A Classical Model of Education, Growth and Distribution

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Abstract

We develop a classical macroeconomic model to examine the growth and distributional consequences of education. Contrary to the received wisdom, we show that human capital accumulation is not necessarily growth-inducing and inequality-reducing. Expansive education policies may foster growth and reduce earning inequalities between workers, but only by transferring income from workers to capitalists. Further, the overall effect of an increase in education depends on the actual characteristics of the educational system and on the nature of labor market relations. We argue that the model can shed light on some recent stylized facts on growth, distribution and education for the US.

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1 Introduction

The role of education – and what is called human capital – has received a great deal of attention in the literature on economic growth in recent decades. The contribution of education to skill formation, and the resultant division of the labor force into high- and low-skilled workers, has also been widely examined in the literatures on income distribution and international trade. This analysis has been conducted empirically and theoretically, and also entered popular discussions. Neoclassical models of growth (including endogenous growth models) see education as promoting growth by making the productivity of labor increase more rapidly, and improving income distribution by increasing wages, although different rates of skill formation – through education – between different groups are sometimes argued to exacerbate income inequality. Surprisingly, however, education has received little or no attention in theories of growth and distribution in the classical tradition, despite the obvious relevance it has for the dynamics of capitalist economies. A recent text on growth theory written mainly from a classical perspective, by Foley and Michl (1999) has no discussion of the role of education. We are, in fact, unaware of any heterodox dynamic model of growth and distribution which analyzes the role of education.[1]

This gap is all the more surprising given the fair amount of attention being given to education in broader political economy discussions. While much of this discussion is in relation to the ideological role of education, some of it also relates to growth. For instance, some commentators have associated the relative neglect of the education sector with the profit squeeze and the decline in productivity growth after the period of the so-called Golden Age of Capitalism which ended in the late 1960s (Glyn et. al. 1990).

The purpose of this paper is to fill this gap in the literature and develop a simple classical model of growth and income distribution in which the primary role of education is to convert low-skilled workers into high-skilled workers, and with which broader political-economy considerations may also be addressed.

[1] To the best of our knowledge, the only exceptions are a post-Keynesian model in Dutt (2010) and our earlier paper on classical growth theory which contains a simplified model but focuses on the contributions of the classical economists on education.

The classical approach to *growth* has thus far focused mainly on the role of capital accumulation due to saving by capitalists (see Foley and Michl 1999, for an exposition), and due to technological change brought about by the response of capitalist firms to labor market conditions (see, e.g., Duménil and Lévy 2003; Flaschel 2009).[2] It is not obvious what effect education would have on growth and distribution according to this tradition. Given the existence of unemployed workers, increases in education, by increasing labor productivity, need not increase output and its growth. Nor, in the presence of unemployment, do they necessarily increase wages and improve income distribution.[3] Indeed, the classical framework has not been concerned with the effect of education and skill formation on *distribution*, focusing instead on the functional distribution of income between wages and profits and its consequences for the dynamics of capital accumulation,[4] with some attention being given to landlords, rent and natural resources in a Ricardian vein (as in Pasinetti's 1960 classic model or, more recently, in Foley and Michl 1999; Foley 2003; and Petith 2008), to managers (Tavani and Vasudevan 2014),[5] and to financial capitalists - following Marx's distinction between interest payments and profit income (Dutt 1989). How will the entry of education and human capital affect the dynamics of distribution? Will limited access to education widen the inequality between people who obtain higher levels of education and those who do not? What are the broader, political-economy effects of education?

[2]There are, of course, many differences regarding the analysis of growth and distribution of different economists who have collectively been referred to as classical economists, such as Smith, Ricardo, Malthus, Mill and Marx, and there are also different views on the analysis of a particular classical economist. We do not enter into a discussion of these issues but, rather, use a model that many have found to capture some essential characteristics common to the classical economists.

[3]Marx, for instance, argued that education would not have a positive effect on wages when the educated labor became unemployed due to the introduction of machinery because the skilled unemployed would bid down wages competing for the remaining jobs (Marx and Engels 1975-2005, Vol. VI, p. 427).

[4]See, for example, the classic analyses by Goodwin (1967) and Foley (1986). More recent contributions include Flaschel (2009). The focus on the capitalist/workers divide is motivated by the centrality of exploitation theory in the Marxian approach. For a thorough, recent analysis of the relation between exploitation and distributive issues, see Veneziani (2007).

[5]The conception and role of managerial labor is quite different from our notion of high-skilled workers. For managers perform purely supervisory work and their prerogatives are not linked to skills or education.

Although this paper attempts to fill a gap in the literature on classical models, our motivation is not purely theoretical or exegetical, We believe that models in the classical tradition can shed light on some important stylized facts of the dynamics of growth and distribution in advanced economies, which are at least prima facie inconsistent with the standard models.

1.1 Stylized facts

To be specific, our analysis tries to incorporate and explain some features of the US economy in the past four decades, which have been characterized by the rise of the knowledge economy and the predominance of neoliberal policies.

Consider distribution first: the income shares of capital and labor have remained roughly constant from 1975 to 2011, with the share of wages in national income averaging 0.64 with a standard deviation of 0.011 and the share of corporate business profits before tax averaging 0.076 with a standard deviation of 0.02 (Federal Reserve Economic Data, Federal Reserve Bank of St Louis). At the same time, earning inequalities within the labor force have significantly increased: the ratio of the hourly wage of College-educated workers to the hourly wage of workers with just a high school degree has moved from around 1.43 in the mid-1970s to 1.76 in 2007 (The State of Working America 2008-10, Table 3.15). Similarly, the income share of the top decile of the income distribution has moved from around 32% in the mid-1970s to 46% in 2010, with most of the gains actually accruing to the top 1% (The State of Working America 12th Edition, Figure 2AA).

Concerning growth and capital accumulation, two features of the dynamics of the US economy stand out. The growth rate in per capita GDP has averaged 1.78% during 1975-2013 (World Development Indicators, World Bank). This is much lower than the average growth rate in the previous years characterized by the so-called Keynesian compromise. But, perhaps more importantly, despite significant fluctuations, there is no sign of an acceleration in growth in the last four decades. If anything, the dynamics of the growth rate show a slightly declining trend.

Trends in education are equally clear, showing an increasingly educated workforce, and a seemingly more open education system that is more accessible, for example, to women and minorities. Total fall enrolment in higher education has seen a continuous and marked upward trend from 11.1 million in 1975 to 21 million in 2010, with an acceleration in the last two decades. At the same time, the percentage of women enrolled saw a remarkable 11 percentage point increase from 46% in 1975 to 57% in 2010.[6] Similarly, in the past four decades the percentage of high school completers going to College has increased significantly across the whole population but particularly marked upward trends can be seen in the black and hispanic populations with both groups moving from around 45% in 1972 to, respectively, 69.5% and 59.3% in 2009.

The developments in education in the last four decades seem at odds with the stylized facts on growth and inequality. Based on endogenous growth theory, a significant increase in human capital should lead to a higher growth rate of the economy, ceteris paribus. Similarly, the increase in the supply of high-skilled labor, and a wider access to education, should reduce skill differentials and have equalizing effects, ceteris paribus. The standard way to reconcile these stylized facts is precisely to deny the empirical relevance of the ceteris paribus clause. In particular, the marked increase in earning inequalities is explained by skill-biased technical change which has led the demand for educated workers to increase even more sharply than supply, thereby raising the skill differential (Katz and Murphy 1992).

This is not the place for a thorough discussion of the vast literature on earning inequalities, but it is worth noting that the empirical evidence on the skill-biased technical change hypothesis is inconclusive. Standard human capital variables such as experience and education usually explain between one eighth and one third of earning inequalities, and a large part of earning inequalities occur within groups of equally educated and experienced workers – even controlling for other observable individual characteristics (see Card and Lemieux 1996; Lemieux 2006). To be sure, one may argue that these results are due to data limi-

[6] Source: U.S. Department of Education, National Center for Education Statistics, Opening Fall Enrollment in Higher Education, 1965; Higher Education General Information Survey (HEGIS), "Fall Enrollment in Institutions of Higher Education" surveys, 1966 through 1985; Integrated Postsecondary Education Data System (IPEDS), "Fall Enrollment Survey" (IPEDS-EF:86-99); and IPEDS Spring 2001 through Spring 2011, Enrollment component.

tations, and any residual variance in earning inequality can be attributed to unobservable individual skills. Yet Egger and Grossman (2004) have shown that cognitive skills remain of limited value to explain the variance of wage differentials among workers with the same level of education.[7] More generally, there is growing empirical evidence suggesting that supply-and-demand factors alone cannot explain observed variations in wage inequality in advanced economies, and institutions, government policies, and the distribution of power among key actors in the labor market have a central explanatory role (e.g. Blau and Kahn 1996 and the contributions in Freeman and Katz 1996).[8]

From this perspective, given the presence of significant imperfections in both product and labor markets, one important question concerns the factors influencing wage setting procedures and institutions, including collective organizations, bargaining power, social norms, expectations, and so on. In particular, in a general bargaining-theoretic framework, one striking – and well-known – empirical fact is the marked decline in all indicators of workers' bargaining power and cohesion. Union density remained reasonably stable during 1947-1979 with an average of 24.45% (and a standard deviation of 0.02) but has since steadily declined reaching a minimum of 11.30% in 2003. The drop has been even more marked in the private non-agricultural sector where unions have virtually disappeared and the percentage of workers covered by a collective agreement has moved from 23.6% in 1979 to 7.4% in 2012 (Hirsch and Macpherson 2003 and updates). A large number of empirical studies have proved that unionization is one of the key variables in explaining income and earning distributions.

Therefore, although directed technical change is a key determinant of the dynamics of growth and distribution, and competitive mechanisms and optimal choices play a fundamental role in determining skill differentials, we explore the connections between education,

[7] Focusing on a very detailed dataset collected by the OECD, Franzini and Raitano (2014) have proved that wage differentials remain largely unexplained even including individual information about the details of educational paths, employment history, and other acquired skills (e.g. knowledge of IT).

[8] Empirical studies in different countries indicate that the rapid increase in educational levels in the labor force may have actually been faster than the educational requirements of jobs, leading to tendencies toward overqualification among those employed (Borghans and de Grip 2000).

growth and distribution in a classical model where institutions, bargaining, and social norms are equally important.

The rest of this paper proceeds as follows. Section 2 discusses the theoretical foundations of our classical model of education, growth and distribution. Section 3 describes the formal structure of the model. Section 4 examines the dynamics of the economy with given bargaining conditions. Section 5 generalizes the analysis of the growth and distributive implications of education to economies in which the workers' bargaining power is endogenously determined. Section 6 examines how our model may explain the stylized facts. Section 7 concludes.

2 Education in a classical framework

This section discusses how this paper incorporates education in a classical framework, by considering three key questions: how do we characterize a classical economy? What exactly is the role of education in the economy? What determines changes in the level of education? We examine each question in order to justify our modelling strategy.

2.1 Characteristics of the economy

We characterize the classical economy as having five key features. First, there is limited substitution between capital and labor, the two basic factors of production, so that we can simplify by assuming fixed coefficients for labor and capital in production.[9] In addition to capturing some key tenets of the classical tradition, this assumption is in line with recent empirical evidence casting doubts on the substitutability between labor and capital, and on marginal productivity theory as an explanation of the distribution of income between capital and labor (Foley and Michl 1999; Basu 2009). Second, there are two *basic* classes, capitalists and workers, with capitalists owning capital and receiving income only from

[9] For many classical models this is not a crucial assumption because distribution is assumed to be exogenously fixed (see below) and so profit-maximizing behavior generally implies a particular chosen technique even with capital-labor substitution. When one allows changes in distribution, however, inelastic factor substitution plays a more important role.

profits, and workers receiving income as wage labor.

Third, capitalists save and workers do not, on account of their relatively lower income. Thus, workers receive income only from wages. For simplicity we assume that capitalists save a constant fraction of their income. We abstract from saving by workers (the key assumption is that they save at a lower rate than capitalists) to abstract from the dynamics of worker-owned capital (Foley and Michl 1999). This assumption is also consistent with recent empirical evidence showing that the propensity to save of different income groups is positively correlated with income (Carvalho and Rezai 2016) and the vast majority of wage laborers have zero net wealth (Piketty 2014). Fourth, the economy has unemployed workers, and the distribution of income is affected by the relative bargaining power between capitalist firms and workers.

Finally, all savings are invested. This implies that the economy has no aggregate demand problems, because there is investment demand from the entire income that is not consumed. While this is an appropriate assumption for Smith and Ricardo, who both assumed versions of Say's Law, it is not faithful to either Malthus, who emphasized the problems of over-saving and gluts, or Marx, who clearly recognized that capitalist producers can hoard rather than invest if they have low profit expectations, and this can result in what Marx called a realization problem and reduce output. However, Marx seems not to have stressed these problems in analyzing long-run growth. Without taking a view on such exegetical issues, we ignore effective demand problems and assume that all savings is automatically invested consistently with almost all growth models in the classical-Marxian tradition.[10] Even those in the classical-Marxian tradition, like Duménil and Lévy (1999), who allow aggregate demand to determine output in the short run assume that in the long run the economy produces at the potential level of output with saving equal to investment.

The assumptions on savings behavior and on effective demand imply that an increase in the share of income going to profits has a positive effect on the growth rate of the

[10] See, e.g., Harris (1978), Marglin (1984), Dutt (1990) and Foley and Michl (1999). Dutt (2011) provides a thorough discussion of the role of aggregate demand in the writings of the classical economists and modern classical-Marxian growth theory.

economy. Although disputes on whether advanced economies are indeed *profit-led* (rather than *wage-led*) are far from settled, there is fair amount of evidence suggesting that liberal market economies in which the earning inequalities and skill differentials have increased most dramatically, such as the US and the UK, seem to be profit-led, consistent with our model (Barbosa-Filho and Taylor 2006; Chiarella et al. 2006; Franke et al 2006).[11]

2.2 Role of education in the economy

Education plays a complex and multifaceted role in the economy, and we try to capture some important aspects of it. First, education has a direct role in determining the dynamics of the stock of human capital and knowledge by transforming low-skilled workers into high-skilled workers. The main difference between low- and high-skilled workers, in terms of their functions in the economy, is that the former are simply an input into the production of the final good, while the latter have a more complex role. High-skilled workers also serve as an input in the production of the final good, but as a distinct factor of production from low-skilled workers. In addition, and more importantly, high-skilled workers have a number of other functions: they increase the efficiency of both low- and high-skilled workers through the process of innovation, and they also help in the process of education, as mentors or educators. While low-skilled workers tend to be employed in routine production activities, high-skilled workers are innovators. Unger (2007, pp.96-97), who distinguishes such roles in terms of ideas about the mind, expresses it as follows:

> We know how to repeat some of our activities, and we do not know how to repeat others. As soon as we learn how to repeat an activity we can express our insight in a formula and embody the formula in a machine ... The not yet repeatable part of our activities – the part for which we lack formulas and therefore also machines – is the realm of innovation, the front line of production. In this realm, production and discovery become much the same thing.

[11] There is also evidence that economies have become more profit-led during the last three decades of neoliberal policies (Onaram et al 2011; Carvalho and Rezai 2016).

We therefore assume that education converts workers who could only do repetitive tasks into discoverers and innovators, although they continue to be engaged in some routine production activities that, however, are qualitatively different from those of low-skilled workers.

This formulation of the role of education can be found in the writings of classical economists, including Smith (1776, p.282) and McCulloch (1825, p.122). McCulloch, in fact, emphasized - much more than his contemporaries - the role of education and the diffusion of knowledge in increasing growth through technological change (see O'Brien 1975, p.217).[12] It is now also a fairly standard one in neoclassical growth theory, which stresses that education and the accumulation of human capital increases productivity growth. However, there are differences between the standard neoclassical approach and ours.

The standard view in neoclassical growth models with education (see, e.g., Uzawa 1965; Lucas 1988) is that there is no essential difference between workers who are educated and those who are not. Raw (or unskilled) labor accumulates human capital through education and becomes skilled labor, with a higher productive power. One worker becomes more than a worker in efficiency units, and therefore receives a higher wage. In this approach, workers are qualitatively the same, and can be shifted between the educational system and actual goods production. This is in contrast to our approach, in which education converts workers into high-skilled workers who are qualitatively different.

Our approach is closer to the neoclassical trade-theoretic literature, which takes low- and high-skilled labor to be qualitatively different and distinct inputs. Models in this vein are used to examine, for example, the implications of trade liberalization for the relative wages of skilled and unskilled workers and the actual increase in wage differentials in developed countries such as the US (see, e.g., Stokey 1991; Wood 1994).[13]

In our model, however, in addition to being qualitatively different *inputs* used in producing the final good, high- and low-skilled workers also have qualitatively different *roles* in the economy. In this respect our approach shares some characteristics with some other models

[12] See O'Donnell (1985) for a review of classical ideas on education, and Dutt and Veneziani (2011) for a more systematic discussion of classical ideas on the role of education in growth and distribution.

[13] For an earlier trade-theoretic model analysing the dynamics of learning, inequality and growth, without explicit consideration of human capital, in the classical tradition, see Dutt [18, 21].

with different types of labor. Galor and Zeira (1993), for example, consider two types of technologies, one which uses capital and skilled workers and the other which uses no capital and unskilled labor; thus skills enable workers to work with capital while unskilled workers cannot do so. In other models the roles are different because high-skilled workers produce differentiated intermediate goods, whereas low-skilled workers only "assemble" the intermediate goods to produce the final good (see Dutt 2005). We combine the two approaches – allowing both low- and high-skilled workers to be inputs into the production of a single good – but also allowing only high-skilled workers to have a role in inducing productivity growth and in the process of education as family members, teachers and mentors.

The economic relevance of education, however, does not reduce to its influence on the dynamics of human capital and technological progress. Authors working in the classical and Marxian tradition have long emphasized the multidimensional role of education in influencing the growth and distributive outcomes of capitalist economies. This literature incorporates broad issues such as the role of education in determining social outcomes as the product of conflict and/or cooperation. On the one hand, Marxist and radical scholars have emphasized the role of education in weakening the position of workers by dividing them into groups based on their level of education, in creating and strengthening the perception of upward socio-economic mobility and thereby increasing tolerance for income inequality, indoctrination and socialization, and easing the process of the extraction of effort (and hence labor productivity and profits) (see, among the many others, Giddens 1973; Bowles and Gintis 1975, 1976).

On the other hand, some classical economists viewed education in a more positive light. Education, McCulloch (1825, p.134) believed, would show workers "how closely their interests are identified with those of their employers, and with the preservation of tranquility and good order". Smith (1776, p.782) argued that education could play a key role in creating a more informed and discerning polity by contrasting the tendency of the increasing division of labor to make workers "as stupid and ignorant as it is possible for a human creature to become". Even Marx (1867, p.453) saw education as "the *only* method of producing *fully developed human beings*", although he was thinking not of education in the form actually existing in his time but of an ideal system, arguably in a classless society.

In section 5 below we explore some of these broader issues by endogenizing the bargaining power of workers and analyzing the implications of different education systems on workers' attitudes and therefore on growth and distribution.

2.3 Dynamics of education

In our approach the rate of change in the number of people educated, or the rate of education for short, depends positively on the stock of high-skilled workers (to capture the influence of more educators and mentors); on the wage of high-skilled workers relative to that of low-skilled workers; and on the access to education which captures factors such as the degree of openness of the education system and the availability of educational loans.

By assuming that the wage differential affects the rate of education, our model is consistent with standard approaches emphasizing the relevance of individual preferences (reflected, for instance, in their rate of time preference) and the returns to schooling in the accumulation of human capital. However, it stresses other factors, such as the degree of access to education, and the wage differential may reflect increases in the opportunity to obtain education because of subsidies provided by businesses that react to the relative cost of educated workers. Our approach is therefore less specific than the standard one, but we consider this lack of specificity to be a virtue because it opens up space for other, oft-neglected determinants of the spread of education.

3 Structure of the model

In this section we set out the basic classical model with two kinds of labor – high- and low-skilled, the quantities employed of which are H_P and L, and which receive real wages w_H and w_L. We use the symbol H_P to distinguish between high-skilled labor employed in production and research and development from those employed as educators, denoted by H_E. The ratio of skilled to unskilled wage, the *skill premium*, is

$$\sigma = w_H/w_L. \tag{1}$$

With capital, we therefore have three inputs in the economy, which correspond to three classes in society: capitalists who own physical capital, high-skilled workers each with one unit of high-skilled labor and low-skilled workers each with one unit of low-skilled labor. We examine a closed economy in which the government taxes to finance education but has no other fiscal functions.

Technology is as follows. There is only one sector producing one good which can be used both for consumption and for (capital) investment. Production uses fixed coefficients input-output relations with capital and a mixture of high- and low-skilled labor as inputs into production. The productivity of high- and low-skilled labor is given at a point in time by A_H and A_L, respectively, and the maximum output that can be produced by a unit of capital is k. To be specific, the production function of the standard firm is:

$$Y = \min[kK, f(A_L L, A_H H_P)], \tag{2}$$

where Y is the output of the good, K is the amount of capital, and f is homogeneous of degree one, which is consistent with the fixed coefficients structure. This function is in line with standard classical assumptions in rejecting the substitutability between labor and capital, but in principle it allows for substitutability between the two types of labor. In the rest of this paper, for the sake of analytical convenience, and without significant loss of generality, we assume a CES specification for the f function.

Assumption 1 (A1). $Y = min\{kK, [(A_L L)^\rho + (A_H H_P)^\rho]^{\frac{1}{\rho}}\}$, with $\rho < 0$.

The CES structure is very general and is widely used in the literature on skill differentials and human capital (see Autor et al. 1998; Acemoglu 2002a, 2002b). We assume limited substitutability between the two types of labor. Although the received view is that $\rho > 0$, recent empirical studies raise doubts on this finding and suggest that elasticity of substitution is below one (Card et al. 1999; Skott and Slonimczyk 2010), a view which we believe to be more plausible. Further, a negative value of ρ makes the analytical results starker and avoids some unnecessary technicalities. However *all* of the main conclusions of the paper continue to hold if $\rho < 0.5$, which encompasses the standard estimates of the elasticity of substitution.

Next, we assume that high-skilled workers are more productive at all t, and that their productivity advantage remains constant over time. Formally:

Assumption 2 (A2). There is a scalar $\mu \geq 1$, such that $A_H = \mu A_L$, all t.

This assumption encompasses the special case with $\mu = 1$, at all t, and allows one to analyse the effect of increases in productivity differentials on growth, distribution, and the relative composition of the labor force. The assumption that μ is constant should be taken as an approximation and a first step to a more complete analysis. Yet, this seems reasonable (if not necessary) in a steady state, such that if any loss of generality occurs, this only has to do with the analysis of the transition path.

Given A1 and A2, the optimal demands for high-skilled and low-skilled labor by profit-maximizing, perfectly competitive firms (we consider one representative firm, with all firms being identical) are as follows.

$$H_P^D = \frac{kK}{\left[\left(\frac{w_H}{w_L}\right)^{-\frac{\rho}{\rho-1}} \mu^{\frac{\rho}{\rho-1}} + 1\right]^{\frac{1}{\rho}} A_L \mu} = \frac{b(\sigma)K}{A_L}, \tag{3}$$

where $b(\sigma) = k\mu^{-1}\left[\sigma^{\frac{\rho}{1-\rho}} M + 1\right]^{-\frac{1}{\rho}}$ and $M = \mu^{\frac{\rho}{\rho-1}}$. Similarly,

$$L^D = \frac{kK}{\left[1 + \left(\frac{w_H}{w_L}\right)^{\frac{\rho}{\rho-1}} \mu^{-\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}} A_L} = \frac{c(\sigma)K}{A_L}, \tag{4}$$

where $c(\sigma) = k\left[\sigma^{-\frac{\rho}{1-\rho}} M^{-1} + 1\right]^{-\frac{1}{\rho}}$. Given A1, it follows that $b' < 0$ and $c' > 0$, where a prime symbol indicates a derivative. Further, as σ tends to zero, $b(\sigma)$ tends to infinity and as σ tends to infinity, $b(\sigma)$ tends to k/μ, and the function b is inelastic for all σ. The function $b(\sigma)$ is shown in Figure 1, where $\underline{b} = k/\mu$.

The government is assumed to employ a fraction, ε, $\varepsilon \in [0, 1)$, of the total supply of high-skilled workers, H^S, which is given at a point in time, and pay them the wage for high-skilled workers, w_H. The rest of the high-skilled workers are available for employment in the private market for high-skilled workers.

The markets for the two kinds of workers are as follows. Low-skilled workers are in unlimited supply, and along standard classical-Marxian lines we assume that the real wage of these workers is determined by the relative bargaining power of low-skilled workers and firms (see, e.g., Goodwin 1967; Marglin 1984; Dutt 1990; Foley and Michl 1999; Duménil and Lévy 2003). We parameterize this state in terms of the real wage of low-skilled workers relative to their efficiency factor, A_L, so that given bargaining conditions, an increase in A_L results in a proportionate increase in w_L. The market for high-skilled workers is flexprice, and the skill premium adjusts in response to the excess demand for high-skilled workers, given the supply of these workers at a point in time, denoted as $(1-\varepsilon)H^s$, and given w_L. The low-skilled worker wage serves as a reference point, and given the skill premium, a high w_L increases w_H proportionately. Formally:

Assumption 3 (A3). There exists a given positive scalar, λ, such that $w_L = \lambda A_L$. Further, given H^s, at any t, σ solves $(1-\varepsilon)H^s = b(\sigma)K/A_L$.

Given the assumptions on the labor market, in what follows we use the symbols H and L, to denote the quantities of high- and low-skilled workers employed in production or education. The level of λ is determined by the relative bargaining power of low-skilled workers, and a key determinant of income distribution.

We formalize the relationship between the use of high-skilled labor and labor productivity growth by assuming that the growth rate of labor productivity of high-skilled workers depends positively on the amount of high-skilled labor in efficiency units as a ratio of the stock of capital (used as a scaling factor representing the size of the productive economy).[14] Without significant loss of generality, we adopt a simple linear functional form, and denoting rates of growth by overhats, we assume that:[15]

[14] We are assuming that high-skilled workers and educators both contribute to technological change, a plausible assumption. If educators contribute to technological change only indirectly by educating low-skilled workers, we can make technological change depend on $(1-\varepsilon)H$, which will not change our results as long as ε is constant.

[15] It is important to stress that *all* of our results continue to hold under more general functional forms and

Assumption 4 (A4). There exist positive scalars τ_0 and τ_1 such that

$$\widehat{A}_H = \tau = \tau_0 + \tau_1 \frac{(A_H H)}{K}. \tag{5}$$

Since all firms are identical, A_H can be thought of the average productivity of high-skilled workers. Thus, although there may be externalities involved, they are not required.[16]

Because A_H and A_L are in general different, in principle equation (5) would not be sufficient to describe the behavior of labor productivity over time. By A2, however, it follows that $\hat{A}_L = \hat{A}_H$, all t, and we can write

$$\hat{A}_L = \tau_0 + \tau_1 \mu \frac{(A_L H)}{K}. \tag{5a}$$

In other words, we conceptualize innovations as non-rival products of learning-by-doing processes and innovation activity by high-skilled workers with an immediate spillover to low-skilled workers, or as high-skilled workers developing new methods of production which increase low-skilled worker productivity.[17]

Low-skilled labor is converted into high-skilled labor through the process of education as specified in the following Assumption.

Assumption 5 (A5). The supply of high-skilled labor H changes over time according to

$$\frac{dH}{dt} = \theta g(\sigma) \varepsilon H, \tag{6}$$

assuming weaker effects of knowledge on productivity.

[16] Consistent with Kaldor's (1961) famous observation, we are assuming that technical progress increases the productivity of labor inputs but not of capital inputs.

[17] Similar specifications are adopted in the standard literature on education, innovation and economic growth where technical change depends on the time spent in education (see, for example, the classic papers by Uzawa 1965 and Lucas 1988). The main difference is that we assume that *both* educators *and* high-skilled workers contribute to technical change. Our analysis is also similar to that of Aghion and Howitt (1998) who assume that the (expected) rate of arrival of innovations depends on the level of employment in the research and development sector. Their model, however, does not take education into account.

where θ is a scalar, $g : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is convex and continuously differentiable, and there exists a value $\sigma_{min} \geq 1$ such that $g(\sigma) = 0$ for all $\sigma \leq \sigma_{min}$ and g is strictly increasing for all $\sigma > \sigma_{min}$.

Thus, the change in the stock of high-skilled workers depends, first, on the demand for education which, in turn, depends positively on the skill premium, which increases the 'return' to education. Second, it depends on the stock of educators. Non-educator high-skilled workers can also positively affect the change in the stock of high-skilled workers by increasing the support for, and access to, education (for instance, a higher stock implies a higher number added from high-skilled worker families), but we do not explicitly model this; it does not affect our results qualitatively. Third, it depends on a parameter, θ, which captures the openness of the education system, either through government policy or through the degree of exclusivity of the education system and also, indirectly, the functioning of credit markets, in their role of financing education. Easier access to low-cost public education and greater access to student loans and grants, and a more open private education system which is less elitist on the basis of class and income would all increase θ. Observe that we allow the parameter θ to be negative or zero: this would correspond to a backward economy, or to a dysfunctional education system in which knowledge and skills are not transmitted, and the stock of human capital is stationary or even decreasing.

To be sure, many different factors determine the influence of the education system (and more generally the transmission of knowledge in a society) on the creation of skills. We regard the parameter θ as a parsimonious way to model such influences, and thus potentially the role of public policy in the creation of skills. It can be seen as a black box, a convenient way of modeling the multifaceted influence of education on the dynamics of human capital.

We do not want to unnecessarily restrict our analysis and, apart from some mild regularity conditions, do not specify an explicit functional form for g. The only theoretical restriction concerns the definition of σ_{min}: A5 incorporates the intuition that no one seeks education if the wage premium falls below a certain level. This seems rather reasonable at a theoretical and empirical level.[18]

[18] Interestingly, this assumption can also be related to Smith's (1776, pp.118-19) explanation of the "dif-

We make the following assumption about consumption and saving behavior.[19]

Assumption 6 (A6). Workers – both high- and low-skilled – do not save, but consume their entire income; capitalists save a fixed fraction, s, of their profits.

The pre-tax income of profit recipients, or capitalists, is given by

$$Y_C = Y - w_L L - w_H(1-\varepsilon)H.$$

We assume that the government finances its educational expenditure, devoted entirely to the payment of wages of educators (abstracting from non-wage costs for simplicity), by taxing profits, keeping a balanced budget. Thus, the *net* income of capitalists is

$$rK = Y_C - w_H \varepsilon H = Y - w_L L - w_H H, \tag{7}$$

where r is the profit rate net of taxes. Hence, total consumption expenditure is

$$C = (1-s)rK + w_L L + w_H H. \tag{8}$$

This implies that saving is given by the standard equation,

$$S = srK. \tag{9}$$

Finally, regarding investment, we have the following.

Assumption 7 (A7). Saving and investment, I, are identically equal.

Capitalists save in order to invest, so that saving and investment are always equal. This version of Say's law is a standard assumption of models in the classical tradition, as discussed earlier. Equation (9) and A7 imply

$$I = srK. \tag{10}$$

ference between the wages of skilled labour and those of common labour" and to Mill's (1848, pp.386-7) analysis of the relation between skills and wages.

[19]We abstract from consumer debt, since it is not central in a classical analysis of the long-run dynamics of education and growth. For a recent, detailed analysis of household debt, see Mason and Jayadev [50].

Further, there is no effective demand problem, so that, given the existence of unemployed low-skilled workers, we have

$$Y = kK. \tag{11}$$

This macroeconomic condition justifies the microeconomic profit-maximizing decision made by each firm to produce at full capacity, as noted earlier.

We examine the determinants of growth and distribution in the economy in terms of a number of different variables. The rate of overall economic growth can be conveniently measured by $\hat{K}$, which is equal to $\hat{Y}$. Since we do not track the overall supply of labor (which may be taken to be growing at an exogenous rate), $\hat{K}$ also proxies the growth rate of *per capita* output.

Distribution can be measured in terms of the income shares of the three classes: $\lambda c(\sigma)/k$ for low-skill workers, $\sigma\lambda b(\sigma)/k(1-\varepsilon)$ for high-skilled workers and r/k for capitalists. The distribution of income between different workers can be measured by $\nu = w_H H/w_L L$ which, using equations (2)-(4), can be written as

$$\nu = \frac{\sigma^{-\frac{\rho}{1-\rho}} \mu^{\frac{\rho}{1-\rho}}}{1-\varepsilon}$$

which is increasing in σ. Thus intra-workers inequalities can be equivalently examined in terms of ν and σ. In examining the well-being of high- and low-skilled workers we will also focus on the growth rates of employment, that is, $\hat{H}$ and $\hat{L}$, as well as the growth rates of wages, $\hat{w}_H$ and $\hat{w}_L$.

Another variable of interest is the skill composition of employed workers (a proxy of the skill composition of the labor force), which we capture with the variable H/L. Given A3, $H/L = b(\sigma)/c(\sigma)(1-\varepsilon)$, so that in the long run the skill composition of employed labor is strictly decreasing in σ.

4 Education, growth and distribution with constant bargaining conditions

We examine the dynamics of the economy by considering two runs, for now assuming that λ, the distributional parameter, is exogenously given. In the short run, K, H and A_L are fixed, and the model solves for Y, L, σ, r and I from equations (3), (4), (7), (10), and (11). The profit rate is given by

$$r = k - \frac{w_L c(\sigma)}{A_L} - \sigma \frac{w_L b(\sigma)}{A_L(1-\varepsilon)} \tag{12}$$

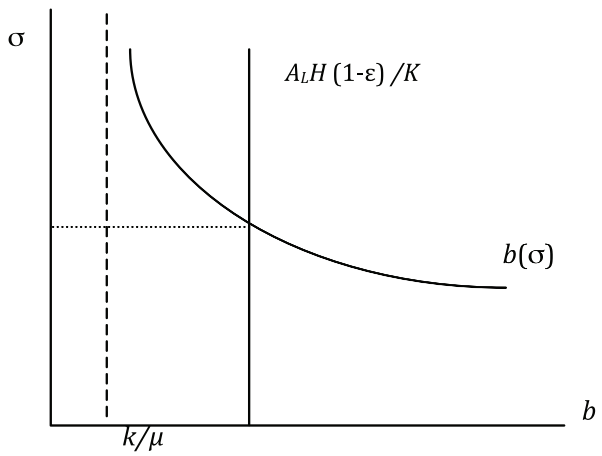

Figure 1: Determination of the skill premium in the short run.

Let $h = A_L H/K$. The short-run equilibrium value of σ is found as shown in Figure 1, and is given by

$$\sigma = \sigma(h) \equiv b^{-1}(A_L H/K) = \mu \left[\left(\frac{k}{\mu h(1-\varepsilon)} \right)^{\rho} - 1 \right]^{\frac{1-\rho}{\rho}}. \tag{13}$$

By A1 $\sigma(h)$ is strictly decreasing and strictly convex for all $h \in (k/\mu(1-\varepsilon), \infty)$.

In the long run K, H and A_L can change. Assuming away the depreciation of capital without loss of generality, the change in capital stock is given by

$$dK/dt = I, \tag{14}$$

and changes in H and A_L are governed by equations (6) and (5a).

We examine the time path of the economy in the long run by focusing on the state variable $h = A_L H/K$. Because the growth rate of h is given by

$$\hat{h} = \hat{A}_L + \hat{H} - \hat{K}, \tag{15}$$

we can substitute from equations (5a), (6), and (11) through (14), to obtain

$$\hat{h} = \tau_0 + \tau_1 \mu h + \theta \varepsilon g(\sigma(h)) - s\left[k - \lambda c(\sigma(h)) - \sigma(h)\lambda h\right]. \tag{16}$$

The economy is defined as being in long-run equilibrium when $\hat{h} = 0$. Proposition 1 proves the existence of multiple long-run equilibria.

Proposition 1 *Assume A1-A7. Suppose that*

$$\tau_0 + \frac{\tau_1 k}{1-\varepsilon}\left[M+1\right]^{-\frac{1}{\rho}} < sk\left[1 - \lambda\mu^{-1}\left[M+1\right]^{-\frac{1}{\rho}}\left[M + \frac{1}{1-\varepsilon}\right]\right]. \tag{KTB}$$

Then there are two long-run equilibria: E_1 with $\sigma_1 > 1$ and E_2 with $\sigma_2 < 1$. E_1 is dynamically stable, whereas E_2 is dynamically unstable.

Proof. By equation (16), it follows that

$$\frac{d\hat{h}}{dh} = \tau_1\mu + \theta g'(\sigma)\sigma'(h) + s\lambda\sigma(h)\left[\varepsilon - (1-\rho)\frac{\left(\frac{k}{\mu(1-\varepsilon)h}\right)^{\rho}}{\left(\frac{k}{\mu(1-\varepsilon)h}\right)^{\rho} - 1}\right].$$

Therefore, by A1 as $h \to k/\mu(1-\varepsilon)$, and σ tends to infinity, then $\hat{h} \to \infty$ and $\frac{d\hat{h}}{dh} \to -\infty$, whereas as $h \to \infty$, and σ tends to zero, then $\hat{h} \to \infty$ and $\frac{d\hat{h}}{dh} \to \tau_1\mu$. If the condition in the antecedent of the proposition is true, this implies that at $h = b(1)\left(1-\varepsilon\right)^{-1} = k\mu^{-1}\left[M+1\right]^{-\frac{1}{\rho}}\left(1-\varepsilon\right)^{-1}$, $\hat{h} < 0$, and thus by continuity there are at least two points $h_1 = b(\sigma_1)(1-\varepsilon)^{-1}$ and $h_2 = b(\sigma_2)(1-\varepsilon)^{-1}$, where $h_2 > h_1$, $\sigma_1 > 1$ and $\sigma_2 < 1$, such that $\hat{h} = 0$. Furthermore, by A1 and A5, it follows that $\hat{h}$ is strictly convex and so there exist exactly two such points. Stability follows in the usual manner noting that $\hat{h} > 0$ for all $h < h_1$ and $h > h_2$, whereas $\hat{h} < 0$ for all $h_1 < h < h_2$. ■

Although Proposition 1 identifies two equilibria, only one of them, E_1, is economically meaningful because it is dynamically stable and has a skill premium greater than one. Therefore in what follows we focus on E_1.[20]

Condition (KTB) is sufficient (but not necessary) for the existence of a long-run equilibrium and it depends on the conditions of capital accumulation (K), technical change (T) and bargaining (B) in the economy. It is more likely to hold the lower τ_0, τ_1, λ, and the higher s, k. If technical progress is too strong, or if profits and capital accumulation are too slow, then the dynamics of innovation may dominate and lead the economy onto an explosive path.[21]

Proposition 2 is the main result of this section. It analyzes the effect of education policies on growth, employment, and distribution.[22]

Proposition 2 ***(Education, growth and distribution)***. *Assume A1-A7. Suppose that (KTB) holds and $\sigma_1 > \sigma_{min}$. At E_1, an increase either in θ or in ε implies that in equilibrium the income share of low-skilled workers increases relative to that of high-skilled*

[20] We note in passing that at E_1, Kaldor's (1961) celebrated stylized facts are verified: constant factor shares, increasing labor productivity, and a positive rate of capital accumulation.

[21] Along this explosive path eventually H no longer increases, but h keeps increasing as technological change occurs faster than capital accumulation. This kind of self-reinforcing dynamics of technological innovation with a constant stock of education, however, is unlikely to occur in practice, and the τ function is likely to flatten out, so that (provided τ_0 is not too high) a stable equilibrium is attained.

[22] Proposition 2 holds under the assumption that $\sigma_1 > \sigma_{\min}$. If $\sigma_{\min} \geq \sigma_1 > 1$, then A5 implies that the change in θ has no effects on the economy, which is caught in a low-skill trap. Intuitively, if there are no people who wish to get educated because the skill premium is too low, increasing access to education has no effect. Interestingly, however, this does not mean that there are no effective government policies on education and skill creation: Proposition 2 implies that if the government expands the public sector by increasing the fraction of high-skilled workers that it employs, ε, this has a beneficial effect on growth and reduces intra-workers inequalities. Yet, this has no effect on the growth rate of human capital, which remains stationary. The existence of a low-skill trap is consistent with results in the standard growth literature, such as Galor and Tsiddon (1997).

workers, but so does the profit share in national income. Furthermore, the skill composition of employed labor, the growth rate of the economy, and the growth rate of technical progress all increase. Finally, there exists a $\sigma^* > 0$ such that if $\sigma_1 > \sigma^*$ then a sufficiently small increase either in θ or in ε yields an increase in the growth rate of human capital and employment.

Proof. 1. Consider θ first. Because $\sigma_1 > \sigma_{min}$, by equation (16) it follows that an increase in θ yields an upward shift of the $\hat{h}$ schedule. Since (KTB) holds, then there exists a long run equilibrium at ${\sigma'}_1 < \sigma_1$. The first part of the proposition then follows noting that $h, H/L, \tau, \widehat{K}$, and r are decreasing functions of σ.
To prove the second part note, first, that at a long-run equilibrium $\hat{H} = \hat{K} - \hat{A}_L$. Therefore for small changes around the equilibrium

$$\frac{d\hat{H}}{dh} = s\lambda\sigma(h)\left[(1-\rho)\frac{k^\rho}{k^\rho - (\mu(1-\varepsilon)h)^\rho} - \varepsilon\right] - \tau_1\mu.$$

Hence, by A1 as $h \to \frac{k}{\mu(1-\varepsilon)}$, we have $\frac{d\hat{H}}{dh} \to \infty$ and as $h \to \infty$ $\frac{d\hat{H}}{dh} \to -\tau_1\mu$, and provided ε is not too large, $\frac{d^2\hat{H}}{dh^2} < 0$. Therefore there is a unique cut-off value h^* such that for all $h < h^*$, $\frac{d\hat{H}}{dh} > 0$ and for all $h > h^*$, $\frac{d\hat{H}}{dh} < 0$, which yields the desired result.
2. Consider next ε. The desired argument follows in a similar fashion noting that, for any given h, an increase in ε yields an increase in $\sigma(h)$ and $c(\sigma(h))$, which implies in turn an upward shift in the $\hat{h}$ schedule for all h. ■

The dynamics of the economy can be examined graphically. Figure 2 shows the growth rates of the main variables, as functions of h, under condition (KTB). The first two terms on the right hand side of equation (16) show the growth rate of labor productivity, A_L, which increases with h. The third term represents the growth rate of the number of skilled workers, H: as h increases, this rate falls as the skill premium falls. The last term represents the growth rate of the capital stock, K. An increase in h implies that the skill premium falls, so that the rate of profit rises because total labor costs decrease. We add up the rates of growth of A_L and H to obtain the $\hat{A}_L + \hat{H}$ curve, which first decreases and then eventually increases with h. The long-run equilibria are determined at $h_1 = b(\sigma_1)(1-\varepsilon)^{-1}$ and $h_2 = b(\sigma_2)(1-\varepsilon)^{-1}$, where $k/\mu(1-\varepsilon) < h_1 < h_{min}$, and $h_{min} = b(\sigma_{min})(1-\varepsilon)^{-1}$.

The long-run equilibrium h_1 is stable: starting from $h < h_1$, for instance, $\hat{A}_L + \hat{H} > \widehat{K}$, so that h increases till it reaches h_1. The long-run equilibrium h_2, instead, is unstable: if $h > h_2$ initially, h increases indefinitely, with physical and human capital growing very slowly, or not at all, and the growth rate of technical change dominating the dynamics.

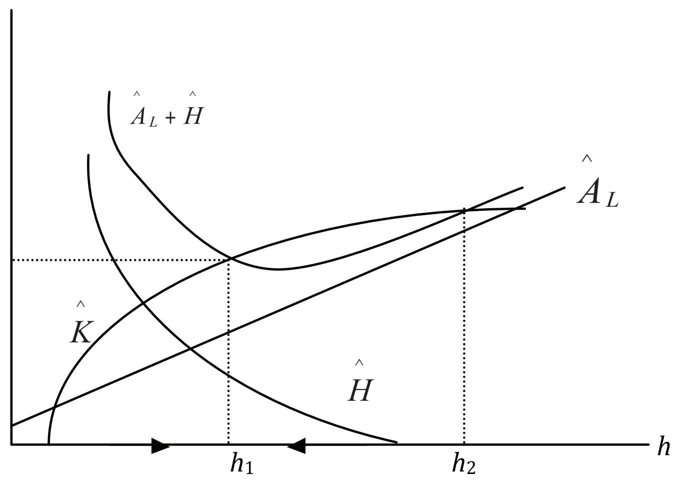

Figure 2: The long-run dynamics and equilibrium.

Focusing on the stable long-run equilibrium E_1, we can consider the effects of an increase in the "openness" of the education system. An increase in θ shifts the $\hat{H}$ and $\hat{A}_L + \hat{H}$ curves upwards, making them steeper, at all values h such that $k/\mu(1-\varepsilon) < h < h_{min}$. The long-run equilibrium growth rates of capital accumulation and labor productivity increase, as can be verified from the figure.

Consider next the effects on distribution. The low-skilled income share, $\lambda c(\sigma)/k$, decreases because the number of low-skilled workers employed is a positive function of σ, given the bargaining parameter and the productivity of capital. The high-skilled income share is given by $\sigma(h)\lambda h/k(1-\varepsilon)$. When θ increases, the long-run equilibrium level of h

increases, but the rise in h is proportionately less than the fall in σ, given the inelasticity of the $b(.)$ function, so that $\sigma(h)h$ falls. Indeed, the overall wage share decreases and both the rate of profit and the profit share, r/k, rise. Thus income is redistributed from workers to capitalists. This, of course, is why the rate of capital accumulation in the economy is speeded up, as we saw earlier. However, intra-workers inequalities decline as high-skilled workers lose more in relative terms than the low-skilled as their relative share falls.

The effects on workers' welfare are also interesting. The growth rate of the real wage of both types of workers rises with a higher rate of technological change. Further, note that the growth rate of low-skilled employment is $\hat{L} = \hat{c}(\sigma) + \hat{K} - \hat{A}_L$. Since in long-run equilibrium $\hat{c}(\sigma) = 0$, and $\hat{H}= \hat{K} - \hat{A}_L$, then $\hat{H} = \hat{L}$, and so the growth rates of the two types of labor coincide in long-run equilibrium. For low levels of h_1 a small increase in θ causes a rise in the growth rate of human capital and employment. So the positive direct effect on the education system dominates the negative indirect effect via the decrease in the long run wage differential. For high levels of h_1, the opposite is true. Thus, for low h_1 the condition of low-skilled workers, in terms of their real wage level and growth rate and employment growth, is unequivocally improved.[23] High-skilled workers, instead, lose out in the sense that the equilibrium wage differential decreases. But they do gain from the increase in the steady state growth rates of the high-skilled real wage and employment.

Although our main focus is on education, it is interesting to analyze the effect of an increase in the productivity differential between high- and low-skilled labor, μ, which captures the effect of skill-biased technical change in our model.[24] Such a change decreases the demand for both types of labor at a given h, due to the limited substitutability between them, thus increasing the profit rate (since firms maximize profits), shifting the $\widehat{K}$ curve

[23] Another potential benefit for the low skilled derives directly from increased access to education leading to a change of status, and of wage, even though this effect is difficult to quantify.

[24] We note in passing that a sufficiently small improvement in the conditions of technical change, (τ_0, τ_1), has the same qualitative effects as an improvement in the openness of the educational system except that the growth rate of human capital and employment definitely declines. Instead an increase in the capitalists' savings rate, s, raises the skill premium leading to an increase in intra-workers inequality and a decrease in the profit share. Therefore, interestingly, it has an ambiguous effect on the growth rate of the economy and a negative effect on the growth rate of technical change.

upwards. The decrease in the demand for high-skilled labor lowers σ at a given h, decreasing $\hat{H}$ and shifting its curve down. However, the increase in μ increases $\hat{A}_L$ for a given h shifting its curve upwards. Although the rate of capital accumulation must increase, overall the effects of skill-biased technical change are, interestingly, unclear. If the effect on $\widehat{K}$ and $\hat{H}$ dominates, then skill-biased technical change leads to an increase in the skill premium and intra-worker inequality, and a decrease in the profit share. However, if technological change responds strongly to h, and the effect on $\hat{A}_L$ dominates, skill-biased technical change may lead to a *reduction* in the skill premium, and an increase in the profit share, contrary to the received wisdom.

Given our focus on growth and distribution, a natural question to ask at this point would concern the effect of a change in the workers' bargaining power. However, the analysis of a purely exogenous perturbation of the class struggle parameter, λ, is not very informative. Given the relevance of education, the existence of two separate groups of workers with potentially conflicting interests, and the endogenous determination of the skill composition of labor, it is important to analyze the interplay of education and labor market conditions in determining growth and distribution.

5 Education, growth and distribution with variable conditions of class struggle

We have so far assumed a constant low wage-productivity ratio, λ, given by bargaining conditions. This assumption may be questionable, given the strong evidence of short-term fluctuations in income shares driven by distributive conflict.[25] To examine the possible implications of changes in λ we assume that low-skilled workers have a target ratio between wages and productivity which they try to achieve by pushing up their real wage, but they are not fully able to increase their real wage at the same rate as productivity growth.

[25] The empirical literature on distributive cycles is too vast for a comprehensive list of references. We refer to recent work by Barbosa-Filho and Taylor (2006), Mohun and Veneziani (2008) and Fiorio et al (2013), and the contributions discussed therein.

Formally:[26]

Assumption 8 (A8). The real wage of low-skilled workers changes according to

$$\widehat{w}_L = \delta_1(\lambda^* - \lambda) + \delta_2\tau, \tag{17}$$

where $\delta_1 > 0$, $\lambda^* = \lambda^*(\theta)$, $\delta_2 = \delta_2(\theta)$, $1 > \delta_2 > 0$, and $\delta_2(.), \lambda^*(.)$ are continuous functions.

The value λ^* is the target ratio of low-skilled workers, which is taken to depend on the educational access parameter, and may be interpreted as reflecting normative considerations (e.g. criteria of 'just pay'). The value δ_2 reflects workers' bargaining strength and cohesion, and thus their ability to claw back the benefits of productivity increases. It may be objected that (17) concerns the dynamics of the *low-skilled* workers' wage in relation to a measure of *low-skilled* labor productivity, but it is not clear why a more open education system should affect the bargaining strength of workers who do *not* actually access the education system. We believe that the effect permeates into the entire working class, including those who do not actually make use of the education. For, the openness of higher education is related to changes in the quality of basic education which affects all workers. Further, the productivity-wage relation captured by λ also affects the base wage of high-skilled workers. Last but not least, education affects workers' solidarity and the ability of workers of both types to maintain their wages in line with productivity increases; and there are "externalities" from the education of high-skilled workers to low-skilled ones.

While we assume that educational access influences λ^* and δ_2, we do not specify exactly how it affects workers' attitudes in bargaining and conflict. Below, we consider different scenarios depending on the nature of education.

A8 states that the growth rate of the real wage depends positively on the extent to which the actual ratio λ falls short of the target, and on the growth of labor productivity, since workers demand and receive at least part of the fruits of higher labor productivity.[27]

[26] Equation (18) can be interpreted as the reduced form of a more complex dynamic model with bargaining and optimizing agents, along the lines, for example, of Tavani (2013).

[27] These dynamics could be made to depend on inflationary dynamics of price and money wage changes, but we abstract from these complications for simplicity. See, for instance, Dutt (1990) for a discussion.

Since, from the definition of λ we have

$$\widehat{\lambda} = \widehat{w}_L - \tau, \tag{18}$$

substituting equations (5a) and (17) into (18) we get

$$\widehat{\lambda} = \delta_1(\lambda^* - \lambda) - (1 - \delta_2)(\tau_0 + \tau_1 \mu h). \tag{19}$$

Thus, an increase in h, by increasing the rate of productivity growth, reduces the growth rate of the wage-productivity ratio because workers are unable to increase their real wage to capture the full gains from productivity growth. An increase in the wage-productivity ratio reduces its rate of increase because workers are closer to their target.

Equations (16) and (19) form a dynamic system involving the two state variables h and λ. Equation (19) implies that the $\widehat{\lambda} = 0$ isocline is:

$$\lambda = \lambda^* - \frac{1 - \delta_2}{\delta_1}(\tau_0 + \tau_1 \mu h). \tag{20}$$

Consider the $\widehat{h} = 0$ isocline. Consistent with the analysis in the previous section, we assume that there exists *some* positive value of λ such that (KTB) holds. Formally:

$$\tau_0 + \tau_1 k \left[M + 1\right]^{-\frac{1}{\rho}} (1 - \varepsilon)^{-1} < sk. \quad (KTB_0)$$

(KTB_0) ensures the existence of a stationary value of h with positive real wages and a skill premium greater than one if $\lambda = 0$. Then there exists a value of λ, denoted as $\overline{\lambda}$, that solves $\tau_0 + \frac{\tau_1 k}{1-\varepsilon} \left[M + 1\right]^{-\frac{1}{\rho}} = sk \left[1 - \bar{\lambda} \mu^{-1} \left[M + 1\right]^{-\frac{1}{\rho}} \left[M + \frac{1}{1-\varepsilon}\right]\right]$. By Proposition 1, we know that for all $\lambda \in [0, \overline{\lambda})$, there exist two values (h_1, h_2), with $h_1 < \overline{h} = b(1)(1 - \varepsilon)^{-1} = k\mu^{-1} \left[M + 1\right]^{-\frac{1}{\rho}} (1 - \varepsilon)^{-1} < h_2$, such that $\widehat{h} = 0$, whereas if $\lambda = \overline{\lambda}$, then $h = \overline{h} = b(1)(1 - \varepsilon)^{-1}$ implies $\widehat{h} = 0$. We also know that, for all $\lambda \in [0, \overline{\lambda})$, at h_1, $\frac{d\hat{h}}{dh} < 0$ whereas at h_2, $\frac{d\hat{h}}{dh} > 0$ and the $\widehat{h} = 0$ isocline is first increasing and then decreasing in h.[28] From equation (16) we see that an increase in λ reduces the rate of profit and the rate of accumulation by

[28] Indeed, the argument in Proposition 1 can be generalized to prove the existence of $\widetilde{\lambda} \geq \overline{\lambda}$ such that for all $\lambda \in [0, \widetilde{\lambda})$, there exist two equilibria (h_1, h_2) such that $h_1 < h_2$ and at h_1 $\frac{d\hat{h}}{dh} < 0$ whereas at h_2, $\frac{d\hat{h}}{dh} > 0$. It can also be proved that the downward sloping part of the $\widehat{h} = 0$ isocline is concave in λ.

increasing the payments to both kinds of workers, and hence increases $\widehat{h}$. At h_1, the effect of h on physical and human capital accumulation is stronger than the effect on technological change, and an increase in h reduces $\widehat{h}$, so that the $\widehat{h}=0$ isocline is positively sloped. At h_2, the opposite holds, and the $\widehat{h}=0$ isocline is negatively sloped.

Let $\lambda^{max}=\lambda^{*}-\frac{1-\delta_2}{\delta_1}\tau_0$ and $h^{max}=\frac{\delta_1}{\tau_1\mu(1-\delta_2)}\lambda^{*}-\frac{\tau_0}{\tau_1\mu}$ denote the intercepts of the $\widehat{\lambda}=0$ isocline. Let $h_1(0)$ and $h_2(0)$ denote the two values of h, with $0<h_1(0)<h_2(0)$, such that $\widehat{h}=0$ when $\lambda=0$: by Proposition 1 they exist, and $h_1(0)=b(\sigma_1)(1-\varepsilon)^{-1}$ with $\sigma_1>1$ and $h_2(0)=b(\sigma_2)(1-\varepsilon)^{-1}$, with $\sigma_2<1$. By A5, $h_2(0)=(sk-\tau_0)/\tau_1\mu$.

Proposition 3 provides sufficient conditions for the existence of economically meaningful long-run equilibria.

Proposition 3 ***(Long-run equilibria)***. *Assume A1-A8. Suppose that (KTB$_0$) holds and* $0<\lambda^{max}<\overline{\lambda}$.
(i) If $h^{max}\geq h_2(0)$, *then there exist two long-run equilibria:* E_1 *is stable with* $\sigma_1>1$ *and* E_2 *is unstable with* $\sigma_2<1$. *At* E_1 *the wage-productivity ratio is higher, and the stock of human capital is lower than at* E_2.
(ii) If $h_1(0)\leq h^{max}<h_2(0)$, *there exists a unique stable long-run equilibrium with* $\sigma_1>1$.

Proof. 1. Claim (i). Suppose that $0<\lambda^{max}<\overline{\lambda}$ and $h^{max}\geq h_2(0)$. The existence of the two equilibria follows noting that given the monotonicity of the two isoclines and the concavity of the downward sloping part of the $\widehat{h}=0$ isocline, the two curves intersect twice: once in the increasing part of the $\widehat{h}=0$ isocline and once in its decreasing part.
2. In order to investigate the stability properties of the two equilibria consider the Jacobian of the dynamic system

$$\frac{d\widehat{h}}{dh}=\tau_1\mu+\theta g'(\sigma)\sigma'(h)+s\lambda\sigma(h)\left[\varepsilon-(1-\rho)\frac{m(h)}{m(h)-1}\right] \quad \frac{d\widehat{h}}{d\lambda}=s\left[c\left(\sigma(h)\right)+\sigma(h)h\right]>0$$

$$\frac{d\widehat{\lambda}}{dh}=-(1-\delta_2)\tau_1\mu<0 \qquad \frac{d\widehat{\lambda}}{d\lambda}=-\delta_1<0$$

where $m(h)=\left(\frac{k}{\mu(1-\varepsilon)h}\right)^{\rho}$. Since the $\widehat{h}=0$ isocline is positively sloped at E_1 and negatively sloped at E_2, the upper left entry is negative in the former case and positive in the latter. Therefore in the neighborhood of E_1 the Jacobian has a negative trace and a positive determinant, satisfying the conditions for stability. In a neighborhood of E_2, instead, the determinant of the Jacobian is negative, so that the equilibrium is a saddle-point.

3. The claims concerning the equilibrium values of the main variables follow from step 1 and Proposition 1 above.

4. The proof of claim (*ii*) follows from obvious modifications of steps 1-3. ■

Remark: Under (KTB_0), there always exist combinations of the parameters such that the conditions $0 < \lambda^{max} < \overline{\lambda}$ and either $h_1(0) \leq h^{max} < h_2(0)$ or $h^{max} \geq h_2(0)$ can both hold.

Observe that both λ^{max} and h^{max} are monotonically increasing in λ^*, δ_1, and δ_2. Therefore, if conditions in the labor market are particularly conflictive, it may happen that the $\widehat{\lambda} = 0$ isocline lies entirely above the $\widehat{h} = 0$ isocline and no equilibrium exists. In this case, the economy settles on a path with ever-increasing h which takes the economy, eventually, to $\lambda = 0$. It is unlikely, however, that the workers' bargaining power goes to zero; and the parameters in equation (17) would likely change, shifting the $\widehat{\lambda} = 0$ isocline down, thereby producing a stable interior equilibrium.

Figure 3 shows the long-run dynamics of this model.[29] The long-run equilibria occur at the intersections of the $\widehat{h} = 0$ and $\widehat{\lambda} = 0$ curves. The economically meaningful equilibrium E_1 is (asymptotically) stable, and the economy may converge directly or cyclically to it, as can be seen from the arrows.[30] The equilibrium E_2 instead is unstable. The possibly cyclical movement around E_1 is quite interesting and consistent with classical models of growth with distributive conflict such as Goodwin's (1967) seminal contribution.

We may now analyze the effect of education in the economy. Consider first the $\widehat{h} = 0$ isocline. Proposition 2 implies that an increase in θ shifts (a portion of) the upward sloping part of the $\widehat{h} = 0$ curve to the right, as shown by the dotted line, because it increases $\widehat{h}$ at given values of h and λ, for all h such that $\sigma = \sigma(h) > \sigma_{min}$.

The effect of an increase in θ on the $\widehat{\lambda} = 0$ isocline depends on the nature of education and on its effect on the workers' bargaining position. If education facilitates the self-development

[29] Figure 3 has been drawn assuming the $\widehat{h} = 0$ isocline to be strictly concave. This is for the purposes of illustration and none of the results depend on it.

[30] If the $\widehat{\lambda} = 0$ locus intersects the $\hat{h} = 0$ curve twice in its negatively-sloped segment, cycles are more likely.

of individuals, making them more conscious of their rights and nature as social beings, and increases workers' solidarity, then the functions $\lambda^*(\theta)$ and $\delta_2(\theta)$ may be increasing in θ. In this case, the $\widehat{\lambda} = 0$ isocline tends to shift to the right and become flatter. The combined effect on the two curves can be summarized in the next proposition.

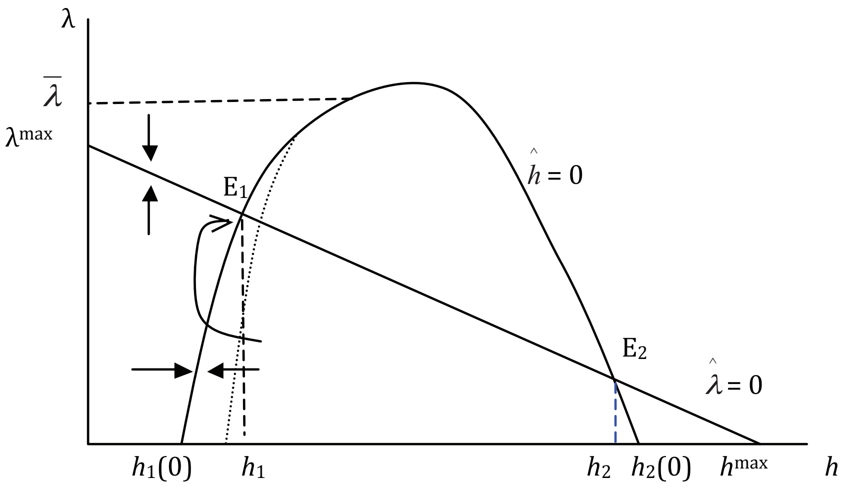

Figure 3: Long-run dynamics and equilibria of the model with variable λ.

Proposition 4 ***(Progressive role of education)***. *Assume A1-A8. Suppose that* (KTB_0) *holds. Suppose that* $0 < \lambda^{max} < \overline{\lambda}$, $h^{max} \geq h_1(0)$, *and* $\lambda^*(\theta), \delta_2(\theta)$ *are increasing in* θ. *Then at a long-run equilibrium with* $\sigma_1 > \sigma_{min}$, *a small increase in* θ *yields a decrease in intra-worker inequality, whereas the skill composition of the labor force and the growth rate of technological progress increase. Further, if education has a strong effect on workers' values, beliefs and expectations, their bargaining strength increases. Yet the profit share and the growth rates of the economy, human capital, and employment may decrease or increase.*

If instead education is important in inculcating an ideology of resignation and moderation, in increasing the tolerance for inequality and creating the perception of greater upward mobility than actually exists, or in undermining workers' solidarity emphasizing differences in educational attainment, then the functions $\lambda^*(\theta)$ and $\delta_2(\theta)$ may be decreasing in θ. In this case, the $\widehat{\lambda} = 0$ isocline will tend to shift to the left and to become steeper. The overall effect can be summarized in the following proposition.

Proposition 5 ***(Education as ideology)***. *Assume A1-A8. Suppose that (KTB_0) holds. Suppose that $0 < \lambda^{max} < \overline{\lambda}$, $h^{max} > h_1(0)$, and $\lambda^*(\theta)$ and $\delta_2(\theta)$ are decreasing in θ. Then at a long-run equilibrium with $\sigma_1 > \sigma_{min}$, a small increase in θ yields a decrease in workers' bargaining strength. Further, if education has a strong effect on workers' values, beliefs and expectations, the wage differential increases. The profit share and the growth rates of the economy, human capital, and employment may decrease or increase.*

Several observations are in order, concerning the effect of education in complex economies with bargaining. First, if education has a significant effect on workers' attitudes and beliefs, more proactive education policies have an impact on income distribution *within the working class*. If education is progressive, workers' bargaining power increases, wage differentials decrease (as an effect of increased workers' solidarity) and the steady state growth rates of high- and low-skilled workers' real wages increase, thanks to the higher growth rate of technological progress. The opposite holds if education deepens any cleavages within the working class.

Perhaps surprisingly, however, the effect on the distribution of income *between workers and capitalists* – and thus on growth – is not obvious, because the bargaining strength of the working class as a whole and wage differences within the working class may move in opposite directions. For example, if education is progressive, the skill differential and intra-workers inequalities tend to decrease, which tends to reduce the share of wages in national income. If education has a strong effect in influencing workers' attitudes, this may be compensated by an increase in workers' bargaining power, which increases the wages of *both* types of workers. Yet the overall effect on the profit rate and the profit share depends on the relative strength of the two effects.[31]

[31] If education has a small effect on workers' attitudes, then an increase in the openness of the system may lead to a *decrease* in λ which leads to an unambiguous increase in the profit rate and in the profit share.

6 Back to the stylized facts

The previous analysis shows that a classical approach produces some interesting and novel insights on the growth and distributional consequences of education. It may be objected, however, that the full model does not allow us to derive definite conclusions on the effects of education on growth and distribution, which would depend on the characteristics of the educational system and on a number of social and institutional factors. We see this apparent theoretical indeterminacy as an advantage of our model. Theoretically, it seems inappropriate to analyse the effects of "education" as if it was an objectively defined, neutral notion and abstracting completely from its interaction with the broader structure of the labor market and employment relations. Empirically, noting the substantial international differences concerning their educational systems, labor market institutions, and so on, our approach allows one to formulate testable claims that are appropriate for given countries. Our analysis, for example, suggests an empirically verifiable hypothesis to explain the stylized facts of the US economy since 1975.

To use our analysis to understand the trends in growth and income distribution we note that while in *quantitative* terms education has expanded, the *nature* of education in the US (and other countries) has changed over the last several decades. Although changes in the nature of education are difficult to demonstrate with hard data, qualitative judgements made by observers of education systems from a variety of perspectives, as well as some broad quantitative indicators, are revealing and suggest that over the past few decades education has taken an increasingly regressive character.

The class-based nature of schooling in the US has long been highlighted by radical and Marxian scholars, who have stressed both the type of education that is provided and the unequalizing effect of education due to its differential treatment of students from different classes (in addition to the different use students can make of educational opportunities because of family circumstances, something that has been discussed more widely). In their seminal work, for example, Bowles and Gintis (1975, 1976) have drawn attention to the fact that schools, colleges and universities socialize "students to accept beliefs, values and forms of behavior on the basis of authority, rather than the students' own critical judgments of their interests" (Bowles and Gintis, 2002, p.12). They have also examined how these qualities can be traced to a variety of factors, including the role of the capitalist class in influencing the formation and growth of educational institutions and their curricula. Baran and Sweezy (1966, pp.333-4) have discussed how high schools in the US differentiate between the "academically talented" and others. They cite the view of Harvard University President James Conant who, after a comprehensive inquiry into the state of the educational system felt that the former, who comprised between 15 to 20 percent of all high-school students, "should be given a break: they should be more challenged, their program of academic subjects should be intensified and broadened." To the remaining 80 to 85 per cent, Conant's approach is quite different. His prescription for the plebs is "meaningful sequences of courses leading to the development of marketable skills ... and while the students enrolled should also get some instruction in English, social studies and the like, no undue stress is to be placed in their curricula on academic subjects".

From a liberal perspective, Nussbaum (2010), drawing on the experiences of the US and other countries, points out that "[r]adical changes are occurring in what democracies teach the young ... Thirsty for national profit, nations, and their systems of education, are heedlessly discarding skills that are needed to keep democracies alive. If this trend continues, nations all over the world will soon be producing generations of useful machines, rather than complete citizens who can think for themselves, criticize tradition, and understand the significance of another person's sufferings and achievements" (p.2). She continues that "[t]he student's freedom of mind is dangerous if what is wanted is a group of technically trained obedient workers to carry out the plans of elites who are aiming at foreign investment and technological development" (p.21).

A trend towards a shift in student interest towards career goals and financial issues is discernable from the available data, although the extent to which the education system contributes to this or results from it is difficult to determine. According to the US Department of Education's Higher Education General Information Survey, the share of undergraduate business majors in total undergraduate degrees awarded increased from 13.7 per cent in

1970-71 to 22.8 per cent in 1990-91 and has remained more or less in the low 20s since then. The American Freshman Survey, which has collected data from 1967, shows that for entering first year students in the US, when asked about "objectives considered to be essential or very important" the percent who checked "developing a meaningful philosophy of life" fell from 87 in 1967 to 45, and those who checked "being very well off financially" increased from 40 to 82. It is arguable that these trends in the nature of the education system can explain – at least in part – the incorrect perceptions about the extent of inequality especially in the US. For instance, Norton and Ariely (2011) show that in contrast to the belief of US survey respondents that the top 20 per cent of people in the US owned 59 per cent of all the wealth and the bottom 40 per cent owned 10 per cent, the top 20 per cent actually owned 84 per cent and the bottom 40 owned 0.3 per cent.

All this may explain why the remarkable expansion in education occurred in the last four decades has not produced the expected growth-inducing and inequality-reducing effects suggested by our basic model, and predicted by standard neoclassical macro models. Granting that education has taken up an increasingly regressive and ideological character, an increase in education has led – as predicted in Proposition 5 – to a decrease in workers' bargaining power, with the breakdown in collective organization and worker solidarity (forcefully shown in the decrease in unionization rates). Our hypothesis is that the breakdown in worker solidarity has led to an increase in intra-workers inequalities which has offset the decrease in skill differential due to the increase in the supply of high-skilled workers. Moreover, the combined effect of the decrease in workers' overall bargaining power and the increase in skill differential has been a relatively stable profit share over the period, which in turn explains (in a profit-led economy such as the US) the relatively flat trend in the economy's growth rate.

7 Conclusion

This paper has developed a classical model to examine the growth and distributional consequences of education. In the model an expansion of education allows more low-skilled workers to become high-skilled and, in terms of broader political economy considerations, it can affect bargaining conditions. From a theoretical viewpoint, this paper has thus attempted to fill a lacuna in the literature on the classical approach, which has neglected the formal analysis of the effects of education and skill formation on distribution and growth, an issue which seems a key feature of contemporary capitalist knowledge-based economies.

The model suggests that in a profit led, class-divided, economy with significant unemployment, an increase in education has a positive effect on growth (and on intra-workers inequalities). Yet, unlike in standard growth models this is not due to an improvement in labor productivity but to a worsening of distribution between workers and capitalists. Education policies may lead to an increase in the profit share, and in the growth rate of the economy, either because an increase in the stock of educated workers lowers the reward to skills (provided that substitutability between two types of workers is not too high), or because education policies can affect workers' norms and beliefs, and thus indirectly their bargaining position.

Regarding income distribution, our classical model analyzes how the expansion of education affects the distribution of income between three classes, that is, capitalists, high-skilled workers and low-skilled workers. The expansion of education has the short-run effect of reducing the skill premium and given limited substitution between high- and low-skilled workers, this increases the profit share (and thus the rate of capital accumulation), given the workers' bargaining power. It also increases the ratio of the effective amount of high-skilled workers to capital, which increases productivity growth for both kinds of workers. When we allow the state of class struggle to change endogenously the distributional effects of an expansion of education depend on the nature of the education system. A progressive education tends to increase workers' bargaining power and solidarity, and to reduce *intra-class* inequalities. The opposite holds if education plays an ideological role of socializing people into accepting large inequalities. Interestingly, however, the effect of education on inter-class inequalities cannot be determined a priori: the movement of the profit share depends on the relative strength of the various effects.

In other words, contrary to the received wisdom, in profit-led economies characterized by labor unemployment and wage bargaining, education and human capital accumulation do not yield unambiguously positive effects on distribution. In the presence of several departures from the abstract model with perfectly competitive markets, optimizing agents, and perfect substitutability between inputs of production, more complex and encompassing measures (e.g., improving education access *and* undertaking proactive policies against unemployment) are needed to obtain equitable growth outcomes than a straightforward expansion in education provision.

To be sure, the model developed here is simple and various assumptions can be relaxed to explore further issues. Several simple extensions of the models may be particularly interesting: allowing high-skilled workers to save and hold capital, and thereby have mixed class interests (this case is examined in Dutt and Veneziani 2016); allowing the wage premium to change slowly with the possibility that some high-skilled workers find low-skilled jobs (being chosen above low-skilled workers); introducing different levels of education (such as primary, secondary, and higher education); and allowing aggregate demand issues to enter into the distribution of output and growth. Other broader questions regarding the role of classes can also be raised. Will the spread of education blur the distinction between workers and capitalists by allowing workers to become capitalists (with human capital)? Or will the classical distinction between workers and capitalists continue to have a central role to play? We leave these issues for further research.

References

[1] Acemoglu, D. (2002a) “Directed Technical Change”, *Review of Economic Studies*, 69, 781-809.

[2] Acemoglu, D. (2002b) “Technical Change, Inequality and the Labor Market”, *Journal of Economic Literature*, 40, 7-72.

[3] Aghion, P. and Howitt, P. (1998) *Endogenous growth theory*, Cambridge, MA: MIT Press.

[4] Autor, D., Katz, L., and Krueger, A. (1998) "Computing Inequality: Have Computers Changed the Labor Market?", *Quarterly Journal of Economics*, 113, 1169-1214.

[5] Baran, P. and Sweezy, P. (1966) *Monopoly capital*, New York and London: Monthly Review Press.

[6] Basu, D. (2009) "A simple test of the neoclassical view of production, technical change and distribution", *Metroeconomica*, 61, 593-621.

[7] Blau, F. and Kahn, L. (1996) "International Differences in Male Wage Inequality: Institutions versus Market Forces", *Journal of Political Economy*, 104, 791-836.

[8] Bowles, S. and Gintis, H. (1975) "The problem with human capital theory – a Marxian critique", *American Economic Review*, 65, May, 74-82.

[9] Bowles, S. and Gintis, H. (1976) *Schooling in capitalist America*, New York: Basic Books.

[10] Bowles, S. and Gintis, H. (2002) "Schooling in capitalist America revisited", *Sociology of Education*, 75, 1-18.

[11] Borghans, L. and de Grip, A. (eds.) (2000) *The Overeducated Worker? The Economics of Skill Utilization,* Cheltenham, UK: Edward Elgar.

[12] Card, D. and Lemieux, T. (1996) "Wage Dispersion, Returns to Skills, and Black-White Wage Differentials", *Journal of Econometrics*, 74, 319-61.

[13] Card, D., Kramarz, F., and Lemieux, T. (1999) "Changes in the relative structure of wages and employment: a comparison of the United States, Canada, and France", *Canadian Journal of Economics*, 32, 843-877.

[14] Carvalho, L. and Rezai, A. (2016) "Personal Income Inequality and Aggregate Demand", *Cambridge Journal of Economics*, forthcoming, doi:10.1093/cje/beu085.

[15] Chiarella, C., Franke, R., Flaschel, P., and Semmler, W. (2006) *Quantitative and Empirical Analysis of Nonlinear Dynamic Macromodels*, Cambridge, UK: Cambridge University Press.

[16] Duménil, G. and Lévy, D. (1999) "Being Keynesian in the short term and classical in the long term", *Manchester School*, 67, 684-716.

[17] Duménil, G. and Lévy, D. (2003) "Technology and distribution: historical trajectories à la Marx", *Journal of Economic Behavior and Organization*, 52, 201-233.

[18] Dutt, A.K. (1986) "Vertical trading and uneven development", *Journal of Development Economics*, 20, 339-359.

[19] Dutt, A.K. (1989) "Accumulation, distribution and inflation in a Marxian-Post Keynesian model with a rentier class", *Review of Radical Political Economics*, 21, 18-26.

[20] Dutt, A.K. (1990) *Growth, distribution and uneven development*, Cambridge, UK: Cambridge University Press.

[21] Dutt, A.K. (1992) "The Origins of Uneven Development: The Indian subcontinent", *American Economic Review*, 82, 146-50.

[22] Dutt, A.K. (2005) "Globalization, South-North migration and uneven development", in P. Gangopadhyay and M. Chatterji, eds., *Economics of Globalization*, Aldershot, UK: Ashgate, 115-129.

[23] Dutt, A.K. (2010) "Keynesian growth theory in the 21^{st} century", in P. Arestis and M. Sawyer, eds., *Twenty-first Century Keynesian Economics*, Houndmills and New York: Macmillan Palgrave, forthcoming.

[24] Dutt, A.K. (2011) "The role of aggregate demand in classical-Marxian models of economic growth", *Cambridge Journal of Economics*, 35, 357-82.

[25] Dutt, A.K. and Veneziani, R. (2011) "Education, growth and distribution: classical-Marxian economic thought and a simple model", *Cahiers d'économie politique*, 61, 157-185.

[26] Dutt, A.K. and Veneziani, R. (2016) "Education and human capitalists in a classical-Marxian model of growth and distribution", unpublished, University of Notre Dame, USA and Queen Mary University of London, UK.

[27] Egger, H. and Grossmann, V. (2004) "Non cognitive Abilities and Within-Group Wage Inequality", *IZA Discussion Paper*, n. 1024.

[28] Fiorio, C.V., Mohun, S., and Veneziani, R. (2013) "Social Democracy and Distributive Conflict in the UK, 1950-2010", WP 6, UMass Amherst.

[29] Flaschel, P. (2009) *Topics in classical Micro- and Macro-economics*, New York and Berlin: Springer.

[30] Foley, D.K. (1986) *Understanding Capital*, Cambridge MA: Harvard University Press.

[31] Foley, D.K. (2003) "Endogenous technical change with externalities in a classical growth model", *Journal of Economic Behavior and Organization*, 52, 167-189.

[32] Foley, D.K. and Michl, T.R. (1999) *Growth and distribution*, Cambridge, MA: Harvard University Press.

[33] Franke, R., Flaschel, P., and Proano, C. (2006) "Wage-price dynamics and income distribution in a semi-structural Keynes-Goodwin model", *Structural Change and Economic Dynamics*, 17, 452-465.

[34] Franzini, M. and Raitano, M. (2014) "Wage gaps and human capital: looking for an explanation", mimeo, University of Rome I.

[35] Freeman, R. and Katz, L. (eds.) (1996) *Differences and Changes in Wage Structures*, Chicago: University of Chicago Press.

[36] Galor, O. and Tsiddon, D. (1997) "The distribution of capital and economic growth", *Journal of Economic Growth*, 2, 93-124

[37] Galor, O. and Zeira, J. (1993) "Income distribution and macroeconomics", *Review of Economic Studies*, 601, 35-52.

[38] Giddens, A. (1973) *The class structure of the advanced societies*, London: Hutchison and Co.

[39] Glyn, A., Hughes, A., Lipietz, A., and Singh, A. (1990) "The rise and fall of the Golden Age", in S.A. Marglin and J. Schor, *The Golden Age of Capitalism. Reinterpreting the Postwar Experience*, Oxford: Clarendon Press.

[40] Goodwin, R.M. (1967) "A Growth Cycle", in C.H. Feinstein, ed., *Socialism, Capitalism and Economic Growth*, Cambridge: Cambridge University Press.

[41] Harris, D.J. (1978) *Capital accumulation and distribution*, Stanford, CA: Stanford University Press.

[42] Hirsch, B.T. and Macpherson, D.A. (2003) "Union Membership and Coverage Database from the Current Population Survey: Note", *Industrial and Labor Relations Review*, 56, 349-54

[43] Kaldor, N. (1961) "Capital accumulation and economic growth", in F.A. Lutz and D.C. Hague, eds., *The theory of capital*, New York: St Martin's Press.

[44] Katz, L., and Murphy, K. (1992) "Changes in Relative Wages, 1963-87: Supply and Demand Factors", *Quarterly Journal of Economics*, 107, 35-78.

[45] Lemieux, T. (2006) "Increasing Residual Wage Inequality: Composition Effects, Noisy Data, or Rising Demand for Skill?", *American Economic Review*, 96, 461-498.

[46] Lucas, R.E. (1988) "On the mechanics of economic development", *Journal of Monetary Economics*, 22, 3-42.

[47] Marglin, S.A. (1984) *Growth, distribution and prices*, Cambridge, Mass.: Harvard University Press.

[48] Marx, K. (1867) *Capital*, Vol. 1, Moscow: Progress Publishers, 1948.

[49] Marx, K. and Engels, F. (1975-2005) *Collected Works*, Moscow: Progress Publishers, London: Lawrence and Wishart, and New York: International Publishers.

[50] Mason, J.W. and Jayadev, A. (2014) "Fisher Dynamics in US Household Debt, 1929–2011", *American Economic Journal: Macroeconomics*, 6, 214-234.

[51] McCulloch, J.R. (1825) *The Principles of Political Economy*, 5^{th} edition, Edinburgh: Adam and Charles Black, 1864.

[52] Mill, J.S. (1848) *Principles of Political Economy with some of their applications to Social Philosophy*, in *Collected Works of John Stuart Mill,* volume 2, Toronto: University of Toronto Press, 1965.

[53] Mohun, S. and Veneziani, R. (2008) "Goodwin Cycles and the U.S. Economy, 1948-2004", in P. Flaschel and M. Landesmann, eds., *Mathematical Economics and the Dynamics of Capitalism*, London: Routledge.

[54] Norton, M. and Ariely, D. (2011) "Building a better America - One wealth quintile at a time", *Perspectives on Psychological Science*, 6, 9-12,

[55] Nussbaum, M.C. (2010) *Not for profit! Why democracy* needs *the humanities*, Princeton, NJ: Princeton University Press.

[56] O'Brien, D.P. (1975) *The classical economists*, Oxford: Clarendon Press.

[57] O'Donnell, M.G. (1985) *The educational thought of the classical political economists*, Lanham, New York and London: University Press of America.

[58] Onaram, O., Stockhammer, E., and Grafl, L. (2011) "Financialisation, income distribution and aggregate demand in the USA", *Cambridge Journal of Economics*, 35, 637-661.

[59] Pasinetti, L.L. (1960) "A mathematical formulation of the Ricardian system", *Review of Economic Studies*, 27, 78-98.

[60] Petith, H. (2008) "Land, technical progress and the falling rate of profit", *Journal of Economic Behavior and Organization*, 66, 687-702.

[61] Piketty, T. (2014) *Capital in the 21st Century*, Cambridge, Ma: Harvard University Press.

[62] Skott, P. and Slonimczyk, F. (2010) "Employment and Distribution Effects of the Minimum Wage", WP 13, U.Mass. Amherst.

[63] Smith, A. (1776) *An Inquiry into the Nature and Causes of the Wealth of Nations*, Volume 1, R.H. Campbell and A.S. Skinner, eds., *The Glasgow Edition of the Works and Correspondence of Adam Smith,* Vol. 2, Oxford: Oxford University Press, 1976.

[64] Stokey, N.J. (1991) "Human capital, product quality and growth", *Quarterly Journal of Economics*, 106, 587-616.

[65] Tavani, D. (2013) "Bargaining over productivity and wages when technical change is induced: implications for growth, distribution, and employment", *Journal of Economics*, 109, 207-244.

[66] Tavani, D. and Vasudevan, R. (2014) "Capitalists, workers, and managers: Wage inequality and effective demand", *Structural Change and Economic Dynamics*, 30, 120-131.

[67] Unger, R. (2007) *Free trade re-imagined*, Princeton, NJ: Princeton University Press.

[68] Uzawa, H. (1965) "Optimum technical change in an aggregative model of growth", *International Economic Review*, 6, 18-31.

[69] Veneziani, R. (2007) "Exploitation and time", *Journal of Economic Theory*, 132, 189-207.

[70] Wood, A. (1994) *North-South Trade, Employment and Inequality*, Oxford: Oxford University Press.

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