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Common and country specific factors in the distribution of real wages.

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Abstract

We use a dynamic factor model to consider if real wage growth in the US, UK and Germany at different percentiles of the distribution can be explained by factors that are common across countries or specific to each country. Our results suggest that common factors explain a large proportion of the movement in wages when considering the left tail of the distribution indicating that shocks that are common across countries are important for low wage households.

Key words: Household wages, dynamic factor model.

JEL codes: C5, E1, E5, E6

1 Introduction

Wage inequality has risen over the last three decades in industrialised countries such as the US and the UK. This trend has been attributed to a wide range of factors. These include domestic economic policy (see for e.g. Coibion *et al.* (2017) and Mumtaz and Theophilopoulou (2017)), institutional factors such as labour market rigidities (e.g. Machin and Reenen (2007)) and globalisation (e.g. Epifani and Gancia (2008)). The behaviour of wages has been under fresh scrutiny by policy makers in recent years with their level displaying a persistent decline in the aftermath of the great recession.

In this paper, we contribute to the investigation of the factors behind the evolution of the real wage distribution by adopting a cross-country perspective. In particular, we use a dynamic factor model to consider if real wage growth in the US, UK and Germany at different percentiles of the distribution can be explained by factors that are common across countries or specific to each country. Our approach is related to the analysis in Otrok and Pourpourides (2008). However, while Otrok and Pourpourides (2008) investigate the presence of wage factors within the US, our interest lies in estimating the importance of factors that drive cross-country co-movements.

Our results suggest that common factors explain a large proportion of the movement in wages when considering the left tail of the distribution. This suggests that shocks that are common across countries are important for low wage households. Country-specific factors make a moderate contribution throughout the distribution. There is some evidence that this contribution is larger towards

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the right tail of the distribution. These results suggest that international economic developments should be taken into account when designing policies to combat wage inequality. From a theoretical perspective, these empirical results highlight the importance of incorporating transmission of international shocks when modelling the distribution of wages.

The paper is organised as follows. Section 2 presents the empirical model and describes the data set. The main results are presented in section 3 while section 4 concludes.

2 Empirical model and data

2.1 Empirical model

As explained below, our data set comprises a pseudo panel of average real wage growth in the deciles of the distribution covering the US, UK and Germany. Collecting the data set into a $T \times M$ matrix X_{it}, we postulate the following model:

$$X_{it} = B_i^C F_t^C + B_i^W F_t^W + v_{it} \tag{1}$$

where $i = 1, 2, .., M$ and $t = 1, 2, .., T$. Equation 1 states that average wage growth is affected by a set of K^C country-specific unobserved factors F_t^C and a set of K^W unobserved factors F_t^W common across countries, while the error term v_{it} captures the unobserved idiosyncratic components. The factor loadings on the country and common factors are denoted by B_i^C and B_i^W.

Each of these unobserved factors are assumed to follow $AR(P)$ processes. For ease of notation we collect these factors in the $T \times \left(3 \times K^C + K^W\right)$ matrix F_t. The kth column of this matrix is described by the process:

$$F_{kt} = c_k + \sum_{j=1}^{P} b_{kj} F_{kt-j} + \sigma_k^{1/2} e_{kt}, e_{kt} \tilde{} N(0, 1) \tag{2}$$

Similarly, the M idiosyncratic components are described by $AR(q)$ process:

$$v_{it} = \sum_{j=1}^{q} d_{ij} v_{it-j} + h_i^{1/2} e_{it}, e_{it} \tilde{} N(0, 1) \tag{3}$$

Equation 1 implies that:

$$var(X_{it}) = \left(B_i^C\right)^2 var\left(F_t^C\right) + \left(B_i^W\right)^2 var\left(F_t^W\right) + var(v_{it})$$

This equation can be used to estimate the contribution of the world and country factors to the variance of each series. The contribution of the world and country factors are:

$$S^W = \frac{\left(B_i^W\right)^2 var\left(F_t^W\right)}{\left(B_i^C\right)^2 var\left(F_t^C\right) + \left(B_i^W\right)^2 var\left(F_t^W\right) + var(v_{it})} \tag{4}$$

$$S^C = \frac{\left(B_i^C\right)^2 var\left(F_t^C\right)}{\left(B_i^C\right)^2 var\left(F_t^C\right) + \left(B_i^W\right)^2 var\left(F_t^W\right) + var(v_{it})} \tag{5}$$

Estimates of S^W and S^C can be used to infer if the variance of wage growth has been driven by country specific factors or events that are common across countries. Similarly, a comparison of the ith data series X_{it} with the 'fitted values' $\hat{X}_{it}^{W} = B_i^W F_t^W$ and $\hat{X}_{it}^{W+C} = B_i^C F_t^C + B_i^W F_t^W$ can be used to assess how the factors contribute to the changes in wage growth over time.

This dynamic factor model is estimated using a Gibbs sampling algorithm which is fairly standard and described in detail in the technical appendix. One noteworthy aspect is the fact that we have to deal with a number of missing values in X_{it}. We treat these missing values as unknown parameters and extend the Gibbs algorithm to sample from their conditional posterior distribution. The model is subject to the usual scale and sign identification problems affecting factor models. First the scale of the factors is unidentified. We fix the scale of the factors by assuming that $\sigma_k^2 = 1$. Second, the sign of the factors and factor loadings is not identified seperately. However, this is not an issue for our applications below as that do not require an estimate of the factors and their loadings individually, but use either the square of the loadings or the product of the factors and their loadings.

As our data is sampled at an annual frequency, we fix the lag lengths P and q to 1. The choice of the number of common and country-specific factors is a key specification issue. We consider values of K^W and K^C up to 3 and compute the deviance information criterion (DIC) for each specification.[1] As discussed in Spiegelhalter *et al.* (2002), the DIC rewards fit while penalising model complexity with smaller values of DIC preferred. In our case the DIC is minimised for $K^W = K^C = 3$.

2.2 Data

For each country, we collect wages using household surveys. For the US, the data is obtained from the Consumer Expenditure Survey (CEX). Wages are defined as the amount of salary income received by all the members of the household over the past twelve months before deductions (code: FSALARYM). We collect this variable for every interview quarter and obtain an average each year in the sample. For the UK, our data source is the Family Expenditure Survey (FES). The variable of interest is defined as gross wage (code: P008). This refers to take home pay including deductions such income tax and national insurance contributions. This variable is summed over all members of each household in the sample. For Germany, the data is obtained from the German Socio-Economic Panel (GSEP) and produced by the Deutsches Institut Für Wirtschaftsforschung (DIW) where the variable is coded as I11103$$. The variable we use is defined as the sum of total family income from labor earnings. Labor earnings include wages and salary from all employment including training, self-employment income, bonuses, overtime, and profit sharing.

For each country, household level wages are deflated using the CPI and top and the bottom percentile are removed to ameliorate the influence of extreme values. Then household wages in each year are divided into nine percentile groups: $P_1 = [\leq 10th], P_2 = [> 10th\& \leq 20th], P_3 = [> 20th\& \leq 30th], ..., P_9 = [> 90th]$. We calculate the average wage in each group, giving us a time-series for real wages by decile running from 1984 to 2014. Our final data set consists of the growth rate of these decile specific real wages.

3 Results

The main results from the variance decomposition are summarised in Table 1. The table presents the contribution of the world and country factors to the variance of decile specific wage growth. It is interesting to note that for groups P_1 to P_4, the world factor plays an important role in determining

[1] The maximum number of factors is fixed to 3 to limit the number of unobserved state variables to a manageable number.

	US		UK		DE	
	World	Country	World	Country	World	Country
P_1	42.5 $(27.1, 58.4)$	22.7 $(11.5, 37.1)$	36.6 $(20.6, 53.1)$	26.9 $(13.5, 42.9)$	34.6 $(19.1, 52.5)$	38.3 $(22.9, 52.6)$
P_2	55.2 $(41.8, 67.5)$	25.1 $(14.8, 37.2)$	62.4 $(48.2, 75.9)$	29.9 $(17, 43.4)$	46.8 $(30.6, 63.9)$	49.7 $(32.7, 65.7)$
P_3	57.1 $(44.5, 67.9)$	26.4 $(16.5, 37.0)$	56.3 $(40.5, 73.5)$	40.1 $(23.1, 55.6)$	59.9 $(43.8, 75.3)$	39.1 $(23.7, 55.2)$
P_4	54.5 $(40.8, 66.7)$	36.5 $(25.1, 49.7)$	57.4 $(37.5, 78.3)$	39.5 $(18.5, 59.4)$	71 $(56.2, 83.1)$	27.9 $(15.8, 42.7)$
P_5	40.7 $(28, 53.3)$	48.2 $(34.4, 62.2)$	56.8 $(37.8, 78.4)$	38.9 $(17.7, 58)$	66.8 $(52, 81.2)$	31.8 $(17.4, 46.4)$
P_6	31.6 $(19.6, 44)$	59.6 $(47.1, 71.4)$	58.0 $(40.8, 78.6)$	38.7 $(17.9, 56.3)$	66.5 $(52, 80.5)$	32.8 $(18.7, 47.3)$
P_7	25.9 $(14.8, 39.4)$	71.8 $(58.2, 82.7)$	64.3 $(47.8, 83.3)$	33.4 $(14.3, 49.9)$	63.7 $(49.1, 78.1)$	35.1 $(20.8, 49.8)$
P_8	19.6 $(9.4, 32.9)$	75.5 $(62.5, 86.2)$	57 $(41.1, 73.4)$	36.2 $(19.6, 52.8)$	59.5 $(44.6, 74.1)$	38.5 $(24.1, 53.3)$
P_9	15.7 $(6.8, 27.2)$	39.2 $(20.7, 61.1)$	31.4 $(17.8, 46.5)$	36.6 $(21.3, 52.4)$	54.4 $(39.3, 69.7)$	39.6 $(24.9, 54.4)$

Table 1: Contribution of World and Country factors to the variance of wage growth. 68 percent error bands in parenthesis

the fluctuations in wage growth for all countries. The magnitude of this contribution is estimated to be large – i.e. close to 50 percent in almost all cases. Moreover, barring the results for groups P_1 and P_2 in Germany, the median contribution of the world factor exceeds that of the country factor. This suggests that world developments are particularly important for wage growth below the median.

Results are more heterogenous across countries when groups P_6 to P_9. Consider the results for the US. The contribution of the country factor to the variance of the wage growth is estimated to be substantially larger than the contribution of the world factor. This is in contrast to the left tail of the distribution where common or world economic developments matter more than the country factor. For the UK, a similar pattern is seen for the top group P_9 where the contribution of the country factor is somewhat larger than the contribution of the world factor. For groups P_6 to P_8, however, the contribution of the world factor is more important. Similarly, the variance of the right tail in the German wage growth distribution is largely explained by the world factor.

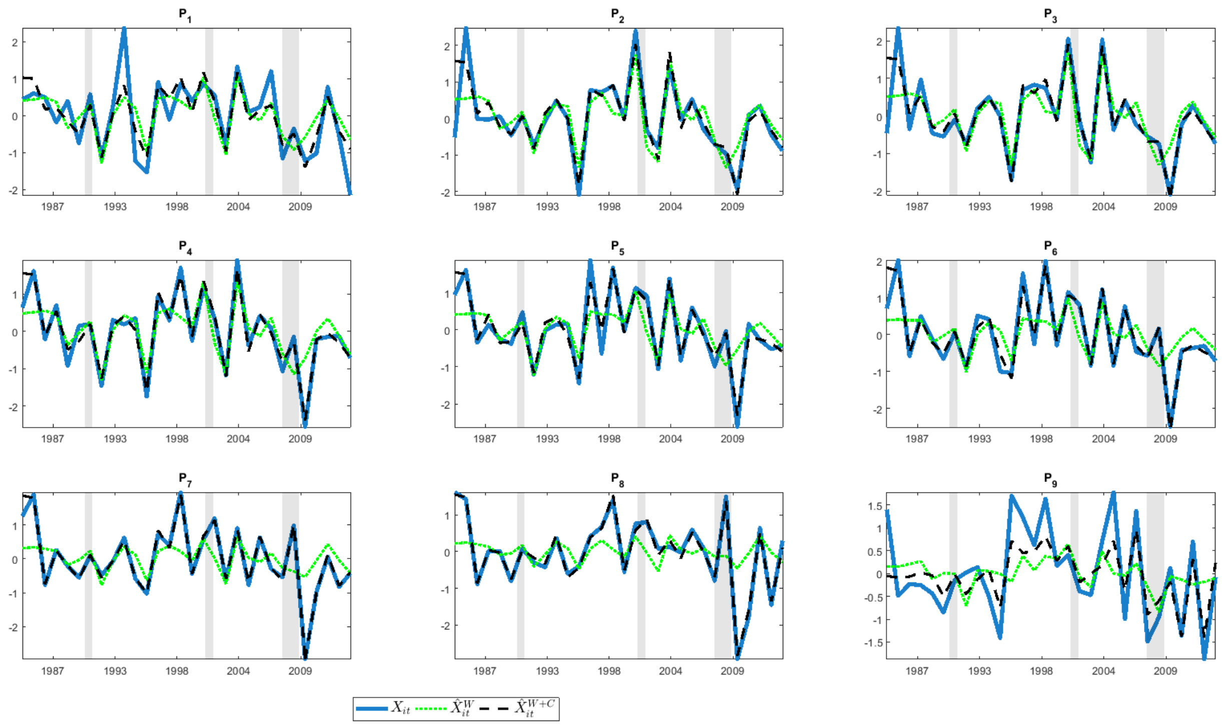

Figure 1: US real wage growth X_{it}, the contribution of the world factor X_{it}^{W} and the contribution of the world and country factors X_{it}^{W+C}. Shaded areas represent NBER recession dates.

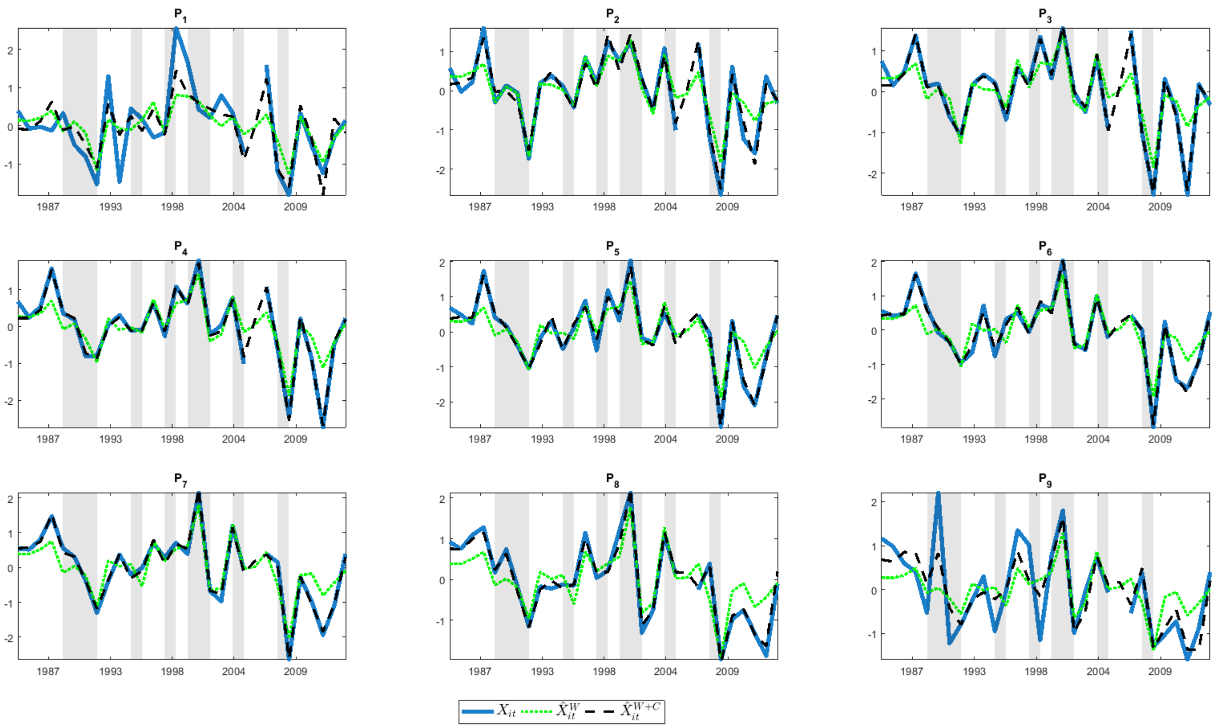

Figure 2: UK real wage growth X_{it}, the contribution of the world factor X_{it}^{W} and the contribution of the world and country factors X_{it}^{W+C}. Shaded areas represent OECD recession dates.

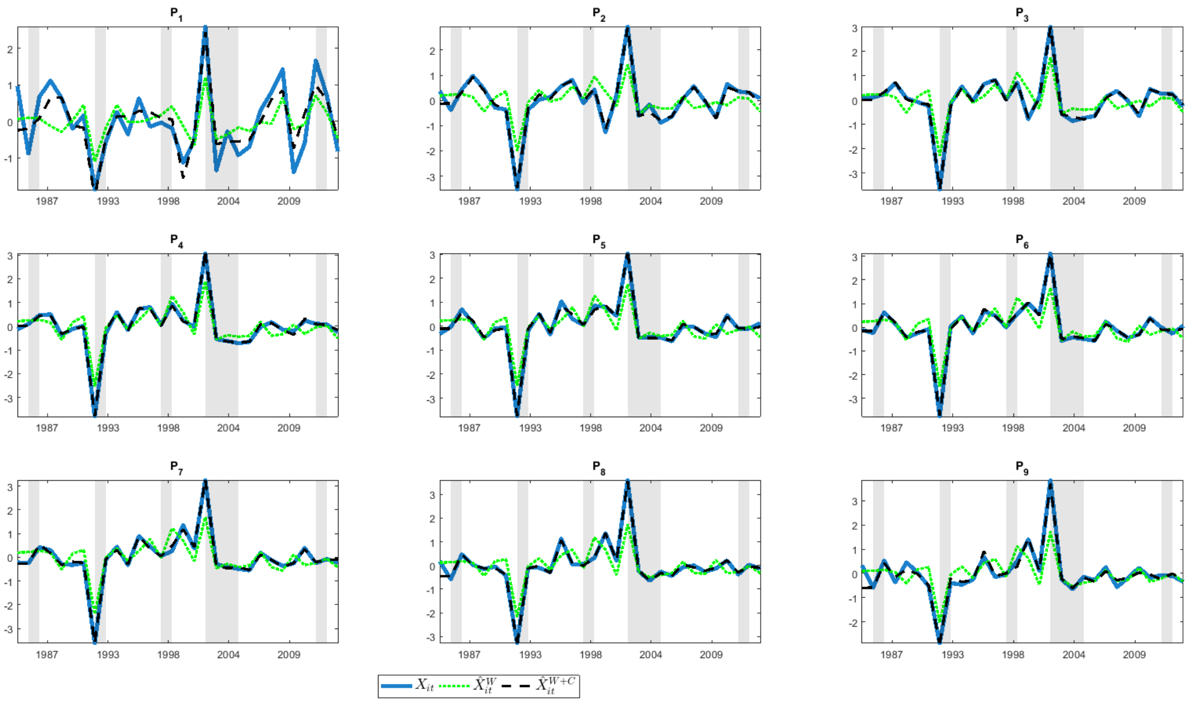

Figure 3: German real wage growth X_{it}, the contribution of the world factor X_{it}^{W} and the contribution of the world and country factors X_{it}^{W+C}. Shaded areas represent OECD recession dates.

Figure 1 shows the growth of real wages in the different groups for the US along with the fitted values $\hat{X}_{it}^{W} = B_i^W F_t^W$ and $\hat{X}_{it}^{W+C} = B_i^C F_t^C + B_i^W F_t^W$. For most groups, the mid 1980s were a period of high wage growth that declined as the 1990s approached. The difference between $\hat{X}_{it}^{W}$ and $\hat{X}_{it}^{W+C}$ over this period indicates that this change was driven to a large extent by country specific conditions. The mid 1990s saw large declines in wage growth. For the lower percentile groups, this was largely driven by the world factor with $\hat{X}_{it}^{W}$ and $\hat{X}_{it}^{W+C}$ overlapping. However, country specific conditions appear to be more important for the higher percentile groups over this period. Wage growth was also supressed in the aftermath of the recessions in 2000 and 2007. In terms of contributions a similar pattern is apparent during these recessions. For lower percentile groups, the world factor makes an important contribution with country-specific conditions more important towards the right tail of the distribution.

For the UK (see Figure 2), country-specific conditions appear to be important in driving the increase in wage growth during the late 1990s and mid-2000's for group P_1. It is interesting to note that for the remaining groups, country-specific factors played an important role towards the end of the sample when wages displayed a sharp decline. In contrast, the the world factor made the largest contribution to the fall in wages after the 2008 recession. Figure 3 shows that real wages in Germany display large movements in the aftermath of the ERM crisis and around the period when the Euro was adopted at the end of the 1990s. In both cases, the bulk of the movement is explained by the world factor. However, note that country-specific conditions also made a contribution to wage movements during the second episode with their impact largest towards the right tail of the distribution.

4 Conclusions

In this note we use a dynamic factor model to decompose movements in real wages at the household level into components specific to each country in our sample and those that capture common shocks. The estimates suggest that factors that are common across countries play an important role for real wages. This is particularly true for the wages of households that lie towards the left tail of the distribution. For the US, country-specific factors remain highly important for high wage households.

In future work it would be useful to examine if similar results hold for household level income and expenditure. It would also be of interest to expand the set of countries and investigate if regional factors play an important role.

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A Appendix: The Gibbs sampling algorithm

Consider the following dynamic factor model:

$$X_{it} = B_i F_t + v_{it} \tag{6}$$

$$F_{kt} = c_k + \sum_{j=1}^{P} b_{kj} F_{kt-j} + e_{kt}, e_{kt} \tilde{} N(0,1) \tag{7}$$

$$v_{it} = \sum_{j=1}^{q} d_{ij} v_{it-j} + h_i^{1/2} e_{it}, e_{it} \tilde{} N(0,1) \tag{8}$$

Note that some observations in X_{it} are missing and are treated as unknown parameters. We denote $\beta_k^F = \begin{pmatrix} c_k \\ b_{k1} \\ . \\ b_{kP} \end{pmatrix}$, $\beta_i^v = \begin{pmatrix} d_{i1} \\ . \\ d_{iq} \end{pmatrix}$, $F_t = \{F_t^C, F_t^W\}, B_i = \begin{pmatrix} B_i^C \\ B_i^W \end{pmatrix}$.

A.1 Priors and starting values

1. $P(B)$: The prior for the factor loadings is Normal $N(B_{i,ols}, I)$. $B_{i,ols}$ denotes an initial estimate of the factor loadings obtained by using principal components as an initial estimate of the factors F_t.
2. $P(h_i)$: The prior is inverse Gamma $IG(h_{i,0}, T_0)$ where $h_{i,0} = 0.001, T_0 = 2$.
3. $P(\beta_k^F)$: This prior is normal with mean $\beta_0^F = \begin{pmatrix} 0 \\ 0.9 \end{pmatrix}$ and $V_0^F = I_2$.
4. $P(\beta_i^v)$: This prior is normal with mean $\beta_0^v = 0.9$ and $V_0^v = 1$.
5. Finally, the initial condition for the factors F_0 are set using the initial values of the principal components $\hat{F}_t$ and the variance V_0^F is an identity matrix.

A.2 Gibbs algorithm

The Gibbs algorithm samples from the following conditional posterior distributions:

1. $H\left(\beta_k^F|\Xi^-\right)$: Here Ξ^- denotes all remaining parameters. Given a draw for the factors F_t, the transition equation 7 can be written as

$$Y_{kt}^* = B_k X_{kt}^* + e_{kt}, e_{kt}\tilde{\ }N(0,1)$$

where $Y_{kt}^* = F_{kt}, X_{kt}^* = [c, F_{kt-1}, ..F_{kt-P}]$. The conditional posterior is normal with mean M and variance V where:

$$\begin{aligned} V &= \left(\left(V_0^F\right)^{-1} + X_{kt}^{*\prime}X_{kt}^*\right)^{-1} \\ M &= V\left(\left(V_0^F\right)^{-1}\beta_k^F + X_{kt}^{*\prime}Y_{kt}^*\right) \end{aligned}$$

2. $H\left(B_i|\Xi^-\right)$: Given a draw for the factors and the autoregressive coefficients β_i^v, the observation equation 6 is a series of regressions with serial correlation. The variables are first transformed to remove the serial correlation:

$$X_{it}^* = B_{it}F_t^* + e_{it}, e_{it}\tilde{\ }N(0,h_i)$$

where

$$\begin{aligned} X_{it}^* &= X_{it} - \sum_{j=1}^{q} d_{ij}X_{it-j} \\ F_t^* &= F_t - \sum_{j=1}^{q} d_{ij}X_{it-j} \end{aligned}$$

The regression coefficients can be drawn from a normal distribution as in step 1 above.

3. $H\left(h_i|\Xi^-\right)$: Given a draw for β_i^v, G_i and the calculated residuals e_{it}, the conditional posterior for h_i is inverse Gamma $IG\left(e_{it}'e_{it} + h_{i,0}, T + T_0\right)$

8. $H\left(F_t|\Xi^-\right)$: Given the remaining parameters, the model can be cast in a multi-variate state-space form. The model in state-space form is:

$$\begin{aligned} X_{it}^{**} &= H_t\tilde{\beta}_t + \tilde{e}_{it} \\ \tilde{\beta}_t &= \mu + f\tilde{\beta}_{t-1} + m_t \\ var\left(\tilde{e}_{it}\right) &= R_t \\ var\left(m_t\right) &= \tilde{Q}_t \end{aligned}$$

where $X_{it}^{**} = X_{it} - \sum_{j=1}^{q} d_{ij}X_{it-j}$ and $\tilde{\beta}_t = \begin{pmatrix} F_t \\ F_{t-1} \end{pmatrix}$. To see the structure of the matrices of the state-space, it is instructive to consider the hypothetical case with $N^C = 2, N = 8, K^C =$

$2, K^W = 1$ where $\tilde{\beta}_t = \begin{pmatrix} F^1_{1t} \\ F^1_{2t} \\ F^2_{1t} \\ F^2_{2t} \\ F^W_t \\ F^1_{1t-1} \\ F^1_{2t-1} \\ F^2_{1t-1} \\ F^2_{2t-1} \\ F^W_{t-1} \end{pmatrix}$. Then these matrices are as follows:

$$
\begin{aligned}
H_t &= [H_{1t}, H_{2t}] \\
H_{1t} &= \begin{pmatrix}
B^1_{11} & B^1_{12} & 0 & 0 & B^W_1 \\
B^1_{21} & B^1_{22} & 0 & 0 & B^W_2 \\
B^1_{31} & B^1_{32} & 0 & 0 & B^W_3 \\
B^1_{41} & B^1_{42} & 0 & 0 & B^W_4 \\
0 & 0 & B^2_{11} & B^2_{12} & B^W_5 \\
0 & 0 & B^2_{21} & B^2_{22} & B^W_6 \\
0 & 0 & B^2_{31} & B^2_{32} & B^W_7 \\
0 & 0 & B^2_{41} & B^2_{42} & B^W_8
\end{pmatrix}
\end{aligned}
$$

$$
H_{2t} = \begin{pmatrix}
-B^1_{11}d_{11} & -B^1_{12}d_{11} & 0 & 0 & -B^W_1 d_{11} \\
-B^1_{21}d_{21} & -B^1_{22}d_{21} & 0 & 0 & -B^W_2 d_{21} \\
-B^1_{31}d_{31} & -B^1_{32}d_{31} & 0 & 0 & -B^W_3 d_{31} \\
-B^1_{41}d_{41} & -B^1_{42}d_{41} & 0 & 0 & -B^W_4 d_{41} \\
0 & 0 & -B^2_{11}d_{51} & B^2_{12}d_{51} & -B^W_5 d_{51} \\
0 & 0 & -B^2_{21}d_{61} & B^2_{22}d_{61} & -B^W_6 d_{61} \\
0 & 0 & -B^2_{31}d_{71} & B^2_{32}d_{71} & -B^W_7 d_{71} \\
0 & 0 & -B^2_{41}d_{81} & B^2_{42}d_{81} & -B^W_8 d_{81}
\end{pmatrix}
$$

$$
R_t = \begin{pmatrix}
h_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & h_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & h_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & h_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & h_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & h_6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & h_7 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & h_8
\end{pmatrix}
$$

$$
\mu = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ 0_{5\times 1} \end{pmatrix}
$$

$$f=\begin{pmatrix} b_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{41} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{51} & 0 & 0 & 0 & 0 & 0 \\ I_{5\times 10} \end{pmatrix}$$

$$\tilde{Q}_t=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0_{5\times 5} \\ 0 & 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 1 & \\ 0_{5\times 10} \end{pmatrix}$$

The Carter and Kohn (1994) algorithm is then used to draw F_t from its conditional posterior.

9. $H\left(X_{it}^m|\Xi^-\right)$: Given a draw for the remaining parameters, the following model describes each series X_{it} :

$$\begin{aligned} X_{it} &= \Gamma_{it}+v_{it} \\ v_{it} &= d_{i1}v_{it-1}+d_{i2}v_{it-2}+d_{i3}v_{it-3}+h_{it}^{0.5}e_{it} \end{aligned}$$

where $\Gamma_{it}=B_{it}F_t$ is known. McCulloch and Tsay (1994) show that the conditional posterior for the missing value X_{it}^m is normal with mean m and variance v :

$$m=\Gamma_{it}+\frac{A+B+C}{\left\|\begin{pmatrix} -1 & d_{i1} & d_{i2} & d_{i3} \end{pmatrix}\right\|^2}$$

$$v=\frac{h_{it}}{\left\|\begin{pmatrix} -1 & d_{i1} & d_{i2} & d_{i3} \end{pmatrix}\right\|^2}$$

where $\|\|$ represents the Eucilidean norm and:

$$A=d_{i1}X_{it-1}^m-d_{i2}\left(d_{i1}X_{it+1}^m-X_{it+2}^m+d_{i3}X_{it-1}^m\right)$$

$$B=-d_{i3}\left(d_{i1}X_{it+2}^m-X_{it+3}^m+d_{i2}X_{it-1}^m\right)$$

$$C=-d_{i1}\left(d_{i2}X_{it-1}^m-X_{it+1}^m+d_{i3}X_{it-2}^m\right)+d_{i2}X_{it-2}^m+d_{i3}X_{it-3}^m$$

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