Nonlinear household earnings dynamics, self-insurance, and welfare

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Abstract

Earnings dynamics are much richer than typically assumed in macro models with heterogeneous agents. This holds for individual-pre-tax and household-post-tax earnings and across administrative (Social Security Administration) and survey (Panel Study of Income Dynamics) data. We study the implications of two processes for household, post-tax earnings in a standard life-cycle model: a canonical earnings process (that includes a persistent and a transitory shock) and a rich earnings dynamics process (that allows for age-dependence of moments, non-normality, and nonlinearity in previous earnings and age). Allowing for richer earnings dynamics implies a substantially better fit of the evolution of cross-sectional consumption inequality over the life cycle and of the individual-level degree of consumption insurance against persistent earnings shocks. Richer earnings dynamics also imply lower welfare costs of earnings risk, but, as the canonical earnings process, do not generate enough concentration at the upper tail of the wealth distribution.

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1   Introduction

Macroeconomic models with heterogeneous agents are ideal laboratory economies to quantitatively study a large set of issues that include household behavior under uncertainty, inequality, and the effects of taxes, transfers, and social insurance reforms.[1] Earnings risk is a crucial source of heterogeneity in these models and its stochastic properties determine how saving and consumption adjust to buffer the impact of earnings shocks on current and future consumption. Appropriately capturing earnings risk is therefore important to understand consumption and wealth inequality, the welfare implications of income fluctuations, and the potential role for social insurance.

With few notable exceptions, most quantitative macroeconomic models adopt earnings processes that imply that persistence and other second and higher conditional moments are independent of age and earnings histories, and that shocks are normally distributed. The *canonical* permanent/transitory process is a popular example.

A growing body of empirical work, though, provides evidence that households' earnings dynamics feature non-normality, age-dependence, and nonlinearities, and devises flexible statistical models that allow for these features. For instance, recent work takes advantage of large, administrative datasets (e.g., W2 confidential Social Security Administration earnings data in Guvenen, Karahan, Ozkan and Song, 2016) and new methodologies applied to survey data sets like the Panel Study of Income Dynamics (PSID) (Arellano, Blundell and Bonhomme, 2017) to show that changes to pre-tax, individual male earnings display substantial skewness and kurtosis and that the persistence of shocks depends both on age and current earnings.

We show that all of these rich dynamics are present not only in individual pre-tax earnings, both in the W2 tax data and the PSID, but also in household, post-tax earnings, which

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are the relevant source of labor income risk at the household level.[2]

Incorporating a flexible earnings process that accounts for these features of the data in a standard life-cycle model is nontrivial. That partly explains why, despite their economic relevance, the implications of these richer earnings dynamics have not received much attention so far.

This paper aims at bridging this gap. Our main contribution is to analyze the effects of richer earnings dynamics on consumption, wealth, and welfare, both in the cross-section and over the life cycle. We use the econometric framework recently proposed by Arellano et al. (2017) that allows to separately identify the distributions of the persistent and transitory components of earnings while allowing for non-normality of shocks, non-linear persistence, and, in general, a rich dependence of the distributions on age and previous earnings. We use PSID data on post-tax, household earnings to estimate two different earnings processes: a richer earnings process along the lines of Arellano et al. (2017) and a "canonical" linear earnings process with a persistent and transitory component and normal innovations, like the one used in Storesletten et al. (2004a). We then compare the implications of the two estimated processes for consumption, wealth, and welfare in the context of a standard life-cycle model of consumption and savings with incomplete markets.

Our main findings are as follows. First, compared to the canonical earnings process, the richer earnings process better fits the observed evolution of consumption inequality over the life cycle. More specifically, under the canonical earnings process, the growth in the variance of consumption substantially overshoots its data counterpart at all ages, while our richer process generates a realistic profile up to ages 50-55, when early and partial retirement start being important. The improved fit is due to the rich features of the earnings data that we model and to the households' precautionary saving response to them. In particular, age-

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[2]These features are consistent with several factors that affect the working lives of individuals. For instance, younger people tend to change jobs more frequently and this implies that the persistence of their earnings is lower. In addition, for most workers, earnings vary little from year to year and shocks are infrequent but can be of large magnitude, such as job loss or a career change, when they happen. This is captured by the high level of kurtosis displayed by earnings changes.

dependent persistence and variance of earnings innovations account for the main share of the improvement of the fit between age 25 and 45, while non-normality and nonlinearity (for instance, the fact that persistence varies with the level of previous earnings) drive the improvement between age 45 and 55.

An alternative, and possibly more intuitive, measure of self-insurance is related to the extent of consumption passthrough of shocks to disposable earnings onto consumption. Our second finding is that the richer earnings process implies a consumption passthrough of persistent earnings shocks broadly consistent with the data. Its value is 0.57 which is within one standard deviation of the point estimate of 0.64 by Blundell, Pistaferri and Preston (2008). Conversely, and in line with the findings in Kaplan and Violante (2010), the canonical process implies a counterfactually high passthrough of 0.86.

Our third finding is that our rich earnings process does not improve the fit of the right tail of the wealth distribution with respect to the canonical earnings process.[3] This is perhaps not so surprising given an established literature, surveyed in De Nardi and Fella (2017), pointing to the fact that accounting for the saving of the rich requires mechanisms—such as a non-homothetic bequest motive, medical-expense risk and entrepreneurship—that go beyond idiosyncratic earnings risk.

Finally, from a normative perspective we find that the welfare costs of earnings risk—as measured by the yearly consumption equivalent—are 1.5 percentage points lower under the richer than under the canonical earnings process. The main reason for this finding is that, while under the canonical process earnings have a permanent, random-walk, component, the richer process implies a lower persistence, particularly in the first part of the working life and at low earnings levels. As a result, life-cycle risk can be more effectively self-insured under the richer earnings process.

An additional contribution of this paper is to propose a simple, simulation-based, method

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[3]In De Nardi, Fella and Paz-Pardo (2016) we show that this conclusion still holds if we estimate a similar richer process on synthetically generated W2 data from the parametric processes proposed in Guvenen et al. (2016). It is thus not related the the issues of lack-of-oversampling and non-participation by higher income people that are usually associated with most survey data sets.

to discretize nonlinear and nonnormal stochastic processes to introduce them in a compu-
tational model. Standard discretization methods used in macroeconomics, such as Tauchen
(1986) and Rouwenhorst (1995), require the continuous process to be approximated to be lin-
ear, typically an AR(1), and, in the case of Tauchen (1986), normal.[4] Our method applies to
any, otherwise unrestricted, age-dependent, first-order Markov process. It relies on simulat-
ing a panel of individual earnings histories using the continuous process to be approximated
and estimating an age-specific, first-order Markov chain on it. This is achieved by discretizing
the simulated marginal distribution of earnings at each age—e.g. into percentiles—and by
replacing the (heterogeneous) values of earnings in each rank percentile with their median.
The associated, age-specific transition matrix is then obtained by computing the proportion
of observations transiting from each percentile rank of the earnings distribution at age t to
that at age $t + 1$. The result is a non-parametric representation of the process that follows a
Markov chain with an age-dependent transition matrix and a fixed number of age-dependent
earnings states.[5]

Our paper is related to the empirical literature on earnings dynamics[6] as well as the
macroeconomic literature on the relationship between income and consumption inequality
over the life cycle. Deaton and Paxson (1994) is the seminal empirical contribution. Storeslet-
ten et al. (2004a), Guvenen (2007), Primiceri and Van Rens (2009), Huggett, Ventura and
Yaron (2011) and Guvenen and Smith (2014) analyze lifetime inequality from the perspective
of the standard, incomplete markets model as we do here. Within this literature, many of
the consequences of richer earnings processes on consumption, savings and welfare in struc-
tural models are still unexplored, with few exceptions. Castañeda et al. (2003) propose an
"awesome or superstar" shock to earnings that is unlikely to be observed in the data but that

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[4]Fella, Gallipoli and Pan (2017) show how Tauchen (1986) and Rouwenhorst (1995) can be extended to
allow for age dependence. Their method still requires linearity though.

[5]This method can be generalized to allow for Markov processes of order higher than one.

[6]Besides Arellano et al. (2017) and Guvenen et al. (2016), discussed above, it includes Geweke and Keane
(2000), Lillard and Willis (1978), Bonhomme and Robin (2009), Meghir and Pistaferri (2004), Blundell,
Graber and Mogstad (2015), Browning, Ejrnaes and Álvarez (2010), and Altonji, Smith and Vidangos
(2013). Recent developments are discussed in Meghir and Pistaferri (2011).

can help to explain the emergence of super-rich people. Karahan and Ozkan (2013) study the implications of age-dependent persistence and variance of shocks. Civale, Díez-Catalán and Fazilet (2016) study the implications of skewness and kurtosis for the aggregate capital stock in an economy à la Aiyagari (1994).

The rest of the paper is organized as follows. Section 2 describes the main features of the data on earnings dynamics for both individuals and households. Section 3 details the methods we use to estimate the canonical and nonlinear earnings processes and their implications. Section 4 explains the discretization procedure we propose to tractably introduce rich nonlinear earnings dynamics in a quantitative life-cycle model. Section 5 presents the model and its calibration. Section 6 discusses the consumption, wealth, and welfare implications of the two earnings processes that we consider, and decomposes the determinants of their differences. Section 7 concludes. Appendix A discusses key features of the PSID data, our sample selection, and earnings definition. Appendix B explains the procedure we use to compute the variances of earnings and consumption by age. Appendix C details the fit of our nonlinear earnings process to important features of the data and shows the robustness of our results to alternative discretization procedures.

2   Earnings data and their features

Recent empirical literature has called into question the established view that (log-)earnings dynamics are well approximated by a linear model of which the canonical random-walk permanent/transitory model (Abowd and Card, 1989) with normal innovations is a popular example. Linear models imply that persistence and other second and higher moments are independent of earnings histories. Instead, Guvenen et al. (2016) and Arellano et al. (2017) document that, contrary to the implications of the canonical model, individual pre-tax earnings display both substantial deviations from log-normality and non-linearity.

Guvenen et al. (2016) use confidential Social Security Administration (W2) tax data to establish these facts. The W2 data set has both advantages and disadvantages compared to the PSID data (and household survey data sets more generally). Regarding its advantages, the W2 data set has a large number of observations, is less likely to be contaminated by measurement error, and is not affected by top-coding and differential survey responses. Thus, it could provide better information on the top earners to the extent that they do not respond to surveys but do pay taxes on all of their earnings. An important disadvantage of the W2 data set is that it is collected at the individual level and lacks the information to identify households and thus to construct household earnings.

The latter is an important shortcoming. In the U.S., the majority of adults are married, 95% of married couples file their income taxes jointly, and taxation of couples and singles is different. Therefore, one needs to know the earnings of both people in a household in order to compute disposable earnings. In this respect household survey data sets that keep track of household structure, like the PSID, have a distinct advantage. This is particularly important if, as we do here, one wants to understand the implications of earnings risk for consumption insurance, which requires taking into account that households and taxes provide insurance against earnings shocks. For such a purpose, *disposable household earnings* is the relevant variable of interest.

The data used in this paper are from the Panel Study of Income Dynamics (PSID), 1968-1992. Our sample consists of households who are in the representative core sample, whose head is between the ages 25 and 60. Given the paper's focus on the implications of earnings risk for consumption insurance, our main variable of interest is *disposable, household* labor earnings, although we also discuss the properties of individual, pre-tax labor earnings for the purpose of comparison with some closely related work (e.g. Arellano et al., 2017; Guvenen et al., 2016).

Disposable, household labor earnings are defined as the sum of household labor income
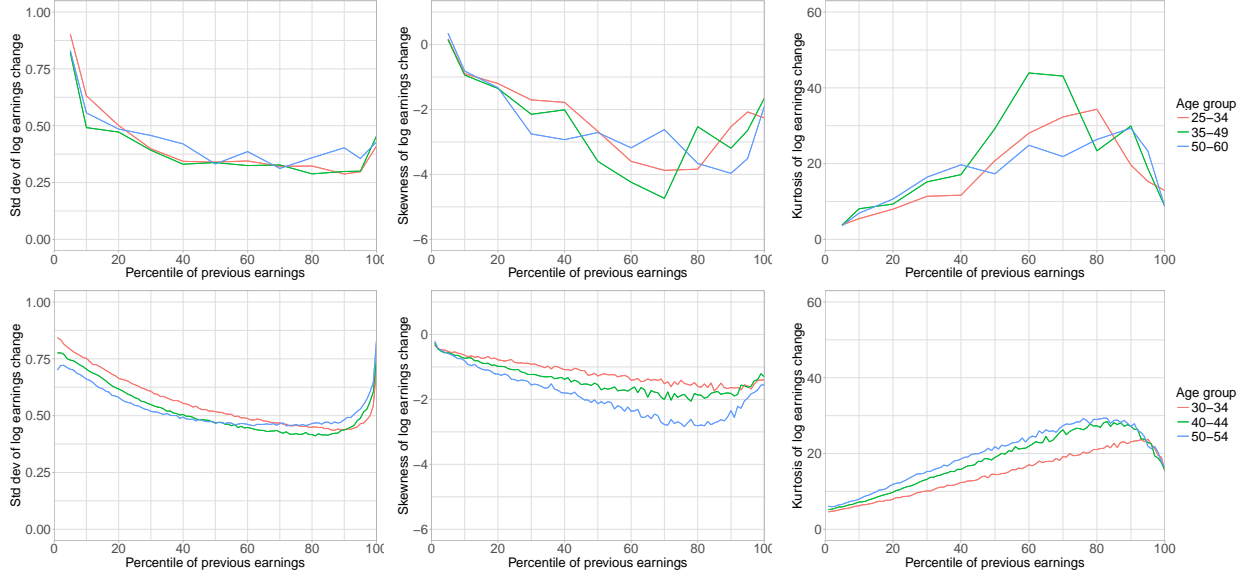
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Figure 1: Standard deviation, skewness, and kurtosis of male pre-tax earnings growth in the PSID (top panel) and W2 (bottom panel)

and transfers, such as welfare payments, net of taxes paid. [7]

We adjust our earnings measure for demographic differences by regressing log earnings on year and age fixed effects and family composition. We use the residuals from these regressions in the analysis below.

2.1   Individual pre-tax earnings in the PSID and the W2 data

We now turn to comparing the properties of *individual pre-tax* earnings data in the PSID with those in the W2 data reported by Guvenen et al. (2016).

Figure 1 compares the second to fourth moments of the W2 data and the PSID. The top panel of Figure 1 plots the conditional standard deviation, skewness and kurtosis (measured as third and fourth standardized moments) of *individual pre-tax* log earnings growth in the PSID by age and decile of previous earnings.[8] The bottom panel of the same figure, taken from Guvenen et al. (2016), reports the same moments, by age and percentile of previous

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[7]Appendix A contains a more detailed description of the PSID data we use, our definition of household earnings and how we estimate taxes on labor following Guvenen and Smith (2014).

[8]For comparability with Guvenen et al. (2016), we report moments for households whose head is a male. All moments are very close to those including female head of households.

earnings, for their W2 data.

Comparing these two sets of figures shows that, overall, the moments in the PSID data are both qualitatively and quantitatively close to those computed from the W2 data. More specifically, the conditional standard deviation of *individual pre-tax* log earnings growth is U-shaped across all age groups, declining until the 40th percentile and increasing again from the 90th onwards. The increase is more pronounced in the W2 for the top percentiles likely reflecting the coarser partition of the distribution in the PSID data. The most notable difference is the much higher variance at all percentiles above the 20th in the W2 data.

The figures also show that in both datasets *individual pre-tax* log earnings growth has strong negative skewness and very high kurtosis, and that these moments depend both on age and previous earnings. The skewness is more negative for individuals in higher earnings percentiles and for individuals between 35 and 45 years of age. This indicates that individuals face a larger downward risk as they approach middle age.[9] The comparison of the implications of the two data sets also reveals that, if anything, there is more negative skewness in the PSID data than in the W2 data, except perhaps at the lowest earnings percentiles.

The kurtosis of *individual pre-tax* log earnings growth is hump-shaped by earnings percentile, and increases until age 35-45 to then decrease thereafter. Even for kurtosis, the maximum value is higher in the PSID, 40, against 30 in the W2 data (compared to 3 for a standard normal distribution).

The top and bottom panels of Figure 2 reveal that the levels and profiles of skewness and kurtosis of *individual pre-tax* log earnings growth are similar in the two datasets also when looking at report robust measures that exclude outliers (Kelly skewness and Crow-Siddiqui kurtosis). The main difference is a higher level of kurtosis in the W2 data once the outliers have been discounted.

Taken together, these moments provide strong evidence against the standard assumption of a log-normal and linear earnings process in the PSID data as well as the W2 data for

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[9]Graber and Lise (2015) account for this kind of earnings behavior in the context of a search and matching model with a job ladder.
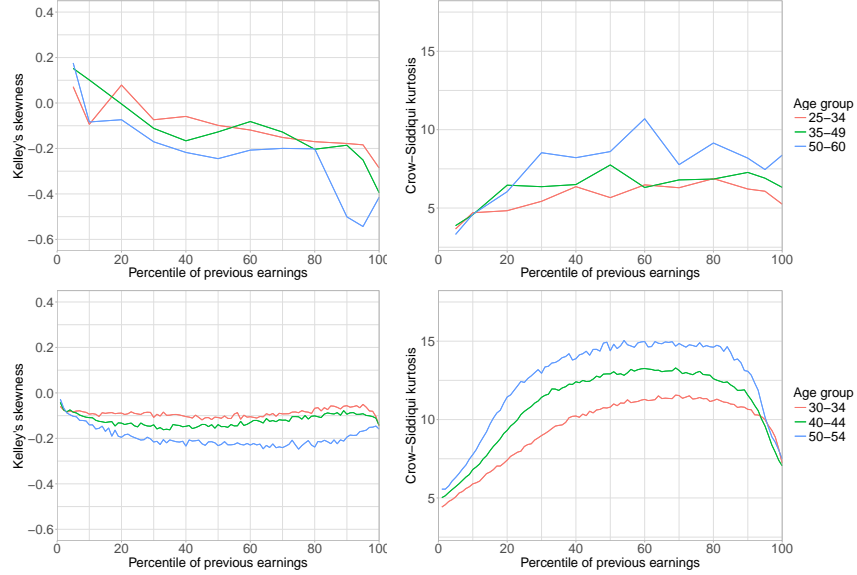
Figure 2: Kelly skewness and Crow-Siddiqui kurtosis of male pre-tax earnings growth in the PSID (top panel) and W2 (bottom panel)

individual pre-tax log earnings growth.

2.2 Individual pre-tax and household disposable earnings in the PSID

Now that we have shown that the implications of the W2 and PSID data for nonlinear earnings dynamics are remarkably similar, we turn to contrasting the properties of *individual pre-tax* and *disposable household* earnings in the PSID.

Figures 3 and 4 compare the relevant moments for *individual pre-tax* log earnings growth (top panel) versus *disposable household* log earnings growth (bottom panel) in the PSID. Comparing the two sets of figures reveals that, as one might have expected, disposable household earnings display much lower variance, skewness, and kurtosis. More specifically, the standard deviation of disposable household earnings is 0.8 times as large at the lower end of the distribution of previous earnings, the skewness is half as large at the higher end of the distribution of previous earnings, and the kurtosis is half as large at its peak. Thus, households and taxes perform an important insurance role in buffering individuals from pre-
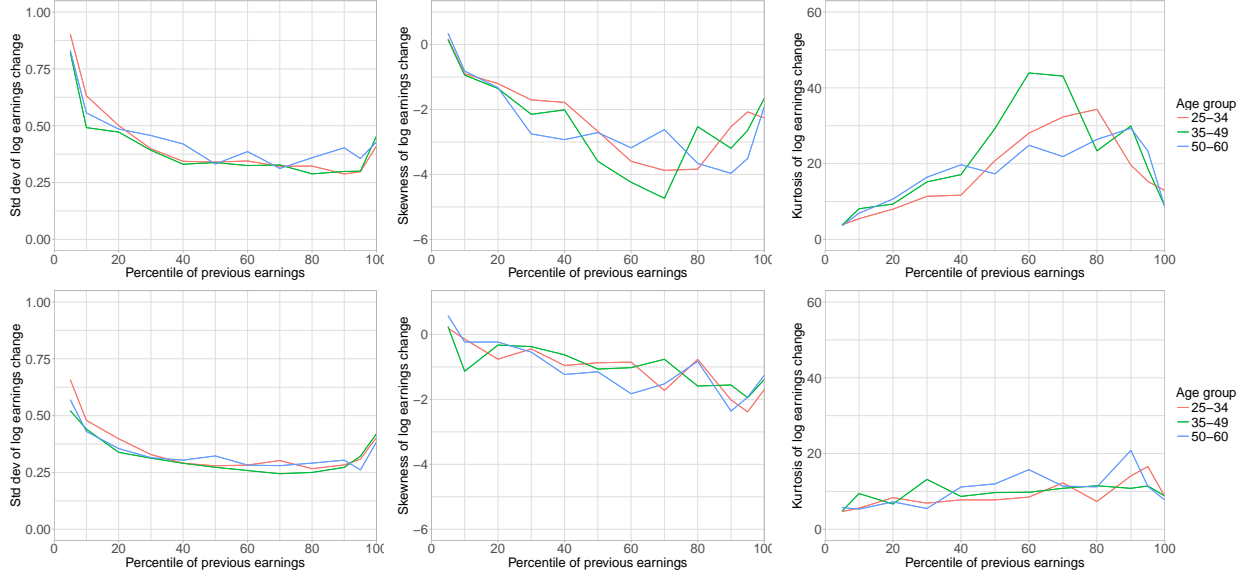
10

Figure 3: Standard deviation, skewness, and kurtosis of male, pre-tax log earnings growth (top panel) and disposable household log earnings growth (bottom panel) in the PSID.

tax earnings changes (as shown by Blundell, Pistaferri and Saporta-Eksten (2016)). This has to be taken into account when considering the economic implications of earnings shocks.

The above discussion has shown that, even after taking into account the insurance implied by pooling at the household level and the tax and welfare system, labor earnings display features that contrast with the *age-independence*, *normality* and *linearity* (independence of variance, skewness and kurtosis of previous earnings realizations) implied by the canonical earnings process.

The same is true of another aspect on nonlinearity, nonlinear persistence, that has been documented by Arellano et al. (2017) using *pre-tax* earnings from the PSID. Figure 5 shows how this same feature is prominent also for *disposable* household earnings. It reports earnings persistence as a function of both the previous- and current-earnings rank in our PSID sample. In line with Arellano et al.'s (2017) findings, we also find that earnings persistence is lower (about 0.6) when previous earnings are highest and the current earning shock is lowest and when previous earnings are lowest and the current earning shock is highest (0.4).
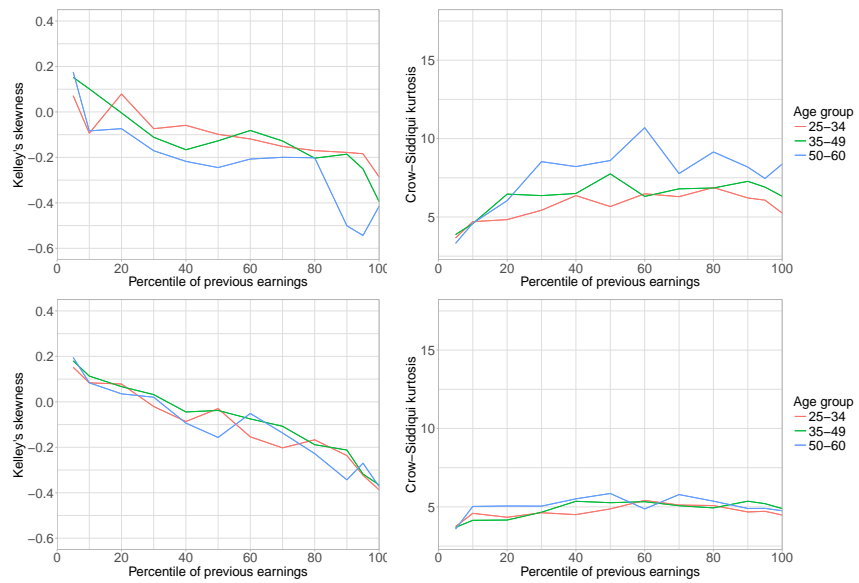
Figure 4: Kelly skewness and Crow-Siddiqui kurtosis of male, pre-tax log earnings growth (top panel) and disposable household log earnings growth (bottom panel) in the PSID.
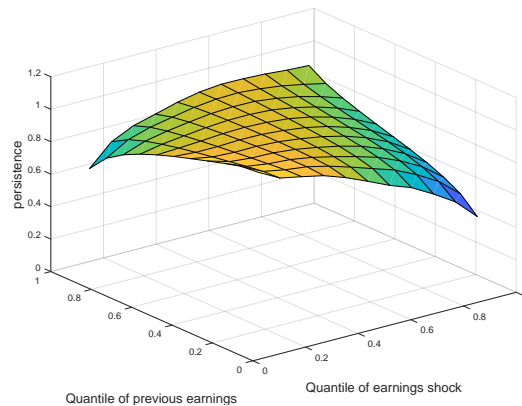

Figure 5: Persistence in log-earnings as a function of previous earnings rank and the rank of the shock received in the current period. PSID data.

3 Earnings processes and their estimation

3.1 Earnings processes

We start by introducing the canonical linear model of earnings dynamics used in macroeconomics before presenting its nonlinear generalization in Arellano et al. (2017).

Consider a cohort of households indexed by i and denote by $t = 1, \ldots, T$ the age of the household head. Let y_{it} denote the logarithm of (residual) disposable household earnings for household i at age t which can be decomposed as

$$y_{it} = \eta_{it} + \varepsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T \tag{1}$$

where η and ε are assumed to have an absolutely continuous distribution. The first component, η_{it}, is assumed to be *persistent* and follows a first-order Markov process. The second component, ε_{it}, is assumed to be *transitory*, have zero mean, be independent over time and of η_{is} for all s.

The *canonical (linear) model* used in macroeconomics is described by

$$\eta_{i,t} = \rho \eta_{i,t-1} + \zeta_{it}, \tag{2}$$

$$\eta_{i1} \stackrel{id}{\sim} N(0, \sigma_{\eta_1}), \quad \zeta_{it} \stackrel{iid}{\sim} N(0, \sigma_\zeta), \quad \varepsilon_{it} \stackrel{iid}{\sim} N(0, \sigma_\varepsilon). \tag{3}$$

Thus, the persistent component η_{it} is an autoregressive process of order one with the innovation ζ_{it} independent of $\eta_{i,t-1}$, while the transitory component ε_{it} is white noise.

Equations (2)-(3) impose three types of restrictions

1. *Age-independence* (stationarity) of the autoregressive coefficient ρ and of the shock distributions, which imply age-independence of the second and higher moments of the conditional distributions of both the transitory and the persistent component. This is clearly at odds with the strong age-dependence in figures 1-4.

2. *Normality* of the shock distributions, which is inconsistent with the negative skewness and high kurtosis discussed above.

3. *Linearity* of the process for the persistent component. Linearity implies: (a) the additive separability of the right hand side of equation (2) into the conditional expectation—the first addendum—and an innovation ζ_{it} independent of $\eta_{i,t-1}$, and (b) the linearity of the conditional expectation in $\eta_{i,t-1}$. Under separability, deviations of η_{it} from its conditional expectation are just a function of the innovation ζ_{it}. As a consequence, all conditional centered second and higher moments are independent of previous realizations of η. This is clearly inconsistent with the dependence of the moments reported in figures 1-5 on previous earnings realizations.

The evidence discussed in Section 2.1 is what motivates us to consider a more general process that relaxes the above three restrictions while maintaining the first-order Markov assumption for η. The question of how to easily introduce a richer and yet tractable earnings process in a structural model is non-trivial and part of what we propose in this paper.

We proceed in two steps. First, we use the quantile-based panel data method proposed by Arellano et al. (2017) to estimate a non-parametric model that allows for age-dependence, non-normality and nonlinearity, and that can be applied in datasets of moderate sample size like the PSID. This step gives us quantile functions for two components of earnings, a persistent one and a transitory one. Second, we use the two quantile functions to simulate histories for the two earnings components and proceed to estimate, for each of them, a discrete Markov-chain approximation, which can then be easily introduced in a structural model (the latter is discussed in detail in Section 4).

Let $Q_z(q|\cdot)$, the conditional quantile function for the variable z, denote the qth conditional quantile of z.[10] The process for η can be written in a very general form by replacing equation

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[10]Intuitively, the conditional quantile function is the inverse of the conditional cumulative density function of the variable z mapping from the $(0,1)$ interval into the support of z. Namely, $z_q = Q_z(q|\cdot)$ satisfies $P[z \leq z_q|\cdot] = q$, where $P[\cdot|\cdot]$ denotes the conditional probability.

(2) with

(4) $$\eta_{it} = Q_\eta(v_{it}|\eta_{i,t-1}, t), \quad v_{it} \overset{iid}{\sim} U(0,1), \; t > 1.$$

Intuitively, the quantile function maps random draws v_{it} from the uniform distribution over $(0,1)$ (cumulative probabilities) into corresponding random (quantile) draws for η. In the linear case in equation (2) the quantile function specializes to the linearly separable form $Q_\eta(v_{it}|\eta_{i,t-1}, t) = \rho\eta_{i,t-1} + \phi^{-1}(v_{it}; \sigma_\zeta)$, where $\phi^{-1}(v_{it}; \sigma_\zeta)$ is the inverse of the cumulative density function of a normal distribution with zero mean and standard deviations σ_ζ. So, age-independence, normality and linearity can be seen as restrictions on the quantile function in equation (4).

In particular, one way to understand the role of nonlinearity is in terms of a generalized notion of persistence

(5) $$\rho(q|\eta_{i,t-1}, t) = \frac{\partial Q_\eta(q|\eta_{i,t-1}, t)}{\partial \eta_{i,t-1}}$$

which measures the persistence of $\eta_{i,t-1}$ when it is hit by a shock that has rank q. In the canonical model, $\rho(q|\eta_{i,t-1}, t) = \rho$, independently of both the past realization of $\eta_{i,t-1}$ and of the shock rank q. Instead, the general model allows persistence to depend both on the past realization $\eta_{i,t-1}$, but also on the realization on the sign and magnitude of the shock to it. Basically, in the nonlinear model shocks are allowed to wipe out the memory of past shocks or, equivalently, the future persistence of a current shock may depend on future shocks.

Of course, a similar unrestricted representation can be used for the transitory component ε_{it} and the initial condition η_1, with the only difference that they are not persistent.

σ_ε^2	$\sigma_{\eta_1}^2$	σ_ζ^2	ρ
0.0675	0.2363	0.0059	1

Table 1: Estimates for the canonical earnings process.

3.2 Estimating the canonical linear earnings process

We estimate the canonical process for residual earnings in equations (1)-(3) using GMM.[11] Table 1 reports our results. As common in the literature, we find that the persistent component has a unit root. For this reason, though we have allowed for individual fixed effects at the estimation stage, their variance cannot be identified separately from the variance of the initial condition $\sigma_{\eta_1}^2$ and we have normalized it to zero.

3.3 Estimating the nonlinear earnings process

Following Arellano et al. (2017), we parameterize the quantile functions for the three variables as low order Hermite polynomials

$$(6) \qquad Q_\varepsilon(q|age_{it}) = \sum_{k=0}^{K} a_k^\varepsilon(q)\psi_k(age_{it})$$

$$(7) \qquad Q_{\eta_1}(q|age_{i1}) = \sum_{k=0}^{K} a_k^{\eta_1}(q)\psi_k(age_{i1})$$

$$(8) \qquad Q_\eta(q|\eta_{i,t-1}, age_{it}) = \sum_{k=0}^{K} a_k^\eta(q)\psi_k(\eta_{i,t-1}, age_{it})$$

where the coefficients a_k^i, $i = \varepsilon, \eta_1, \eta$, are modelled as piecewise-linear splines on a grid $\{q_1 < \ldots < q_L\} \in (0,1)$.[12] The intercept coefficients $a_0^i(q)$ for q in $(0, q_1]$ and $[q_L, 1)$ are specified as the quantiles of an exponential distribution with parameters λ_1^i and λ_L^i.

If the two earnings components ε_{it} and η_{it} were observable one could compute the polynomial coefficients simply by quantile regression for each point of the quantile grid q_j. To

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[11]Appendix A.3.4 provides more information about our estimation method.

[12]Following Arellano et al. (2017), we use tensor products of Hermite polynomials of degrees (3,2) in $\eta_{i,t-1}$, and age for $Q_\eta(q|\eta_{i,t-1}, age_{it})$ and second-order polynomials in age for $Q_\varepsilon(q|age_{it})$ and $Q_{\eta_1}(q|age_{i1})$.
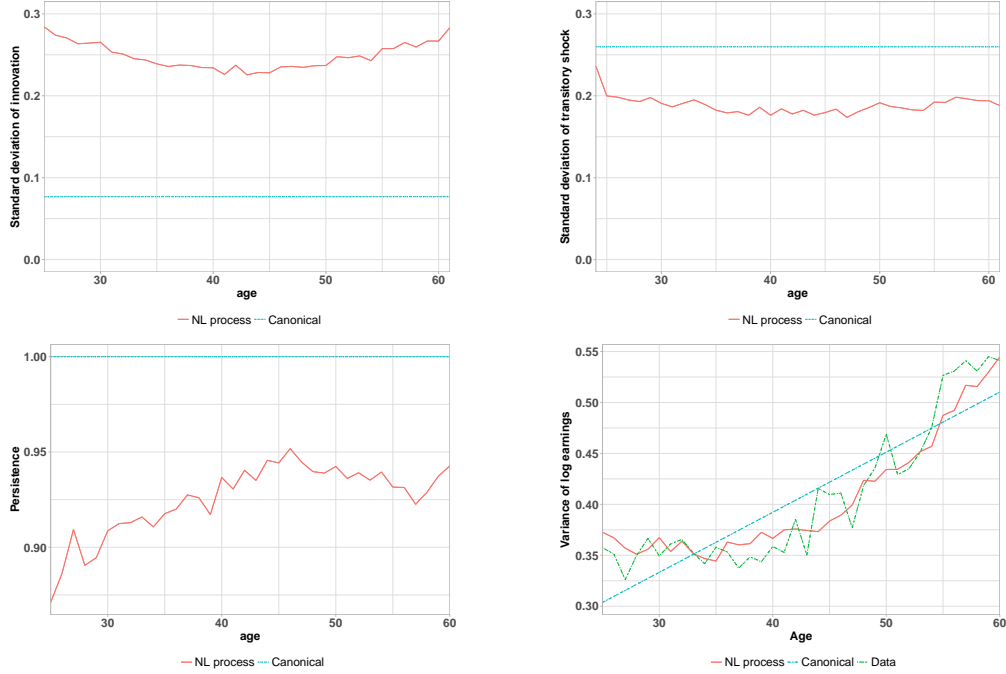
Figure 6: Age dependence of second moments: nonlinear vs canonical process. Top left, standard deviation of the innovation to the persistent component. Top right, standard deviation of the transitory shock. Bottom left, autocorrelation of the persistent component. Bottom right, cross-sectional variance of log earnings.

deal with the latent earnings components, the estimation algorithm starts from an initial guess for the coefficients and iterates sequentially between draws from the posterior distribution of the latent persistent components of earnings and quantile regression estimation until convergence of the sequence of coefficient estimates.

3.4 Comparing the implications of the nonlinear and canonical earning processes

To understand the economic implications of the nonlinear and canonical earnings processes, it is useful to compare their implications in terms of (a) age-dependence of second moments; (b) non-normality; (c) nonlinearity.

Starting from the **age-dependence** of second moments, the upper panel of Figure 6 plots the age profile of the standard deviations of the shocks to the persistent and transitory

components of earnings. Both are age-independent by construction in the canonical process. The standard deviation of shocks to the persistent component is substantially higher for the nonlinear process and follows a U-shaped pattern by age. In contrast, the standard deviation of the transitory component of the nonlinear process displays little age variation and is lower in the nonlinear than in the canonical model. The bottom left panel of Figure 6 reports the age-profile of the first-order autocorrelation of the persistent earnings component for the two processes. In the nonlinear earnings process it is lower than in the canonical case for all ages, but it does increase between age 25 and 45. Given these differences, it is not surprising that the nonlinear process provides a substantially better fit of the age profile of the cross-sectional earnings dispersion, which we display in the bottom right panel of Figure 6[13]. More specifically, the canonical earnings process cannot capture the convex shape of the cross-sectional variance of earnings by age while the nonlinear process provides an extremely close fit, thanks to the combination of increasing persistence and declining variance of the persistent component over the ages 25 to 45. It is also apparent that the canonical model requires a low variance of the persistent shocks relative to the transitory ones to match the relatively low rate of growth of the cross-sectional variance of earnings over the life-cycle. Figure 7 displays more evidence on age-dependence, which also manifests itself in the skewness and kurtosis of the shocks.

Turning to **non-normality**, Figure 7 reports skewness and kurtosis for the innovation to the transitory (top panel) and persistent component of earnings (bottom panel) by age and highlights that the earnings data display deviations from normality (the turquoise line) by age. However, they also highlight limited skewness but much larger kurtosis than a normal distribution.

Turning to **nonlinearity**, Figure 8 plots the standard deviation of shocks to the innovation to the persistent component of earnings by previous earnings, while the right panel plots the persistence measure in equation (5)—namely the correlation between the percentile

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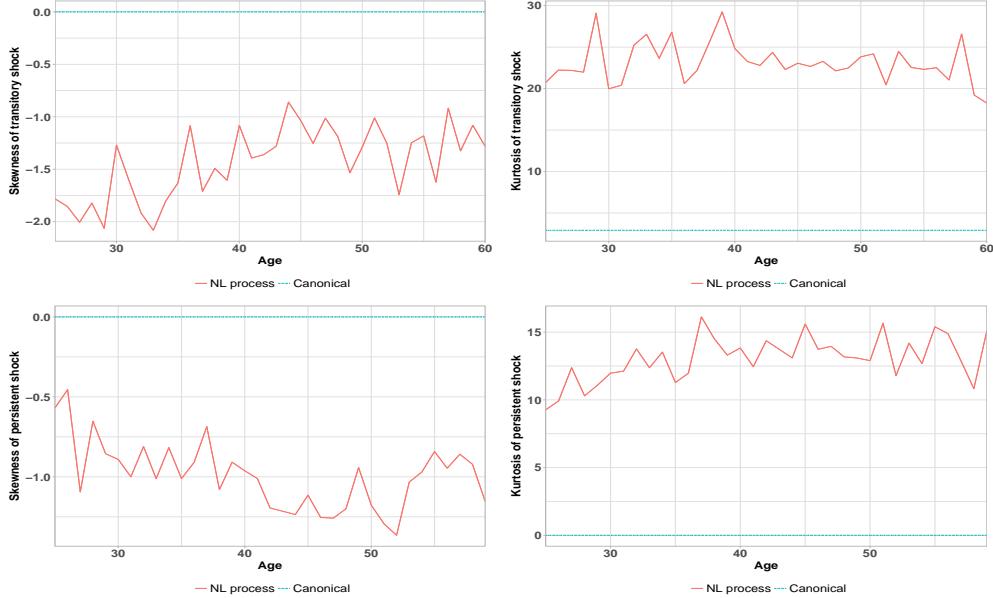[13]See Appendix B for details on the computation of this variance.

Figure 7: Skewness and kurtosis (by age) of the innovations to (a) the transitory component of earnings (top) and (b) the persistent component of earnings (bottom).

of η_{t-1} and of the innovation to it—averaged by age. In the right panel of this figure, we do not plot the persistence of the canonical model (which is constant at 1) for picture clarity. These two panels clearly illustrate that the constant variance and persistence implied by the canonical process are strongly at odds with the highly nonlinear patterns in Figure 8 and the features of the observed data.

4 The discretized nonlinear earnings process

To use the estimated process (1)-(3) in the life-cycle model, we discretize it using an age-dependent Markov chain.

We start by simulating a large set of histories for the persistent and transitory component of earnings. For each component in the simulated sample, we estimate a Markov chain of order one, with age-dependent state space $Z^t = \{\bar{z}_1, \dots, \bar{z}_N\}$, $t = 1, \dots, T$ and an age dependent transition matrices Π^t, of size $(N \times N)$. That is, we assume that the dimension N of the state space is constant across ages but we allow the set of states and the transition
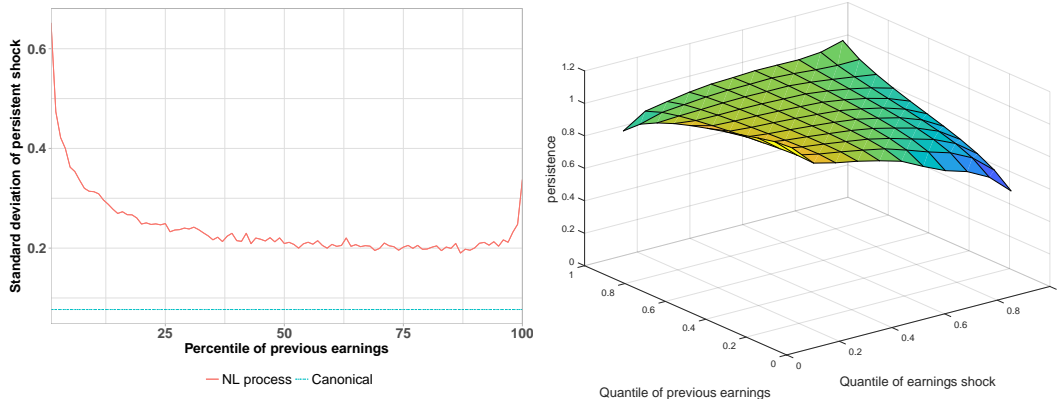
Figure 8: Standard deviations of persistent shocks by previous earnings (left) and nonlinear persistence of the persistent component, by quantile of previous earnings and quantile of shock received in the current period (right).

matrices to be age-dependent.

We determine the points of the state-space and the transition matrices at each age in the following way.

1. At each age, we order the realizations of each component by their size and we group them into N bins. Due to the limited sample size of the PSID, we want to strike a balance between a rich approximation of the earnings dynamics by earnings level (a large number of bins) and keeping the sample size in each bin sufficiently large. In our main specification we report the results for bins representing deciles, with the exception of the top and bottom deciles, that we split in 5. Thus, bins 1 to 5 and 14 to 18 include 2% of the agents at any given age, while bins $n = 6, \ldots, 13$ include 10% of the agents at any given age. This implies a total of 18 bins.

2. The points of the state space at each age t are chosen so that point z_t^n is the median in bin n at age t. Kennan (2006) proves that setting the gridpoint at the median of the bin (in the specific case of equally-sized bins) and attributing a weight of $1/N$ to each of the N bins constitutes the best discrete approximation of an arbitrary distribution.

3. The initial distribution at model age 0 is the empirical distribution at the first age we consider.

4. The elements π^t_{mn} of the transition matrix Π^t between age t and $t+1$ are the proportion of individuals in bin m at age t that are in bin n at age $t+1$.

Allowing for an age-dependent Markov chain allows to capture the non-constancy of moments of the earnings distribution over the life-cycle. The flexible form of the transition matrix allows to capture nonlinearities as a function of current earnings. The use of this kind of transition matrices is well established in the literature. Krueger and Perri (2003) use them to study the welfare consequences of an increase in earnings inequality. Studies of income mobility (e.g. Jäntti and Jenkins (2015)) and consumption mobility (e.g. Jappelli and Pistaferri (2006)) rely on them to analyze intra- or inter- generational mobility across relative rankings in the distributions. In this paper, instead, we are interested in capturing movements across earnings levels.

5   The model

The model is based on Huggett (1996)'s paper. There is no aggregate uncertainty. The economy is populated by overlapping generations of individuals who are equal at birth but receive idiosyncratic shocks to earnings throughout their working lives. We restrict attention to stationary equilibria.

5.1   Demographics

Each year, a positive measure of agents is born. People start life as workers and work until retirement at age T^{ret}. The population grows at rate n.

An agent of age t faces a positive probability of dying $(1 - s_t)$ by the end of the period, where s_t denotes the one-period survival probability for an agent of age t. Agents die with probability one by age T.

5.2 Preferences and technology

Preferences are time separable, with a constant discount factor. The intra-period utility is CRRA: $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$.

Agents are endowed with one indivisible unit of labor which they supply inelastically at zero disutility. Their earnings are subject to random shocks and follow the process described by equations (1)-(3).

5.3 Markets and the government

Asset markets are incomplete. Agents cannot borrow and can only invest in the risk-free asset at an exogenous rate of return r. There are no annuity markets to insure against the risk of premature death. As a result, there is a positive flow of accidental bequests in each period. We assume these are lost to the economy and thus are not received by any individual or the government.

Retired individuals receive an after-tax pension p from the government until they die. The pension is a function of the last realization of their earnings.

5.4 The household's problem

In any given period, a t-year old agent chooses consumption c and risk-free asset holdings for the next period a', as a function of the relevant state vector. The optimal decision rules for consumption and savings solve the dynamic programming problems described below.

(i) Agents of working age $t < T_{ret}$ solve the recursive problem

$$(9) \qquad V(t, z, \eta) = \max_{c,a'} \left\{ u(c) + \beta s_t E_t V(t+1, z', \eta') \right\}$$

$$\text{s.t. } a' = z - c, \quad a' \geq 0,$$

$$z = (1+r)a + \eta + \varepsilon,$$

where z denotes total cash at hand.[14]

(ii) From the retirement age T^{ret} to the terminal age T agents no longer work and live off their pension p and accumulated wealth. Their value function satisfies:

$$(10) \qquad W(t, z, p) = \max_{c,a'} \left\{ u(c) + s_t \beta W(t+1, z', p) \right\}$$

$$\text{s.t. } a' = z - c, \quad a' \geq 0,$$

$$z = (1+r)a + p.$$

The agent's pension p enters the state vector because it is a function of the agent's earnings pre-retirement. The terminal value function $W(T, a, p)$ is equal to zero (agents do not derive utility from bequests).

The definition of equilibrium is standard.

5.5   The model calibration

The model period is one year. Agents enter the labor market at age 25. The retirement and terminal ages are $T^{ret} = 60$ and $T = 85$. The population growth rate n is set to 1.2% per year. The survival probabilities s_t are from Bell, Wade and Goss (1992).

The coefficient of relative risk aversion is set to 2, a standard value. The risk-free rate is 6% and the discount factor β is calibrated to match a wealth to income ratio of 3.1. It equals 0.944 under the canonical earnings process and 0.927 under the nonlinear one.

As described in Section 2, our earnings processes are based on disposable earnings, hence we do not explicitly include taxation in the model.[15] In both cases, we impose the same average income profile, which we estimate from our PSID sample.

We discretize the two earnings processes as follows. In the case of the canonical earnings

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[14]The choice of state vector does not require separately keeping track of the transitory component of earnings ε which is independently distributed over time.

[15]Appendix A provides more details about the earnings definition.

process, whose estimates we report in Table 1, we discretize the persistent component using the modified version of the Rouwenhorst method for non-stationary processes proposed by Fella et al. (2017). We use 18 gridpoints at each age. We use 8 grid for the transitory, i.i.d. component. In the case of the nonlinear earnings process, we apply the procedure described in Section 4.

The social security pension benefit p are a function on the last realization of disposable earnings $y_{ret} = \eta_{ret} + \varepsilon_{ret}$. The function is meant to mimic the US system and is based on Kaplan and Violante (2010). Namely, the replacement rate is: (a) 90 percent for the fraction of the last earnings below 0.18 of cross-sectional average gross earnings, (b) 32 percent for the fraction between 0.18 and 1.10, and (c) 15 percent for the fraction above 1.10. Benefits are then (very slightly) scaled up proportionately so that a worker that makes average earnings is entitled to a 45 percent replacement rate.

6   Consumption, wealth, and welfare implications

This section studies the model's implications for consumption under the canonical and non-linear earnings processes and compares them to U.S. consumption data. To do so, we first analyze the growth in consumption dispersion over the working life and then turn to measuring self-insurance insurance as proposed by Blundell et al. (2008). Finally, we compare the implications of these earnings processes for wealth inequality and welfare.

6.1   Consumption inequality over the working life

We start by studying the rise in cross-sectional consumption dispersion over the lifecycle. Following Deaton and Paxson (1994) and Storesletten, Telmer and Yaron (2004b), it is common to interpret it as a measure of risk sharing.

A number of studies analyze the variance of (log) equivalized, household consumption in the U.S. by regressing its variance across households in different age-year groups on age and
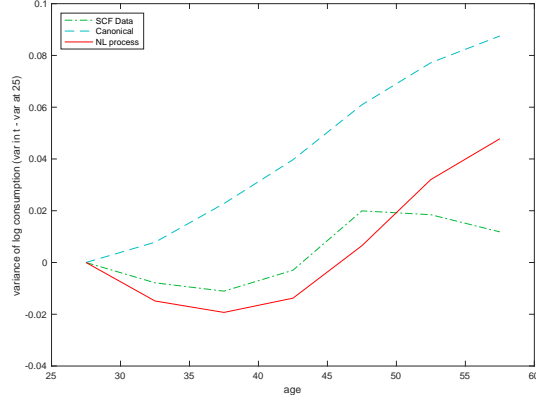
Figure 9: Growth in the cross-sectional variance of log consumption, data and implications of two earnings processes.

time dummies or age and cohort dummies. The coefficients of the age dummies are then used as a measure of the age profile of cross-sectional consumption dispersion. The green line in Figure 9 plots the age profile of the cross-sectional variance of (log) equivalized nondurable consumption between ages 25 and 60 computed from the CEX during the period 1980-2007, controlling for time effects and using the same data and procedure as Heathcote, Perri and Violante (2010). Given the relatively small sample size, we group observations in 5-year age groups. The series are normalized so that each starts at zero at age 27, which is the midpoint of the first 5-year age group (25–29). The dashed and solid lines plot the increase in the variance of consumption generated by the model under the canonical and nonlinear earnings processes, respectively.[16]

Because the increase in consumption inequality over the working period is informative about peoples' ability to insure against earnings risk, it provides a useful benchmark against which to assess the ability of the model to capture the degree of insurability of earning shocks in the data. The canonical earnings process fails to match both the overall growth and the shape of the profile of consumption dispersion. Its overall growth rate is more than double that in the data and its profile is monotonically increasing. Conversely, in the data, consumption dispersion dips between age 25 and 47. The nonlinear process, instead, matches

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[16]We perform this comparison recalibrating beta so as to keep the wealth to income ratio constant across earnings processes.

well the overall growth in consumption dispersion and captures the non-monotonic pattern in the first part of the life cycle. The one part that it misses is the flattening out after age 47. The finding that the estimated richer earnings processes implies a profile of consumption dispersion in line with the data is remarkable. Standard models with linear earnings processes (see Storesletten et al. ($2004a$)) generate a profile similar to the one implied by the canonical earnings process in Figure 9, and thus overstate the rate of growth of consumption dispersion, unless the process for earnings has an idiosyncratic deterministic time trend, or Heterogeneous Income Profile (Guvenen, 2007; Primiceri and Van Rens, 2009). Intuitively, heterogeneity in individual, life-cycle trend growth generates a substantially smaller rise in consumption dispersion because the individual-specific trend growth is known to consumers but not to the econometrician. Huggett et al. (2011) show that heterogeneity in earnings growth rates can be also generated by the endogenous response of human-capital investment over the life cycle to heterogeneity in initial human capital levels.

Our findings suggest a novel explanation: the age profile of cross-sectional consumption dispersion can be generated by the response of saving to the richer earnings dynamics that we consider, without resorting to heterogeneity in income profiles. It should also be noted that allowing for heterogeneity in income profiles cannot generate (cfr. Guvenen, 2007; Primiceri and Van Rens, 2009) the strong non-monotonicity that characterizes the consumption data (green line in Figure 9).

As we have discussed in Section 3.1, our rich earnings process deviates from the canonical linear process along three main dimensions: (1) age-dependence, (2) non-normality, and (3) nonlinearity. To understand the contribution of each of these factors to the growth of consumption dispersion over the life cycle, we conduct a series of counterfactual experiments, simulating the model under progressively richer stochastic processes for earnings.

We start by restricting the functional form of the earnings process to be the sum of an AR(1) plus a white noise component, as in the canonical process, but allowing for both age-dependent persistence and variance of shocks (as in Karahan and Ozkan (2013)), as well

as non-normality of their distributions. Compared to the fully general nonlinear earnings process, this one imposes linearity in $\eta_{i,t-1}$; namely, that persistence and other second and higher conditional moments are independent of $\eta_{i,t-1}$. We estimate this process on our PSID data, following the procedure described in Section 3.1 for the nonlinear process, but restricting the quantile function for the persistent component in equation (4) to be linear in its past value.

To further disentangle the effect of the age dependence of persistence and variance from that of non-normality, we perform two set of simulations using the restricted estimates that we have just described. In the first one, we simulate earnings using the estimated persistence and variances but drawing shocks from a normal distribution. In the second experiment, we simulate earnings using the estimated distribution (i.e. quantile function), that also allows for non-normality. We discretize each of the resulting processes using the method in Section 4. The recalibrated value of the discount factor equals 0.926 in the economy with normal shocks and 0.927 in the other one.

Figure 10 plots the cross-sectional variance profiles reported in Figure 9, with the addition of the two profiles implied by (a) only age-dependence and (b) age-dependence together with non-normality.

The solid dark blue line in Figure 10 corresponds to the case of an age-dependent linear process with normal innovations. Compared to the canonical case, allowing for age dependence substantially improves the fit of consumption dispersion in the first part of the life cycle, but counterfactually implies an even larger growth rate of consumption dispersion from age 43 onwards. The net effect for the age-dependent earnings process is an overall rate of growth in consumption dispersion between ages 25 and 60 that is three percentage points higher than in the canonical case.

The intuition behind the above finding is the following. Allowing for age-dependence implies that the estimated process for earnings matches the age-profile of the cross-sectional earnings variance in the bottom right panel of Figure 6; namely, relatively flat until age
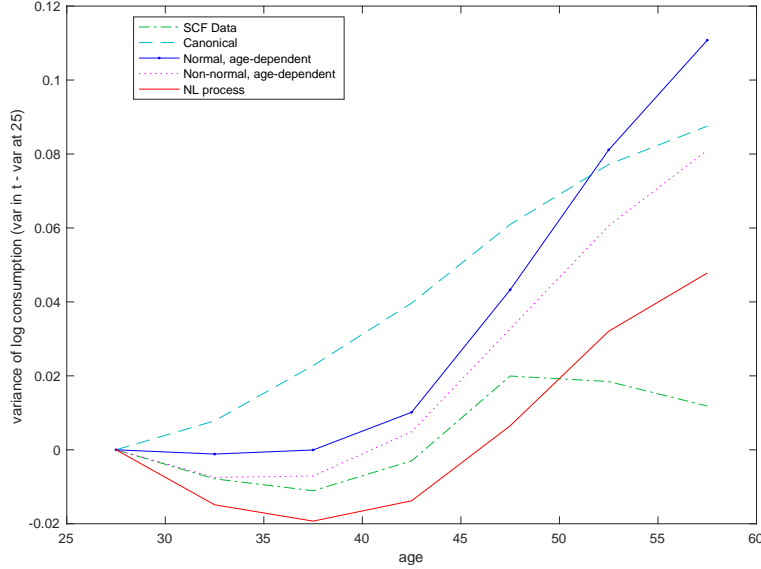
27

Figure 10: Growth in the cross-sectional variance of log consumption. Contribution of (1) age-dependence, (2) non-normality and (3) nonlinearity to the growth of the cross-sectional variance of log consumption.

43 but growing at a rate substantially above its working-life average afterwards. The forces underpinning this pattern are: (a) the U-shaped profile of the variance of the persistent component of earnings; and (b) a persistence below one that increases until age 45 but flattens out afterwards (see Figure 6). Compared to the canonical process with a unit root and constant shock variance, the interaction of these two forces implies that self-insurance through precautionary saving is more effective and, as a consequence, the growth in consumption dispersion is lower until middle age. In the second half of the working life, though, the increase in the variance of the persistent earnings shocks reduces the ability to self-insure and results in a substantial increase in consumption dispersion. This is confirmed by comparing the age profile of average wealth reported in the left panel of Figure 11 under the canonical (turquoise curve) and age-dependent earnings process with normal shocks (blue curve). Though the aggregate wealth-to-earnings ratio is the same in the two economies, average saving is higher before and lower after age 50 in the economy with age-dependent earnings process.

We now turn to the linear process with the same (age-dependent) first and second moments as above but with non-normal innovations. The dashed pink line in Figure 10 plots

the associated age profile of variance. Compared to the normal case, the rate of growth of the consumption variance is everywhere lower. The difference is particularly pronounced towards the end of the working life. To understand the mechanism at work, it is important to understand the impact of negative skewness and kurtosis on precautionary saving and the wealth distribution. Civale et al. (2016) study the issue in an Aiyagari economy and show that, everything else equal, negative skewness reduces both the cross-sectional mean and dispersion of wealth while kurtosis increases both.

The effect of higher kurtosis is in line with intuition. By increasing the probability of tail events higher kurtosis increases precautionary saving for all agents and therefore the mean and variance of the wealth distribution. The effect of negative skewness, though, is less intuitive. Basically, for a distribution to have higher negative skewness keeping the other moments constant, some probability mass has to move towards the top of the distribution. Wealthy agents are not sensitive to left skewness but, confronted with a higher probability of positive shocks, save less. Conversely, agents who are close to the borrowing constraint are more sensitive to skewness than to the higher probability of positive shocks and save more. In the aggregate, the response of wealth-rich agents dominates that of the wealth-poor and average wealth falls. More intuitively, so does the variance of wealth holdings. Comparing the solid blue and dashed red lines in Figure 11 reveals that, in our model, the net effect of negative skewness and kurtosis hardly affects the life-cycle profile of average wealth (left panel), but substantially reduces the rate of growth of the variance of wealth holdings (right panel) compared to the case with normal shocks. This fall in wealth dispersion accounts for the fall in consumption dispersion in Figure 10 when skewness and kurtosis of shocks are introduced.

Finally, comparing the dashed pink and red solid lines in Figure 10 shows allowing for nonlinearity brings the overall fit of life-cycle inequality closest to the data, compared to all of the earnings processes that we consider. Figure 8 provides insight into the effects of earnings nonlinearities. For individuals with previous earnings realizations below the median, positive
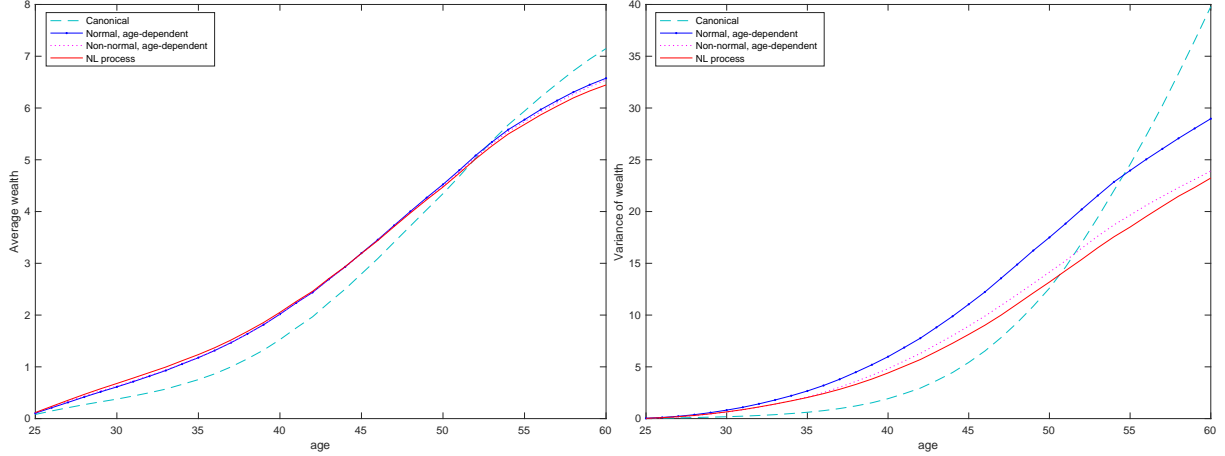
29

Figure 11: Cross-sectional average wealth (left) and variance of wealth holdings (right), by age and earnings process.

shocks (above the median) reduce the persistence of previous earnings. This implies that the memory of previous bad realizations is erased and effectively reduces earnings risk, even at a constant variance. Intuitively, in the full nonlinear process it is much more likely to change from a history of bad realizations to a history of good earnings realizations by means of one single shock (that could be thought of as joining a new company, or starting a new career). In the canonical earnings process, the fact that earnings are a random walk and that all past realizations are permanent makes such a transition much more difficult. This effect is present in reverse, though much less pronounced, for individuals with last earnings realizations in the top two deciles for which very negative shocks (below the first percentile) reduce earnings more than additively (this could be thought of as health shocks, for instance). The net effect is to increase the overall insurability of bad shocks and reduce growth of consumption dispersion over the life-cycle, bringing it much closer to the data, particularly for ages up to 50.

None of our earnings processes captures the flattening out in the variance of consumption that we measure after age 47 because the variance of earnings in the data keeps increasing. Our structural model misses two aspects of the data that could be important in this regard. The first one is early retirement. For retirees, income is mainly composed of Social Security

payments and does not vary much. Thus, consumption is no longer exposed to earnings fluctuations and medical expense risk is not very high until well into retirement age, as shown by De Nardi, French and Jones (2010). The second one is the role of durables and housing, that become substantial by that age and might affect both measured consumption (we only look at nondurable consumption) and one's ability to self-insure.

6.2   Measuring self-insurance against earnings shocks

An alternative, and possibly more intuitive, measure of self-insurance is related to the extent of pass through from shocks to disposable earnings onto consumption. Blundell et al. (2008) propose estimating consumption insurance coefficients on persistent and transitory earning shocks by positing the following equation

(11) $$\Delta c_{it} = (1 - \psi^p)\nu_{it} + (1 - \psi^{tr})\varepsilon_{it} + \xi_{it},$$

where $\nu_{it} = \eta_{it} - E[\eta_{it}|t, \eta_{i,t-1}]$ denotes the innovation to the persistent component of earnings and ε_{it} the transitory component. The insurance coefficients with respect to persistent (ψ^p) and transitory (ψ^{tr}) shocks

(12) $$\psi^p = 1 - \frac{\text{cov}(\Delta c_{it}, \nu_{it})}{\text{var}(\nu_{it})}, \quad \psi^{tr} = 1 - \frac{\text{cov}(\Delta c_{it}, \varepsilon_{it})}{\text{var}(\varepsilon_{it})}$$

capture the fraction of the variance of either type of shock that does not translate into movements in consumption. Similarly, one can compute age-specific insurance coefficients ψ_t^p, ψ_t^{tr} where moments are computed only over agents of age t.

To compute the insurance coefficients implied by our model, we simulate a panel of working lives under both the benchmark and nonlinear processes and compute the associated consumption c_{it} and insurance coefficients in equation (12) on the simulated data.

Computing the coefficients in equation (12) within the model is straightforward since the shocks are observable. In contrast, estimating them from the data requires identifying the two

types of earning shocks at the individual level. Blundell et al. (2008) propose an identification strategy under the assumption that earnings follow the canonical linear process (1)-(3). The estimators for the insurance coefficients based on the BPP methodology are given by

$$(13) \qquad \psi^p_{BPP} = 1 - \frac{\text{cov}(\Delta c_{it}, y_{i,t+1} - y_{i,t-2})}{\text{cov}(\Delta y_{it}, y_{i,t+1} - y_{i,t-2})}, \ \ \psi^{tr}_{BPP} = 1 - \frac{\text{cov}(\Delta c_{it}, \Delta y_{i,t+1})}{\text{cov}(\Delta y_{i,t}, \Delta y_{i,t+1})}.$$

As pointed out by Kaplan and Violante (2010), comparing the coefficients in equation (13) estimated within the model to the estimates in Blundell et al. (2008) conveys information on the degree of shock insurability in the model relative to the data.

The coefficients in equation (13), though, may provide biased estimates of the true coefficients in equation (12) to the extent that the identification assumption on which they are based is violated. The assumption can be violated for two reasons. First, if earnings do not follow the canonical linear process in equation (1)-(3). This is obviously true in the more flexible cases we consider. Second, as pointed out by Kaplan and Violante (2010), even if earnings follow a canonical linear process the ψ^p_{BPP} estimator may be biased whenever consumption does not equal permanent income, as is the case in the presence of a precautionary saving motive.[17] For this reason, we compute both types of coefficients. Table 2 reports their values under the alternative income processes.

Columns 1 and 2 in Table 2 report the coefficients in equation (13). As a reference, the first row reports the estimates by Blundell et al. (2008)—respectively 0.36 for permanent and 0.95 for transitory shocks—on the PSID using similar data to ours.[18] The corresponding values for the model, when earnings follow the canonical earnings process, are 0.14 and 0.88, which confirms the finding by Kaplan and Violante (2010) that the extent of self-insurance of permanent earnings shocks implied by the model is substantially lower than

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[17]Formally, the bias is present whenever present consumption responds to past persistent income changes, which implies that $\text{cov}(\Delta c_{it}, y_{i,t+1} - y_{i,t-2})$ is a biased estimator of $\text{cov}(\Delta c_{it}, \nu_{it})$. Kaplan and Violante (2010) show that this is indeed the case in a life-cycle model similar to ours with a canonical earnings process and occasionally-binding borrowing constraints.

[18]Blundell et al. (2008) conduct their analysis using disposable household earnings for continuously married coupled headed by a male head.

Process/Coefficients	ψ^p_{BPP}	ψ^{tr}_{BPP}	ψ^p	ψ^{tr}
	Data: BPP (2008)			
Canonical (S.E. in parenthesis)	0.36	0.95	–	–
	(0.09)	(0.04)		
	Model			
Canonical	0.14	0.88	0.30	0.91
Nonlinear process	0.43	0.81	0.46	0.89
Normal, age-dependent	0.41	0.82	0.46	0.88
Non-normal, age-dependent	0.41	0.82	0.45	0.84

Table 2: Insurance coefficients

the degree of insurance in the data. On the other hand, the estimates for the model with a nonlinear earnings process imply an insurance coefficient for persistent shocks of 0.46 which is substantially more in line with, and even marginally larger than, the BPP estimate.

From a qualitative perspective, this result is very much in line with our findings in Section 6 that agents are more able to self-insure against income fluctuations when earnings follow the nonlinear process than in the canonical case. Interestingly, our finding that allowing for a richer earnings process implies a substantially different estimate of the insurance coefficient for persistent shocks is confined to *disposable* household earnings. Using the same earnings process we use here, Arellano et al. (2017) estimate an average insurance coefficient for persistent shocks to *pre-tax* household earnings between 0.6 and 0.7 which is in line with an estimate of 0.69 in Blundell et al. (2008) under the identifying assumption that earnings follow the canonical process. As discussed in Blundell et al. (2008), the nearly double magnitude of the insurance coefficients with respect to *pre-tax* rather than *disposable* earnings is due to the of insurance implied by the tax and transfer system.

Turning to the insurance coefficient for transitory shocks in column 2, it may seem surprising that it is higher under the canonical than under the nonlinear earnings process. As pointed out in Kaplan and Violante (2010), though, the intuition is that the increased insurability of persistent shocks induces households to shift the use of savings from the smoothing of transitory shocks to the smoothing of persistent shocks.

Columns 3 and 4 in Table 2 report the estimates of the *true* insurance coefficients in equation (12) within the model. Comparing them to the BPP estimates in columns 1 and 2 reveals that the downward bias of the insurance coefficient for persistent shocks implied by the BPP procedure is sizeable (0.14 against 0.3) in the case of the canonical income process but small (0.43 against 0.46) for the nonlinear process. The intuition is that, as pointed out by Kaplan and Violante (2010), the bias is exacerbated in an economy in which the borrowing constraint is occasionally binding. As discussed above, when earnings follow the nonlinear process shocks are more insurable, and precautionary saving larger. For this reason, the economy spends less time close to the borrowing constraint and the bias is lower.

Finally, the last two lines reports the same coefficient for the case with age dependence and normal shocks and the one that also allows for non-normality. Comparing the three set of estimates reveals that the feature that drives the better match of the insurance coefficient for persistent shock estimated by BPP is essentially the age dependence of the earnings process. This is consistent with the finding in Karahan and Ozkan (2013) that the (true) insurance coefficient for persistent shocks in a life-cycle economy with an age-dependent earnings process with normal shocks is 0.38.[19]

While Table 2 reports the average insurance coefficients, Figure 12 plots the true insurance coefficient for persistent shocks ψ_t^p at each age. The coefficients are increasing with age, as: (a) wealth is accumulated; and (b) the fall in the residual working life reduces the effective shock persistence. The degree of insurability at all ages but the last working age is substantially higher under the nonlinear earnings process than under the canonical one. For the same reason, the age profile of the coefficients is substantially flatter in the former case. In line with the discussion above, most of the difference is due to the age-dependence of earnings. It is only from age 45 onwards that the coefficients are marginally higher under the nonlinear process than under the age-dependent earnings process with normal shocks.

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[19]The earnings process used by Karahan and Ozkan (2013) is similar to our age-dependent process with normal shocks. Their estimate of 0.38 for the true coefficient ψ^p is in the ballpark of our estimate of 0.46 in Table 2.
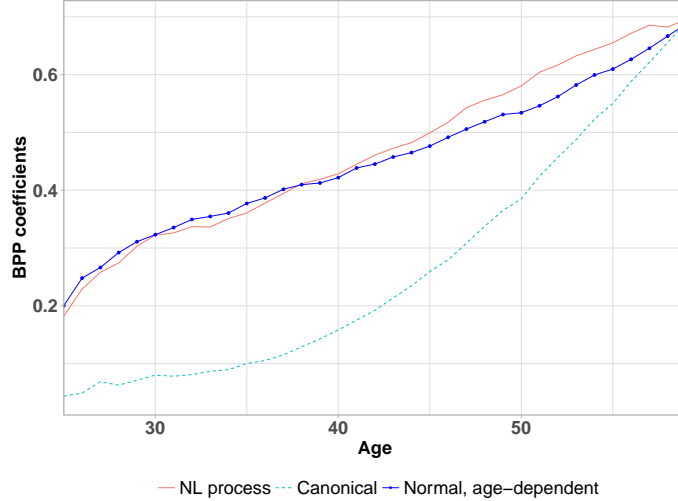
34

Figure 12: Partial insurance coefficients on persistent shocks, ψ_t^p, by age

	Wealth Gini	Percentage wealth in the top					
		1%	5%	20%	40%	60%	80%
Data (SCF 1989)	.79	30	54	81	94	99	100
Model: Canonical	.62	9	27	63	85	96	99.5
Model: Nonlinear process	.59	7	24	59	84	95	99.3

Table 3: Wealth distribution

6.3   Wealth

Table 3 compares the implied wealth distribution of the canonical and nonlinear earnings processes with data from the U.S 1989 SCF (Kuhn and Ríos-Rull, 2015).

As known in the literature (see Quadrini and Ríos-Rull (2014), Cagetti and De Nardi (2008), and De Nardi and Fella (2017)), the model with a canonical earnings process is unable to generate the substantial level of wealth concentration that we observe in the data. For instance, the top 1% of agents holds about 30% of total wealth in the data, while the corresponding share is only holds 9% in the model. Comparing the second and third rows in the table reveals that allowing for nonlinear earnings does not improve the fit of the wealth the distribution. If anything it marginally reduces the degree of wealth concentration at the

	Welfare cost
Canonical process	28.91%
Nonlinear process	27.35%

Table 4: Consumption measure of welfare costs

top.[20] One may think that this may be due to the nature of the PSID data, which top-codes earnings and does not oversample the rich. However, De Nardi et al. (2016) conduct a similar exercise using synthetically-generated W2 Social Security Administration tax data, which do not suffer from those top-coding and lack of oversampling, and they imply very similar results for the concentration of wealth at the top. As pointed out by De Nardi and Fella (2017), non-homothetic preferences for bequests, entrepreneurship, and medical-expense risk are crucial for life-cycle models to be able to account for top wealth concentration.

6.4   Welfare

The differences in the evolution of the variance of log consumption and the pass-through of income shocks to consumption show that income risk affects households in a different way in the two economies. A natural question is to which extent these differences affect welfare.

To measure welfare, Table 4 displays the constant fraction of consumption that households are willing to give up to live in a world with no income uncertainty; i.e., a world where earnings are equal to the common and deterministic average earnings profile. We compute this measure under the veil of ignorance (before people enter the labor market and draw the first earnings realization) and, for comparability, we keep the discount factor the same for both processes and fix it to its calibrated value for the nonlinear process.

The nonlinear process features larger variance of shocks, negative skewness and high kurtosis, but also nonlinear persistence, that, as we have discussed in Section 6.1, improves shock insurability. Viceversa in the canonical model, the lower variance of shocks at all ages after the first one is counteracted by their high persistence (unit root) and the higher

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[20]We target a wealth to income ratio of 3.1, but this has little effect on wealth concentration.

variance of the initial condition. The net effect of all these forces is that overall risk is higher under the canonical process. In particular, households be willing to give up 28.91% of their consumption in every state to eliminate earnings risk under the canonical earnings process compared to 27.35% under the nonlinear earnings process.

7   Conclusions

We estimate a richer stochastic process for *household disposable earnings* featuring a transitory and persistent component and allowing for age-dependence, non-normality and nonlinearity. We use a standard life-cycle model with incomplete markets to compare the implications of our richer process to those of canonical permanent/transitory linear process with age-independent, normally-distributed shocks. Our main findings are as following. Compared to the canonical process, the richer process implies a much better fit of the growth in cross-sectional consumption dispersion over the life cycle and a degree of self-insurance of persistent earnings shocks in line with the empirical estimates in Blundell et al. (2008). It also implies smaller welfare costs of earnings fluctuations. In terms of wealth inequality, we find that the two earnings processes have similar implications, including at the upper tail of the wealth distribution.

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A Appendix: PSID data

A.1 The PSID

The Panel Study of Income Dynamics (PSID) follows a large number of U.S. households over time and reports information about their demographic characteristics and sources of income. The PSID was initially composed of two major subsamples. The first of them, the SRC (Survey Research Center) or core subsample, was designed to be representative of the U.S. population and is a random sample itself. The second, the SEO (Survey of Economic Opportunity) subsample, was created to study the characteristics of the most deprived households. Later, Immigrant and Latino subsamples were also added to the PSID.

From 1968 to 1997, the survey was yearly. After 1997, it started having a biennial structure. We only consider the SRC or core subsample because the SEO oversamples the poor. After dropping the SEO and Latino samples we are left with a random sample, which makes computations simpler since weights are not needed (Haider, 2001) [21].

A.2 Sample selection

Since the model period is one year, we restrict ourselves to the yearly part of the survey, and focus on the years 1968-1992. In our main results we do not consider the period 1993-1997 because the procedure via which information was collected was substantially redesigned (with the introduction of computer-based surveys) and there were changes in some of the variable definitions we rely on (for instance, asset income of other family members is no longer available, and wife labor income is redefined). We have verified that results are not sensitive to including these five years.

Following standard practice in the literature, we only consider individuals between ages 25 and 60. This also allows us to have a relatively large amount of data per age group, which

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[21]It must be taken into account that the weighting of our final dataset can be affected by attrition and by the fact that we are neglecting observations of yearly income under $ 1500 (expressed in 2015 dollars)

is necessary for our binning procedure.

Unlike many other papers, but similarly to Krueger, Mitman and Perri (2016) we consider all households, whether or not male-headed. We do not impose any restrictions regarding e.g. family composition or its changes, as we consider that, once we have properly equivalized earnings, all remaining changes due to family composition shocks are also possible sources of income risk that we wish to capture.

We deflate values to 2013 dollars, and only keep observations above $1500. This is also in accordance with standard practice in the literature, where observations below a minimum earnings threshold are dropped (De Nardi (2004) or Guvenen et al. (2016), for instance).

A.3   Income definition

Our main income definition is post-tax equivalized household earnings. We obtain it by computing nonfinancial household earnings in the PSID, estimating and using a tax function to predict post-tax earnings, and finally running a regression on the number of family members for the purposes of equivalization.

A.3.1   Nonfinancial tax income

We construct nonfinancial pre-tax income closely following Guvenen and Smith (2014). Namely, before the 1976 wave we construct it by subtracting head+wife taxable income (which includes asset income) from total family income and then adding back earnings for the head and the wife. Between 1976 and 1983, we construct it by subtracting asset income of head and wife from total family income. Asset income is formed of farm income, business income, rent and interests, with the addition of gardening and roomers income (from 1978). Between 1984 and 1992, asset income of family members other than head and wife becomes available, so we subtract that as well.

We keep top-coded observations, but drop the very small number (8) of households who, probably due to measurement error, would have nonfinancial income below zero.

A.3.2 Tax function

To obtain nonfinancial **disposable** income, we run a regression of the federal income tax variable (which is available in the PSID until 1990) on nonfinancial income and its square, and asset income and its square. This also follows Guvenen and Smith (2014). We use the estimated coefficients to predict post-tax labor income.

A.3.3 Equivalization

We then regress log post-tax nonfinancial disposable income on year fixed effects and a dummy for the number of family members in the household, and keep the residuals. Age fixed effects are controlled for when we implement the Arellano et al. (2017) procedure, but could otherwise be controlled for at this stage.

To finally implement the Arellano et al. (2017) procedure, we create a sample with all sets of subsequent three-year observations (without replacement: once an observation in the PSID sample is in a 3-year set in our sample we drop it). This implies that we are also dropping all of those households that do not have three consecutive valid income observations in the PSID.

A.3.4 Estimating the canonical earnings process

In Storesletten et al. (2004a) (and in many other papers in the literature, e.g. Krueger et al. (2016)) the earnings process is estimated by fitting a parametric process to the variance of earnings profile that we observe in the data. The standard way is to compute the variance of earnings by age-cohort-year cells, and then get the coefficients of a regression of those on either age and year or age and cohort. For consistence with our approach and with the consumption data we rely on, we use the one that controls for year effects (see discussion below).

We follow a GMM procedure in which we minimize the distance of the estimated process

to the profile of variances and first-order autocovariances of earnings over the life cycle[22].
The weighting matrix is the identity matrix.

The canonical earnings process in equations (1)-(3) implies (for $t > 1$)

$$(14) \qquad y_{it} = \rho^{t-1}\eta_{i1} + \sum_{j=2}^{t} \rho^{t-j}\zeta_{ij} + \varepsilon_{it}$$

from which

$$(15) \qquad var(y_{it}) = \rho^{2(t-1)}\sigma_{\eta_1}^2 + \sum_{j=2}^{t} \rho^{2(t-j)}\sigma_{\zeta}^2 + \sigma_{\varepsilon}^2.$$

and

$$(16) \qquad cov(y_{it}, y_{i,t+1}) = \rho^{2t-1}\sigma_{\eta_1}^2 + \sum_{j=2}^{t} \rho^{1+2(t-j)}\sigma_{\zeta}^2$$

follow, allowing to identify moments.

B   Appendix: Computation of the variances of log earnings and log consumption.

We estimate the canonical earnings process described in Section 3.2 by matching the variance and first autocovariance of log earnings.

To compute the variance of log earnings, which we report in Figure 6, we use the procedure described in Kaplan (2012) (Appendix C.3), controlling for year effects. More specifically, we take log disposable and equivalized labor income $\tilde{y}_{it}$, where i indexes the household and

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[22]We describe in Appendix B how we compute these variances.

t is the age of its head, and run the regression

(17)
$$\tilde{y}_{it} = \beta'_t \mathfrak{D}_t + \beta'_d \mathfrak{D}_d + y_{it},$$

where $\mathfrak{D}_t$ and $\mathfrak{D}_d$ are matrices with columns corresponding to a full set of age and year (date) dummies, respectively. The vectors β_t and β_d are the corresponding coefficients and y_{it} the earnings residuals0.[23]

We compute the variance of y_{it} by age group as

(18)
$$Var_t(y) = \frac{1}{D} \sum_{d=1}^{D} \left(\sum_{i=1}^{N_{d,t}} \frac{y_{it}^2}{N_{d,t}} \right),$$

where D is the number of years in the dataset, and $N_{d,t}$ is the numerosity of each age-year cell. This implies that the variance of earnings at age t weighs equally the corresponding conditional variances of earnings in each year.

We also compute the variance of y_{it} by age group controlling for cohort instead of year effect, s using the coh01ort counterpart of equation (18)

(19)
$$Var_t(y) = \frac{1}{K(t)} \sum_{k=1}^{K(t)} \left(\sum_{i=1}^{N_{k,t}} \frac{y_{it}^2}{N_{k,t}} \right),$$

where $K(t)$ is the number of cohorts containing individuals of age t and $N_{k,t}$ is the numerosity of each cohort-age cell.[24] This approach weighs the conditional variances from each cohort equally.

Under both approaches, we obtain very similar age profiles (Figure 13) and parameter estimates for the canonical process (Table 5).

Turning to consumption, we compute the variance of log consumption using data from the

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[23]As described in Appe0ndix A, we use the earnings residuals from equations (17) to estimate our earnings processes.

[24]The residuals used in equations (18) and (19) are the same. Given that year, age and cohort are linearly dependent, the residuals from equation (17) are the same that would obtain from projecting onto age and cohort dummies.
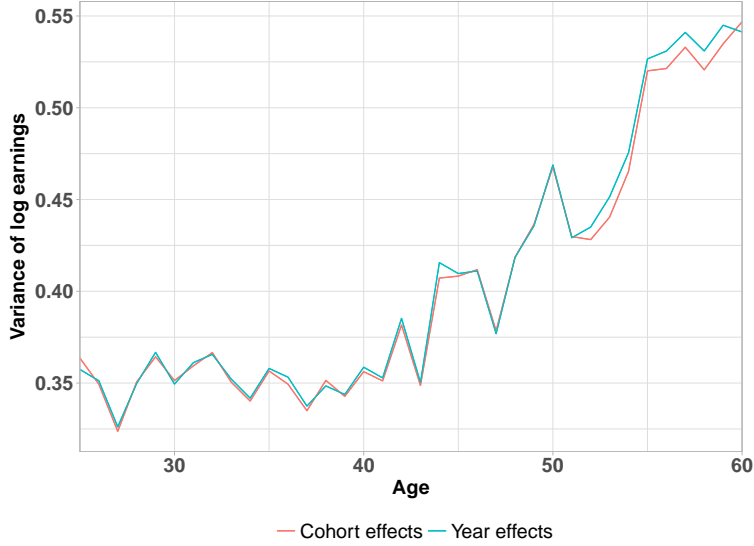
Figure 13: Cross-sectional variance of log earnings over the life cycle, cohort effects vs year effects

	σ_ε^2	$\sigma_{\eta_1}^2$	σ_ζ^2	ρ
Year effects	0.0675	0.2363	0.0059	1
Cohort effects	0.0675	0.2304	0.0059	1

Table 5: Estimates for the canonical earnings process: cohort vs. year effects

CEX for the period 1980-2007. Nondurable consumption includes food, clothing, gasoline, household operation, transportation, medical care, recreation, tobacco, and education.

We perform our computations of the variance of consumption using two different methods. First we apply to the same procedure proposed by Kaplan (2012) that we use for earnings. Second, for comparability, we also compute it as in Heathcote, Perri and Violante (2010) (HPV10). Figure 14 shows that the implied variances of consumption by age are very similar.

More specifically concerning our computations to mimic HPV10, we use (log) equivalized nondurable consumption for households whose head is between ages 25 and 60 and form 5-year age groups. More specifically, we first compute the variance of equivalized consumption by age-cohort-year cells. The coefficients by age are then obtained by running a regression of those variances on age and year dummies, and reporting the former (which are by construction differences with respect to the first age group). The implied profile is extremely

similar to that obtained if we were to use the same procedure as we used for earnings (see Figure 14).
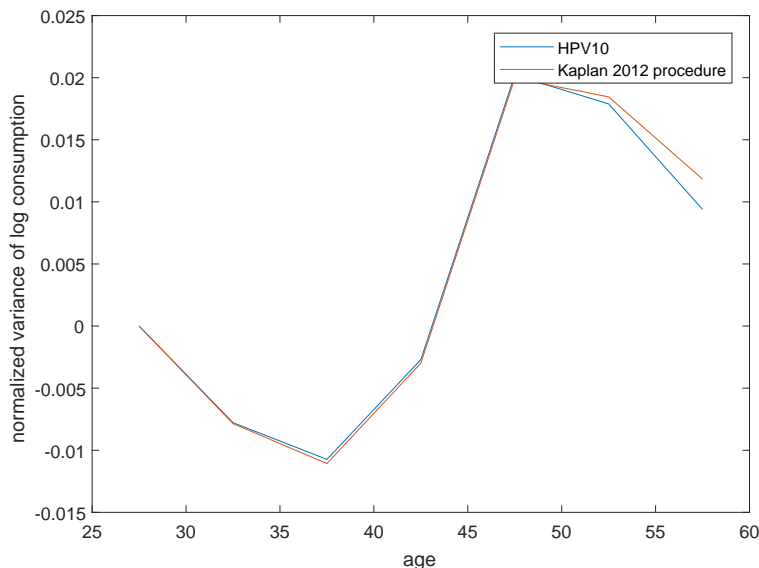

Figure 14: Cross-sectional variance of consumption over the life cycle, HPV10 vs Kaplan (2012).

C Appendix: Fit of the earnings process to the data and robustness of the discretization

To sum up, our procedure for earnings requires two steps:

1. Apply the Arellano et al. (2017) decomposition to *PSID data* to obtain the *persistent* and *transitory* components of earnings. This is described in Sections 3.1 and 3.3.

2. Discretize the simulated persistent component and transitory component using an age-dependent Markov chain to obtain the *discretized persistent* and *discretized transitory* components. This is described in Section 4.

In this appendix we show that (1) the central features of the data that we are interested in replicating are preserved by the discretized process that we use in the structural model

and (2) that our main findings are robust to increasing the number of grid points for the two components of earnings.

C.1 Fit of the earnings process to the data

C.1.1 Conditional moments of earnings changes

Figure 15 plots the second, third, and fourth standardized moments of earnings changes. The top panel refers to our PSID sample (and thus replicates the bottom panel of Figure 3). The central panel displays the same measures computed on earnings (i.e. the sum of the *persistent* and *transitory* components), simulated using the estimated nonlinear process associated with the quantile functions in (6)-(8). The bottom panel reports the corresponding moments computed on earnings simulated using our discrete approximation of such continuous processes.

The polynomial quantile functions and their discretization smooths the skewness and kurtosis graphs (which are very noisy and affected by outliers). Yet, the main patterns of the data (negative skewness, large kurtosis and variation over previous earnings) are preserved.

Figure 16 zooms in on just the estimated *persistent component* of earnings and compares its features of the *persistent component* (top) with those its discretized counterpart. It suggests that our flexible discretization, despite only having 18 bins per age group, captures these feature of the data.

Figure 17 conducts the same comparison for the estimated transitory component and shows that our discretization reproduces the observed age-dependence, high negative skewness, and large kurtosis. The 8-gridpoints discretization (top panel) generates much larger kurtosis than that of a normal distribution but falls a bit short of that in the data because it cannot accurately capture the effect of outliers. A finer, 16-gridpoints, discretization (bottom panel), which has additional bins on the tails, does match the kurtosis in the data. In Appendix C.2, we describe this alternative discretization more in detail and we show it does not make a difference for our results from our structural model.
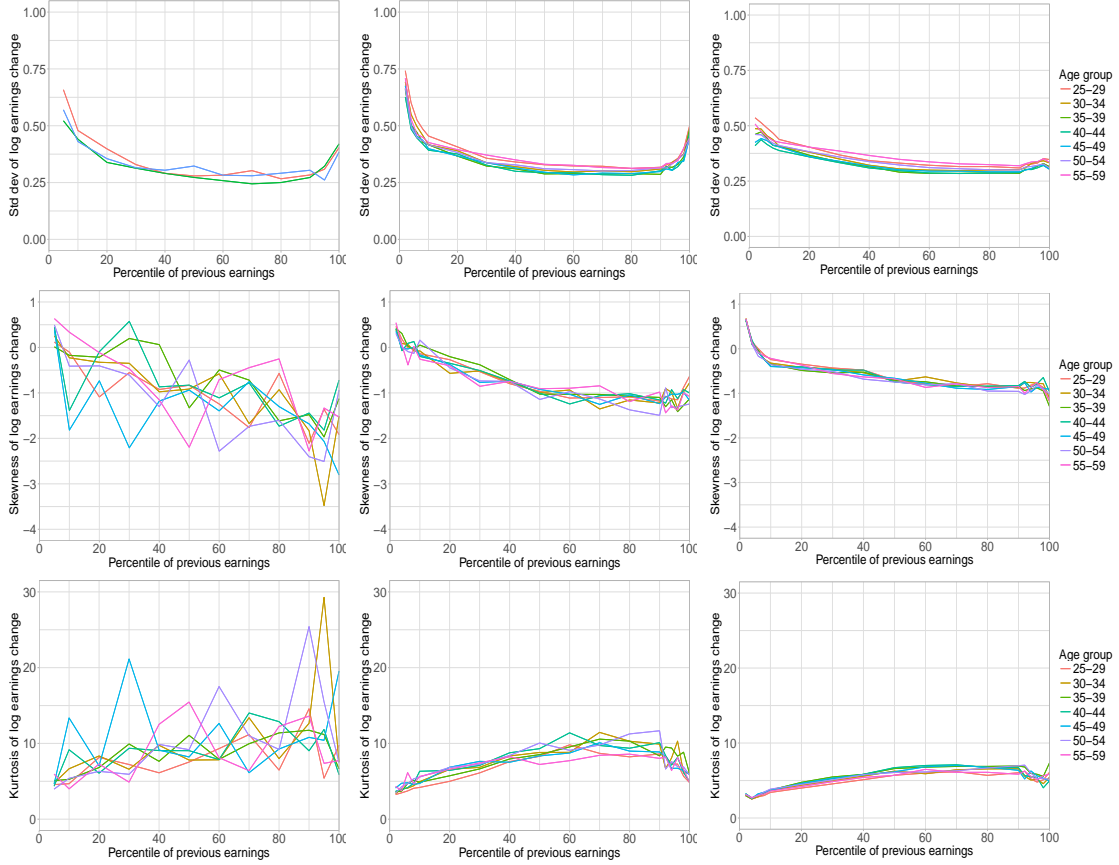
Figure 15: Conditional moments of earnings changes. From top to bottom: standard deviation, skewness, kurtosis. Left: PSID data; center: simulated earnings using estimated nonlinear process; right: simulated earnings using discretized nonlinear process.

Finally, Figure 18 reports persistence by previous-earnings (percentile τ_{init}) and current-shock (percentile τ_{shock}) rank. The top left panel refers to the PSID data, the top right panel to earnings simulated using the discretized process (persistent plus transitory component), the bottom left panel to the estimated *persistent component*, and the bottom right panel to its discretized counterpart. The discretization makes the graph for nonlinear persistence less smooth, but it preserves most of its important features. Namely, earnings are less persistent for high earners who receive a very bad shock and and low earners who receive a very good shock, while they are most persistent for high earners who receive a good shock and low earners who receive a bad shock.
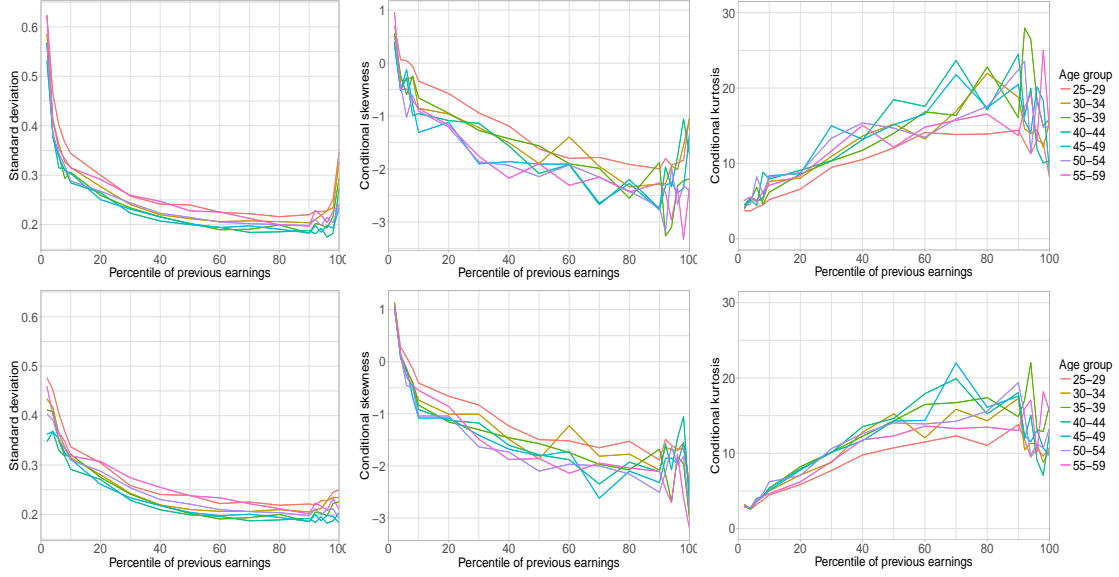
Figure 16: Conditional moments of earnings changes (persistent component). From left to right: standard deviation, skewness, kurtosis. Top: persistent component; bottom: discretized persistent component.

C.1.2 Unconditional moments of persistent earnings

Figure 19 plots the *unconditional* moments of the persistent earnings distribution, as opposed to the conditional moments of earnings changes in the previous sections. Our discretization captures very well their levels and variations by age, but similarly to Figure 17 we need a finer discretization to match the very high levels of kurtosis in the data. We describe this finer discretization in Section C.2, where we also show that it generates very similar results to our main discretization.
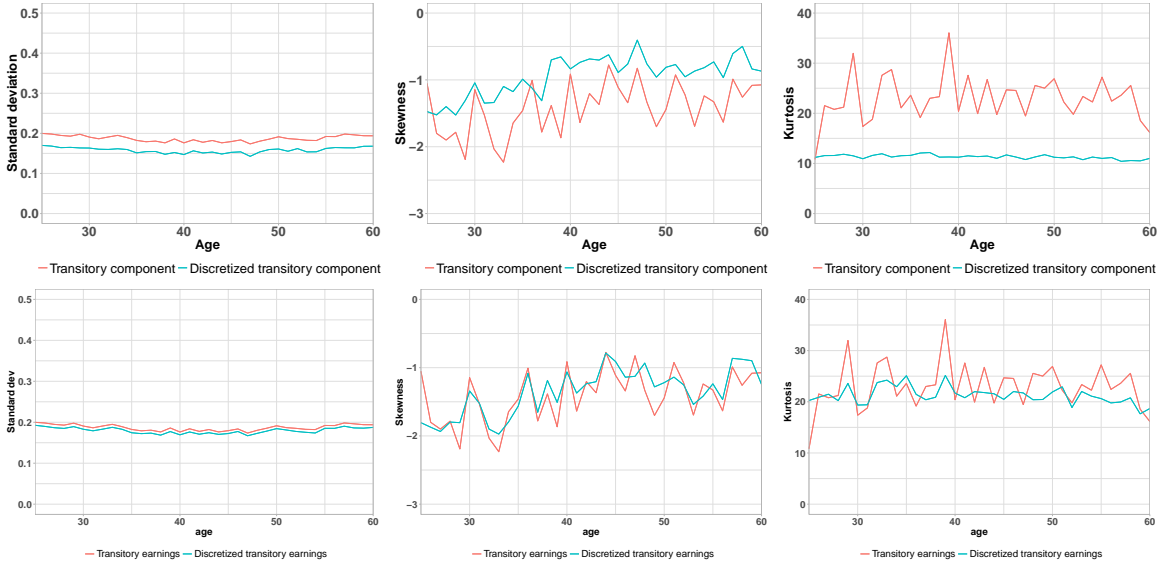
Figure 17: Moments for the transitory shock. From left to right, standard deviation, skewness and kurtosis (top: main 8-gridpoint discretization; bottom: 16-gridpoint discretization).
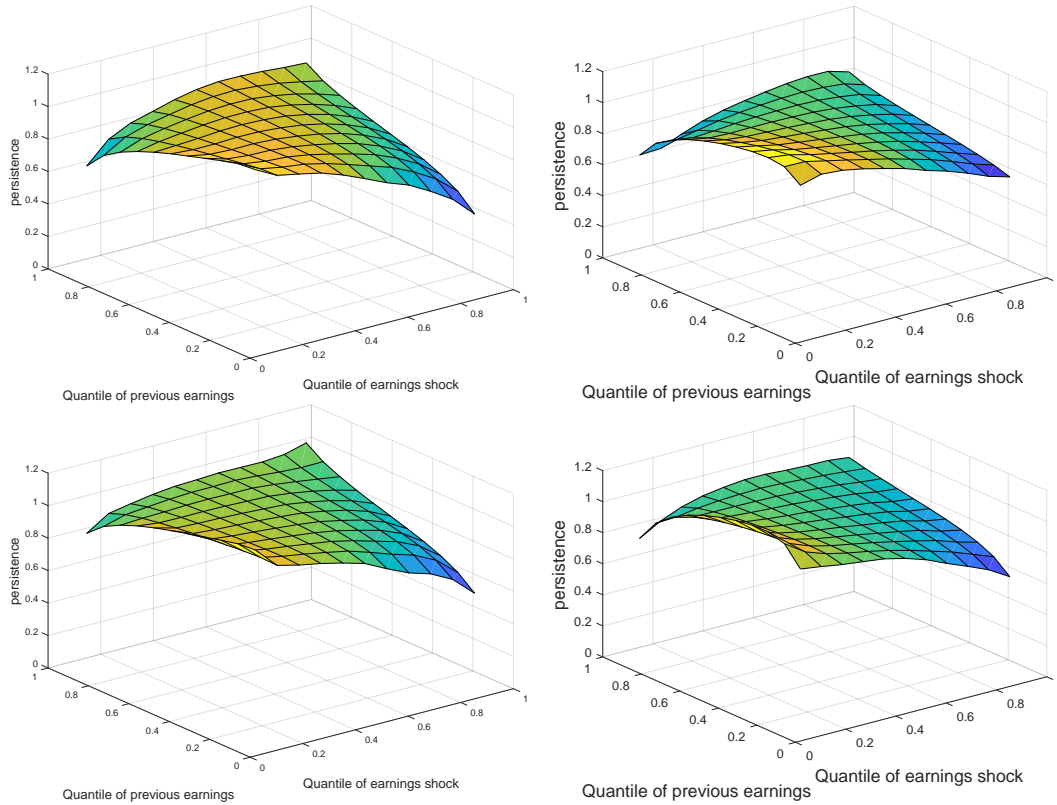

Figure 18: Nonlinear persistence by quantile of previous earnings and quantile of the shock received in the current period (top left, PSID data; top right, persistent component; bottom left, discretized persistent + transitory component; bottom right, discretized persistent component).
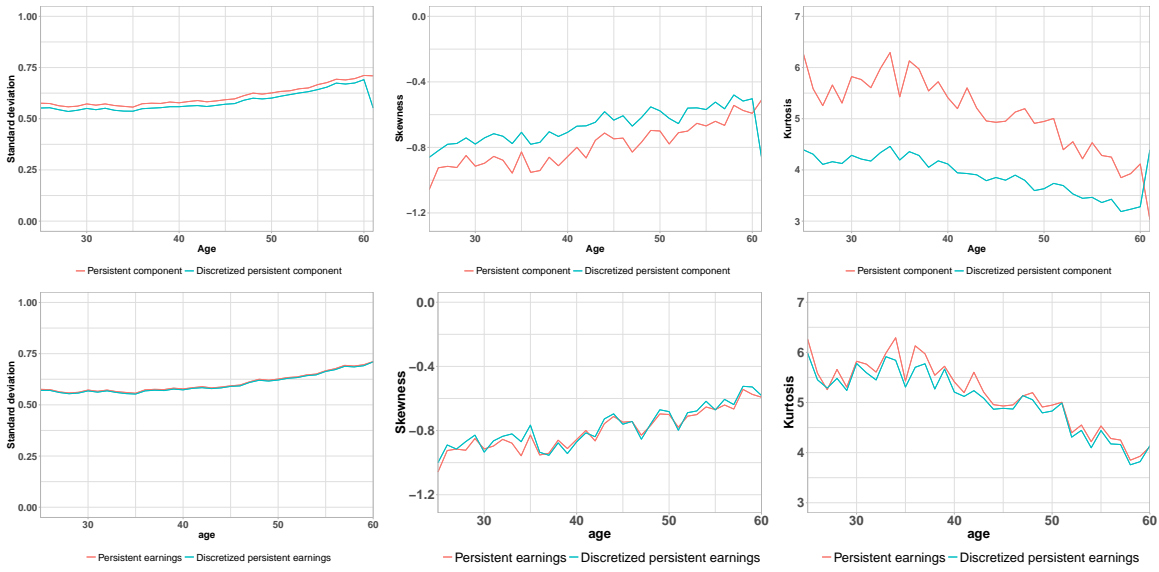
Figure 19: Unconditional moments of the persistent earnings distribution: from left to right, standard deviation, skewness and kurtosis (top: main 18-gridpoint discretization; bottom: 36-gridpoint discretization).

C.2 Robustness to the number of earnings gridpoints

In our main results, at each age, we discretize the persistent component of earnings with 18 gridpoints and the transitory one with 8 gridpoints, as described in Section 4. These are the smallest grid sizes beyond which adding additional grid points makes little difference for the economic implications discussed in Section 6.

In this section, we report results on simulating our model with finer grids and show that our results are robust to this changes. The finer discretization has the advantage that it fits some moments of the data better (like the unconditional moments of the transitory and persistent earnings shocks, as seen in Figures 17 and 19).

Figure 20 and Table 6 show the growth in the variance of log consumption and the BPP coefficients under finer discretization for the transitory component. More specifically, the transitory component is divided into, respectively, 8 gridpoints as in our main results (corresponding to the bottom 2.5%, next 2.5%, next 5%, next 40%, next 40%, next 5%, next 2.5% and top 2.5%) and 16 gridpoints (bottom 0.1%, next 0.4%, next 0.5%, next 2%, next 2%, next 5%, four quintiles, next 5%, next 2%, next 2%, next 0.5%, next 0.4% and finally top 0.1%). The differences between the 8- and 16- gridpoints specification are very small.

Figure 21 and Table 7 show the results for alternative discretizations of the *persistent component*. Namely, it compares our main results with a 36-gridpoints discretization which adopts a very thin division of the bottom and top percentiles (for the top, there is a bin for the top 0.05%, following 0.05%, 0.2%, 0.2% and 0.5% and symmetrically for the bottom), percentiles for the rest of the top and bottom 5% and groups of 5% for the rest of the persistent earnings distribution. Differences are, again, minor.
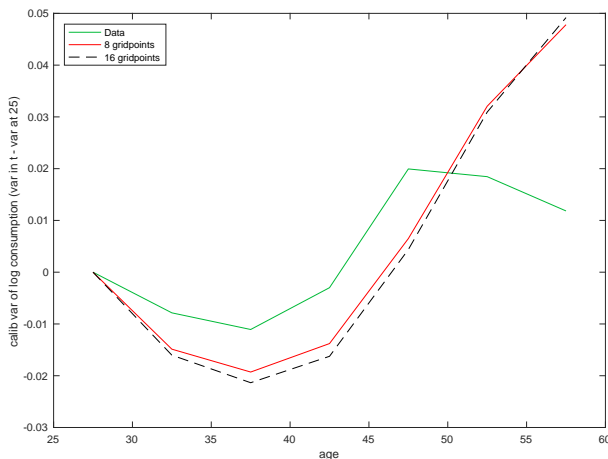
Figure 20: Growth in the variance of log consumption, different discretizations for the transitory component.

Coefficients	ψ^p_{BPP}	ψ^{tr}_{BPP}	ψ^p	ψ^{tr}
Data	0.36	0.95	–	–
NL process	0.43	0.81	0.46	0.89
NL process, 16 gridpoints	0.42	0.82	0.46	0.88

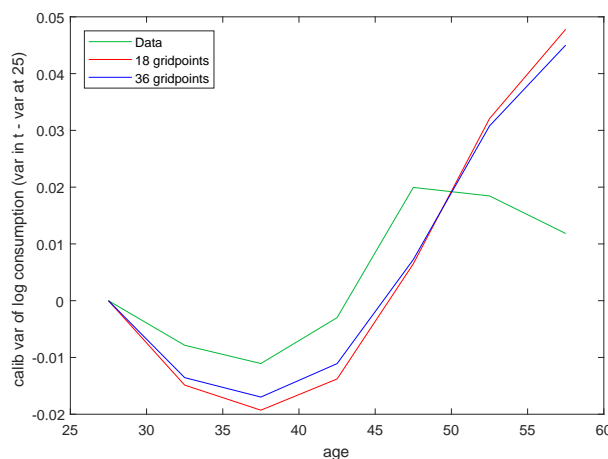*Table 6:* BPP coefficients, different discretizations for the transitory component.

Figure 21: Growth in the variance of log consumption, different discretizations for the persistent component.

Coefficients	ψ^p_{BPP}	ψ^{tr}_{BPP}	ψ^p	ψ^{tr}
Data	0.36	0.95	–	–
NL process	0.43	0.81	0.46	0.89
NL process, 36 gridpoints	0.41	0.82	0.44	0.9

Table 7: BPP coefficients, different discretizations for the persistent component.

C.3 Alternative discretization of the canonical process

As described in Section 5.5, for our main results we discretize the canonical process using the modified version of the Rouwenhorst method for non-stationary processes proposed by Fella et al. (2017). Here we show that our findings that the nonlinear process provides a substantially better fit are not due to using to different discretizations for the canonical and the nonlinear processes. To this effect, we discretize the canonical process by taking the parametric estimates in Table 1, simulating a panel of earnings histories and applying the same the same age-varying Markov chain procedure as we followed for the NL process.

Figure 22 and Table 8 show the implied consumption profiles and BPP coefficients. Both discretizations give rise to qualitatively similar results. Under the alternative discretization, the canonical earnings process overshoots the growth in the variance of log consumption over the life-cycle by an even larger amount.

C.4 Results without persistent-transitory decomposition

In Section 3.1 we describe the flexible earnings process that we estimate, which is based in the persistent-transitory decomposition proposed by Arellano et al. (2017). However, an alternative, computationally less costly choice is to apply directly our Markov-chain flexible discretization method to the raw data. Figure 23 and Table 9 provide the results of applying that simpler method to our PSID sample.

Neglecting the persistent-transitory decomposition implies a substantial underestimation of the growth of the variance of log consumption over the life cycle and, consistently, an overestimation of the ability of households to self-insure against earnings shocks. This can partially reflect the existence of measurement error in the PSID, and provides further support to the procedure we follow in our main results. With administrative data, where the measurement error issue is smaller, we also the expect differences in implications when we take our the transitory shock to be smaller.
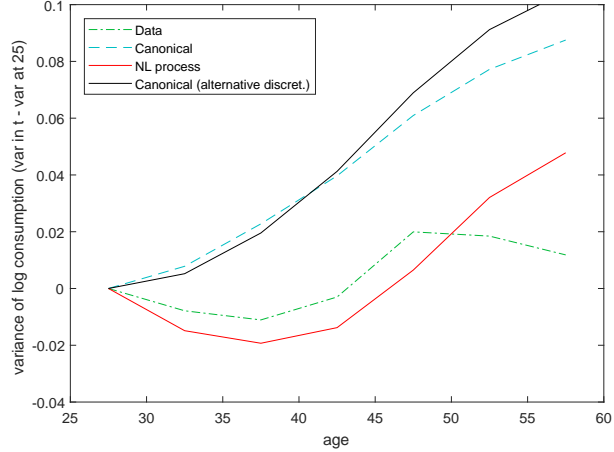
Figure 22: Growth in the variance of log consumption, different discretizations for the canonical process.

Coefficients	ψ^p_{BPP}	ψ^{tr}_{BPP}	ψ^p	ψ^{tr}
Data	0.36	0.95	–	–
Canonical	0.14	0.88	0.30	0.91
Canonical, alternative discretization	0.2	0.88	0.34	0.91

Table 8: BPP coefficients, different discretizations for the canonical process.

C.5   Average consumption

Figure 24 plots average log consumption by age from our life cycle model under the two earnings processes that we consider, the canonical and nonlinear one. (We pick the discount factor in each economy to generate the same wealth to income ratio that we use for our main results.) In the data, consumption peaks between ages 45-55 (Attanasio, Banks, Meghir and Weber (1999)). Thus, even though we impose that average labor earnings by age are the same for both earnings processes, our nonlinear implies that consumption peaks at an age closer the one in the data than the one implied by the canonical earnings process.
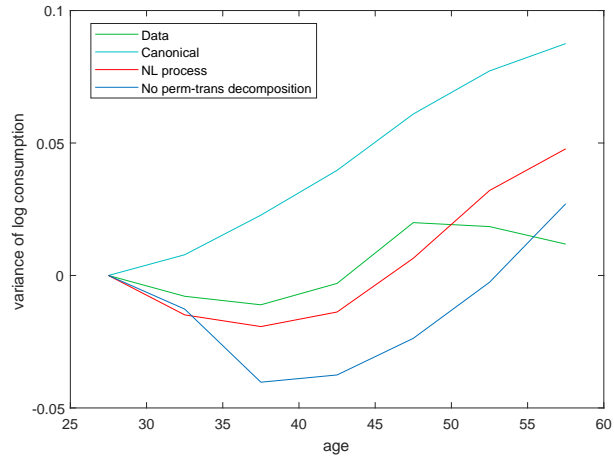
Figure 23: Growth in the variance of log consumption, no permanent-transitory decomposition.

Coefficients	ψ^p_{BPP}	ψ^{tr}_{BPP}	ψ^p	ψ^{tr}
Data	0.36	0.95	–	–
NL process	0.43	0.81	0.46	0.89
NL process, no pers-transitory	0.54	–	0.57	–

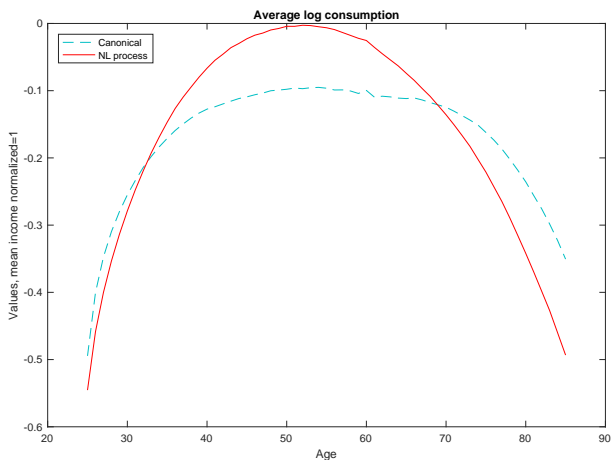*Table 9:* BPP coefficients, no permanent-transitory decomposition.

Figure 24: Average cross-sectional log consumption. Income is normalized so that its mean value in levels is 1.

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