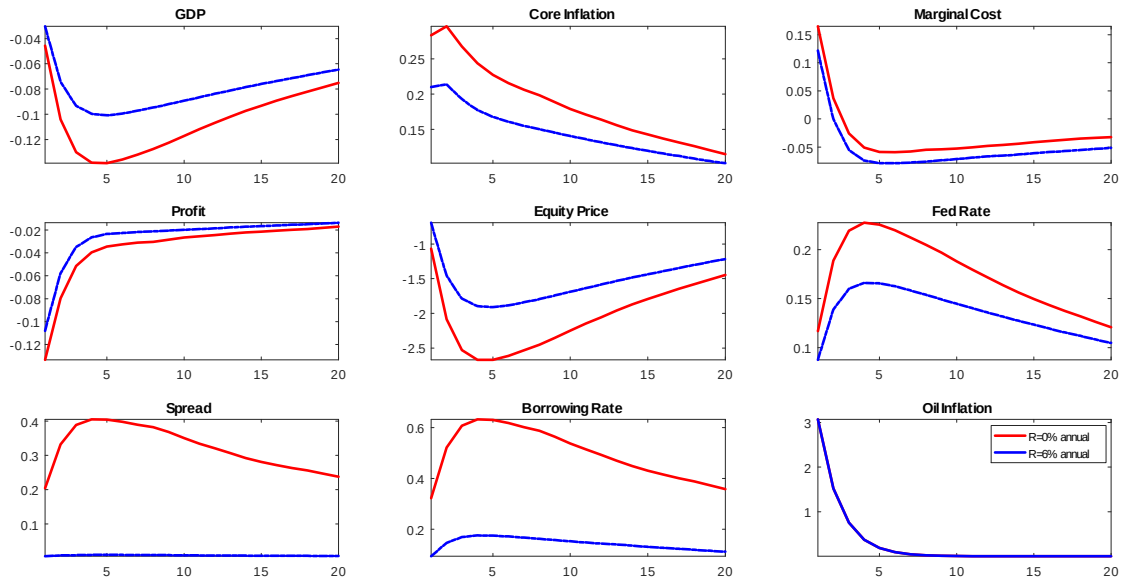

Figure 10: The figure plots IRFs to an exogenous shock to oil price inflation. The red line plots IRFs when annual interest rate equals 0%. The blue line plots IRFs when annual interest rate equals 6%.

equal the sum of discounted value of firms' expected profit, equity prices fall as well.

However, the implication of oil price shock for equity prices depend significantly on where the economy is on the credit spread function. In line with empirical results in this paper, equity prices decline significantly more in a low inflationary and, therefore, low interest rate environment. When the policy rate is at 0%, an increase in the policy rate increases credit spread by significantly more. As a result, borrowing costs increase substantially. Since firms use borrowing to finance their working expenditure, higher borrowing costs increase firms' marginal costs and, consequently, decrease firms' profit by more. On the other hand, when policy rate is at 6%, a further increase in the policy rate has a limited effect on the credit spread and borrowing costs increase by relatively less. Therefore, equity prices decline almost twice as much under a low inflation and low interest rate environment than otherwise.

4 Conclusions

This paper uses a panel TVAR model to estimate the regime-dependent impact of oil shocks on stock prices. The use of a hierarchical prior in the proposed model allows us to estimate the impulse responses for the

average OECD country making use of both the time-series and cross-section dimension of the data. Our estimates indicate strong evidence of non-linearity in the response of stock prices to oil supply shocks. In particular, adverse oil supply shocks have a negative impact on stock prices when oil inflation is low. In contrast, this effect is statistically equal to zero in the regime characterised by higher oil price inflation. Using a simple DSGE model, we suggest that the explanation for this result may be tied to the behaviour of credit spreads. When oil inflation and aggregate inflation is low, lower policy rates encourage firms to get highly leveraged. An adverse oil shock in this situation leads to large increase in spreads, cutting profits and equity prices. In contrast, at higher rates of oil/aggregate inflation, spreads are less affected by the oil shock, ameliorating the impact on the stock market.

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Non-linear effects of oil shocks on stock prices.

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Abstract

Appendix

1 Estimation

$$
\begin{aligned}
Z_{it} &= \left(c_{1i} + \sum_{j=1}^{P} b_{1i,j} Z_{it-j} + u_{it}\right) S_{it} + \\
&\quad \left(c_{2i} + \sum_{j=1}^{P} b_{2i,j} Z_{it-j} + u_{it}\right) (1 - S_{it})
\end{aligned} \tag{1}
$$

where $S_{it} = 1 \Longleftrightarrow z_{it-d_i} \leq z_i^*$.
Here $i = 1, 2, ..M$ denotes the M countries included in the panel.
The covariance of errors is defined as

$$
\begin{aligned}
var\left(u_{it}\right) &= \Sigma_{it} = S_{it} \odot \Sigma_{1i} + (1 - S_{it}) \odot \Sigma_{1i} \\
\Sigma_{1i} &= A_{1i}^{-1} H_{1i} A_{1i}^{-1\prime} \\
\Sigma_{2i} &= A_{2i}^{-1} H_{2i} A_{2i}^{-1\prime}
\end{aligned}
$$

where A_{1i}, A_{2i} are lower triangular and $H_{1i} = diag\left(h_{1i}\right), H_{2i} = diag\left(h_{2i}\right)$ are diagonal matrices with the variances of the orthogonal shocks $\left(h_{1i} = [h_{1i}^{(1)}, .., h_{1i}^{(N)}], h_{2i} = [h_{2i}^{(1)}, .., h_{2i}^{(N)}]\right)$ on the main diagonal. Here $\odot$ denotes element by element multiplication

1.1 Priors

Collect the slope coefficients in the following $\bar{K} \times 1$ vectors $\beta_{1,i} = vec\left([b_{1i,1}, .., b_{1i,P}]\right)$ and $\beta_{2,i} = vec\left([b_{2i,1}, .., b_{2i,P}]\right)$. Denote the vectorised non-zero and non-one elements in A_{1i}, A_{2i} as a_{1i}, a_{2i}. The model assumes the following hierararchical priors

$$
\begin{aligned}
p\left(\beta_{1,i} | \bar{\beta}_1, \lambda_1\right) \tilde{} N\left(\bar{\beta}_1, \lambda_1 \Lambda_i\right) \\
p\left(\beta_{2,i} | \bar{\beta}_2, \lambda_2\right) \tilde{} N\left(\bar{\beta}_2, \lambda_2 \Lambda_i\right)
\end{aligned}
$$

where $\bar{\beta}_1$ and $\bar{\beta}_2$ are the (weighted) cross-sectional average coefficients in the two regimes and Λ_i is set according to the Minnesota procedure. The parameter λ controls the degree of pooling in the model. As $\lambda \to 0$ the heterogeneity across states declines. In order to set the variances Λ_i, we use dummy observations as in Banbura *et al.* (2010), setting the overall prior tightness parameter to 1.

The prior for λ_1, λ_2 is assumed to be inverse Gamma:$p\left(\lambda_k\right) \tilde{} IG\left(S_0, V_0\right)$ where $S_0 = 0$ and $V_0 = -1$ and $k = 1, 2$. As discussed in Gelman (2006), this prior corresponds to a uniform prior on the standard deviation.

Similarly, a hierararchical prior is set for a_{1i}, a_{2i}:

$$
\begin{aligned}
p\left(a_{1,i} | \bar{a}_1, \delta_1\right) \tilde{} N\left(\bar{a}_1, \delta_1 \Xi_i\right) \\
p\left(a_{2,i} | \bar{a}_2, \delta_2\right) \tilde{} N\left(\bar{a}_2, \delta_2 \Xi_i\right)
\end{aligned}
$$

where $\bar{a}_1$ and $\bar{a}_2$ are weighted cross-sectional averages and Ξ_i equals a matrix with $10 \times abs\left(a_{i,ols}\right)$ on the main diagonal. $a_{i,ols}$ represents the non-zero and non-one elements of preliminary estimate of the contemporaneous impact matrix obtained via OLS. The degree of pooling is controlled by δ.

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The prior for δ_1, δ_2 is assumed to be inverse Gamma: $p(\delta_k) \tilde{} IG(s_0, v_0)$ where $s_0 = 0$ and $v_0 = -1$ and $k = 1, 2$.
We also assume a hierarchical prior for the threshold z_i^*. That is:

$$p(z_i^* | \bar{z}, \varpi) \tilde{} N(\bar{z}, \varpi \Psi_i)$$

Here $\bar{z}$ denotes the weighted cross sectional average of the threshold level and the degree of pooling is controlled by ϖ. Ψ_i is set equal to $abs(median(z_{it-d})) \times 10$. The prior for ϖ is $IG(\tilde{s}_0, \tilde{v}_0)$ where $\tilde{s}_0 = 0$ and $\tilde{v}_0 = -1$

We assume a normal prior for the intercepts: $p(c_K) \tilde{} N(c_0, \Lambda_C)$ where Λ_C is a diagonal matrix with 10,000 on the main diagonal. The prior for h_{1i}, h_{2i} is inverse Gamma: $IG(h_0, v_h)$ where $h_0 = 0.1$ and $v_h = 1$.

1.2 Gibbs Sampling Algorithm

The Gibbs algorithm is based on Jarocinski (2010). It draws from the following conditional posterior distributions (Ξ^* denotes all remaining parameters):

1. $G(\beta_{1,i} | \Xi^*)$. Denote the observations for country i in regime 1 as This conditional posterior is normal: $N(M, V)$ where

$$\begin{aligned} V &= \left((\lambda_1 \Lambda_i)^{-1} + \Sigma_{1i}^{-1} \otimes X_{1,i}' X_{1,i} \right)^{-1} \\ M &= V \left((\lambda_1 \Lambda_i)^{-1} \bar{\beta}_1 + \Sigma_{1i}^{-1} \otimes X_{1,i}' (Y_{1,i} - c_{1,i}) \right) \end{aligned}$$

   where $Y_{1,it}$ and $X_{1,it}$ denote the left and the right hand side of the VAR model for country i with data selected for regime 1, i.e. over periods when $S_{it} = 1$. The number of observations in the regime are denoted by $T_{1,i}$.

2. $G(c_{1,i} | \Xi^*)$. The conditional posterior is normal: $N(m, v)$ where

$$\begin{aligned} v &= \left(\Lambda_C^{-1} + \Sigma_{1i}^{-1} \otimes x_{1,i}' x_{1,i} \right)^{-1} \\ m &= v \left(\Lambda_C^{-1} c_0 + \Sigma_{1i}^{-1} \otimes x_{1,i}' vec \left(Y_{1,i} - X_{1,i} \tilde{\beta}_{1,i} \right) \right) \end{aligned}$$

   where $x_{1,i} = 1_{T_{1,i} \times 1}$ and $\tilde{\beta}_{1,i}$ denotes $\beta_{1,i}$ reshaped to be comformable with $X_{1,i}$.

2. $G(a_{1i} | \Xi^*)$. Denote the residuals in regime 1 as $E_{1,it} = Y_{1,i} - X_{1,i} \tilde{\beta}_{1,i} - c_{1,i}$. Then the model in regime 1 can be written as $A_{1i} E_{1,it} = H_{1i}^{1/2} U_{1,it}$ where $U_{1,it}$ is $N(0, 1)$. This is a system of linear equations. The kth equation is $E_{1,it}(k) = -\alpha E_{1,it}(-k) + H_{1i}^{1/2}(K) U_{1,it}(k)$ where k in the parenthesis denotes the kth column while $-k$ denotes columns 1 to $k-1$ and α denotes the relevant elements of a_{1i}. The draw from this conditional posterior thus requires drawing the coefficients of a series of linear regressions. As is well known, the conditional posterior is normal with mean and variance M^* and V^* :

$$\begin{aligned} M^* &= \left(\left(\delta_1 \Xi_i^{(k)} \right)^{-1} + \frac{1}{H_{1i}(k)} E_{1,it}(-k)' E_{1,it}(-k) \right)^{-1} \left(\left(\delta_1 \Xi_i^{(k)} \right)^{-1} \bar{a}_1^{(k)} + \frac{1}{H_{1i}(k)} E_{1,it}(-k)' E_{1,it}(k) \right) \\ V^* &= \left(\left(\delta_1 \Xi_i^{(k)} \right)^{-1} + \frac{1}{H_{1i}(k)} E_{1,it}(-k)' E_{1,it}(-k) \right)^{-1} \end{aligned}$$

   Note that the superscript (k) denotes the fact that priors corresponding to the parameters of the kth equation are used.

3. $G(h_{1i} | \Xi^*)$. As discussed in step 3, the model in regime 1 can be written as $A_{1i} E_{1,it} = \tilde{U}_{1,it}$ where $\tilde{U}_{1,it} \tilde{} N(0, h_{1i})$. The conditional posterior is inverse Gamma with posterior scale parameter $\tilde{U}_{1,it}' \tilde{U}_{1,it} + h_0$ and degrees of freedom $T_{1,i} + v_h$.

5. $G(\beta_{2,i} | \Xi^*)$. The form of the conditional posterior is as defined in step 1.

4. $G(c_{2,i} | \Xi^*)$. The form of the conditional posterior is as defined in step 2.

5. $G(a_{2,i} | \Xi^*)$. The form of the conditional posterior is as defined in step 3.

6. $G(h_{2,i} | \Xi^*)$. The form of the conditional posterior is as defined in step 4.

9. $G(z_i^* | \Xi^*)$. The threshold value is drawn using a Metropolis Hastings step. We draw candidate value of $z_{i,new}^*$ from $z_{i,new}^* = z_{i,old}^* + \Psi_i^{1/2} \epsilon$, $\epsilon \sim N(0, 1)$. The acceptance probability is given by $F\left(Z_{it} \middle| z_{i,new}^*, \Xi^*\right) / F\left(Z_{it} \middle| z_{i,old}^*, \Xi^*\right)$, where $F(.)$ denotes the posterior density: $F(Z_{it} | z_i^*, \Xi^*) \propto f(Z_{it} | z_i^*, \Xi^*) p(z_i^* | \bar{z}, \varpi)$ where $f(.)$ is the likelihood function of the VAR model for country i. Note that the likelihood function is simply the product of the likelihood in the two regimes. The scale Ψ_i is chosen to ensure that the acceptance rate is between 20% and 50%.

7. $G(d_i | \Xi^*)$. Chen and Lee (1995) show that the conditional posterior for d is a multinomial distribution with probability $f(Y_t | d_i, \Xi^*) / \sum_{d_i=1}^{d_{i,\max}} f(Y_t | d_i, \Xi^*)$, where $d_{i,\max}$ denotes the maximum delay allowed for.

11. $G(\lambda_1 | \Xi^*)$. The form of the conditional posterior is inverse Gamma with scale parameter $\sum_{i=1}^{M} (\beta_{1,i} - \bar{\beta}_1) \Lambda_i^{-1} (\beta_{1,i} - \bar{\beta}_1)' + S_0$ and degrees of freedom $\left(M \times \bar{K}\right) + V_0$

12. $G\left(\lambda_2|\Xi^*\right)$. The form of the conditional posterior is as defined in step 6.

13. $G\left(\bar{\beta}_1|\Xi^*\right)$. By the Bayes Theorem, $G\left(\bar{\beta}_1|\beta_1,\lambda_1\right) \propto p\left(\beta_1|\bar{\beta}_1,\lambda_1\right) p\left(\bar{\beta}_1\right)$ with $\beta_1 = [\ \beta_{1,1},\beta_{1,2},..,\beta_{1,N}]$. This density is normal as $p\left(\beta_1|\bar{\beta}_1,\lambda_1\right)$ is normal and product of the normal priors for each i. With a flat prior for $\bar{\beta}_1$ this density is given by $N\left(\bar{M},\bar{V}\right)$:

$$\begin{aligned}\bar{V} &= \left(\frac{1}{\lambda_1}\sum_{i=1}^{M}\Lambda_i^{-1}\right)^{-1}\\ \bar{M} &= \bar{V}\left(\frac{1}{\lambda_1}\sum_{i=1}^{M}\Lambda_i^{-1}\beta_{1,i}\right)\end{aligned}$$

14. $G\left(\bar{\beta}_2|\Xi^*\right)$. The form of the conditional posterior is as defined in step 13 above.

15. $G\left(\delta_1|\Xi^*\right)$. As in step 11, this conditional posterior is inverse Gamma with scale parameter $\sum_{i=1}^{M}\left(a_{1,i}-\bar{a}_1\right)\Lambda_i^{-1}\left(a_{1,i}-\bar{a}_1\right)' + s_0$ and degrees of freedom $\left(M\times\left(\frac{N\times(N-1)}{2}\right)\right)+v_0$

16. $G\left(\delta_2|\Xi^*\right)$. The form of the conditional posterior is as defined in step 15 above.

17. $G\left(\bar{a}_1|\Xi^*\right)$. The form of the conditional posterior is as defined in step 13. The conditional posterior is normal $N\left(\bar{m},\bar{v}\right)$ where:

$$\begin{aligned}\bar{v} &= \left(\frac{1}{\delta_1}\sum_{i=1}^{M}\Xi_i^{-1}\right)^{-1}\\ \bar{m} &= \bar{v}\left(\frac{1}{\delta_1}\sum_{i=1}^{M}\Xi_i^{-1}a_{1,i}\right)\end{aligned}$$

18. $G\left(\bar{a}_2|\Xi^*\right)$. The form of the conditional posterior is as defined in step 17 above.

8. $G\left(\varpi|\Xi^*\right)$. As above, this variance has an inverse Gamma conditional posterior with scale parameter $\sum_{i=1}^{M}\left(z_i^*-\bar{z}\right)\Psi_i^{-1}\left(z_i^*-\bar{z}\right)'+\tilde{s}_0$ and degrees of freedom $M+v_0$.

9. $G\left(\bar{z}|\Xi^*\right)$. Given the normal prior on z_i^*, the conditional posterior of $\bar{z}$ is normal $N\left(m_z,v_z\right)$ where:

$$\begin{aligned}v_z &= \left(\frac{1}{\varpi}\sum_{i=1}^{M}\Psi_i^{-1}\right)^{-1}\\ m_z &= v_z\left(\frac{1}{\varpi}\sum_{i=1}^{M}\Psi_i^{-1}z_i^*\right)\end{aligned}$$

1.2.1 A Monte-Carlo experiment

In order to test the algorithm and the code we conduct a simple Monte-Carlo experiment. The follow DGP is used:

$$\begin{aligned}Z_{it} &= \left(c_{1i}+\sum_{j=1}^{2}b_{1i,j}Z_{it-j}+u_{it}\right)S_{it}+\\ &\left(c_{2i}+\sum_{j=1}^{2}b_{2i,j}Z_{it-j}+u_{it}\right)\left(1-S_{it}\right)\end{aligned} \tag{2}$$

where $Z_{it}=[Y_{i,t},X_{i,t},D_{it}]$ and $S_{it}=1$ if $Y_{i,t-1}\leq Y_i^*$ for $i=1,2,...,N$. We assume that the number of cross-sections is $N=10$.

The shock variances are defined as:

$$\begin{aligned}var\left(u_{it}\right) &= \Sigma_{it}=S_{it}\odot\Sigma_{1i}+\left(1-S_{it}\right)\odot\Sigma_{1i}\\ \Sigma_{1i} &= A_{1i}^{-1}H_{1i}A_{1i}^{-1\prime}\\ \Sigma_{2i} &= A_{2i}^{-1}H_{2i}A_{2i}^{-1\prime}\end{aligned}$$

The non-zero and non-one elements of A_{1i},A_{2i} denoted by a_{1i},a_{2i} are generated from $N\left(\bar{a}_1,0.001\right)$ and $N\left(\bar{a}_2,0.001\right)$, respectively where:

$$\bar{a}_1=\begin{pmatrix}0.2\\0.1\\-0.2\end{pmatrix},\bar{a}_2=-\bar{a}_1$$

we assume that $H_{1i}=H_{2i}=1$.

The VAR coefficients in each regime are generated from the following normal distributions: $N\left(\bar{\beta}_1, 0.001\right)$ and $N\left(\bar{\beta}_2, 0.001\right)$

$$\bar{\beta}_1 = \begin{pmatrix} 0.7 & -0.1 & -0.1 \\ 0.1 & 0.7 & 0.1 \\ -0.1 & 0.1 & 0.7 \\ 0.01 & 0.01 & 0.01 \\ 0.02 & 0.02 & 0.02 \\ -0.01 & -0.01 & -0.01 \end{pmatrix}$$

and

$$\bar{\beta}_2 = \begin{pmatrix} 0.7 & 0.1 & 0.1 \\ -0.1 & 0.7 & -0.1 \\ 0.1 & -0.1 & 0.7 \\ 0.01 & 0.01 & 0.01 \\ 0.02 & 0.02 & 0.02 \\ -0.01 & -0.01 & -0.01 \end{pmatrix}$$

Finally, we assume that the threshold Y_i^* is drawn from Normal distribution: $N\left(Y^*, 0.001\right)$ where $Y^* = -0.3$.

We generate 400 observations for each cross section discarding the first 100 to remove the influence of initial conditions. Overall 100 panel datasets are generated and the Gibbs sampling algorithm described above is employed to estimate the model using 5,000 iterations. For each of the 100 Monte-Carlo iterations we compute the (linear) impulse response to a unit shock in each regime using the posterior draws of the average coefficients $\bar{\beta}_1, \bar{\beta}_2$ and the average contemporaneous impact matrices $\bar{A}_{1i}, \bar{A}_{2i}$.

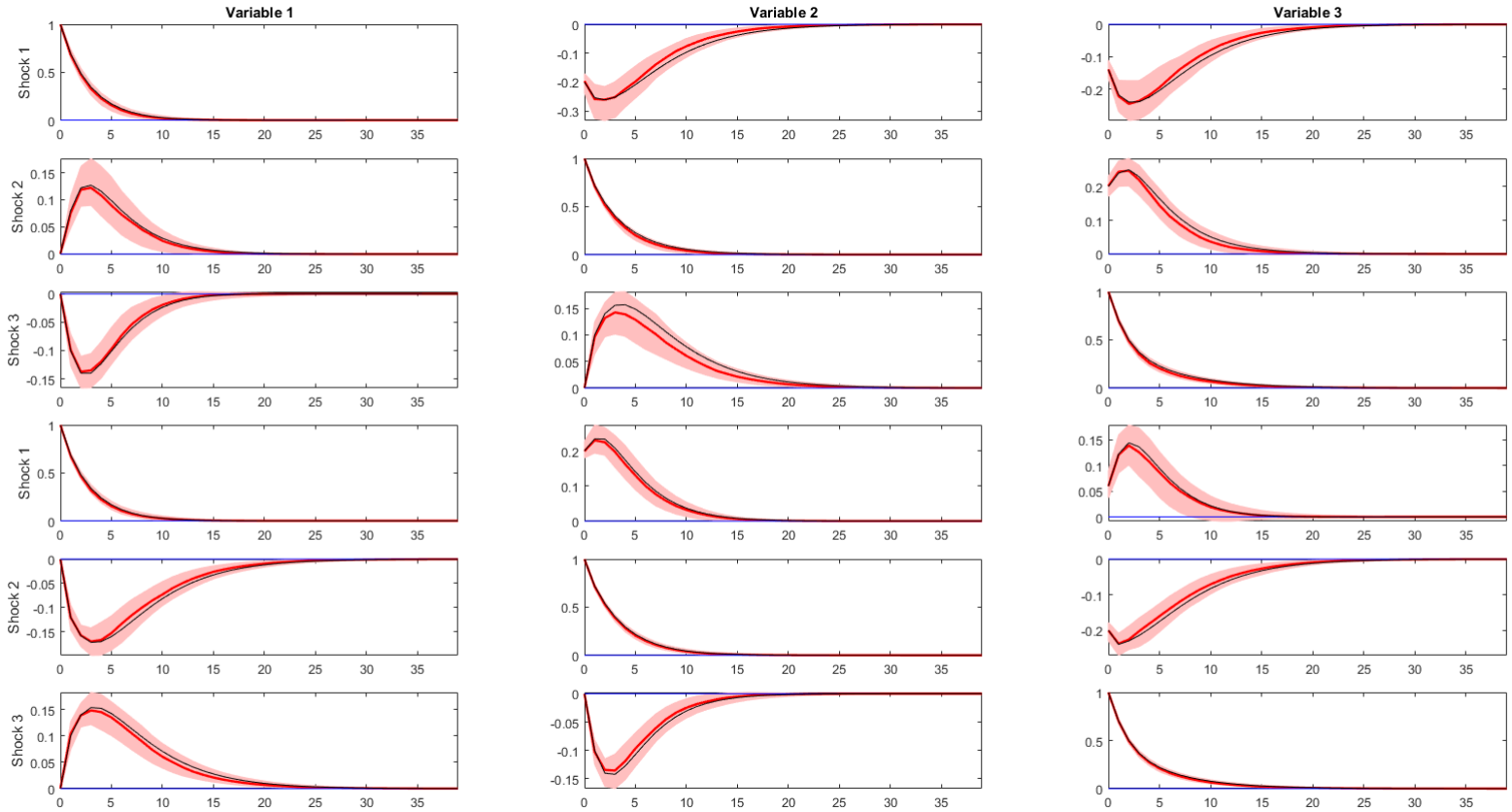

Figure 1: Monte-Carlo results: The black lines represent the true impulse response. The red line and the shaded area is the Monte-Carlo median estimate and the 1 SD error band. The top three rows of the figure show the estimated response in regime 1. The bottom three rows show the same response in regime 2.

Figure 1 presents the estimated impulse responses in the two regimes along with their counterpart in the DGP. The estimates are close to the true responses suggesting that the algorithm and code work fairly well.

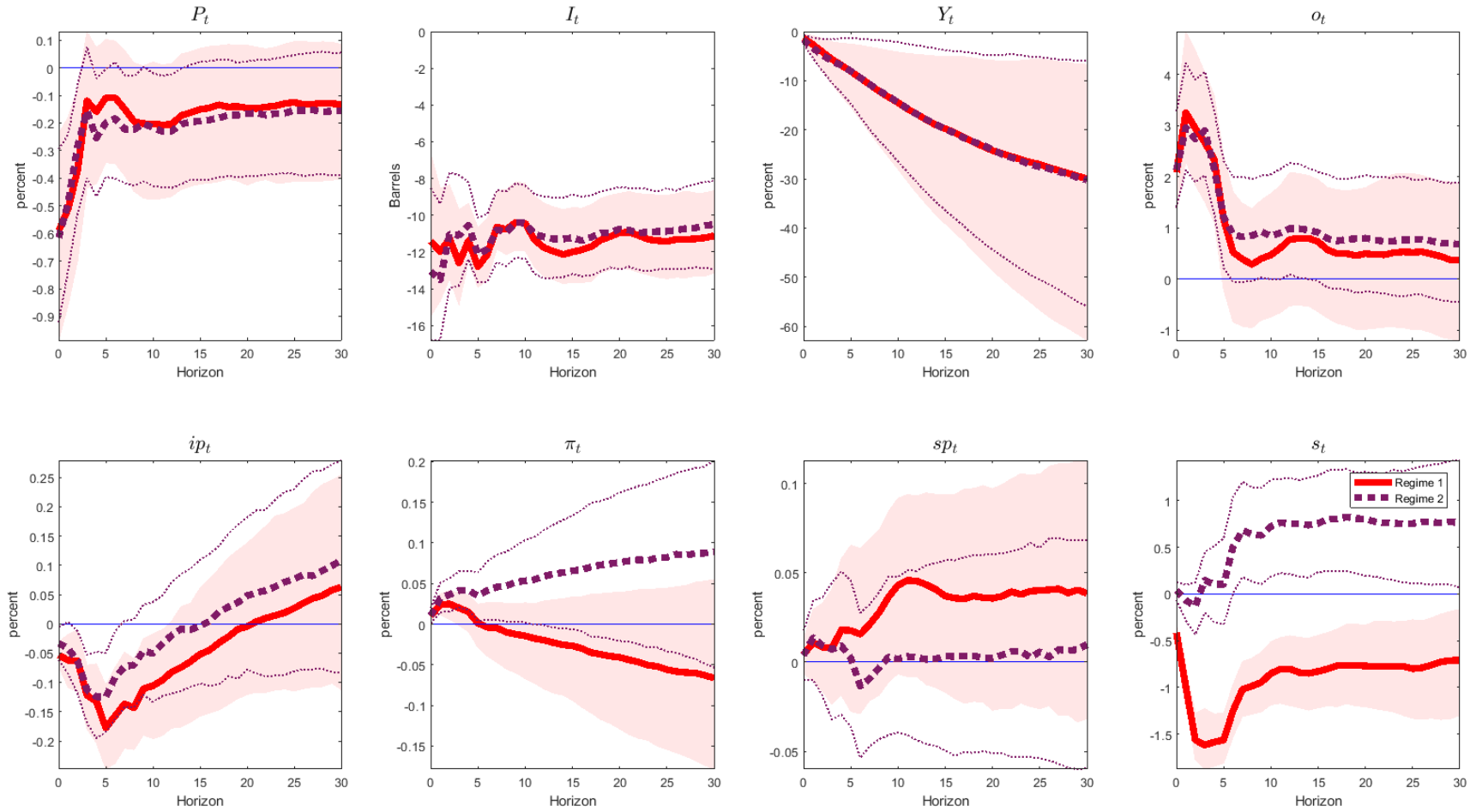

Figure 2: Response to Oil Supply shocks. Sample truncated to 2007.

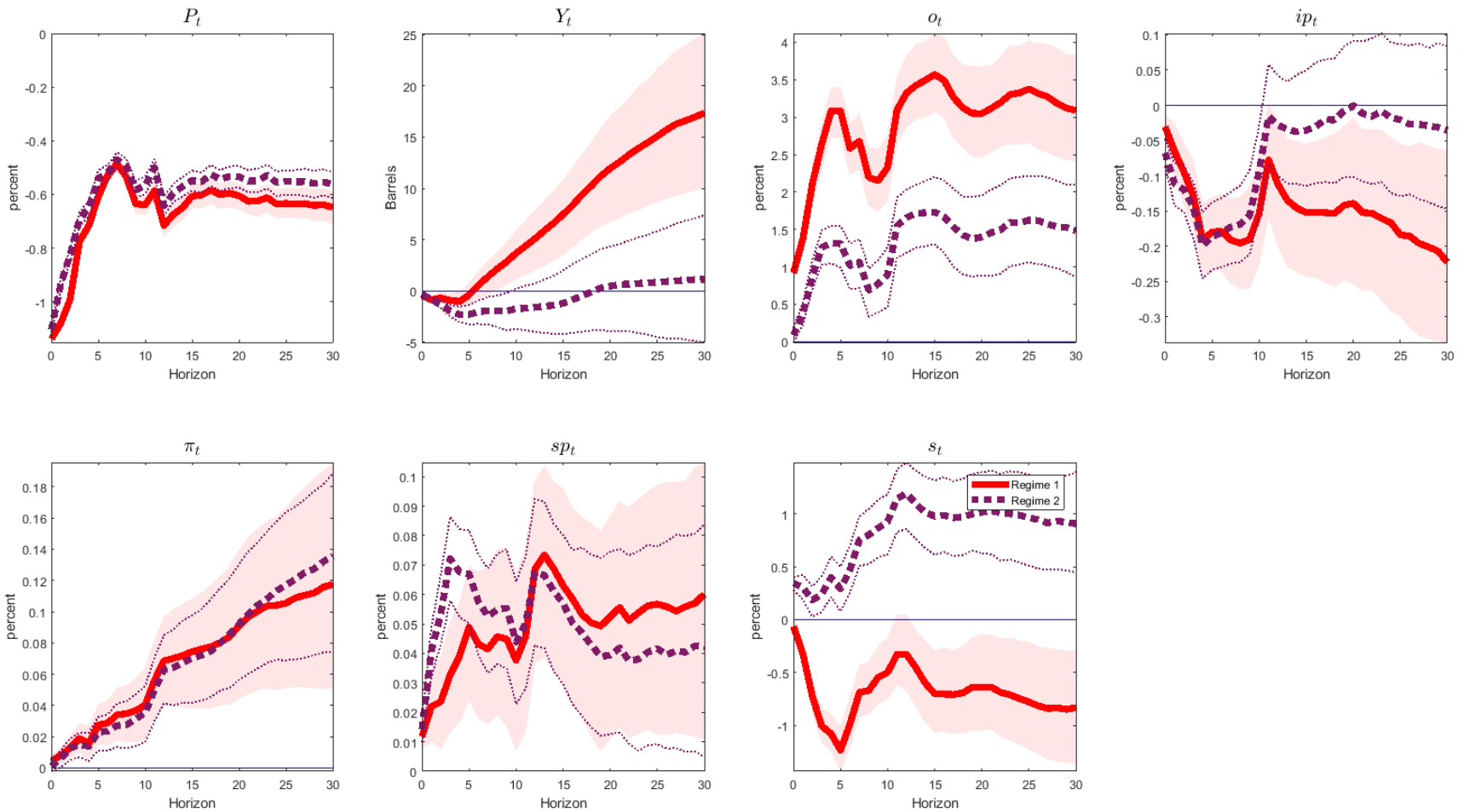

Figure 3: Response to Oil supply shocks using a Cholesky decomposition

2 Results based on a version of the model with the sample truncated to 2007

Figure 2 presents the responses to oil supply shocks. The response of stock prices to this shock is negative in the first regime. This is in contrast to the second regime, when the response is close to zero and mildly positive. This is supportive of the benchmark results.

3 Results based on a Cholesky decomposition

Following, Kilian and Park (2009)oil supply shocks, oil demand shocks and speculative demand shocks are identified via a Cholesky decomposition with the variables ordered as P_t, Y_t, o_t, s_t.[1] As discussed in Kilian and Park (2009), this ordering is consistent with the presence of a vertical oil supply curve and a downward sloping oil demand curve, with the remaining shock to oil prices attributed to speculative demand. Figure 3 shows that the response of stock prices to oil supply shocks is qualitatively similar to the benchmark case.

4 DSGE MODEL EQUATIONS

4.1 Households

$$\lambda_t = (c_t - hc_{t-1})^{-\sigma} - \beta h(c_{t+1} - hc_t)^{-\sigma} \tag{3}$$

$$\lambda_t = \beta\lambda_{t+1}\frac{R_t}{\pi_{t+1}} \tag{4}$$

$$c_{n,t} = (1-\alpha)p_{n,t}^{-\eta}c_t \tag{5}$$

$$c_{o,t} = \alpha p_{o,t}^{-\eta}c_t \tag{6}$$

$$1 = (1-\alpha)p_{n,t}^{1-\eta} + \alpha p_{o,t}^{1-\eta} \tag{7}$$

$$w_t = \Psi\frac{N_t^{\varphi}}{\lambda_t} \tag{8}$$

4.2 Finished Goods Firms

$$Y_t = \left(A_tN_t\right)^{1-\mu}\left(O_t\right)^{\mu} - \Phi \tag{9}$$

$$mc_t = \left(\frac{1}{1-\mu}\right)^{1-\mu}\left(\frac{1}{\mu}\right)^{\mu}\left(\frac{\bar{w}_t}{A_t}\right)^{1-\mu}\left(\bar{p}_{o,t}\right)^{\mu} \tag{10}$$

$$N_t = \left(\frac{1-\mu}{\mu}\right)\left(\frac{\bar{p}_{o,t}}{\bar{w}_t}\right)^{\gamma}O_tA_t{}^{\gamma-1} \tag{11}$$

$$\bar{w}_t = \left\{1 - \psi^n + \psi^n[R_t + \phi(\hat{R}_t)]\right\}w_t \tag{12}$$

$$\bar{p}_{o,t} = \left\{1 - \psi^o + \psi^o[R_t + \phi(\hat{R}_t)]\right\}p_{o,t} \tag{13}$$

$$\phi(\hat{R}_t) = \kappa + \sqrt{\frac{\hat{R}_t^{1-\epsilon} - \gamma}{1000 * (1-\epsilon)}} \tag{14}$$

[1] This alternative model is estimated using these four variables only.

4.2.1 Nonlinear New Keynesian Philips Curve

$$g_{1,t} = \lambda_t \frac{mc_t}{p_{n,t}} Y_t^d + \beta\zeta E_t \left[\frac{\pi_{n,t}^{\iota}}{\pi_{n,t+1}} \right]^{-\theta} g_{1,t+1} \tag{15}$$

$$g_{2,t} = \lambda_t \Pi_{n,t}^* Y_t^d + \beta\zeta E_t \left[\frac{\pi_{n,t}^{\iota}}{\pi_{n,t+1}} \right]^{1-\theta} \frac{\Pi_{n,t}^*}{\Pi_{n,t+1}^*} g_{2,t+1} \tag{16}$$

$$\theta g_{1,t} = (\theta - 1) g_{2,t} \tag{17}$$

$$Y_t = \nu_{p,t} Y_t^d \tag{18}$$

$$\nu_{p,t} = \zeta_p \left(\frac{\pi_{n,t-1}^{\iota}}{\pi_{n,t}} \right)^{-\theta} \nu_{p,t-1} + (1 - \zeta_p)(\Pi_{n,t}^*)^{-\theta} \tag{19}$$

$$1 = (1 - \zeta_p)(\Pi_{n,t}^*)^{1-\theta} + \zeta_p \left(\frac{\pi_{n,t-1}^{\iota}}{\pi_{n,t}} \right)^{1-\theta} \tag{20}$$

$$\pi_{n,t} = \frac{p_{n,t}}{p_{n,t-1}} \pi_t \tag{21}$$

4.2.2 Real Equity Price

$$p_t^{eq} = \beta M_{t+1} (p_{t+1}^{eq} + \pi_{t+1}^{eq}) \tag{22}$$

$$\pi_t^{eq} = p_{n,t} Y_t^d - \bar{w}_t N_t - \bar{p}_{o,t} O_t \tag{23}$$

$$M_t = \frac{\lambda_{t+1}}{\lambda_t} \tag{24}$$

4.3 Oil Price

$$p_{o,t} = \frac{p_{o,t-1}}{\pi_t} \pi_{o,t} \tag{25}$$

$$\pi_{o,t} = \rho_o \pi_{o,t-1} + \epsilon_{o,t} \tag{26}$$

4.4 Monetary Policy

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_r} \left[\left(\frac{\pi_{n,t}}{\bar{\pi}} \right)^{r_\pi} \left(\frac{Y_t}{Y^{ss}} \right)^{r_y} \right]^{1-\rho_r} \tag{27}$$

4.5 Market Clearing

$$c_t = p_{n,t} Y_t - p_{o,t} O_t \tag{28}$$

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