Aggregate Consequences of Credit Subsidy Policies: Firm Dynamics and Misallocation

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Abstract

Government policies that attempt to alleviate credit constraints faced by small and young firms are widely adopted across countries. We study the aggregate impact of such targeted credit subsidies in a heterogeneous firm model with collateral constraints and endogenous entry and exit. A defining feature of our model is a non-Gaussian process of firm-level productivity, which allows us to capture the skewed firm size distribution seen in the Business Dynamics Statistics (BDS). We compare the welfare and aggregate productivity implications of our non-Gaussian process to those of a standard AR(1) process. While credit subsidies resolve misallocation of resources and enhance aggregate productivity, increased factor prices, in equilibrium, reduce the number of firms in production, which in turn depresses aggregate productivity. We show that the latter indirect general equilibrium effects dominate the former direct productivity gains in a model with the standard AR(1) process, as compared to our non-Gaussian process, under which both welfare and aggregate productivity increase by subsidy policies.

Keywords: misallocation, collateral constraints, firm dynamics, firm size
JEL Classifications: E22, G32, O16

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1 Introduction

Government policies that attempt to alleviate credit constraints faced by small and young firms are widely adopted across countries; provision of subsidized credit is a prime example of this.[1] Despite the popularity of such targeted industrial policies, quantitative studies on their macroeconomic effects are scarce.[2] Subsidized credit helps small and young firms achieve efficient and larger scales of production, which also resolves misallocation of resources and enhances aggregate productivity. However, increased factor prices in equilibrium reduce the number of firms in production, which in turn depresses aggregate productivity. The relative magnitudes of each channel—the direct productivity gains and the indirect general equilibrium effects—depend on the underlying distribution of firms and their financial status. Thus, whether long-run effects are productivity enhancing or not is ambiguous.

In this paper, we offer a general equilibrium analysis of such targeted credit subsidy policies by extending a heterogeneous firm model with collateral constraints and endogenous entry and exit. In particular, we employ a Pareto-distributed firm productivity process to capture the skewed firm size distribution seen in the Business Dynamics Statistics (BDS).[3] We compare the welfare and aggregate productivity implications of such a non-Gaussian process to those of a standard AR(1) process, a common assumption used in the literature.[4] Our main finding is that credit subsidy policies *decrease* aggregate productivity in a model using the AR(1) process. This contrasts with our benchmark model with a non-Gaussian process, wherein we find that credit subsidy policies *increase* aggregate productivity.

A model with a standard AR(1) process cannot capture the right tail of firms in positively skewed firm size distributions. Therefore, this model misses firms that could grow substantially if collateral constraints are lifted, for which there are large potential gains from credit subsidy policies. In the absence of such firms with substantial growth potential, the direct effect from credit subsidy policies is relatively small and is dominated by the negative indirect effects that come from declines in the total number of firms. It also follows that, even a firm-level productivity is assumed to follow an AR(1) process, a model without endogenous extensive margin fluctuations can improve its aggregate productivity by credit subsidy policies as the number of

[1] For example, the U.S. Small Business Administration (SBA) provides a guarantee for funding when entrepreneurs want to start or expand small businesses under Advantage Loans 7(a). The volume of loan guarantees in this program has been steadily growing in recent years (USD 24 billion in 2016, compared to USD 14 billion in 2007).

[2] For example, Guner, Ventura, and Xu (2008), Buera, Moll, and Shin (2013), and Buera, Kaboski, and Shin (2017) study the aggregate implications of policies aimed at specific groups of firms or entrepreneurs.

[3] In 2012, the share of employment at large firms was 51.6 percent while the employment share of small firms was 16.7 percent in the United States (Statistics of U.S. Businesses, 2012).

[4] See Khan and Thomas (2008) and Bloom (2009) for the seminal works.

firms will be constant by construction.

Our departure from Gaussian firm-level productivity shocks is essential to capturing the positively skewed firm size distribution that is observed in the data. More importantly, it matters for results from a quantitative evaluation of targeted credit policies such as ours. In this regard, we share the spirit of Buera, Kaboski, and Shin (2017), who studied the macroeconomic effects of microfinance programs. In a model of entrepreneurs, they found cleansing effects as the main driver distinguishing general equilibrium results from their partial equilibrium results. We differ in that we study a model of firms and find that the number of production units matters more than the average productivity of firms in the economy.

Our model builds on a standard heterogeneous firm model. The model has three key ingredients. First, we employ a bounded Pareto distribution for firm-level productivity. It is well-known that the empirical distribution of firm size in employment is highly skewed. That is, small firms dominate the business population, while large firms account for the largest fraction of aggregate employment. Our specification of firm productivity successfully replicates the empirical firm size distribution in the model economy, coupled with decreasing-returns-to-scale production technology. Second, we allow endogenous entry and exit by firms.[5] This element is essential to reproducing the empirical patterns of firm dynamics, including substantially lower survival rates among young firms. In addition, it allows us to examine the importance and influence of extensive margins in aggregate productivity. Without this component, our result that credit subsidy policies may reduce aggregate productivity in the long run does not emerge. Third, forward-looking collateral constraints are added, in the spirit of Kiyotaki and Moore (1997). This, together with the second ingredient, leads to an empirically consistent distribution of firm age.[6] The presence of this collateral constraint hinders immediate firm growth upon entry.[7] This helps us to have a substantial number of small firms in the economy. Moreover, we can also generate the right pattern of firm lifecycle dynamics; newborn firms gradually build up capital stock over time by relying on loans.

We use this model to study the aggregate implications of targeted credit subsidy policies in a general equilibrium environment. Credit subsidies in our model are lump-sum cash transfers

[5] For example, Khan and Thomas (2013), Buera and Moll (2015), and Catherine, Chaney, Huang, Sraer, and Thesmar (2017) do not have endogenous exit margins in their models. Buera, Kaboski, and Shin (2011, 2017) study the importance of extensive margins in models of entrepreneurs. Midrigan and Xu (2014) also explored this in their extended version of the model.

[6] The importance of firm age has been studied in the following seminal papers: Davis, Haltiwanger, and Schuh (1996), Dunne, Roberts, and Samuelson (1989), and Evans (1987). More recently, Fort et al. (2013) emphasized the age dimension of firm heterogeneity during the Great Recession. Hsieh and Klenow (2014) examined the lifecycle of plants in Mexico and India and resource misallocation.

[7] See Bahaj, Foulis, and Pinter (2016) for microeconometric evidence on the importance of collateral constraints on firm investment.

from households to targeted firms.[8] We show that there are four effects, one direct and three indirect, following a targeted credit subsidy in our model. The first one is a direct effect that emerges from the fact that small and young firms can achieve an efficient scale of production by receiving a subsidy. This direct effect stems from alleviating capital misallocation across firms due to credit constraints. In line with the findings in previous studies (e.g., Buera, Kaboski, and Shin (2011), Khan and Thomas (2013), Midrigan and Xu (2014)), we show the quantitative importance of misallocation in explaining aggregate productivity. The second effect is an indirect general equilibrium effect. Increased demand for capital and labor—from the recipients of the subsidy—will raise factor prices. Higher factor prices depress the scale of production of untargeted firms. The third effect is also indirect. Increased factor prices, in equilibrium, lead less-productive incumbents to exit, and these firms are replaced by productive potential entrants (*cleansing effects*).[9] The last effect is that there are fewer firms in operation because of higher costs of production, which in turn depresses aggregate productivity.[10]

While we focus on targeted credit subsidy policies, there is a variety of government support schemes for smaller businesses.[11] Hence, the argument made in this paper is also relevant to many policy debates around the world. Nonetheless, the recent literature has documented that young firms start small and grow fast; however, they also tend to fail at greater rates. They are especially vulnerable to negative shocks such as recessions and natural disasters. Moreover, credit constraints compound such adverse shocks; these factors, among others, lead to the observed high exit rates among young firms.[12] Given its pervasiveness, we focus on a subsidized credit policy that alleviates difficulties in external borrowing for small and young firms, as in Buera, Moll, and Shin (2013).

Backing our policy experiment results, there are two main mechanisms affecting the allocative efficiency of resources, one across incumbents (e.g., Restuccia and Rogerson (2008)), and one across entering and exiting firms.[13] The role of financial frictions in generating resource misal-

[8] See Catherine et al. (2017) for a similar application on the impact of an investment tax credit.

[9] We examine the cleansing effect of industrial policies, while the business cycle literature has investigated the cleansing effect of recessions. See Osotimehin and Pappada (2015) for a recent application.

[10] Under the decreasing-returns-to-scale, the number of production units in a model is positively associated with the level of measured total factor productivity, holding aggregate factor inputs fixed. See Khan, Senga, and Thomas (2016) for similar channels at work.

[11] Beginning in January 2011, the Obama Administration promoted a set of entrepreneur-focused policies, including the Jumpstart Our Business Startups (JOBS) Act. The White House (2012) claims that these policies allow *"Main Street small businesses and high-growth enterprises to raise capital from investors more efficiently, allowing small and young firms across the country to grow and hire faster."* The targets of the policy initiatives include finance, education, red tape, innovation, and market opportunities for entrepreneurs.

[12] For job creation by firm age, see Haltiwanger, Jarmin, and Miranda (2016). Collier et al. (2016) discuss the role of credit constraints.

[13] Seminal empirical works on the allocative efficiency include Hsieh and Klenow (2009) and Bartelsman, Haltiwanger, and Scarpetta (2013).

location and its aggregate implications have been studied by Khan and Thomas (2013), Buera and Moll (2015), Buera, Kaboski, and Shin (2011), and Midrigan and Xu (2014), among others. More recently, Catherine, Chaney, Huang, Sraer, and Thesmar (2017) studied misallocation in a model of collateral constraints and found a positive productivity gain from lifting collateral constraints. Our policy exercises are distinct as we allow endogenous firm entry and exit, which leads to our new finding that the long-run effect on aggregate productivity from alleviating collateral constraints may be negative.

Our focus here is on the role of policies in alleviating micro-level distortions. Previous studies, in contrast, have looked at policies that distort the allocation of resources. Guner, Ventura, and Xu (2008) considered a model with size-contingent regulations and used their calibrated version of the model to show sizable welfare losses. Gourio and Roys (2014) also studied the aggregate implication of a French policy that distorts the size distribution of firms, showing the important role of such policy distortion in explaining aggregate productivity.[14]

We present our theory in Section 2. We calibrate the theoretical model in Section 3. We then examine the steady state of the benchmark model in Section 4. In Section 5, we analyze the impact of industrial policies, and Section 6 investigates the role of extensive margins and skewed firm size distribution. Section 7 concludes the paper.

2 Model

Time is discrete in infinite horizon. There are a large number of heterogeneous firms producing a homogeneous good. Firms are subject to persistent shocks to individual productivity and face collateral constraints. These, together with endogenous entry and exit, yield substantial heterogeneity in production. Households are identical and infinitely-lived. We abstract from aggregate uncertainty and consider a stationary industrial equilibrium.

In the following sub-sections, we present our model economy in detail and examine the optimization problem of firms, followed by the household problem and the definition of recursive competitive equilibrium. We then characterize firm-level decisions in a tractable way to render our counterfactual exercises feasible. In particular, we summarize two firm-level states, capital and borrowing, by using a single-state variable defined as cash-on-hand.

[14] See Garicano, Lelarge, and Van Reenen (2016) and Braguinsky, Branstetter, and Regateiro (2011) for econometric applications of size-dependent policies.

2.1 Firms

2.1.1 Production and Financial Friction

The economy consists of a continuum of firms. Each firm owns its predetermined capital stock, k, and hires labor, n. The production technology is described by $y = \epsilon F(k, n)$, where $F(\cdot)$ exhibits decreasing-returns-to-scale (DRS) and ϵ represents firm-level productivity. We assume that $\epsilon \in \mathbf{E} \equiv \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{N_\epsilon}\}$ follows a Markov chain with $\pi^\epsilon_{ij} \equiv Pr(\epsilon' = \epsilon_j | \epsilon = \epsilon_i) \geq 0$ and $\sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} = 1$ for each $i = 1, 2, \ldots, N_\epsilon$, where primes denote future values for notational convenience.

Not all firms are able to finance their desired investment due to financial frictions. All debt is priced at q, and each firm faces a borrowing limit on this one-period discount debt. This borrowing constraint restricts the amount of new debt level, b', not to exceed a firm's collateral. Based on the idea of limited enforceability of financial contracts, we assume that the firm's future period capital, k', serves as collateral for current borrowing. Therefore, for a firm choosing k' in the current period, the collateral constraint is given by $b' \leq \theta k'$, where the financial parameter, θ, captures financial frictions at the economy-wide level. Notice that θ is assumed to be common across firms, but the borrowing decision depends endogenously on each firm's state. When θ is close to the real interest rate, $\frac{1}{q}$, the financial market allows firms to invest at their desired level. Note that our specification of collateral constraints is forward-looking in the spirit of Kiyotaki and Moore (1997), while we abstract from their feedback channel from asset prices. In the presence of collateral constraints, firms gradually accumulate capital. Once they achieve a capital level consistent with their expected productivity, they can also accumulate financial savings, $b' < 0$. This implies that a borrowing constraint is binding for some but not all firms in a given period. In this way, the model reproduces the observed heterogeneity in the reliance on external borrowing.

2.1.2 Entry and Exit

We model firm entry and exit based on the standard approaches in the literature.[15] In each period, incumbent firms may exit the economy either by an exogenous probability or by their endogenous decisions. Due to the financial frictions at the firm-level, individual states of productivity, capital, and borrowing jointly affect the latter endogenous exit decision. Together with endogenous entry decisions by potential entrants, our model features realistic firm dynamics.

[15] Hopenhayn (1992) is the seminal work in this literature with industry dynamics driven by firms' endogenous entry and exit. Clementi and Palazzo (2016) modify the timing of entry in the Hopenhayn model to investigate the business cycle implications of firm dynamics. We follow the similar approach of Clementi and Palazzo, while introducing the exogenous exit as employed in Khan and Thomas (2013).

At the beginning of each period, firms are informed of their respective status of exit which takes place after production, and the fixed probability of exit, $\pi_d \in (0,1)$, is common across firms.[16] The remaining firms after the exogenous exit need to pay ξ_o units of output in order to continue operation in the next period. This fixed cost of operation creates a binary exit decision. If a firm does not pay this cost, it has to exit the economy permanently which implies its value is zero. Thus, only the firms continuing to the next period make intertemporal decisions about investment and borrowing after paying ξ_o.[17] This endogenous exit margin of firm dynamics enables relatively less-profitable firms to endogenously choose to exit, which can potentially reduce the degree of resource misallocation arising from financial frictions in the model.

We further assume that there is a fixed measure, M^e, of potential entrants in each period. The potential entrants are uniformly distributed over their initial capital and debt combination, (k_0, b_0), and the initial productivity of a potential entrant, ϵ_0, is randomly drawn from the ergodic distribution of $\epsilon \in \mathbf{E}$.[18] When a potential entrant decides to enter, it needs to pay a fixed entry cost, ξ_e, in units of output. In this way, we are able to set up a simple binary problem of endogenous entry. Note that firm entry takes place at the end of a period, and actual entrants start operating in the next period, given their initial state, (k_0, b_0, ϵ_0).

2.1.3 Timing and Firm Distribution

At the beginning of a period, an incumbent firm is identified by its individual state vector, (k, b, ϵ); the predetermined capital, $k \in \mathbf{K} \subset \mathbb{R}_+$; the amount of debt carried from the previous period, $b \in \mathbf{B} \subset \mathbb{R}$; and the current period idiosyncratic productivity level, $\epsilon \in \mathbf{E}$. We summarize the distribution of firms by a probability measure, $\mu(k, b, \epsilon)$, which is defined on a Borel algebra $\mathcal{S} \equiv \mathbf{K} \times \mathbf{B} \times \mathbf{E}$.

We illustrate the timing of the model in a given period as shown in the following diagram. Given the current state, (k, b, ϵ), an incumbent firm maximizes the sum of its current and future expected discounted dividends, considering its possible exit within each period. At the beginning of a period, the firm realizes its exogenous exit status after production. Once production is completed, all firms make payments for wage bill and existing debt, and then π_d share of firms

[16] This is a simple way to avoid the Modigliani-Miller environment where financial frictions are irrelevant when firms survive indefinitely. Further, the exogenous exit assumption helps the model reproduce the empirical distribution of firm age by allowing turnovers from old-large firm groups.

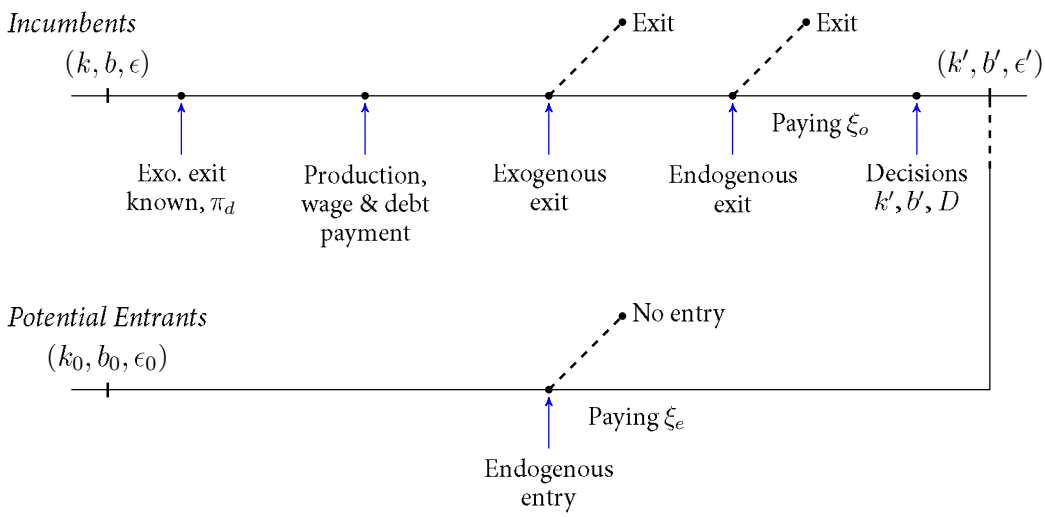
[17] Both exogenous and endogenous exit occur after production is completed in a given period. This ensures that all existing debt is repaid by the exiting firms.

[18] k_0 and b_0 are jointly drawn from a uniform distribution with its density, $\int_0^{\bar{k}_0}\int_0^{\bar{b}_0}(\bar{k}_0 \cdot \bar{b}_0)^{-1}(d[k_0 \times b_0])$. We assume that $\bar{k}_0$ corresponds to a fraction, χ_e, of the capital choice without financial frictions at the medium productivity value of ϵ, and set $\bar{b}_0 = \theta_e \bar{k}_0$, where θ_e is the maximum leverage of potential entrants. Later in our calibration, we target the relative employment size of entrants in the US.

on $\mu(k, b, \epsilon)$ disappears exogenously. The remaining firms decide whether to continue to the next period by paying the fixed operation cost, ξ_o. Conditional on staying, firms undertake intertemporal decisions on investment, i, and borrowing, b', while determining their current dividend payments, D. The capital accumulation of each firm is standard, $i = k' - (1-\delta)k$, with $\delta \in (0, 1)$.

Meantime, potential entrants draw their initial state, (k_0, b_0, ϵ_0), and decide whether to enter the economy by paying the fixed entry cost, ξ_e. Given its initial state, an entering firm starts operating in the next period, along with the continuing incumbents in the above. Markets are perfectly competitive, so firms take the wage rate, w, and the discount debt price, q, as given.

Timing within a Period



2.1.4 Firm's Problem

Given $\epsilon_i \in \mathbf{E}$, let $v^0(k, b, \epsilon_i)$ be the value of a firm at the beginning of the current period, before its survival from exogenous exit is known. Accordingly, define $v^1(k, b, \epsilon_i)$ as a surviving firm's value, before making its decision to pay the operation cost ξ_o. Finally, if the firm decides to continue to the next period, its value is given by $v(k, b, \epsilon_i)$. Then the firm's optimization problem can be recursively defined by using v^0, v^1, and v.

$$\begin{aligned} v^0(k, b, \epsilon_i) = \pi_d \cdot \max_n \left[\epsilon_i F(k, n) - wn + (1-\delta)k - b\right] \\ + (1 - \pi_d) \cdot v^1(k, b, \epsilon_i) \end{aligned} \tag{1}$$

$$v^1(k, b, \epsilon_i) = \max\left\{0, -\xi_o + v(k, b, \epsilon_i)\right\} \tag{2}$$

In equation (1), the firm takes the possibility of exogenous exit into account. In case it is destined to exit by π_d, the firm maximizes its liquidation value at the end of the period without dynamic decisions. In case of surviving, on the other hand, the firm makes a binary decision over the value of zero, *(endogenous exit)*, and the value of $-\xi_o + v(k, b, \epsilon_i)$, *(stay)*, in equation (2). The value of continuing to the next period is given by $v(k, b, \epsilon_i)$ as below.

$$v(k, b, \epsilon_i) = \max_{n,k',b',D} \left[D + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij}^{\epsilon} v^0(k', b', \epsilon_j)\right] \tag{3}$$

subject to

$$0 \leq D \equiv \epsilon_i F(k, n) - wn + (1-\delta)k - b - k' + qb'$$

$$b' \leq \theta k'$$

In (3), the firm optimally chooses its labor demand, n, future capital, k', and new debt level, b', to maximize the sum of the firm's current dividends, D, and the beginning-of-the-period value, $v^0(k', b', \epsilon_j)$, in the next period. Current period dividends are the residual defined in the firm's budget constraint, and we restrict them to be non-negative. Firms discount their future expected values using $\beta \in (0, 1)$.[19] Financial frictions are introduced in a collateral constraint on the firm's new borrowing in the above maximization problem.

Lastly, we define the value of a potential entrant with (k_0, b_0, ϵ_0) as v^e.

$$v^e(k_0, b_0, \epsilon_0) = \max\left\{0, -\xi_e + \beta v^0(k_0, b_0, \epsilon_0)\right\} \tag{4}$$

The potential entrant makes a binary decision to pay or not to pay the entry cost ξ_e. Once it enters, the firm starts operation in the next period given its initial state, as indicated in the value of entry in (4).

[19] We also use β to denote households' subjective discount factor in the next subsection. Abstracting from aggregate uncertainty, there is no stochastic discount factor in the above firm's problem. Thus, the equilibrium discount debt price, q, is equal to β.

2.2 Households and Equilibrium

2.2.1 Representative Household

We assume that there is a unit measure of identical households in the economy. In each period, households earn their labor income by supplying a fraction of their time endowment. Period utility is given by $U(C, 1-N)$, and households discount future utility by a subjective discount factor, β. The representative household holds a comprehensive portfolio of assets; firm shares of measure λ and non-contingent discount bonds ϕ. It maximizes lifetime expected discounted utility by choosing the quantities of aggregate consumption demand, C^h, and labor supply, N^h, while adjusting its asset portfolio. The household value, V^h, is defined as below.

$$V^h(\lambda, \phi) = \max_{C^h, N^h, \lambda', \phi'} \Big[U(C^h, 1-N^h) + \beta V^h(\lambda', \phi') \Big] \tag{5}$$

subject to

$$C^h + q\phi' + \int_{\mathcal{S}} \rho_1(k', b', \epsilon')\lambda'(d[k \times b \times \epsilon])$$
$$\leq wN^h + \phi + \int_{\mathcal{S}} \rho_0(k, b, \epsilon)\lambda(d[k \times b \times \epsilon])$$

We apply the following notation for stock prices. In (5), $\rho_1(k', b', \epsilon')$ denotes the ex-dividend prices of firm shares in the current period, and $\rho_0(k, b, \epsilon)$ is the dividend-inclusive value for current shareholding, λ. Let $\Phi^h(\lambda, \phi)$ be the household's decision for bonds and $\Lambda^h(k', b', \epsilon', \lambda, \phi)$ its choice of firm shares corresponding to the future state (k', b', ϵ').

2.2.2 Recursive Competitive Equilibrium

We consider a stationary industrial equilibrium of the model, where the distribution of firms, $\mu(k, b, \epsilon)$, is time-invariant. In the following, we define recursive competitive equilibrium. For simplicity, we denote the distribution of producing firms as μ^p, the measure of actual entrants as μ^e, and the measure of endogenously exiting firms as μ^{ex}. Whenever possible, we suppress the arguments of functions in the definition below.

A stationary *recursive competitive equilibrium* is a set of functions including prices $\left(w, q, \rho_0, \rho_1\right)$, quantities $\left(N, K, B, D, C^h, N^h, \Phi^h, \Lambda^h\right)$, a distribution $\mu(k, b, \epsilon)$, and values $\left(v^0, v^1, v, v^e, V^h\right)$ that solve the optimization problems and clear markets in the following conditions.

1. v^0, v^1, and v solve equations (1)–(3), and (N, K, B, D) are the associated policy functions for firms.
2. V^h solves (5), and $(C^h, N^h, \Phi^h, \Lambda^h)$ are the associated policy functions for households.
3. The labor market clears, $N^h = \int_{\mathcal{S}} N(k, \epsilon) \cdot \mu^p(d[k \times b \times \epsilon])$.
4. The goods market clears.

$$C^h = \Big(\int_{\mathcal{S}} \Big[\epsilon F(k, N) - (1 - \pi_d)\big(K(k, b, \epsilon) - (1 - \delta)k\big) + \pi_d(1 - \delta)k - \xi_o \Big] \cdot \mu^p(d[k \times b \times \epsilon]) \Big) + \int_{U(0, \bar{k}_0)} (k_0 - \xi_e) \cdot \mu^e(d[k_0 \times b_0]) - \int_{\mathcal{S}} k \cdot \mu^{ex}(d[k \times b \times \epsilon])$$

5. The distribution of firms, $\mu(k, b, \epsilon)$, is a fixed point where its transition is consistent with the policy functions, (K, B), and the law of motion for ϵ.

2.3 Characterizing Firm-Level Decisions

2.3.1 Firm Types

To characterize the firm-level decision rules in the model, it is convenient to first define a subset of firms in the distribution whose decisions are not affected by the collateral constraints in any possible future state. In particular, we follow the approach of Khan and Thomas (2013) to distinguish firm types in our model economy. Note that this distinction is solely for deriving all the intertemporal decisions made by firms and that a firm may alter its type depending on the state variables.

We define a firm as *unconstrained* when it has already accumulated sufficient wealth such that it never experiences binding borrowing constraints in any possible future state. In this case, all the unconstrained firm's Lagrangian multipliers on its borrowing constraints become zero, and the firm is indifferent between dividend payments and retained earnings. On the other hand, the remaining firms in the distribution are defined as *constrained.* Constrained firms may or may not experience binding borrowing constraints in the current period; thus, they choose to pay zero dividends in the current period as the shadow value of retained earnings is greater than that of paying dividends.

First, we derive the optimal static labor choice, which applies to all firms. Labor choices are frictionless and a firm with (k, ϵ) chooses $n = N^w(k, \epsilon)$, which solves the static labor condition, $\epsilon D_2 F(k, n) = w$.

Next, we derive the choice of future capital, k', by the unconstrained firms. The collateral constraint is irrelevant for this type of firm, so we can easily derive their optimal level of $k' = K^w(\epsilon)$ as follows. Let $\Pi^w(k, \epsilon) \equiv \epsilon F(k, N^w) - wN^w$ be the current earnings of a firm with its optimal labor hiring N^w. Given the Markov property for the assumed stochastic processes and the absence of capital adjustment costs, K^w is the solution to the following problem.

$$\max_{k'} \Big[-k' + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij}^{\epsilon} \big(\Pi^w(k', \epsilon_j) + (1-\delta)k'\big) \Big]$$

With the policy functions N^w and K^w, as in Khan and Thomas (2013), the *minimum savings policy*, $b' = B^w(\epsilon)$, is recursively defined by the following two equations.

$$B^w(\epsilon_i) = \min_{(\epsilon_j)_{j=1}^{N_\epsilon}} \Big(\tilde{B}\big(K^w(\epsilon_i), \epsilon_j\big) \Big) \tag{6}$$

$$\begin{aligned} \tilde{B}(k, \epsilon_i) = {} & \epsilon_i F(k, N^w) - wN^w + (1-\delta)k - K^w(\epsilon_i) \\ & + q \min \big\{ B^w(\epsilon_i), \theta K^w(\epsilon_i) \big\} \end{aligned} \tag{7}$$

$\tilde{B}(K^w, \epsilon_j)$ in (6) denotes the maximum level of debt (or the minimum level of saving) that an unconstrained firm can hold at the beginning of the next period in which $\epsilon' = \epsilon_j$ is realized. Having chosen the unconstrained choice of capital, K^w, at the current period, the firm will remain unconstrained in the subsequent periods by definition. The minimum savings policy, B^w, ensures that the firm's debt never exceeds this threshold level, $\tilde{B}$, given all possible realizations of ϵ. Moreover, the threshold function can be retrieved by using B^w and K^w at the current period state with (k, ϵ_i), as in (7). Notice that the minimum operator in (7) reflects the collateral constraint in the firm's problem. In (6), B^w again is determined to have the unconstrained firms unaffected by the constraint over any future path of ϵ.

2.3.2 Cash-on-Hand and Decision Rules

The incumbent firm's problem in (3) is a challenging object because of the occasionally binding constraints for D and b'. In addition, notice that a firm's individual state vector includes two endogenous variables, (k, b), with each having a continuous support. However, levels of k and b of firms do not separately determine the choices of k' and b'. This is due to the absence of any real adjustment costs in the model, and therefore we can collapse these two continuous individual state variables into a newly defined variable called *cash-on-hand.* We define the cash-on-hand,

$m(k, b, \epsilon)$, of a firm with (k, b, ϵ) by using the optimal labor demand N^w.

$$m(k, b, \epsilon) \equiv \epsilon F(k, N^w) - wN^w + (1-\delta)k - b$$

Notice that the decisions of k' and b' made by continuing incumbents determine the level of cash-on-hand in the future period, $m(k', b', \epsilon')$, along with the realization of ϵ'. By using this new state variable, we rewrite the incumbent firm's problem in (1)–(3) using the following values W^0, W^1, and W.

$$W^0(m, \epsilon_i) = \pi_d \cdot m + (1-\pi_d) \cdot W^1(m, \epsilon_i) \tag{8}$$

$$W^1(m, \epsilon_i) = \max\left\{0, -\xi_o + W(m, \epsilon_i)\right\} \tag{9}$$

$$W(m, \epsilon_i) = \max_{k', b', D, m'_j} \left[D + \beta \sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} W^0(m'_j, \epsilon_j)\right] \tag{10}$$

subject to

$$0 \leq D \equiv m - k' + qb'$$

$$b' \leq \theta k'$$

$$m'_j \equiv m(k', b', \epsilon_j)$$

$$= \epsilon_j F\big(k', N^w(k', \epsilon_j)\big) - wN^w(k', \epsilon_j) + (1-\delta)k' - b'$$

Having solved the unconstrained policies K^w and B^w, we define a threshold level of m for each ϵ that distinguishes the unconstrained firms. From the firm's budget constraint in (10), an unconstrained firm pays the current dividends, $D^w = m - K^w + qB^w \geq 0$. It follows that this firm's cash-on-hand is greater than or equal to a certain threshold level, $\tilde{m}(\epsilon) \equiv K^w(\epsilon) - qB^w(\epsilon)$. Therefore, any firm with $m(k, b, \epsilon) < \tilde{m}(\epsilon)$ can be identified as constrained.

Recall that, in any given period, some constrained firms experience currently binding borrowing constraints while the others do not. We call the latter constrained firms *Type-1*, and the firms with currently binding constraints *Type-2*. Notice that Type-1 firms can still adopt the unconstrained capital policy, K^w, but not the minimum savings policy, B^w. The debt policy of Type-1 firms can be easily determined by the zero-dividend policy after substituting $k' = K^w$ in the budget constraint. On the other hand, Type-2 firms can only invest to the extent allowed by their borrowing limits. This constrained choice of capital is derived from the level of cash-on-hand, m, held by Type-2 firms. Specifically, $D = 0$ implies that a Type-2 firm's decision of k' and b' must be feasible, given the firm's available cash-on-hand. Given m, we define the upper bound of a Type-2 firm's capital choice as $\bar{K}(m) \equiv \frac{m}{1-q\theta}$, from the binding collateral constraint. Firms with more cash-on-hand, therefore, can relax this upper bound until they can choose the

unconstrained capital policy, K^w. Notice also that the upper bound, $\bar{K}(m)$, approaches infinity as the financial parameter θ becomes closer to q^{-1}, which illustrates the case of perfect credit markets. Below, we summarize the decision rules of k' and b' by firm type given (m, ϵ).

- Firms with $m \geq \tilde{m}(\epsilon)$ are *unconstrained* and therefore adopt $K^w(\epsilon)$ and $B^w(\epsilon)$.
- For *constrained* firms with $m < \tilde{m}(\epsilon)$, the upper bound for k' is $\bar{K}(m) \equiv \frac{m}{1-q\theta}$.
  - Firms with $K^w(\epsilon) \leq \bar{K}(m)$ are *Type-1* and adopt $k' = K^w(\epsilon)$ and $b' = \frac{1}{q}\big(K^w(\epsilon) - m\big)$.
  - Firms with $K^w(\epsilon) > \bar{K}(m)$ are *Type-2* and adopt $k' = \bar{K}(m)$ and $b' = \frac{1}{q}\big(\bar{K}(m) - m\big)$.

3 Model Parameters

The model is annual, and we assume the standard functional forms of household preference and production technology in the literature. The period utility function of the representative household features the indivisible labor, $U(C, 1-N) = \log C + \psi(1-N)$, following Rogerson (1988) and Hopenhayn and Rogerson (1993). The DRS production function is in Cobb-Douglas form, $F(k, n) = k^\alpha n^\nu$, with $\alpha, \nu > 0$ and $\alpha + \nu < 1$. Given the assumed functional forms, we set the model parameter values to jointly match the key macroeconomic moments and the firm-level heterogeneity observed in the US data. Table 1 reports the parameter values used in our calibration, and Table 2 summarizes the resulting moments from the model along with the corresponding US data.

Table 1 : Parameter Values

Model Parameter Values			
α	0.280	θ	0.820
β	0.960	M^e	0.300
δ	0.069	χ_e	0.0436
ν	0.600	N_ϵ	13
ψ	2.480	ρ_ϵ	0.750
π_d	0.100	ϵ_m	0.393
ξ_o	0.03392	ϵ_M	1.122
ξ_e	0.00991	γ_ϵ	4.420

First, we set the value of the subjective discount factor, β, to imply the long-run real interest rate of about 4 percent per annum. The curvature of labor input in the production function, ν, is set to have the average share of labor income of 0.6, following Cooley and Prescott (1995). We set the preference parameter of labor disutility, ψ, to get the average hours worked of 0.33 at the stationary equilibrium of the model. The annual depreciation rate, δ, is chosen to match the

Table 2 : Moments

Aggregate Moments		
Description	Data	Model
Average hours worked	-	0.333
Measure of firms, $\int \mu \cdot d\mu$	-	1.000
Investment to capital ratio (BEA)	0.069	0.069
Capital to output ratio (BEA)	2.300	2.222
Debt to capital ratio (Flow of Funds)	0.567	0.564
Total exit rate (BDS)	0.110	0.114
Employment share of entrants (BDS)	0.033	0.033

Firm Size Distribution					Firm Age Distribution		
	Employment Share		Population Share			Population Share	
Employees	Data	Model	Data	Model	Age	Data	Model
1 to 4	0.0584	0.0584	0.5506	0.5412	0	0.1236	0.1176
5 to 19	0.1455	0.1455	0.3342	0.3299	1	0.0950	0.0870
20 to 99	0.1814	0.1814	0.0964	0.0911	2	0.0806	0.0766
100 to 499	0.1395	0.1395	0.0153	0.0225	3	0.0697	0.0691
500 to 2499	0.1179	0.1179	0.0026	0.0083	4	0.0613	0.0624
2500+	0.3573	0.3573	0.0009	0.0071	5	0.0548	0.0564
					6 to 10	0.1939	0.2104
SME (1-499)	0.5248	0.5248	0.9965	0.9847	11 to 15	0.1302	0.1271
					16+	0.1910	0.1934
					Young (0-4)	0.4302	0.4127

Note: The empirical firm size and age distributions are calculated from the annual tables in the BDS database. We report the average values from 1977 to 2007 for the size distribution, and from 1993 to 2007 for the age distribution, respectively. SME denotes small-medium sized enterprises.

average aggregate investment to capital ratio of the postwar US economy, while the production parameter, α, is set to be consistent with the average capital to output ratio of 2.3. For the aggregate time series of investment, output, and private capital, we use the Fixed Asset Tables and National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA) between 1954 and 2007. The parameter value in the collateral constraint, θ, is 0.82, to imply the average debt to capital ratio of the non-farm, non-financial businesses in the Flow of Funds from 1954 to 2007.

We set the exogenous exit rate in the model, π_d, at 0.10 and the fixed measure of potential entrants, M^e, at 0.30. We further assume that the maximum leverage of potential entrants, θ_e, is the half of that of incumbents, θ. The rest of model parameter values are calibrated to reproduce the observed size and age distribution of firms and the moments of firm entry and exit in the data. The parameter values of the fixed operation cost, ξ_o, and the entry cost, ξ_e, largely determine the quantitative magnitude of the entry and exit margins, given a value of χ_e which determines the relative size of the largest entrants, $\bar{k}_0$, to that of incumbents. Our calibrated model implies the

average total exit rate of private firms (11 percent) and the relative total employment by entrants to the aggregate (3 percent) close to the BDS data. In addition, the above parameters governing firm dynamics in the model critically affect the resulting shape of firm age distribution. Table 2 shows that our model also successfully matches the average distribution of firm age in the BDS.

In order to replicate the empirical firm size distribution, we assume that the idiosyncratic productivity, ϵ, is drawn from a time-invariant distribution, $G(\epsilon; \epsilon_m, \epsilon_M, \gamma_\epsilon)$, which is a bounded Pareto distribution.[20] In each period, a firm in our model economy retains its previous level of individual productivity with a fixed probability, ρ_ϵ. We set $\rho_\epsilon = 0.75$ to be consistent with the evidence on the persistence of firm-level productivity in the data.[21] The bounds of ϵ support, (ϵ_m, ϵ_M), and the shape parameter, γ_ϵ, of the bounded Pareto distribution are chosen to have both the employment share and the population share in each firm size bin aligned with the corresponding average values reported in the BDS from 1977 to 2007. We discretize ϵ using 13 values in our numerical applications.[22]

Table 2 reports the model-generated firm size distribution and its empirical counterpart, in which we divide the employment size groups by 6 bins. As is well known in the literature, Table 2 shows that the empirical distribution of firm size is highly skewed, where more than 88 percent of firms hire fewer than 20 employees in a given year. We almost perfectly replicate this lower tail of the empirical firm size distribution in our model economy by employing the Pareto-distributed ϵ. By the nature of the collateral constraint that we assumed, moreover, small firms in the size distribution are more likely to be financially constrained when their productivity is expected to rise whereas their cash-on-hand is insufficient.

Before we discuss the quantitative results from the model, we elaborate our procedure of computing the model-generated firm size distribution which is directly taken from Jo (2017). Given the stationary distribution of firms, $\mu(k, b, \epsilon)$, in equilibrium, we begin with constructing a cumulative distribution of employment by using $N^w(k, \epsilon)$. Based on the employment shares across size bins in the BDS, we find the employment threshold, $\bar{n}$, in each firm size group along the above cumulative distribution from the model. We then compute the measure of firms specifically located on each firm size bin which is defined from those employment thresholds. In sum, we first align the model employment shares by firm size to be exactly the same with the corresponding

[20] Jo (2017) shows that the bounded Pareto distribution requires a minimal set of parameters to replicate the empirical firm size distribution.

[21] The constant hazard of resetting productivity is recently employed in the models of production heterogeneity. See Buera, Kaboski, and Shin (2011) or Buera and Shin (2013), for example. The average persistence of ϵ in a long simulation is very close to the value of ρ_ϵ. This falls into the range of the persistence estimates in Foster, Haltiwanger, and Syverson (2008).

[22] Our simulation of the assumed ϵ process results in the unconditional mean of 0.466 and the standard deviation of 0.113. The share of firms with productivity less than the mean value is about 73 percent.

values in the BDS, and then choose the parameter values to generate the model population shares as closely as possible to the data.

4 Results: Steady State

4.1 Firm Heterogeneity and Decisions

We begin by describing the stationary distribution of firms in the model. Figure 1 shows the entire distribution of cash-on-hand, $m(k, b, \epsilon)$, at the steady state. The distribution of m is highly skewed with more than 88 percent of firms holding cash-on-hand of less than 1. This represents the corresponding shape of the underlying firm size distribution in the model, as reported in Table 2 and Figure 2. It follows that our model generates substantial heterogeneity at the firm level that is endogenously determined by the combination of persistent productivity, collateral constraints, and firm entry/exit margins.

More importantly, firms with small cash-on-hand are those most likely to be financially constrained subject to their productivity and borrowing limits, as we discussed in Section 2. To see this more clearly, Figure 3 provides a snapshot of the decision rules of incumbent firms on capital, k', debt, b', and dividends, D, as functions of $m(k, b, \epsilon)$ at a specific value of ϵ. In the figure, the level of cash-on-hand, $m(k, b, \epsilon)$, is on the horizontal axis, and we add the two vertical lines to distinguish the firm types: unconstrained, Type-1, and Type-2. The vertical line near $m = 100$ represents the threshold for being unconstrained, $\tilde{m}(\epsilon)$, and the one near $m = 20$ is the threshold for Type-1 firms. Starting from the right-hand side of the figure, when a firm has survived and accumulated sufficient wealth over time such that $m \geq \tilde{m}(\epsilon)$, it is considered unconstrained. The firm then adopts the unconstrained choices of capital, $K^w(\epsilon)$, and debt, $B^w(\epsilon)$, and starts paying positive dividends. Constrained firms with m less than the above threshold value, in contrast, follow the zero-dividend policy to accumulate their internal savings in order to become unconstrained. Type-1 firms between the two thresholds can still adopt the optimal level of capital, $K^w(\epsilon)$, while gradually reducing debt as their m increases. Lastly, Type-2 firms with small m are only able to invest up to their borrowing limits, so their choice of capital is constrained with positive borrowing. From Figure 1, we observe that these Type-2 firms are concentrated at the lower tail of the cash-on-hand distribution while maintaining positive leverages.

In our model, young firms start relatively small upon entry and then gradually accumulate cash-on-hand over time. These lifecycle dynamics of incumbent firms are illustrated in Figure 4. At age 0, an average firm in our model economy is financially constrained because it is short on collateral for external financing. Thus, the firm keeps raising its external debt level until age

4 and then gradually de-leverages once it can finance the optimal level of investment for $K^w(\epsilon)$, around age 6. In addition, conditional on remaining in the economy, firms still accumulate financial savings even after age 18, so they eventually become eventually unconstrained and pay positive dividends. This is represented by the hump-shaped leverage curve in the lower panel of the figure. Any policy targeting firms in a specific age group will therefore shift the average life-cycle dynamics in Figure 4, which eventually reshapes the entire firm distribution in the model economy. Further, such a policy will also affect the entry and exit margins of firm decision by affecting the equilibrium prices.

Next, we look at the endogenous exit decision by incumbent firms at the steady state of the model. Figure 5 shows the exit choice (=1) over capital and debt at the ergodic distribution of ϵ. Consistent with conventional knowledge, a firm in the model decides to exit when it accumulates relatively larger debt than its existing capital stock. This is because the continuation value of such an over-leveraged firm falls below 0, and the firm thus finds it better to exit before paying the fixed operation cost as illustrated in equation (4). Such vulnerable incumbents largely correspond to Type-2 firms in the model, and, in Figure 5, the exit decision occurs at the margin of positive leverages, which is the case of binding collateral constraints. Young firms with both low productivity and small wealth are easily tempted to this outside option of exit, so we are also able to capture the disproportionately high exit rate among those firms. This endogenous margin of firm dynamics in our model naturally entails a cleansing mechanism that drives unprofitable firms out of the economy, which is in contrast to the case with only exogenous exit.

Lastly, Figure 6 presents the entry decisions by potential entrants. Recall that the potential entrants are uniformly distributed over (k_0, b_0) at a given level of initial productivity, ϵ_0, which is drawn from a bounded Pareto distribution. We calculate the average decision of entry weighted over ϵ_0, and the brightest area in Figure 6 implies entry (=1). In the clear case of entry (the brightest area), a potential entrant is more likely to bear the fixed cost of entry when its initial capital, k_0, is relatively larger than its debt, b_0. It follows that the slope of the entry threshold, where the color map changes into the brightest color, represents the maximum leverage that the actual entrants can take. The area of entry becomes larger when the initial productivity of potential entrants is higher, so the dark colored areas between 0 and 1 illustrate the entry decisions across potential entrants with different ϵ_0. That is, more firms enter the economy in the areas with lower k_0 and higher b_0 as initial productivity increases. Since the share of potential entrants with high ϵ_0 is relatively small, the average entry probability is low at a high initial productivity level, which induces more small potential entrants with large initial debt to enter. Moreover, the slope of the entry threshold in the figure becomes steeper as the initial productivity increases. This implies that the entrants with high ϵ_0 are allowed to take more leverage when they decide to

enter. Such firms have strong growth potential over time, while the persistence and the relative dispersion of the underlying productivity distribution jointly determine the severity of financial frictions at the micro level.

5 Results: Targeted Policies

5.1 Overview of Counterfactual Exercises

We next examine the aggregate consequences of credit subsidies. Our counterfactual policy exercise is to compare the pre-intervention equilibrium described above *(benchmark)* and the post-intervention equilibrium, which is the new equilibrium reached by the economy after implementation of credit subsidy policies. That is, we measure the equilibrium changes in the aggregate economy under each policy relative to the benchmark.

Policy implementation works as follows. Suppose that the government selects a group of target firms and provides them credit subsidies. If a firm faces binding collateral constraint, then the credit subsidizing policy helps it to achieve the optimal level of capital stock, by setting $\theta = q^{-1} > \theta_{ss}$, where θ_{ss} is the value at the benchmark.[23] We assume that the government can exactly identify firms with binding borrowing constraints and help only such firms. Therefore, the actual number of subsidized firms is a subset of each targeted group.[24] We consider two different target groups, small firms (*size-dependent policy*) and young firms (*age-dependent policy*). The *size-dependent policy* selects small-medium sized firms (SME), defined as those hiring fewer than 500 employees.[25] The *age-dependent policy* selects firms with age between 0 and 4. The rest of the firms that are not given credit subsidies may be constrained by the borrowing limits with $\theta = \theta_{ss}$. The other model parameter values remain the same as in the benchmark.

We assume that subsidized capital is financed by lump-sum cash transfers from households. This assumption allows us to isolate and highlight the unintended effects of credit subsidies through general equilibrium price adjustments. Importantly, the total amount of cash transfers will be determined in equilibrium as policies affect the distribution of firms and change the level of unconstrained capital, $K^w(\epsilon)$. To compute the total cash transfers under a certain policy, we identify the firms with binding borrowing constraints and then calculate the gap between

[23] The government subsidizes only the additionally required investment for the targeted firms such that they can obtain an efficient scale of production at a given level of productivity. Catherine et al. (2017) also consider the similar type of counterfactuals. We include the results from modest increases in θ in the appendix.

[24] This is a different approach from that taken by Guner, Ventura, and Xu (2008). They investigate the effects of distortionary taxes on factor input uses. Once a firm falls into the targeted group in their model, its size is always restricted by such policies. In our approach, however, only firms with binding constraints are subsidized.

[25] Our definitions of small and young firms are consistent with those of Fort et al. (2013).

those firms' efficient investment, i, and the constrained level of investment without the policy, i^B.[26] The aggregate cash transfer, rt, is computed by aggregating the gap between i and i^B:

$$rt(p) = \int_{\mu^B} (i - i^B) d\mu^B,$$

where μ^B denotes the measure of subsidized firms with binding borrowing constraints on $\mu(k, b, \epsilon)$.

We measure the aggregate gains, relative to the benchmark economy, in the measured total factor productivity, ΔTFP, under each credit subsidy policy.[27] As will be discussed below, any policy intervention in our model economy involves changes in the equilibrium number of firms. Due to DRS production technology, TFP generally falls in the total number of firms given the same aggregate quantities. To control for the differences in the number of firms across policy regimes, we also measure the average productivity gains per firm, ΔTFP_μ, which can be calculated by re-scaling the aggregate variables by the measure of firms. In addition, note that the required cash transfer in each policy differs when the firm distribution changes in equilibrium. We normalize ΔTFP by the total amount of credit subsidy, $\Delta TFP/rt$, to measure the average TFP gain per unit of credit subsidy. This quantifies how effective each policy is in terms of resolving resource misallocation across firms.

Lastly, we compare the welfare consequences from different policies aimed at resolving financial frictions. Following the standard approach, we measure consumption equivalence variations (CEV) relative to the benchmark economy.[28]

5.2 Entry, Exit, and Aggregate Results

This sub-section presents the results from our counterfactual analysis of the two different targeted policies described above. We examine the aggregate effects of each credit subsidy policy and explain the direct effect and the indirect general equilibrium effects.

As in Table 3, both age- and size-dependent policies largely improve aggregate outcomes in the economy. Specifically, the age-dependent policy for young firms raises aggregate consumption and capital, respectively, by 5.6 and 11.5 percent from the benchmark, while output slightly

[26] Note that the market clearing prices in turn affect the firm distribution in our model economy when θ is adjusted. In particular, changes in $K^w(\epsilon)$ and $\bar{K}(m)$ alter the composition of firm types in equilibrium among unconstrained, Type-1, and Type-2 firms.

[27] Given aggregate output, capital and labor in equilibrium, we calculate TFP as the Solow residual by keeping α and ν values at the benchmark.

[28] Given the functional form of period utility, CEV equates $U(C^0(1+CEV), 1-N^0) = U(C^1, 1-N^1)$ when (C^0, N^0) is from the benchmark. Hopenhayn and Rogerson (1993) use a similar approach to investigate the welfare effects of firing costs.

falls. In the following, we explain that this fall in aggregate output is mainly from the huge decrease in the number of firms after the age-dependent policy is implemented. Aggregate output under the size-dependent policy that targeting small-medium sized firms (SMEs), on the other hand, increases by 2.7 percent from its pre-policy equilibrium. In addition, our welfare measurement (CEV) increases by more than 9 percent under the targeted policies. This is mainly due to the rise in equilibrium consumption and the fall in employment in both economies. Because we abstract from policy-related distortions in the counterfactual exercises, we view these aggregate improvements in Table 3 as the upper bounds of the positive impacts from the credit subsidizing policies.

Table 3 : Aggregate Results from Policy Experiments

Policy Counterfactual: Aggregates			
	Benchmark	Age-dependent (age 0 to 4)	Size-dependent (SMEs)
consumption	100 (0.1876)	105.64	106.33
capital	100 (0.5744)	111.54	113.04
output	100 (0.2584)	99.88	102.71
employment	100 (0.3334)	94.54	96.55
debt	100 (0.3238)	147.56	144.66
cash-on-hand	100 (0.8020)	76.91	83.95
firms (μ)	1.0001	0.5193	0.6802
endo. exit rate	0.0140	0.0430	0.0203
entrants rel. size	0.0331	0.0197	0.0196
cash transfer (rt)	–	0.2282	0.2003
subsidized firms (μ^B)	–	0.0558	0.0555
Type-2 share	0.0992	0.0117	0.0151
ΔTFP(%)	(0.5834)	0.1936	1.3562
$\Delta TFP/rt$	–	0.0049	0.0395
ΔTFP_μ(%)	–	8.3916	6.1551
CEV(%)	–	10.5177	9.4113

Note: In the top panel, we normalize the aggregate quantities to 100 at the benchmark, and the values in the parentheses are the corresponding absolute values. rt is the required cash transfer in each policy. ΔTFP refers to the relative change in the measured total factor productivity from the benchmark. ΔTFP_μ is the productivity gain per firm. CEV denotes the consumption equivalence measure of welfare.

The above aggregate improvements are largely driven by the reduced resource misallocation across firms. In our model, collateral constraints prevent small but productive firms from undertaking investment at their desired level. Insufficient capital in those firms reduces aggregate TFP. Therefore, providing credit subsidies to those firms undoes capital misallocation, which in turn leads to increased aggregate productivity. In Table 3, this first-order impact is shown in the fall in the share of Type-2 firms, thus raising the measured TFP (ΔTFP) following each

targeted policy. The productivity improvement from the age-dependent policy is 0.19 percent relative to the benchmark, and that from the size-dependent policy is 1.35 percent. These gains are obtained by subsidizing firms with binding borrowing constraints, which accounts for about 8 to 10 percent of the entire population (μ^B/μ). We also normalize the productivity gains by the size of cash transfers from households, ($\Delta TFP/rt$). This measures how much improvement in TFP can be achieved per unit of cash transfer, so $\Delta TFP/rt$ implies the effectiveness of each policy in boosting aggregate productivity. We observe that the size-dependent policy leads to higher marginal gains in productivity than the age-dependent policy does.

The number of firms in production (μ) drastically falls following each policy implementation, as reported in Table 3, which will be further discussed below. Since the number of firms also affects the measured TFP in each policy regime, we separately calculate the average productivity gain per firm, ΔTFP_μ. The productivity gain per firm is 8.39 percent under the age-dependent policy and 6.15 percent under the size-dependent policy targeting SMEs. It follows that targeting young firms results in relatively higher productivity gain for an average incumbent, but the associated improvement in measured TFP is rather modest. Therefore, our results from measuring different gains in productivity, whether it is the average per firm or per cash transfer, imply that each targeted policy has an edge over the other. This sheds light on the importance of targeting the group of firms whose marginal gains are large in practice.[29]

In addition to the above direct effects from undoing capital misallocation, providing credit subsidies has indirect effects on the general equilibrium. The equilibrium price changes accompanying the targeted policies involve adjustments in entry and exit margins, which eventually determines the number of firms in the economy. Below, we discuss these indirect effects from the equilibrium price channel.

We begin with the extensive margins of firm entry and exit. In labor markets, due to the boosted aggregate productivity, higher labor input by the subsidized firms leads to an increase in the equilibrium wage rate. The rise in factor price puts more pressure on firms with low profitability to exit. This mechanism is represented by the higher endogenous exit rates under the policies in Table 3, ranging from 2.0 to 4.3 percent in each period.

Moreover, the higher equilibrium wage rate also affects the entry margin of firm dynamics, which either strengthens or weakens the selection among potential entrants. That is, potential firms in the model observe higher profitability under each policy that relaxes borrowing constraints, but the increased cost of production prevents those with low productivity from entering. These two opposing effects lead to an ambiguous prediction on the entry margin following each

[29] The information asymmetry between firms and the government, for example, may further distort credit allocations. We do not consider this case, but the age-dependent policy may partly overcome this issue, given firm registry data on age.

policy, and we show the corresponding overall results quantitatively. Table 3 demonstrates that both targeted policies induce more small firms to enter the economy by paying the fixed entry cost. The average employment size of entrants is about 60 percent (0.0197 and 0.0196) of that in the benchmark (0.0331). This result indicates that the above policies promote entry by less productive firms, because the size of entrants only depends on their initial productivity draw, ϵ_0. Therefore, we quantitatively show that a credit subsidy targeting young or small firms indeed weakens the selection among potential entrants, even with the rise in equilibrium wage rate.

Figure 7 illustrates the change in the entry margin after the size-dependent policy. It clearly shows that entry occurs at any initial capital and debt combination, (k_0, b_0), with positive probability, in contrast to the results shown in Figure 6 for the benchmark economy.[30] Thus, the average size of entrants significantly falls, as shown in Table 3. Moreover, Figure 7 shows that the entry threshold changes as the credit subsidy is implemented. For instance, the capital threshold for the brightest area when $b_0 = 0$ falls from 0.0992 to 0.0949. (see Figures 6 and 7) Since the slopes of the threshold lines under the size-dependent policy also become steeper, it follows that the marginal entrants take on relatively more leverage. Because these entrants are relatively smaller, they are also likely to face tighter collateral constraints in addition to the fixed operating cost in the model. Combined with the higher leverage, this raises the exit hazard of age 0 firms disproportionately, thus contributing to the increased exit rates shown in Table 3.

These changes in the extensive margins in our policy counterfactuals imply that those small entrants with low productivity are continuously replaced by new firms over time. This cleansing of young firms early in their lifecycle leads to improvements in TFP. In sum, our policies targeting only incumbents actively adjust firm entry and exit margins through the general equilibrium channel. This accelerates *the cleansing effect of young firms replacing less productive incumbents.*[31] This yields additional gains from such policies in our model economy.

From the adjustments in the extensive margins, the number of firms falls in equilibrium, which in turn depresses aggregate productivity. Due to decreasing-returns-to-scale production technology at the firm level, the number of producing firms is a non-trivial factor that affects measured aggregate total factor productivity. As already shown in Table 3, the total number of firms decreases by 32 to 48 percent in both policy counterfactuals relative to the benchmark economy. Fewer firms in operation following the targeted policies dampens the aggregate gains in productivity, and this is why we observe only mild improvements in the measured TFP (ΔTFP)

[30] Note that the uniform distribution of potential entrants over (k_0, b_0) also changes as the equilibrium wage changes. This is because we set the upper bound of initial capital, $\bar{k}_0$, to be proportional to $K^w(\epsilon_{med})$. So, the higher wage narrows the support of the uniform distribution while its density rises.

[31] This is a different perspective from that in the literature on creative destruction, for instance, in Caballero and Hammour (1994). In that study, the cleansing effect of recessions comes from replacing low productive-outdated production units with innovative ones.

in the table. The negative impact on TFP due to the fall in the number of firms can be measured by the difference of the relative gains in TFP (ΔTFP) and in the average productivity per firm (ΔTFP_{μ}). The age-dependent policy is relatively more successful in boosting average firm productivity by more than 8 percent, but this gain almost disappears by losing more firms in operation.

So far, we are able to identify both direct and indirect effects of credit subsidy policies targeting small and young firms. As expected, a targeted policy directly resolves the resource misallocation faced by small or young firms. However, such policies also have unintended effects that indirectly emerge from the equilibrium price channel. In particular, the increased price under each policy has three effects at the firm level that in turn affect the aggregate results. First, it reduces the efficient scales of production for all firms. Next, the rise in price also adjusts the firm entry and exit margins to determine the relative size of the cleansing effect resulting from each policy. The last effect comes from the reduction in the total number of firms, which lowers the aggregate productivity gains. Together, these direct and indirect effects jointly quantify the aggregate outcome of a credit subsidy policy. Our results suggest that the unintended effects are quantitatively substantial when such policies are present. In this regard, the endogenous firm entry and exit in our model economy are critical in evaluating the overall effect of the targeted policies. We further investigate and highlight the importance of the extensive margins in a separate section of this paper.

5.3 Firm Dynamics

In our policy analysis, we consider credit subsidizing policies that differ in their targets, either young or small firms, and hence have disparate impacts across firms. It is evident that such policies affect firm-level decisions at different stages of their lifecycle. In this sub-section, we examine how targeted credit subsidies reshape firm dynamics by looking at the evolution of average size and productivity of firms in a cohort over time.

Figure 8 compares the average firm dynamics under different policies. The upper-left panel of the figure shows the typical growth pattern of entrants in our benchmark economy (*blue line with dots*). First, firms, on average, start small. The relative size of entrants is about one-fourth of that of mature firms at age 20. Upon entry, entrants start accumulating capital stock and gradually become larger as they age. As they become older, selection forces unproductive incumbents in an age cohort to exit from the economy. Hence, as shown in the upper-right panel of Figure 8, the average productivity of firms increases over time, whereas the productivity of age-0 firms in our benchmark economy is about 20 percent lower than that of firms at age 20.

Under the age-dependent policy, all firms between age 0 and 4 can achieve optimal capital

investment as collateral constraints are entirely lifted. This is reflected in the upper-left panel of Figure 8, where the policy (*red-dashed line with dots*) allows firms age 1 to 5 to have unconstrained levels of capital stock. To reach unconstrained capital at age 1, entrants take on massive leverage at age 0. Thus, these young firms have the largest borrowing upon entry and then gradually de-leverage over time, conditional on survival (lower-left panel). Due to the large inflow of small entrants under this policy, as discussed in the previous sub-section, the productivity of age 0 firms is slightly lower than that of the benchmark (upper-right panel). In addition, firms older than age 4 become ineligible for the age-dependent policy, so the average size reverts back to the level when borrowing constraints are present. Since those firms were able to accumulate more cash-on-hand under this policy, however, they tend to start saving faster in terms of financial assets after age 5, as illustrated by the steeper slope of de-leveraging over time. This implies that, at age 5, firms are better prepared for idiosyncratic risks due to their lower leverage ratio, which is a relatively sound financial position. Thus, we confirm that our age-dependent policy supports young-productive firms by allowing them to survive longer, similar to *selectively protecting infant firms.* This view is shared by Moll (2014), who distinguishes between the loss from misallocation at the steady state and that during the transition. With persistent productivity shocks, Moll shows that the loss at the steady state is relatively small, while the economy stays longer in transition. In this regard, our policy exercises consider the possibility of expediting the transition process by allowing firms to self-finance.

In contrast, the case of subsidizing SMEs (*solid green line*), also shown in Figure 8, does not deliver the observed patterns of firm growth under the age-dependent policy. This is because the size-dependent policy always allows a small-productive firm to effectively finance its desired investment whenever the firm hits its borrowing constraint, regardless of age. Since most of entrants in our model are small, the policy still accelerates the growth of average firm size and productivity in comparison to the benchmark. This enables young firms to immediately increase their leverage upon entry, but the corresponding de-leveraging pattern after age 5 follows that of the benchmark economy. Moreover, the equilibrium price effects on the entry margin still exist. As in the case of age-dependent policy, the credit subsidies for SMEs induce more entry by smaller firms in equilibrium. This slightly lowers the productivity of firms at age 0 (top-right panel), but it immediately improves to the level of age 20 firms due to the targeted policy. Notice that the top panels in Figure 8 are relative to the values at age 20 in each economy. In fact, the average productivity of a cohort under the size-dependent policy always reaches a higher value than that in the benchmark, starting from age 1. It follows that subsidizing only SMEs is a special case of removing financial frictions in the entire economy, because the collateral constraint we

consider mainly limits borrowing by firms with insufficient cash-on-hand in the model.[32]

6 Extensive Margins and Firm Size Distribution

6.1 Alternative Models

From our counterfactual exercises in a general equilibrium environment, we have shown that directly resolving the resource misallocation faced by small or young firms enhances productivity. However, such direct effects may be offset by the indirect general equilibrium effects. If the total number of firms decreases significantly due to the increased prices in equilibrium, the associated aggregate productivity gain from such policies will be significantly weaker than expected.

This section shows that the aggregate productivity gain from credit subsidy policies could even be negative when firm-level productivity shocks are assumed to be Gaussian, a standard modeling assumption, in contrast to our benchmark model with Pareto-distributed productivity. As we clarify below, the model with a standard AR(1) process cannot capture the right tail of firm size distribution. It follows that such model ignores firms that are currently small due collateral constraints but have strong growth potential once collateral constraints are removed, for which there are large potential gains from policies aimed at relaxing their borrowing limits. In such a case, the direct effect from credit subsidy policies is relatively small and is dominated by the negative indirect effects that arise from declines in the number of firms. It also follows that, even when a firm-level productivity is assumed to follow an AR(1) process, a model without endogenous extensive margins can improve its aggregate productivity by adopting credit subsidy policies as the number of firms remains constant by construction.

To examine the quantitative importance of our joint consideration of skewed firm-level productivity and endogenous extensive margins, we depart from our benchmark economy along two dimensions: log-normally distributed productivity and exogenous entry and exit. In each different model specification, we re-calibrate the model parameter values at the respective steady state to be consistent with the observed moments in Table 2.[33]

For the alternative specification of firm-level productivity, we consider a log AR(1) process for ϵ, which replaces the Pareto distribution. Specifically, we assume $\log \epsilon' = \rho_\epsilon \log \epsilon + \eta'_\epsilon$ with

[32] In the appendix, we show the results from a non-targeted policy which is the case without financial frictions in the model.

[33] With log-normally distributed productivity, the model firm size distribution is not consistent with its empirical counterpart. So, our re-calibration for such models is not intended to completely reproduce the size distribution moments in the lower panel of Table 2. The re-calibrated parameter values under each model specification are available upon request.

$\eta'_\epsilon \sim \mathcal{N}(0, \sigma_\eta^2)$, which is a standard modeling choice adopted by recent studies of heterogeneous firm models. To make the log-normal ϵ process comparable to our benchmark, we simulate the Pareto ϵ process and fit the assumed AR (1) process to set the values of ρ_ϵ and σ_η.[34] Using these two parameters, we discretize the AR(1) firm-level productivity process on a grid of 13 points, as we did in the benchmark parameterization. Although both Pareto and log-normal productivity distributions lead to the same persistence and volatility at the individual firm level, the population shares of firms across productivity grid points are largely different, as shown in the left panel of Figure 10.

For a model with exogenous entry and exit, we simply set the value of π_d, the exogenous exit rate in each period, to be the same as the total exit rate in the benchmark economy. In addition, we allow all potential entrants with mass $M^e = \pi_d$ to start production without paying the fixed entry cost, while matching the observed employment size of entrants in the data. This model environment abstracts from the endogenous cleansing effect of replacing unproductive incumbents with new firms, which we discussed in the previous section. Hence, the model with only exogenous extensive margins leads to a relatively higher population share of Type-2 firms whose collateral constraints are binding at equilibrium without policy interventions. In fact, the model with only exogenous extensive margins has about 29 percent of firms as Type-2, which is more than triple the amount in our benchmark. Thus, any policy targeted toward such firms will deliver quantitatively different aggregate results while the alternative model still captures the same macroeconomic moments. In this regard, we highlight the importance of endogenous changes in extensive margins when credit subsidy policies are introduced.

6.2 Comparison of Aggregate Results

In this sub-section, we report the aggregate results from our policy counterfactuals in the models with different specifications for firm-level productivity and extensive margins. In the following, we focus only on the case of the age-dependent credit subsidy policy. Table 4 presents the results at the post-intervention equilibria; the earlier values from the benchmark economy are given in the first column of the table to facilitate comparisons.

The first and second columns in Table 4 compare the aggregate consequences of the age-dependent policy with and without the endogenous margins of firm entry and exit. In contrast to the benchmark case (*the first column*), the policy counterfactual for the case with exogenous entry and exit (*the second column*) exhibits larger positive changes for all the aggregate variables, apart from the number of firms (μ), which remains constant by construction. Without a de-

[34] Figure 9 illustrates the simulation of the two different productivity processes with the same mean, persistence, and volatility.

Table 4 : Aggregate Results by Model Specification

Policy Counterfactual: Aggregates, Age-dependent Policy				
ϵ process entry/exit	Pareto Endogenous	Pareto Exogenous	Log-normal Endogenous	Log-normal Exogenous
consumption	105.64	109.20	100.83	104.85
capital	111.54	121.29	103.09	109.33
output	99.88	111.14	100.09	105.59
employment	94.54	101.77	99.25	100.72
firms (μ)	0.5193	1.0000	0.9180	1.0000
endo. exit rate	0.0430	–	0.0329	–
entrants rel. size	0.0197	0.0193	0.0321	0.0248
ΔTFP(%)	0.1936	4.2304	-0.3093	2.5471
CEV(%)	10.5177	7.8339	1.4734	4.3098

Note: In the top panel, we first normalize the aggregate quantities to 100 at each steady state by model specification and report their relative changes under the age-dependent policy. ΔTFP refers to the relative change in the measured total factor productivity from each steady state. CEV denotes the consumption equivalence measure of welfare. We use re-calibrated values of α and ψ to compute TFP and CEV in each model specification.

cline in the total number of firms in operation in the economy, the aggregate productivity gain is larger, about 4.23 percent, in contrast to our benchmark case which achieves a 0.19 percent gain.[35] This result highlights the importance of endogenous extensive margins in quantitatively determining the size of macroeconomic impact of credit subsidizing policies. It is also evident that the losses from resource misallocation due to financial frictions can vary across models with different specifications of firm entry and exit, as pointed out in recent studies.[36]

In Table 4, the comparison between the first and the third columns reveals a more striking result; the sign of the aggregate productivity gain under the age-dependent policy changes from positive to negative. Specifically, the credit subsidy for young firms lowers the measured TFP by 0.31 percent relative to its pre-intervention steady state when the assumed firm-level productivity instead follows a log AR(1) process.[37] This is in stark contrast to the results from our benchmark economy where both size- and age-dependent policies lead to positive gains in aggregate productivity. Moreover, it is noticeable that the negative impact on TFP emerges without any

[35] Notice that the results in Table 4 are under a general equilibrium setting, where each economy reaches different levels of equilibrium prices. We show the outcomes from a partial equilibrium setting in the appendix. There, prices are fixed at the steady state and hence there are no negative impacts on untargeted firms. Moreover, the endogenous margins with fixed prices yield a rise in the equilibrium number of firms, which amplifies the productivity gains from a subsidizing policy.

[36] Midrigan and Xu (2014) consider a two-sector model economy with financial frictions existing only in the modern sector. They consider endogenous extensive margins in their extended version of the model and find relatively small losses from resource misallocation, which is consistent with our results.

[37] Similarly, the size-dependent policy in the same model results in a negative productivity gain of 0.25 percent. The negative gains in productivity also emerge from modest increases in θ by 5 to 10 percent.

direct policy distortions because we maintain our assumption of lump-sum cash transfers across all model variations in Table 4. As a result, the aggregate improvement is quite modest in the model with log-normally distributed firm productivity, and the corresponding household welfare increases only by 1.5 percent. Recall that we adopt the Pareto-distributed ϵ process mainly to reproduce the observed dispersion and skewness in firm size distribution in the data.[38] Thus, the above comparison simply illustrates that a model without a realistic firm size distribution may give rise to inaccurate, or potentially wrong, predictions of macroeconomic policies in the long run.

To examine this disparity from the perspective of firm dynamics, we examine how firm dynamics are compared in the above models that differ in firm-level productivity distribution. We calculate the average levels of capital stock and TFP for each age group of firms following the age-dependent policy considered in Table 4. Figure 11 plots the gap between pre- and post-intervention capital and TFP across age cohorts, by model specification. Overall, the dynamics of firm size and productivity display similar patterns between the two different models. The top panel of the figure shows that credit subsidies raise the level of capital held by young firms (age 1 to 7) and then slightly depress it for mature firms. However, the magnitude of such changes is much more pronounced in our benchmark case with Pareto-distributed firm productivity.

Why are the overall policy impacts relatively small in the model with log-normally distributed productivity? As we see in the left panel of Figure 10, the right tail of the ergodic distribution of a log-normal ϵ process does not stretch out enough (the highest ϵ value is below 0.7) in comparison to the Pareto case, which has a maximum ϵ value greater than 1.1. Given the convexity of the optimal capital decision, as shown in the right panel of Figure 10, the right tail of the firm-level productivity distribution matters because the potential gain from a credit subsidy is large when firms with such high productivity are financially constrained. This is indeed the case in our comparison in Table 4, and it leads to the substantial differences in firm dynamics shown in Figure 11. It follows, for a positive aggregate productivity gain, that a newborn firm with huge growth potential but without sufficient funds should be included in the group targeted by credit subsidy policies. Otherwise, as in the alternative model with log-normally distributed ϵ, the negative effect from the decline in the equilibrium number of firms becomes more pervasive.

In sum, we illustrate a case for which the aggregate consequences of an industrial policy may be altered when a calibrated model only targets macroeconomic moments without considering the endogenous extensive margins and the realistic firm size distribution. In other words, introducing such model elements is not only a prerequisite for being consistent with micro-level data

[38] The model economy in column 3 of Table 4 largely fails to generate a realistic firm size distribution at its steady state. The resulting population share of SMEs is 85.7 percent, in contrast to 98.5 percent in our benchmark economy, whereas their total employment share still matches the observed value in the BDS.

before conducting policy counterfactual studies such as ours, but it is also crucial in examining the associated aggregate equilibrium results following a policy intervention. Therefore, our results indicate that the micro-level consistency of a macroeconomic model is vital to quantifying the aggregate outcomes of targeted credit policies. Moreover, this also implies that understanding the firm-level heterogeneity of an economy is essential in designing and implementing such targeted policies in practice, which we believe is missing in micro-evaluations of policies.

7 Concluding Remarks

In this paper, we presented a general equilibrium model of heterogeneous producers with collateral constraints and endogenous entry and exit. By employing a bounded Pareto distribution for firm-level productivity, our model replicates cross-sectional features of firm dynamics, as seen in the BDS. We use this model to quantify the macroeconomic implications of targeted credit subsidy policies. We show that aggregate outcomes of such policies critically hinge on the underlying distribution of firms and their financial status. Subsidized credit alleviates credit constraints faced by small and young firms, which helps them achieve efficient and larger scales of production. This direct effect is, however, either reinforced or offset by indirect general equilibrium effects. In particular, the indirect general equilibrium effects from declines in the number of firms in the economy dominate the direct productivity gains in a model with the standard AR(1) process, while we found a sizable positive productivity gain by subsidy policies in our benchmark model with a bounded Pareto distribution for firm-level productivity. We view this as a cautionary tale about unintended consequences of targeted policies that perform poorly in nurturing economic growth.

Because small and young firms account for a substantial share of employment in the economy, policies that target small and young firms are popular among policymakers. However, research on the aggregate implications of such industrial policies is scarce. We believe that our results demonstrate a potential pitfall in the analysis of industrial policies: failure to consider general equilibrium price movements and micro-level consistency may create misleading public debates on such policies.

Enhancing aggregate productivity in the economy appears to be immediately relevant for many countries. In particular, a variety of targeted industrial policies have been implemented in European and Asian countries. A prime example includes the Moratorium Act in Japan as mentioned above. Quantitative investigations of this specific example using our framework and micro-level company financial data may be useful. This paper, however, did not consider any policy distortion itself, which is also potentially important. This research is left for future study.

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Online Appendix: Not for Publication

In this appendix, we report notable results from our policy counterfactual exercises that are not discussed in the main text.

A Aggregate Results from Alternative Policies

A.1 Non-targeted Credit Subsidy

In Section 5 and 6, we focused on the macroeconomic implications of credit subsidy policies that target small or young firms in an economy. In this subsection, we present the aggregate results from a non-targeted policy that subsidizes all firms in the economy regardless of age or size. Specifically, we consider a case of perfect credit market, where $\theta = q^{-1}$ implies that firms are allowed to achieve their desired investment. Under this perfect credit market setup, we still maintain the endogenous entry and exit and the underlying distribution of firm-level productivity in the model. This exercise will help us understanding the mechanics of the indirect general equilibrium effects and also contrast the results from targeted policies in Table 3.

The second column of Table 5 reports the case when each firm's borrowing is not subject to the collateral constraint at all. The first thing to note is that the results from the non-targeted policy is almost similar to the results under the size-dependent policy (the last column of Table 3). Intuitively, this is because our size-dependent policy is targeting SMEs that account for more than 98 percent of all firms. However, the aggregate gain in measured TFP is slightly lower (0.68%) under the perfect credit market in relative to that from the size-dependent policy (1.36%).

By comparing the average productivity gain per incumbent, ΔTFP_{μ}, we recognize that the above disparity in measured TFP gain is mainly from the indirect general equilibrium effects. First, the selection among potential entrants become more weaker under the non-targeted credit subsidies as represented by a smaller entrants size. Next, the selection among incumbents become stronger because of the increased less-productive entrants, which is shown by the relatively higher exit rate in Table 5. As a result, the equilibrium number of firms under the non-targeted policy gets lower than that with the size-dependent policy. This depresses the aggregate productivity gain, as we already discussed in the main text. The above comparison implies that a policy that entirely removes borrowing limits for all firms in an economy may not always guarantee the efficient outcome, due to the presence of endogenous extensive margins.

Table 5 : Aggregate Results, Perfect Credit

Policy Counterfactual: Aggregates, Perfect Credit		
	Benchmark	Perfect Credit
consumption	100 (0.1876)	106.55
capital	100 (0.5744)	115.34
output	100 (0.2584)	102.13
employment	100 (0.3334)	95.83
debt	100 (0.3238)	153.52
cash-on-hand	100 (0.8020)	78.24
firms (μ)	1.0001	0.5936
endo. exit rate	0.0140	0.0243
entrants rel. size	0.0331	0.0183
Type-2 share	0.0992	0.0000
ΔTFP(%)	(0.5834)	0.6681
ΔTFP_{μ}(%)	–	7.1701
CEV(%)	–	10.2846

Note: In the top panel, we normalize the aggregate quantities to 100 at the benchmark, and the values in the parentheses are the corresponding absolute values. rt is the required cash transfer in each policy. ΔTFP refers to the relative change in the measured total factor productivity from the benchmark. ΔTFP_{μ} is the productivity gain per firm. CEV denotes the consumption equivalence measure of welfare.

A.2 Limited Credit Subsidies

In the main policy analysis of this paper, we entirely remove the borrowing limits of the targeted firms in the model. The existing programs that subsidize credits to small or young businesses, in practice, rather tend to indirectly alleviate the financial concerns of such firms. In this sub-section, we consider alternative targeted credit subsidy schemes that partially relax the collateral constraints. This will illustrate the cases of policy intervention at a moderate degree, which can be more relevant in actual policy design by providing the corresponding outcome.

In particular, we consider cases with different increases in the value of θ for the age-dependent policy: 5%, 10%, and 15% increases from the steady state value of θ.[39] We base our choices of the 5 to 15 percent increases of credit on the observation that the average size of all US commercial and industry loans is about 8 percent larger than that of risky collateralized loans.[40] There is no direct mapping of this empirical observation into the model economy, but our results from 5 or 10 percent cases can be understood as illustrating a counterfactual when the US commercial banks can match the risky loan size to the level of the other loans. The aggregate results are reported in Table 6.

[39]For instance, an increase of 15% implies the value of $\theta = 0.943$ for the targeted firms.
[40]Survey of Terms of Business Lending (quarterly), FRED.

Table 6 : Aggregate Results, Limited Subsidies

Policy Counterfactual: Aggregates, Limited Subsidies				
	Age-dependent Policy			
	(5%)	(10%)	(15%)	($\theta = q^{-1}$)
consumption	100.58	101.25	102.04	105.64
capital	100.89	102.00	103.52	111.54
output	99.92	99.65	99.50	99.88
employment	99.31	98.41	97.48	94.54
debt	104.91	110.90	118.13	147.56
cash-on-hand	96.86	93.02	88.94	76.91
firms (μ)	0.9470	0.8790	0.7993	0.5193
endo. exit rate	0.0158	0.0189	0.0236	0.0430
entrants rel. size	0.0315	0.0297	0.0277	0.0197
cash transfer (rt)	0.0257	0.0571	0.0930	0.2282
subsidized firms (μ^B)	0.0714	0.0682	0.0657	0.0558
Type-2 share	0.0918	0.0814	0.0682	0.0117
ΔTFP(%)	0.0905	0.0575	0.0590	0.1936
$\Delta TFP/rt$	0.0206	0.0059	0.0037	0.0049
ΔTFP_μ(%)	0.7480	1.6191	2.7859	8.3916
CEV(%)	1.1554	2.5915	4.1851	10.5177

Note: In the top panel, we normalize the aggregate quantities to 100 at the benchmark, and the values in the parentheses are the corresponding absolute values. rt is the required cash transfer in each policy. ΔTFP refers to the relative change in the measured total factor productivity from the benchmark. ΔTFP_μ is the productivity gain per firm. CEV denotes the consumption equivalence measure of welfare. Columns with (5%)-(15%) are the results from raising θ for the target respectively by the corresponding amount from its steady state value.

As shown in the table, the aggregates and the associated productivity gains gradually improve as θ increases. Although the overall improvements exhibit monotone patterns, the indirect general equilibrium effects that we discussed earlier become stronger. From the table, this can be represented by the drastic fall in the number of firms as we move to the case of no borrowing limits (last column). Accordingly, it implies that more firms need to be subsidized (μ^B/μ) as θ increases, and the effectiveness of the age-dependent policy ($\Delta TFP/rt$) falls due to the increasing cash transfers required. However, our results in Table 6 imply that limited credit subsidies targeting young or small firms can still lead to a decrease in the resource misallocation arising from financial frictions, while improving the aggregate welfare.

B The Importance of General Equilibrium

So far, we have conducted our policy counterfactual exercises in a general equilibrium (GE) environment. This involves endogenous changes in the equilibrium price of the model, which is

the real wage rate (w). Our focus on the equilibrium price adjustments is nothing novel when compared to the previous studies of firm dynamics with an industrial policy such as Hopenhayn and Rogerson (1993). However, in this section, we attempt to decompose the mechanism behind our results from the GE policy counterfactuals by contrasting them with those from the partial equilibrium (PE) environment. In addition, recent studies such as Buera, Kaboski, and Shin (2017) emphasize the use of standard macroeconomic approaches in analyzing policy effects at the long-run equilibrium, which can also complement the conventional microeconometric approaches.

In fact, the equilibrium wage under each policy is higher than that in the benchmark case. This section compares the aggregate results of a targeted policy in GE and PE of the model. In particular, we illustrate the importance of the GE price effect on firm entry and exit margins by comparing the firm dynamics respectively under GE and PE. We only report the results from the age-dependent policy because the main results still hold under different credit subsidy policies.

Table 7 : Aggregate Results, GE vs. PE

Policy Counterfactual: Aggregates, GE vs. PE			
	Benchmark	Age-dependent (GE)	Age-dependent (PE)
consumption	100 (0.1876)	105.64	100.00
capital	100 (0.5744)	111.54	215.86
output	100 (0.2584)	99.88	194.85
employment	100 (0.3334)	94.54	194.81
debt	100 (0.3238)	147.56	264.33
cash-on-hand	100 (0.8020)	76.91	165.15
firms (μ)	1.0001	0.5193	1.4250
endo. exit rate	0.0140	0.0430	0.0171
entrants rel. size	0.0331	0.0197	0.0208
cash transfer (rt)	–	0.2282	0.3482
subsidized firms (μ^B)	–	0.0558	0.0791
Type-2 share	0.0992	0.0117	0.0170
ΔTFP(%)	(0.5834)	0.1936	5.2860
$\Delta TFP/rt$	–	0.0049	0.0886
ΔTFP_μ(%)	–	8.3916	0.9056
CEV(%)	–	10.5177	-54.3391

Note: In the top panel, we normalize the aggregate quantities to 100 at the benchmark, and the values in the parentheses are the corresponding absolute values. rt is the required cash transfer in each policy. ΔTFP refers to the relative change in the measured total factor productivity from the benchmark. ΔTFP_μ is the productivity gain per firm. CEV denotes the consumption equivalence measure of welfare.

From Table 7, we recognize that the quantitative effects of a credit subsidy policy can be seriously misleading when the equilibrium price adjustments are not considered. The first two

columns of 7 are exactly from Table 3 to facilitate the comparison. The last column of the table reports the aggregate results from the age-dependent policy when the real wage is fixed. Under this PE exercise, the policy increases the aggregate quantities enormously. For instance, the aggregate capital more than doubles under PE. Moreover, the number of firms increases by about 42 percent in relative to the benchmark, which dramatically raises the size of productivity gains in the aggregate: 5.29% under PE and 0.19% under GE. By comparing this result with the gains in average productivity per firm, TFP_{μ}, in the table, we can see that the boosted aggregate productivity under PE largely comes from the increased number of incumbents that are, on average, less productive than those in GE. On the other hand, the welfare of households falls drastically under PE following the age-dependent policy, and this is mainly due to the large increase in aggregate employment while consumption remains constant.

Therefore, Table 7 shows that it is quantitatively important to consider the effects of equilibrium price changes when we evaluate a targeted credit subsidy policy. In PE, we fail to consider the important indirect general equilibrium effects as discussed in the main text: (i) changes in optimal production scale, (ii) firm entry and exit margins, and (iii) equilibrium number of firms.

Figure 12 compares firm dynamics with and without the equilibrium wage adjustment under the age-dependent policy. Clearly, the dynamics of the average firm size and productivity under PE are not much different from those in the GE environment. One noticeable difference, however, is that the levels of capital stock of each age group are larger in PE. With credit subsidies, the equilibrium real wage rate rises and this reduces the optimal capital choice, $K^w(\epsilon)$, and the constrained capital choice, $\bar{K}(m)$. Thus, the average size of firms falls in a GE environment. This is exactly what we previously mentioned as the first indirect effect of the GE price channel.

More importantly, the changes in equilibrium wage affect firm exit and entry margins, which we distinguished as the second indirect effect above. Other things being equal, the higher real wage rate hurts firms with low productivity and high leverage ratio, as reflected in the higher exit rate in the second column of Table 7. In contrast, the case of PE implies that the endogenous margin of exit works only marginally as shown by a slight increase in the exit rate.

In addition, the rise in equilibrium wage may strengthen or weaken the selection among the potential entrants. As discussed earlier, the GE environment yields that it actually lowers the initial productivity of entrants by inducing more unproductive firms to enter. From Figure 12, this effect is reversed under the PE case by raising the initial productivity slightly.[41] Noting that the relative size of entrants is similar across the GE and PE economies after the age-dependent

[41] This is different from what Buera, Moll, and Shin (2013) predict in a similar model environment. They show that credit subsidies improve aggregates in the short run, while distorting the entry into entrepreneurships in the long run due to the fixed subsidy target over time. Our policies, in contrast, depend on firm size or age in each period.

policy, the rise in productivity of age 0 firms indicates that the entry margin is adjusted along the fat-tailed productivity distribution under PE.

The last indirect effect is on the equilibrium adjustments in the number of firms. Given the relatively weaker cleansing effect from the exit margins and the positive selection among potential entrants as discussed above, the total number of firms under the PE environment rather increases (Table 7). This in turn increases the aggregate gains from the size- or age-dependent policies in a misleading way. Our point here is that the GE environment is crucial in more precisely quantifying and evaluating the macroeconomic impact of a targeted credit subsidy policy for firms' external financing.

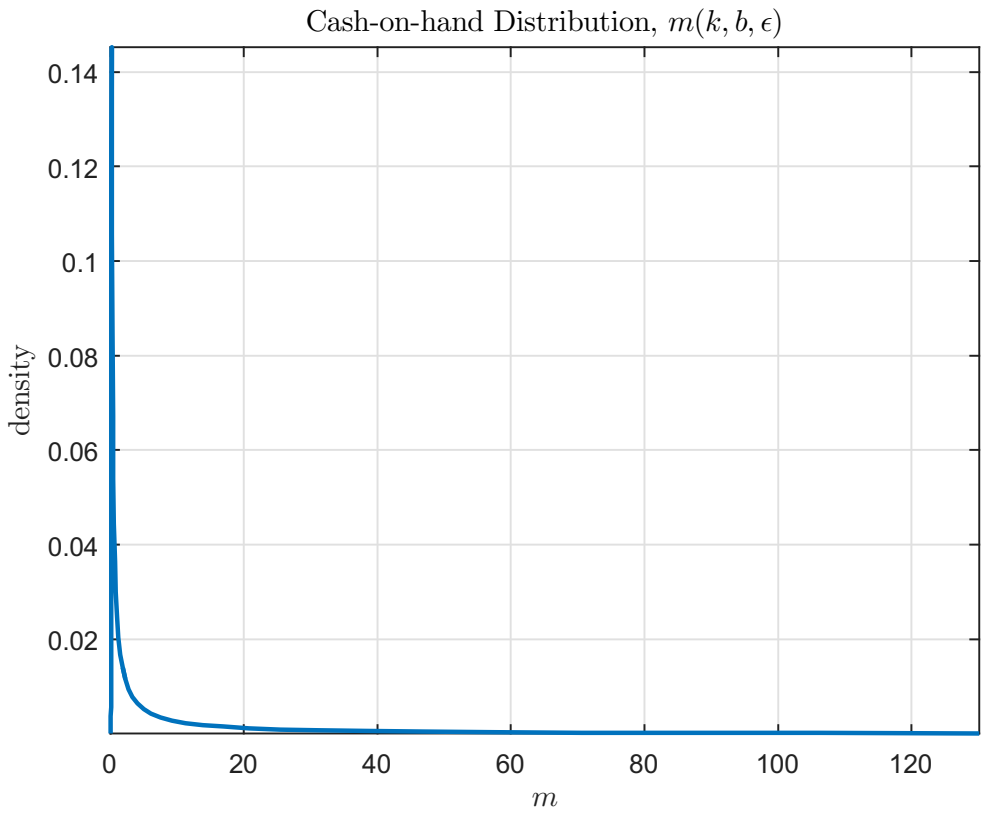

Figure 1 : The stationary distribution of cash-on-hand, $m(k, b, \epsilon)$, at the steady state (*benchmark*) of the economy.

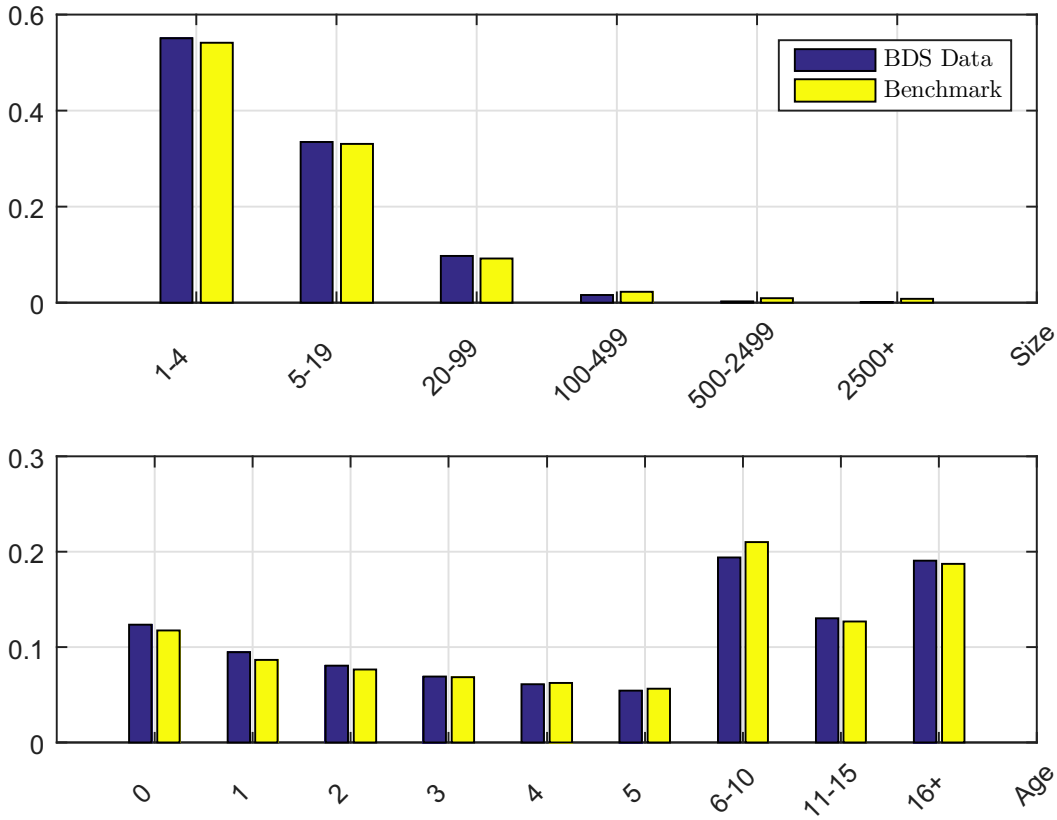

Figure 2 : Distribution of firms over size and age bins.

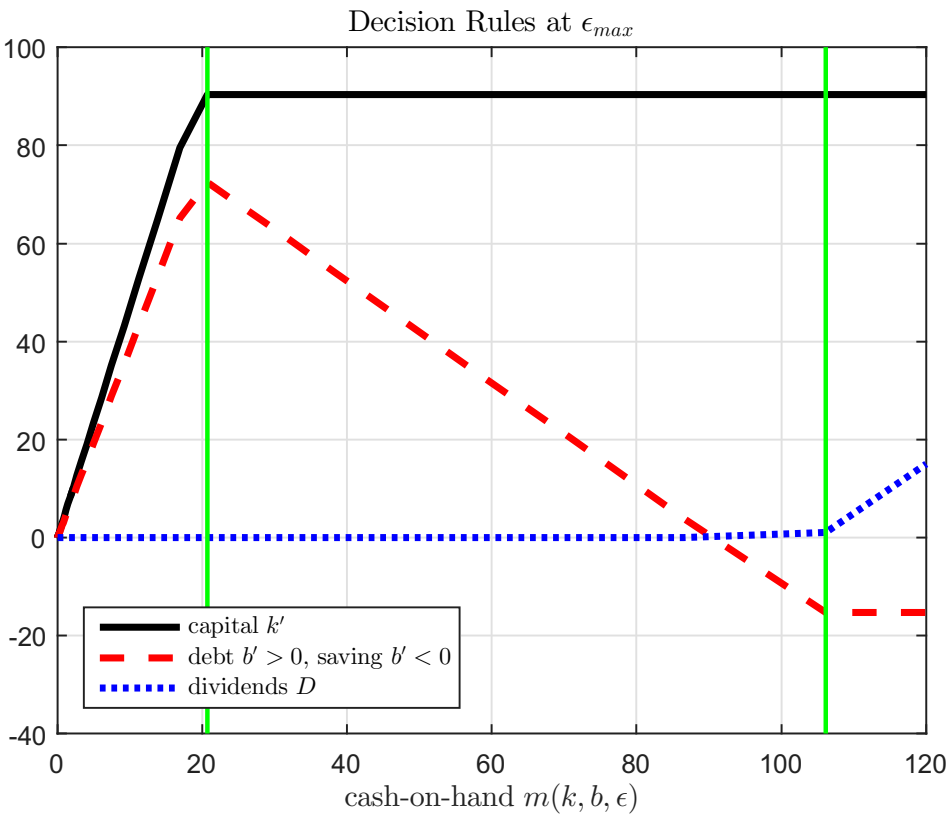

Figure 3 : Choices of capital, debt or financial savings and dividends are plotted for each level of cash-on-hand.

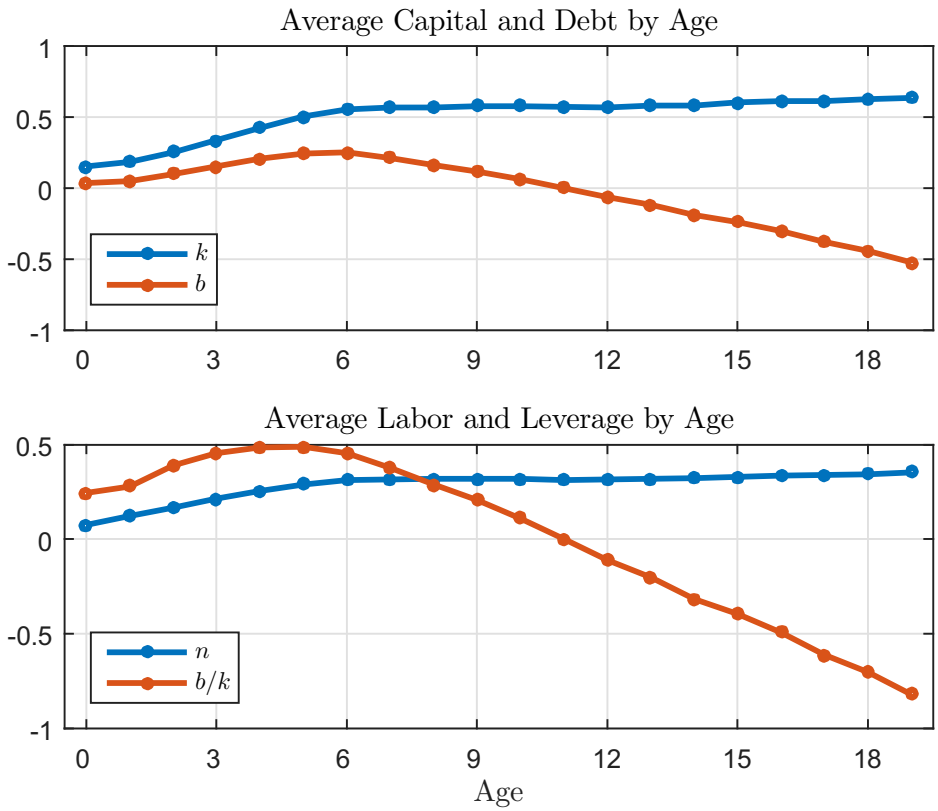

Figure 4 : Cohort average capital, debt or financial savings, labor and leverage ratio are constructed from a simulation of an unbalanced panel of firms.

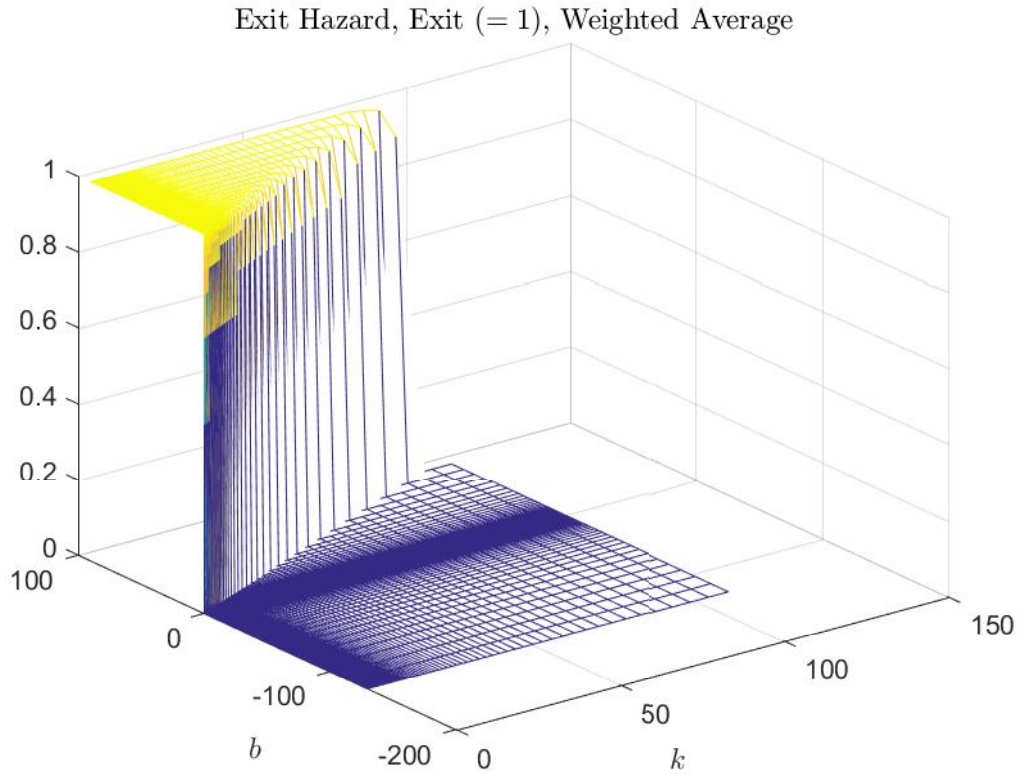

Figure 5 : Capital increases right to left. Debt increases front to back; negative values are financial savings.

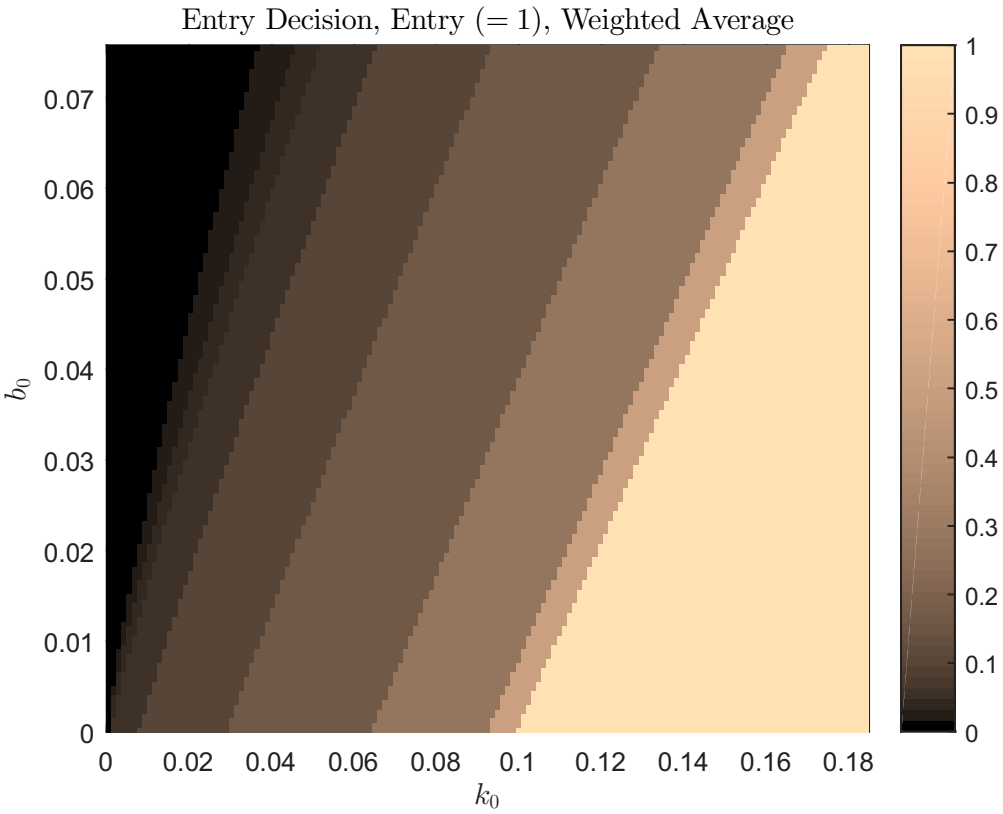

Figure 6 : Entry decision over the initial (uniform) distribution of capital and debt, (k_0, b_0), in the benchmark economy.

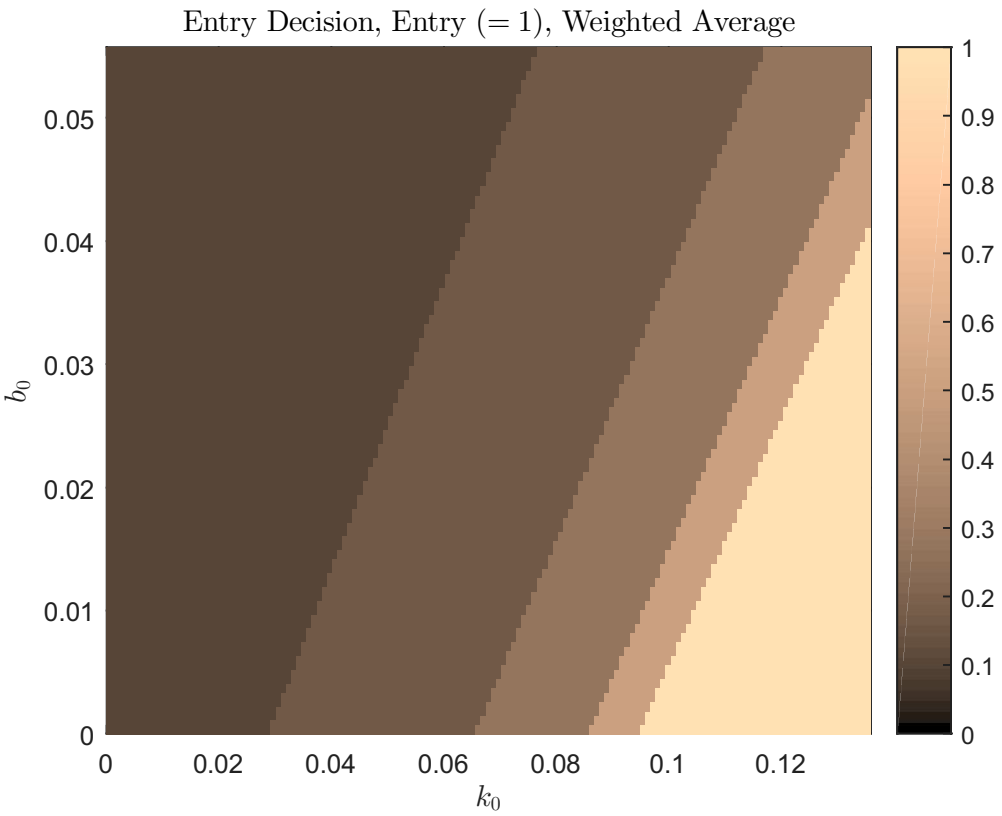

Figure 7 : Entry decision over the initial (uniform) distribution of capital and debt, (k_0, b_0), under the size-dependent policy.

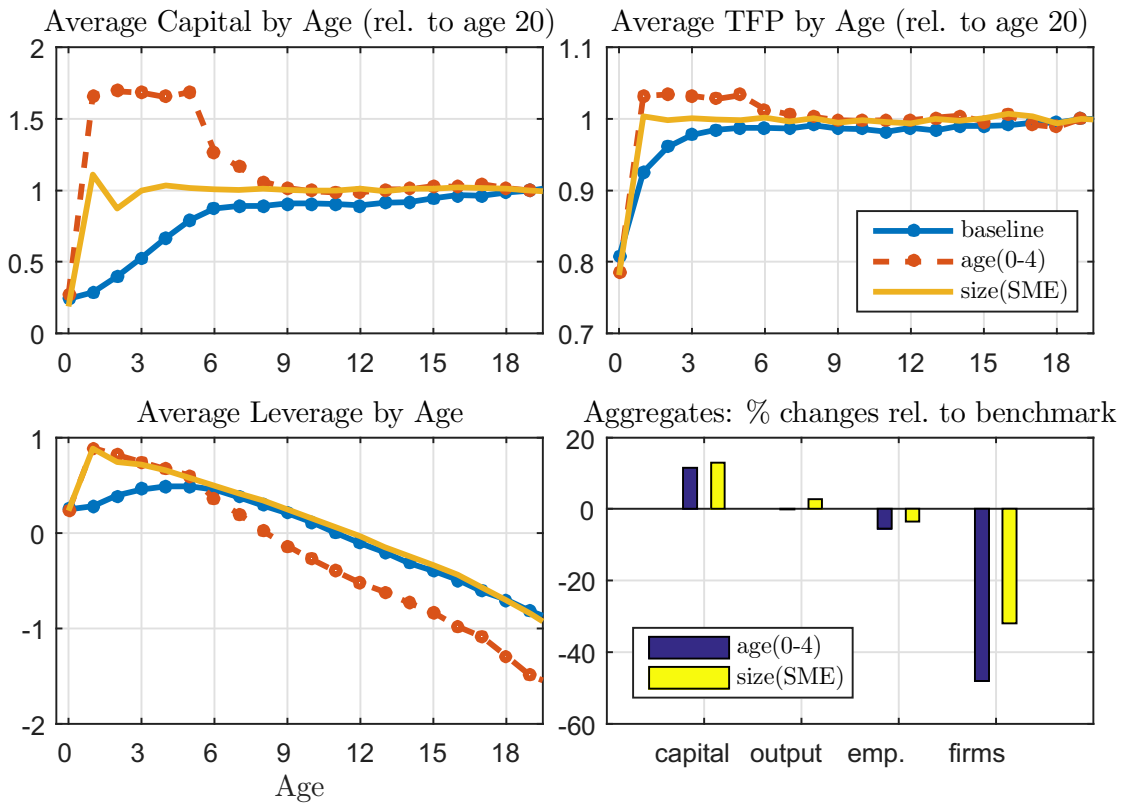

Figure 8 : Cohort average capital, TFP and leverage ratio are constructed from a simulation of an unbalanced panel of firms.

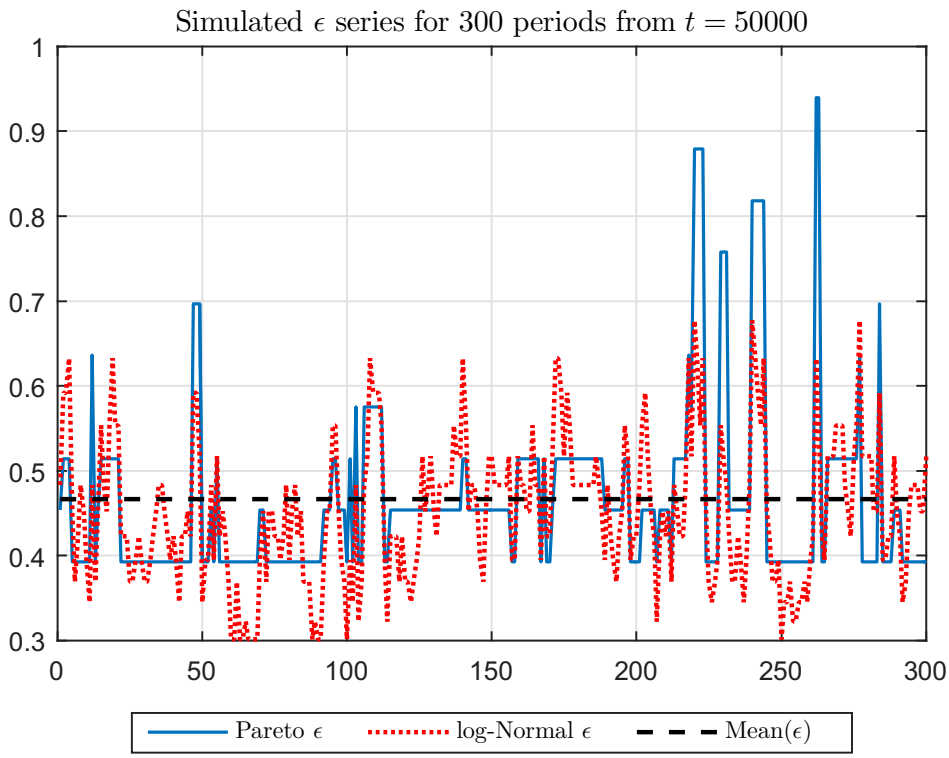

Figure 9 : Moments of the ergodic distribution for both Pareto and log-normal distributions are matched and the moments are: mean 0.466, standard deviation 0.112, coefficient of autocorrelation 0.750. Using the parameterized processes, a panel of firms are simulated and represented simulation paths are plotted in the above figure.

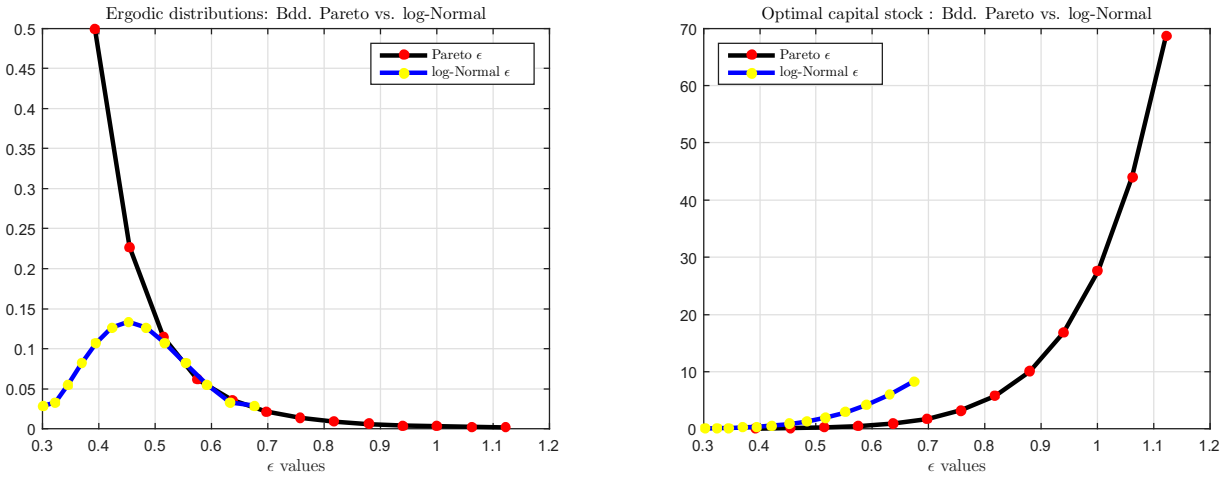

Figure 10 : The mass of firms is plotted for each of 13 productivity grid points on the left panel. The efficient capital choice is plotted against each of 13 productivity grid points on the right panel.

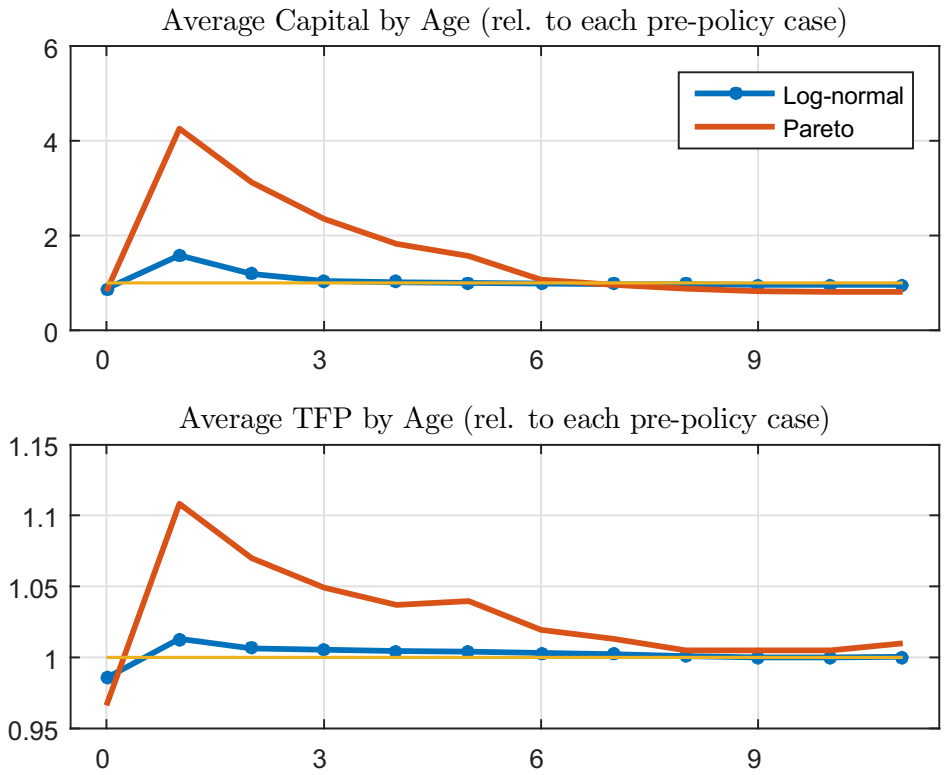

Figure 11 : Cohort average capital and TFP are constructed from a simulation of an unbalanced panel of firms.

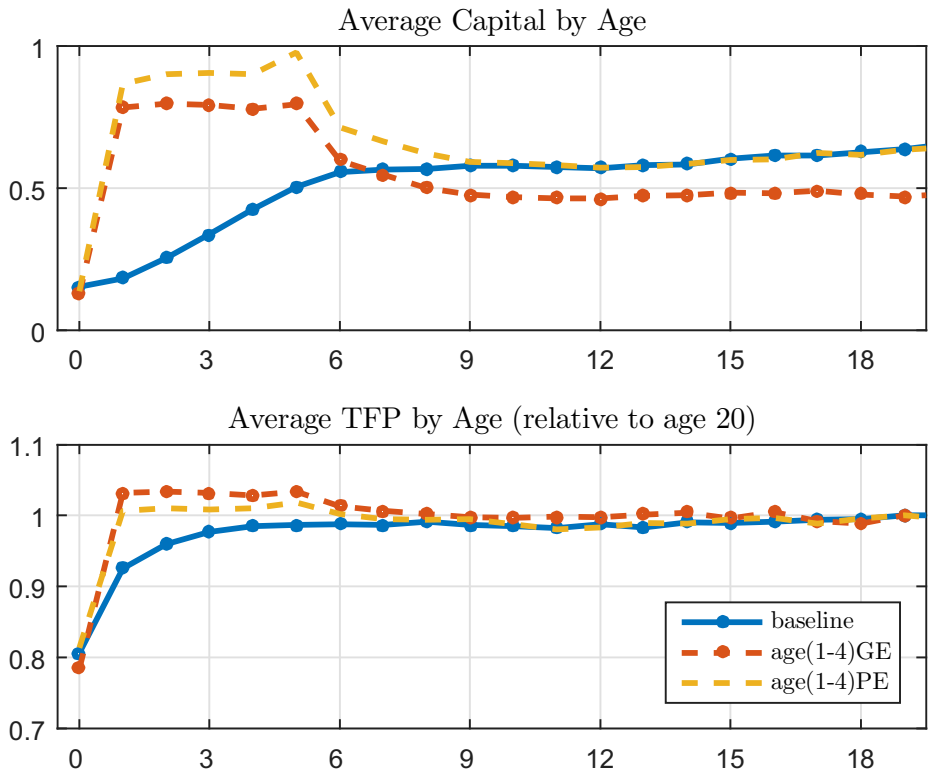

Figure 12 : Cohort average capital and TFP are constructed from a simulation of an unbalanced panel of firms.

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