School of Economics and Finance

On the Mechanics of New-Keynesian Models

Peter Rupert and Roman Šustek

Working Paper No. 784 February 2016 ISSN 1473-0278

Queen Mary
University of London

On the Mechanics of New-Keynesian Models*

Peter Rupert†‡ and Roman Šustek§

February 24, 2016

Abstract

We scrutinize the monetary transmission mechanism in New-Keynesian models, focusing on the role of capital, the key ingredient in the transition from the basic framework to DSGE models. The widely held view that monetary policy affects output and inflation in these models through a real interest rate channel is shown to be misguided. A decline in output and inflation is consistent with a decline, increase, or no change in the real interest rate. The expected path of Taylor rule shocks and the New-Keynesian Phillips Curve are key for inflation and output; the real rate largely reflects consumption smoothing.

JEL Classification Codes: E30, E40, E50.

Keywords: New-Keynesian models, monetary transmission mechanism, real interest rate channel, capital.

*We thank James Bullard, Narayana Kocherlakota, and Wouter den Haan for valuable comments.

†University of California–Santa Barbara; peter.rupert@ucsb.edu.

‡Corresponding author: Peter Rupert, address: Department of Economics, 2127 North Hall, Santa Barbara, CA 93106, U.S.A., tel: +1 805 893-2258, fax: +1 805 893-8830, e-mail: rupert@econ.ucsb.edu.

§Queen Mary University of London and Centre for Macroeconomics; sustek19@gmail.com.

1 Introduction

The New-Keynesian model—a dynamic stochastic general equilibrium (DSGE) model with sticky prices—has become a workhorse in the analysis of monetary policy. It has grown in popularity at tremendous speed both in academia and at central banks around the world. From a basic framework, consisting of an Euler equation, a New-Keynesian Phillips curve, and a Taylor rule, it has quickly grown into a model with many different frictions, adjustment costs, and other features. Whereas the basic framework abstracts from capital, the extended model—building on the real business cycle tradition of Kydland and Prescott (1982) and Long and Plosser (1983)—includes capital and investment as an integral part of the environment. The basic framework is typically used to illustrate optimal monetary policy (e.g., Clarida, Gali, and Gertler, 1999), whereas the extended model—often referred to as a medium-scale DSGE model—is used for practical monetary policy and forecasting (Linde, Smets, and Wouters, 2015, provide a sample of central banks using such DSGE models in policy discussions).

Unfortunately, in our view, widespread understanding of the internal mechanism of the New-Keynesian model has been lost along its fast track to popularity. The existing literature does not help. Textbooks covering the New-Keynesian model (such as Walsh, 2010; Gali, 2015) stop at the basic framework and proceed with a discussion of optimal monetary policy, while research based on the medium-scale DSGE models (represented by, e.g., Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007) starts straight away with the full-blown version. Researchers outside of this field, as well as graduate students, are left on their own to connect the dots, especially when moving from the first to the second strand of the literature.[1]

The purpose of this article, therefore, is to carefully lay out the internal mechanism of the

[1]McCandless (2008) comes closest to bridging this gap, but his treatment of New-Keynesian models is carried out in the context of a model with money and a monetary policy rule formulated as a money growth rule, while most of the New-Keynesian literature follows Woodford (2003) by abstracting from money and formulating monetary policy as a nominal interest rate (Taylor) rule. Woodford (2003), while providing a thorough analysis of many aspects of the New-Keynesian framework, discusses the role of capital only briefly.

New-Keynesian model, demonstrate its behavior under alternative parameterizations, and discuss the role of capital—the key ingredient in the transition from the basic framework to the medium-scale DSGE models. The presence of capital—an endogenous state variable—introduces internal dynamics into the model by allowing households, in the aggregate, to smooth out the effects of fluctuations in output, and thus income, on consumption. We explain the effect of capital on the internal mechanism of the New-Keynesian model in a step-by-step fashion, starting with the basic framework. We argue that the extended model better reveals the underlying transmission mechanism of monetary policy in New-Keynesian models and that the lessons learned in the model with capital apply also to the basic framework.

The motivation behind the development of New-Keynesian models was the desire to introduce the Keynesian real interest rate channel of monetary policy transmission, commonly used in monetary policy debates, into a modern dynamic general equilibrium environment (Ireland, 2015). According to the real interest rate channel, the central bank—controlling the short-term nominal interest rate—has leverage over the ex-ante *real* interest rate because nominal prices are sticky. The real interest rate, in turn, affects consumption and investment decisions of households and firms and thus aggregate demand and output, as in the textbook AS-AD framework. This puts pressure on firms to gradually adjust prices to a new level (see Bernanke and Gertler, 1995; Mishkin, 1996, for a discussion of the real rate channel). A vast empirical literature seems to support this channel. In a nutshell, in response to a positive innovation in a short-term nominal interest rate, (i) the nominal interest rate increases, (ii) output declines, and (iii) inflation declines, but less than output. The ex-ante real interest rate increases as a result of (i) and (iii).[2]

We demonstrate the following properties of the New-Keynesian model: (i) even in the basic three-equation version, generating all of the above impulse-responses depends on parameterization, (ii) consumption smoothing through capital accumulation makes the desir-

[2]The empirical properties are based on impulse-responses from structural VARs (see, e.g., Walsh, 2010, Chapter 1, for an overview). Over time, the literature has achieved great success in matching the size and timing of these responses by extending the basic model in several dimensions (Christiano et al., 2005).

able impulse-responses much harder to obtain, and (iii) the real effects of monetary policy in the model with capital have little to do with the real interest rate channel—a decline in output and inflation in response to a positive monetary policy shock is consistent with a fall, increase, or no change in the ex-ante real interest rate.

The effect of capital on the equilibrium of the New-Keynesian model has a flavor of findings in the asset pricing literature. Jermann (1998) and Tallarini (2000), for instance, show that desirable asset pricing properties established in economies without capital are much harder to obtain in models that allow capital accumulation. Similarly here, the desirable responses to a monetary policy shock that can be established in a New-Keynesian model without capital are much harder to get in a version with capital. Like in the asset pricing literature, sufficiently high capital adjustment costs need to be introduced to reverse this result (infinite adjustment costs replicate the equilibrium of the model without capital). Nevertheless, the lessons learned in the model with capital cast doubt on the usual interpretation of the transmission mechanism, as a real rate channel, even in the basic model.[3]

If not through the real interest rate, how does monetary policy transmit into output and inflation in the New-Keynesian model? We argue that, as a first pass, one can think of the transmission mechanism as follows. The Euler equation for bonds (the Fisher equation) together with the Taylor rule pin down inflation almost in the same way as in a flexible price model—inflation, first and foremost, depends on current and expected future shocks to the Taylor rule. Given the path of the inflation rate, the New-Keynesian Phillips curve then determines output. The real rate largely only reflects the desire and ability of households to smooth consumption in response to changes in income (output). In the model without capital, consumption smoothing in the aggregate is not possible. In order for this to occur in equilibrium, the real interest rate has to adjust. The equilibrium outcome of the model without capital thus makes it appear as if monetary policy affected output and inflation

[3]As a side point, we show that even the basic model does not allow the widespread interpretation as an aggregate supply-demand system. It thus makes little sense to describe the transmission mechanism in the New-Keynesian model using concepts such as aggregate demand and demand shocks, as one may do in the traditional AS-AD paradigm.

through the real interest rate channel. This intuition proves wrong once capital is introduced.

The description of the mechanism is based on a linearized version of the model. We express the endogenous variables as linear functions of state variables and obtain analytical solutions for these functions using the method of undetermined coefficients. We then rely on the analytical solutions as much as possible to describe the workings of the model. When insight from the analytical solutions is limited, numerical analysis is used to explore how the equilibrium functions depend on parameter values.

We do not discuss optimal policy as this is well covered by existing texts. For this reason we are not concerned with the concepts of an output gap and the natural real interest rate. The policy implication of our analysis is that (i) either monetary policy in actual economies does transmit through a real interest rate channel, but then the New-Keynesian model is not suitable for its analysis or (ii) the New-Keynesian model—for its micro-foundations of the price-setting behavior and internal consistency—is a useful description of actual economies, but then policy makers relying on this framework need to rethink the way monetary policy transmits into inflation and real activity.[4]

The paper is divided into three main sections. Section 2 deals with the basic model without capital. Section 3 demonstrates how the key properties of the model change once capital is introduced. Section 4 makes some additional observations, related to the interpretation of the Taylor principle as operating through the real rate channel and the (lack of) AS-AD representation of the model, a common description used in its analysis and applications. Section 5 concludes. Secondary results and derivations are contained in an Appendix.

[4]The New-Keynesian model is usually studied under the assumption that the nominal interest rate can increase or decrease without any constraint. Our analysis is carried out in this tradition. A few recent studies focus on the model's behavior at the zero lower bound. Cochrane (2016), for instance, shows that at the zero lower bound an interest rate peg is stable and inflation is first and foremost determined by the Fisher effect. Bullard (2015) uses this analysis in his discussion of the recent nominal environment in G7 countries. Kocharlakota (2016) takes a different view, arguing that at the zero lower bound, monetary policy can have large real effects even when the Phillips curve is only slightly less than vertical.

2 The basic model without capital

The exposition is based on a common per-period utility function

$$u = \log c - \frac{l^{1+\eta}}{1+\eta}, \quad \eta \geq 0,$$

and the intermediate goods aggregator

$$y = \left[\int y(j)^{\varepsilon} dj \right]^{\frac{1}{\varepsilon}}, \quad \varepsilon \in (0,1).$$

The starting point of our analysis is the system of equations describing the general equilibrium, with the New-Keynesian Phillips curve (NKPC) already in its linearized form, around the zero steady-state inflation rate, a common approximation point. The derivation of this system from first principles is contained, for instance, in Walsh (2010) and Gali (2015). In the spirit of Woodford (2003) and much of the literature that followed, the economy is cashless and monetary policy is formulated as a Taylor-type rule. The general equilibrium is characterized by

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{1}$$

$$\frac{1}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} \frac{1+i_t}{1+\pi_{t+1}} \right), \tag{2}$$

$$y_t = l_t, \tag{3}$$

$$\chi_t = w_t, \tag{4}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{5}$$

$$i_t = i + \nu \pi_t + \xi_t, \tag{6}$$

$$y_t = c_t. \tag{7}$$

Here, c_t is consumption, w_t is a real wage rate, l_t is labor, i_t is a one-period nominal interest rate, π_t is the inflation rate between periods $t-1$ and t, y_t is output, χ_t is the marginal cost, and ξ_t is a mean-zero monetary policy shock. Equation (1) is the consumer's first-order condition for labor, equation (2) is the Euler equation for a one-period nominal bond, equation (3) is a production function, equation (4) gives the marginal cost, equation (5) is the NKPC (for the Rotemberg, 1982, quadratic price adjustment cost specification), equation (6) is the Taylor rule, and equation (7) is the goods market clearing condition. In the NKPC, $\phi \geq 0$ is the Rotemberg cost parameter. Further, $\beta \in (0,1)$ is a discount factor and $\nu > 1$ is the weight on inflation in the Taylor rule. Variables without a time subscript denote steady-state values (the steady-state value of the inflation rate is equal to zero). For the points made in this paper, the exposition is cleaner when the weight on output in the Taylor rule is set equal to zero, as implicitly assumed in equation (6). The main text also deals only with a Taylor rule that responds to current inflation. The appendix derives results for forward- and backward-looking Taylor rules and shows that those alternative specifications do not change the main points made here.

Even though the linearized NKPC is derived for the Rotemberg specification, it is well-known that the same form is obtained also for the Calvo (1983) specification.[5] Namely, under Calvo specification,

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}(\chi_t - \chi) + \beta E_t \pi_{t+1}, \tag{8}$$

where $\theta \in [0,1]$ is the fraction of producers not adjusting prices in a given period. The mapping between Rotemberg and Calvo NKPC is thus

$$\frac{(1-\theta)(1-\theta\beta)}{\theta} = -\frac{1}{\phi(\varepsilon - 1)}.$$

[5] The Calvo specification leads to an aggregation bias that shows up as total factor productivity in the production function (3). This bias, however, disappears once the model is linearized. The Rotemberg specification, on the other hand, leads to a resource cost that shows up in the goods market clearing condition (7). Again, it disappears in a linearized version of the model. For these reasons, the above general equilibrium system abstracts from these two details.

The endogenous variables in the system (1)-(7) are c_t, w_t, l_t,i_t, π_t, y_t, and χ_t. The exogenous variable is the monetary policy shock ξ_t. In the model without capital, the shock is the only state variable. In a linear solution, the dynamics of the endogenous variables are thus fully governed by the exogenous process for the shock; the model parameters affect only the sign and size of the responses of the endogenous variables to the shock.[6]

Eliminating equations (3), (4), and (7) by substitutions for c_t, l_t, and χ_t, the system can be reduced to a four-equation system, which when log-linearized around a nonstochastic steady state (with $y = 1$) becomes

$$\widehat{w}_t = (1+\eta)\widehat{y}_t, \tag{9}$$

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \widehat{i}_t - E_t\pi_{t+1}, \tag{10}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon-1)}\widehat{w}_t + \beta E_t\pi_{t+1}, \tag{11}$$

$$\widehat{i}_t = \nu\pi_t + \xi_t. \tag{12}$$

Here, $\widehat{i}_t \equiv i_t - i$, $\widehat{y}_t \equiv (y_t - y)/y$, and $\widehat{w}_t \equiv (w_t - w)/w$. This system is often reduced further by substituting out w_t from equation (9) and i_t from equation (12), resulting in two equations in two endogenous variables, π_t and y_t,

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu\pi_t + \xi_t - E_t\pi_{t+1} \tag{13}$$

$$\pi_t = \Omega\widehat{y}_t + \beta E_t\pi_{t+1}. \tag{14}$$

Here, $\Omega \equiv -(1+\eta)/[\phi(\varepsilon-1)] = (1+\eta)(1-\theta)(1-\theta\beta)/\theta > 0$, depending on whether Rotemberg or Calvo NKPC is used.

[6] As we are not concerned with optimal policy, we do not proceed to further normalize the variables as deviations from flexible-price levels. To study the model dynamics in response to a monetary policy shock, it is sufficient to work with the deviations from steady state.

2.1 Equilibrium output and inflation

The system (13)-(14) can be solved by, for instance, the method of undetermined coefficients. Assume that the equilibrium decision rule and pricing function are linear functions of the state variable

$$\widehat{y}_t = a\xi_t \quad \text{and} \quad \pi_t = b\xi_t,$$

where a and b are unknown. The guesses are linear, rather than affine, functions of the state as the variables are expressed as deviations from steady state. Suppose that the monetary policy shock follows a stationary AR(1) process

$$\xi_{t+1} = \rho\xi_t + \epsilon_{t+1}, \quad \rho \in [0, 1),$$

where ϵ_{t+1} is an innovation. Substituting the guesses into the system (13)-(14), evaluating the expectations using the AR(1) process, and aligning terms gives unique equilibrium coefficients

$$a = -\frac{1-\beta\rho}{(1-\rho)(1-\beta\rho) + \Omega(\nu - \rho)} < 0, \tag{15}$$

$$b = -\frac{1}{(1-\rho)\frac{1-\beta\rho}{\Omega} + (\nu - \rho)} < 0. \tag{16}$$

It is illustrative to consider two extreme cases of price stickiness. First, suppose that prices are fully flexible ($\theta \to 0$ or $\phi \to 0 \Rightarrow \Omega \to \infty$). In this case

$$a \to 0 \quad \text{and} \quad b \to -\frac{1}{\nu - \rho}.$$

Output in this case is unaffected by the monetary policy shock. The coefficient b is greater in absolute value the more persistent is the shock or the smaller is the weight on inflation in the monetary policy rule.[7] Why is the response of inflation to a positive monetary policy shock negative when prices are fully flexible? To answer this question, it is helpful to rewrite

[7] The equilibrium coefficient for inflation under flexible prices can be alternatively obtained by using, in the method of undetermined coefficients, only equations (10) and (12), with $\widehat{y}_t = 0$.

the monetary policy rule (6) as

$$i_t = (i + \zeta_t) + \nu(\pi_t - \zeta_t), \tag{17}$$

where we have used $\xi_t \equiv -(\nu - 1)\zeta_t$. The shock ζ_t is just a linear transformation of the original shock ξ_t and thus inherits the persistence of the original shock. The two shocks, however, are negatively related. When the policy rule is rewritten as equation (17), the shock ζ_t has an interpretation as an inflation target shock, which helps explain why, when ξ_t increases (ζ_t declines), the inflation rate declines.

Second, suppose that prices are completely fixed ($\theta \to 1$ or $\phi \to \infty \Rightarrow \Omega \to 0$). In this case

$$a \to -\frac{1}{1-\rho} \quad \text{and} \quad b \to 0.$$

Now, inflation is unaffected by monetary policy.[8] Why does output decline in response to a positive monetary shock ξ_t? Output is fully determined by the Euler equation and the monetary policy rule (producers find any output level optimal under $\Omega \to 0$; see the NKPC). With the inflation rate equal to the steady-state value of zero, the combination of equations (10) and (12) yields $E_t\widehat{y}_{t+1} - \widehat{y}_t = \xi_t$. According to this equation, output is expected to be growing as long as the exogenous shock is positive. Because the model is stationary (ξ_t is governed by a stationary AR(1) process), the only way output can grow is if it falls, on the impact of the shock, below its steady state. As the shock converges back to its steady-state level, output gradually grows back to steady state.

Going back to the general solution (15) and (16), observe that Ω can be viewed as a weight shifting the equilibrium coefficients between the fully flexible and completely fixed price solutions. The quantitative effects of θ (and thus Ω) on a and b are demonstrated in the upper-left panel of Figure 1 (the other parameters are $\beta = 0.99$, $\eta = 1$, $\nu = 1.5$, and $\rho = 0.5$; all fairly standard values in the literature).[9]

[8]This solution can be alternatively obtained by using, in the method of undetermined coefficients, only equations (10) and (12), with $\pi_t = 0$.

[9]The advantage of using the Calvo specification for numerical examples is that the literature contains a

The lower-left panel, instead, demonstrates the effect of ρ (for $\theta = 0.7$, a typical value, implying average price stickiness for 3.3 periods). Notice that as $\rho \to 1$, the equilibrium coefficients converge, approximately, to the flexible-price solution (the convergence can be arbitrarily close the closer is β to one). This is because, in the NKPC, output depends on the expected *change* in the inflation rate. Thus, the more persistent inflation is, the smaller is its period-on-period change and thus the smaller is its effect on output.

2.2 The equilibrium nominal and real interest rates

The equilibrium functions for the nominal and ex-ante real interest rates can be derived, respectively, as

$$\widehat{i}_t = \nu\pi_t + \xi_t = (1 + \nu b)\xi_t$$

and

$$E_t\widehat{r}_{t+1} \equiv \widehat{i}_t - E_t\pi_{t+1} = [1 + (\nu - \rho)b]\,\xi_t,$$

where $b < 0$ is given by (16). The equilibrium nominal interest rate consists of two parts: a direct effect of the shock in the Taylor rule and an indirect effect due to the response of monetary policy to the equilibrium inflation rate. While the first effect is positive, the second effect is negative and its absolute value increases with inflation persistence (see the lower-left panel of Figure 1). The overall effect of the shock on the nominal interest rate thus depends on the relative strength of the two effects. The equilibrium ex-ante real interest rate depends on three effects. In addition to the above two effects, it also depends on an expected inflation effect. Like the direct effect of the shock, the expected inflation rate effect is positive ($-\rho b > 0$). Substituting in for b and rearranging terms gives

$$E_t\widehat{r}_{t+1} = \left(1 - \frac{1}{1 + \frac{1-\rho}{\nu-\rho}\frac{1-\beta\rho}{\Omega}}\right)\xi_t, \qquad (18)$$

great amount of estimates of θ. Further, θ can be directly translated into the number of periods prices are on average fixed, as $(1-\theta)^{-1}$.

where the expression on the brackets is positive, as the term in the denominator is greater than one. The two positive effects thus always dominate the negative effect.[10]

The right-hand side panels of Figure 1 show the effects of θ and ρ on the equilibrium nominal and ex-ante real interest rates. Notice that for a significant part of the parameter space (lower θ and higher ρ), the equilibrium coefficient of the nominal interest rate is negative—the indirect effect of the shock, working through the response of monetary policy to the decline in inflation, dominates the direct effect. For such values, declines in output and inflation in response to a positive monetary policy shock are accompanied by a decline in the nominal interest rate. This is in contrast to the desired impulse-responses described in the Introduction.

3 The model with capital

When capital is introduced into the model, the general equilibrium becomes characterized by the following system

$$\frac{w_t}{c_t} = l_t^{\eta}, \tag{19}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \right], \tag{20}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right], \tag{21}$$

$$y_t = k_t^{\alpha} l_t^{1-\alpha}, \tag{22}$$

$$\frac{w_t}{r_t} = \frac{1-\alpha}{\alpha} \left(\frac{k_t}{l_t} \right), \tag{23}$$

$$\chi_t = \left(\frac{r_t}{\alpha} \right)^{\alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha}, \tag{24}$$

$$\pi_t = \Psi \widehat{\chi}_t + \beta E_t \pi_{t+1}, \tag{25}$$

[10] Alternatively, one can see that the ex-ante real rate always responds positively to the ξ_t shock by realizing that output always declines on impact of the shock (a is always negative) and converges back to its steady state from that point on (i.e., it is growing). This can only happen, according to the Euler equation, if the deviation from steady state of the ex-ante real interest rate is positive.

$$i_t = i + \nu\pi_t + \xi_t, \tag{26}$$

$$y_t = c_t + k_{t+1} - (1-\delta)k_t. \tag{27}$$

Here, k_t is capital, r_t is the capital rental rate, and $\delta \in (0,1)$ is a depreciation rate; investment can be defined as $x_t \equiv k_{t+1} - (1-\delta)k_t$. Further, $\Psi \equiv -\chi/[\phi(\varepsilon-1)] = [(1-\theta)(1-\theta\beta)/\theta]\Theta > 0$, where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha/(1-\varepsilon)}$, and $\widehat{\chi}_t \equiv (\chi_t - \chi)/\chi$. The endogenous variables are c_t, w_t, l_t, i_t, π_t, y_t, χ_t, r_t, k_t; the exogenous variables are ξ_t and k_0. Notice that (19), (20), and (26) are the same as before. Further, (22), (24), and (25) are the same as before for $\alpha = 0$. The truly new equations are equations (21) and (23), which add the two new endogenous variables, k_t and r_t. Equation (21) is the Euler equation for capital and equation (23) is a condition for the optimal mix of capital and labor in production; it equates the marginal rate of technological substitution with the relative factor prices.

Under flexible prices, the model collapses into a real business cycle model with two additions, the Euler equation for bonds and the Taylor rule. To see this, note that under flexible prices, $\Psi \to \infty$. The NKPC (25) then implies $\widehat{\chi}_t = 0$, or $\chi_t = \chi$. If, in addition, $\varepsilon = 1$ (perfect competition), $\chi = 1$; see Gali (2015). This is a standard profit maximization condition under perfect competition, stating that the marginal cost is equal to the good's relative price, which is equal to one, as under perfect competition all goods are prefect substitutes. When this condition is used in equation (24), and the resulting equation is combined with (23), we get the standard conditions equalizing marginal products to factor prices

$$r_t = \alpha k_t^{\alpha-1} l_t^{1-\alpha}, \tag{28}$$

$$w_t = (1-\alpha) k_t^{\alpha} l_t^{-\alpha}. \tag{29}$$

These two conditions replace equations (23) and (24) in the above system. Notice that the system becomes recursive: equations (19), (21), (22), (27), (28), and (29) determine c_t, w_t, l_t, y_t, r_t, and k_t, given k_0, independently of ξ_t (in addition, $\chi_t = 1$ from the NKPC). Equations (20) and (26) then determine i_t and π_t, given stochastic paths of c_t and ξ_t.

When prices are completely fixed, $\Psi \to 0$ and the NKPC implies $\pi_t = 0$. The Euler equation for bonds then determines the growth rate of consumption, given ξ_t, but due to the presence of capital (investment) in equation (27), not the growth rate of output.

In what follows, it will be convenient to substitute in for r_t in equation (24) from (23). The marginal cost then becomes

$$\chi_t = \frac{w_t}{1-\alpha}\left(\frac{y_t}{k_t}\right)^{\frac{\alpha}{1-\alpha}}.$$

For $\alpha = 0$ this expression becomes the same as in the model without capital. Further, substitute in for l_t in equation (19) from the production function (22). This gives the first-order condition for labor as

$$\frac{w_t}{c_t} = \left(\frac{y_t}{k_t^{\alpha}}\right)^{\frac{\eta}{1-\alpha}}.$$

Again, for $\alpha = 0$, this condition is the same as in the model without capital.

With the above two substitutions, we can log-linearize the general equilibrium system to get

$$-\widehat{c}_t + \widehat{w}_t = \frac{\eta}{1-\alpha}\widehat{y}_t - \frac{\alpha\eta}{1-\alpha}\widehat{k}_t$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \widehat{i}_t - E_t\pi_{t+1},$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{r}_{t+1},$$

$$\widehat{l}_t = \frac{1}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\widehat{r}_t = r(\widehat{l}_t - \widehat{k}_t + \widehat{w}_t),$$

$$\widehat{\chi}_t = \widehat{w}_t + \frac{\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha}{1-\alpha}\widehat{k}_t,$$

$$\pi_t = \Psi\widehat{\chi}_t + \beta E_t\pi_{t+1},$$

$$\widehat{i}_t = \nu\pi_t + \xi_t,$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t.$$

Here, variables without time subscripts are steady-state values, interest rates are expressed as percentage point deviations from steady state, $\widehat{r}_t \equiv r_t - r$, $\widehat{i}_t \equiv i_t - i$, and all other variables are expressed as percentage deviations from steady state, e.g., $\widehat{c}_t \equiv (c_t - c)/c$. Again, observe that in the absence of capital and its rental rate, and for $\alpha = 0$, the system is the same as in the model without capital. Eliminating $\widehat{r}_t$, $\widehat{\chi}_t$, $\widehat{w}_t$, $\widehat{i}_t$, and $\widehat{l}_t$ we get a final system of four equilibrium first-order difference equations in four endogenous variables $\widehat{c}_t$, $\widehat{y}_t$, $\widehat{k}_t$, and $\widehat{\pi}_t$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \nu\pi_t - E_t\pi_{t+1} + \xi_t, \tag{30}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{31}$$

$$\pi_t = \Psi\left[\frac{\eta+\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha(1+\eta)}{1-\alpha}\widehat{k}_t + \widehat{c}_t\right] + \beta E_t\pi_{t+1}, \tag{32}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t. \tag{33}$$

Here, (30) is the same as in the model without capital, (33) is the same as in the model without capital for $k = 0$, and (32) is the same as in the model without capital for $k = 0$ and $\alpha = 0$. Equation (31) is new. Clearly, capital disrupts the close connection between inflation and output present in the NKPC of the basic model, as here investment implies $\widehat{c}_t \neq \widehat{y}_t$.

3.1 The real effects of monetary policy shocks—a first look

According to the common interpretation of the New-Keynesian model as a real rate channel of monetary policy transmission, a positive shock to the Taylor rule increases the nominal interest rate. Because prices are sticky, the ex-ante real interest rate increases, which suppresses aggregate demand, as consumers postpone consumption and producers cut down on investment. As producers find it costly to adjust prices in face of a decline in demand, they cut production and aggregate output falls. While this interpretation may be observationally equivalent to the mechanism in the basic model—in which a decline in output is always accompanied by an increase in the ex-ante real rate, though not necessarily the nominal rate—it is hard to justify in the model with capital. Here we take a first look at this issue and revisit it in Sections 3.2 and 3.3.

It helps, for the exposition, to make the assumption that the steady-state real interest rate r is small enough to be approximated by zero (under the parameterization below, $r = 0.0351$). Under this assumption, equation (31) implies $\widehat{c}_t = E_t\widehat{c}_{t+1}$; that is, the presence of capital—demonstrated by the Euler equation—allows perfect consumption smoothing across time. Further, as the model is stationary (see below), it then has to be the case that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Otherwise, under $\widehat{c}_t = E_t\widehat{c}_{t+1}$, a given shock would lead to a permanent shift of consumption away from the steady state, which violates stationarity. Equation (30) then determines the equilibrium inflation rate, which is unaffected by the rest of the model and depends only on the monetary policy shock. The solution for the inflation rate is therefore the same as in the model without capital under flexible prices, $\pi_t = -[1/(\nu - \rho)]\xi_t$. The inflation rate thus falls on the impact of the shock and converges back to zero from below. Along this path, $\pi_t - E_t\pi_{t+1}$ is negative. Thus, for β close to one, equation (32) implies that on the impact of the shock output declines, as both $\widehat{c}_t$ and $\widehat{k}_t$ are equal to zero in the impact period (the deviation of consumption is equal to zero for the above reasons; the deviation of capital is equal to zero as the existing capital stock in the impact period is in steady state). From equation (33) then, $\widehat{k}_{t+1}$ has to decline; i.e., the decline in output is fully absorbed by a decline in investment.

Equilibrium inflation in the model with capital is thus first and foremost determined by the Fisherian principle (current and expected future monetary policy shocks and therefore expected future inflation), while equilibrium output is determined by the Keynesian principle (the Phillips curve). While this description provides a simplifying account of the model's dynamics (it is based on $r = 0$ and β close to one), Sections 3.2 and 3.3 show that it is a useful way to think about the model's internal mechanism.

3.2 Equilibrium functions and impulse-responses

The model can be again solved by the method of undetermined coefficients. There are two state variables, k_t and ξ_t. The general solution thus has the form: $\widehat{c}_t = a_0\widehat{k}_t + a_1\xi_t$, $\pi_t = b_0\widehat{k}_t + b_1\xi_t$, $\widehat{y}_t = d_0\widehat{k}_t + d_1\xi_t$, and $\widehat{k}_{t+1} = f_0\widehat{k}_t + f_1\xi_t$. Using these functions in the system (30)-(33) and aligning terms yields a system of eight equations in eight unknowns, a_0,a_1, b_0, b_1, d_0, d_1, f_0, f_1. From equation (30) we get:

$$-a_0 = -a_0 f_0 + \nu b_0 - b_0 f_0, \tag{34}$$

$$-a_1 = -a_0 f_1 - a_1\rho + \nu b_1 - b_0 f_1 - b_1\rho + 1.$$

From equation (31):

$$-a_0 = -(1-r)a_0 f_0 + \frac{r(1+\eta)}{1-\alpha}d_0 f_0 - \frac{r(1+\alpha\eta)}{1-\alpha}f_0, \tag{35}$$

$$-a_1 = -(1-r)a_0 f_1 - (1-r)a_1\rho + \frac{r(1+\eta)}{1-\alpha}d_0 f_1 + \frac{r(1+\eta)}{1-\alpha}d_1\rho - \frac{r(1+\alpha\eta)}{1-\alpha}f_1.$$

From equation (32):

$$b_0 = -\psi\frac{\eta+\alpha}{1-\alpha}d_0 + \psi\frac{\alpha\eta+\alpha}{1-\alpha} - \psi a_0 + \beta b_0 f_0, \tag{36}$$

$$b_1 = -\psi\frac{\eta+\alpha}{1-\alpha}d_1 - \psi a_1 + \beta b_0 f_1 + \beta b_1\rho.$$

And from equation (33):

$$f_0 = \frac{y}{k}d_0 - \frac{c}{y}a_0 + (1-\delta), \tag{37}$$

$$f_1 = \frac{y}{k}d_1 - \frac{c}{y}a_1.$$

The system is recursive, whereby a_0, b_0, d_0, and f_0 can be solved from the four equations (34)-(37), independently of a_1, b_1, d_1, and f_1. The persistence of the shock, ρ, thus has no effect on the equilibrium coefficients loading onto $\widehat{k}_t$. In other words, the internal dynamics of the model are unaffected by the dynamics of the shock. The equilibrium coefficients loading onto $\widehat{k}_t$, however, affect a_1, b_1, d_1, and f_1 and thus the direct responses of the endogenous variables to the monetary policy shock. In other words, the presence of capital affects the responses of the endogenous variables to the shock.

The system of equations (34)-(37) can be rewritten as a fourth-order polynomial, which has up to four distinct roots. From here, we proceed numerically, using the following parameter values: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$ (same as before) and $\delta = 0.025$, $\alpha = 0.3$, and $\varepsilon = 0.83$ (fairly standard values).[11] The persistence parameter of the monetary policy shock is again treated as a free parameter. We consider five values, $\rho \in \{0, 0.1, 0.5, 0.95, 0.995\}$. For these parameter values, the polynomial has a single root within the unit circle, and three roots outside of the unit circle, yielding a unique stationary equilibrium.

Figures 2-6 display responses to a 1 percentage point increase in ξ_t in period $t = 1$ for the above five values of ρ. Interest rates are reported as percentage point deviations from steady state; all other variables as percentage deviations from steady state. The impulse-response functions reveal our conjecture from Section 3.1 that the effects of monetary policy shocks in the model with capital do not propagate through the real interest rate channel. In all cases but $\rho = 0$, output and inflation fall, in response to the positive monetary policy shock, despite a decline in the ex-ante real interest rate. The nominal interest rate also declines in all but the case of no persistence. Experimentation reveals that the nominal interest rate increases, in response to the shock, only for $\rho \in [0, 0.06]$, while both the nominal and real interest rates increase only for $\rho \in [0, 0.04]$.[12] This region is tiny, compared with the model without capital (refer back to the lower panels of Figure 1). Regarding consumption and investment, both variables fall, although consumption falls only little, as expected from our discussion in Section 3.1. As ρ gets closer to one, and inflation becomes more persistent, the response of output converges to close to monetary policy neutrality, as one would expect

[11] The values of k and c are, respectively, 7.0938 and 0.8227. The value of y is again normalized to be equal to one. As mentioned above, the value of $r = 1/\beta - 1 + \delta = 0.0351$.

[12] In all cases, the real interest rate increases above its steady-state level several periods after the impact of the shock due to the decline in capital; once the effect of sticky prices (the NKPC) dies off, the dynamics of the real interest rate become governed by the marginal product of capital, as in a real business cycle model.

from the NKPC.

In sum, only when the shock persistence is close to zero, the model produces, at least qualitatively, the responses consistent with the real rate channel described in the Introduction; the presence of capital dramatically reduces the admissible region for the shock persistence generating such responses.

3.3 The effect of capital on the ex-ante real interest rate

Why does the model have such a hard time producing an increase in the ex-ante real interest rate in response to the monetary policy shock? As in Section 2, it is helpful to write out the equilibrium function for the real interest rate

$$\begin{aligned} E_t\widehat{r}_{t+1} = \widehat{i}_t - E_t\pi_{t+1} &= \nu\pi_t + \xi_t - E_t\pi_{t+1} \\ &= \nu(b_0\widehat{k}_t + b_1\xi_t) + \xi_t - E_t\left(b_0\widehat{k}_{t+1} + b_1\xi_{t+1}\right) \\ &= b_0(\nu - f_0)\widehat{k}_t + [1 + (\nu - \rho)b_1 - b_0 f_1]\,\xi_t. \end{aligned} \tag{38}$$

We focus on the immediate response (from steady state), thus setting $\widehat{k}_t = 0$.

Observe that the equilibrium coefficient loading onto ξ_t consists of four parts. The first three parts, $1+\nu b_1 - \rho b_1$, are the same as in the basic model, even though the value of b_1 may be different. These are, respectively, the direct effect of the shock on the nominal interest rate, the reaction of monetary policy to the equilibrium response of today's inflation to the shock, and the equilibrium response of tomorrow's inflation to the expected value of the shock tomorrow. In the model without capital, the coefficient in the equilibrium function for inflation, for the baseline value of $\rho = 0.5$, is equal to -0.35 (this can be read off from Figure 1). This resulted in the sum of the three effects being positive (in fact, we were able to prove, using the reduced form solution for the coefficient in the equilibrium inflation function, that the sum of the three effects is always positive). Now, however, the equilibrium inflation function is $\pi_t = -0.057\widehat{k}_t - 1.44\xi_t$, with the coefficient on the monetary policy shock equal to -1.44. As a result, the sum of the three effects is negative. Furthermore, there is a fourth effect in the equilibrium coefficient for the real rate, $-b_0 f_1$. The fourth effect is related to the direct role of capital. Specifically, f_1 is the equilibrium response of $\widehat{k}_{t+1}$ to ξ_t and b_0 gives the equilibrium response of π_{t+1} to $\widehat{k}_{t+1}$. The product of f_1 and b_0 thus captures the equilibrium response of expected inflation to today's monetary policy shock occurring through a change in the capital stock. The equilibrium law of motion for capital is $\widehat{k}_{t+1} = 0.936\widehat{k}_t - 1.56\xi_t$. As both f_1 and b_0 are negative ($f_1 = -1.56$ and $b_0 = -0.057$), the product is positive and the capital channel increases inflation expectations, in response to a positive ξ_t shock. The capital channel thus contributes to the decline in the ex-ante real interest rate.

The intuition for the negative response of k_{t+1} to the monetary policy shock is based on consumption smoothing. A decline in output (income) induces consumers to reduce capital to keep consumption as smooth as possible. The intuition for the increase in future inflation in response to a decline in future capital stock is based on the version of the model without price stickiness. A decline in future capital increases future marginal product of capital, and thus future real interest rate, which—through equations (20), (21), and (26)—transmits to higher future inflation.

The capital accumulation channel plays a negative role even when the first three effects generate a positive response of the ex-ante real rate. For instance, in the case of $\rho = 0.1$, which yields $b_1 = -0.7$ and thus a positive sum of the first three effects, the fourth effect counterweights them and the real rate declines (Figure 3). Out of the five cases considered, only in the case of $\rho = 0$ is the sum of the first three effects positive and large enough to dominate the fourth effect (Figure 2).

3.4 Adjustment costs

Suppose that whenever the consumer changes the capital stock, he has to incur a resource cost in terms of foregone real income. We assume the simplest possible form of capital adjustment costs

$$-\frac{\kappa}{2}(k_{t+1}-k_t)^2, \quad \kappa \geq 0.$$

In steady state, the adjustment cost is equal to zero. Further, as the adjustment cost is quadratic, it will not affect the resource constraint of the economy in a log-linear approximation of the model around a steady state. The Euler equation for capital now becomes

$$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \left(\frac{r_{t+1}-\delta}{q_t} + \frac{q_{t+1}}{q_t} \right) \right],$$

where

$$q_t \equiv 1 + \kappa(k_{t+1}-k_t).$$

Notice that for $\kappa = 0$, the Euler equation collapses to the Euler equation in the version without adjustment costs. The expression in the round brackets has an interpretation as a sum of a dividend yield and a capital gain. Denote the capital gain by $G_{t+1} \equiv q_{t+1}/q_t$.

The log-linearized system becomes

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + \nu\pi_t - E_t\pi_{t+1} + \xi_t, \tag{39}$$

$$-\widehat{c}_t = -E_t\widehat{c}_{t+1} + E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right), \tag{40}$$

$$\pi_t = \Psi\left[\frac{\eta+\alpha}{1-\alpha}\widehat{y}_t - \frac{\alpha(1+\eta)}{1-\alpha}\widehat{k}_t + \widehat{c}_t\right] + \beta E_t\pi_{t+1}, \tag{41}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{k}{y}\widehat{k}_{t+1} - (1-\delta)\frac{k}{y}\widehat{k}_t, \tag{42}$$

where

$$\widehat{G}_{t+1} = \widehat{q}_{t+1} - \widehat{q}_t = \overline{\kappa}(\widehat{k}_{t+2} - \widehat{k}_{t+1}) - \overline{\kappa}(\widehat{k}_{t+1} - \widehat{k}_t) \tag{43}$$

is a percentage deviation of capital gains from steady state and $\overline{\kappa} \equiv \kappa k$. Only equation (40) in the above system is different, compared with the system of the previous section. Of course, it coincides with equation (31) if $\kappa = 0$. Combining equations (39) and (40), the ex-ante real interest rate can be written as the sum of capital gains and expected dividend yield

$$\begin{aligned} i_t - E_t\pi_{t+1} &= \nu\pi_t + \xi_t - E_t\pi_{t+1} \\ &= E_t\widehat{G}_{t+1} + rE_t\left(\widehat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\widehat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\widehat{k}_{t+1}\right). \end{aligned} \tag{44}$$

The exposition proceeds, as before, under the simplifying assumption that r is close to zero. Under this assumption, the last term in equation (40) again drops out (this is the dividend term). Now, however, the capital gains term does not allow us to conclude that $\widehat{c}_t = E_t\widehat{c}_{t+1} = 0$. Capital adjustment costs prevent perfect consumption smoothing, resulting in $\widehat{c}_t \neq E_t\widehat{c}_{t+1} \neq 0$. Any drop in output dictated by inflation dynamics and the NKPC has to be, at least partially, accommodated by a drop in consumption. The higher is κ, the more any given change in the capital stock affects capital gains and thus the expected growth rate of consumption. As a result, the higher is κ the more any given change in output is accounted for by a change in consumption, rather than investment. Increasing κ thus brings the model closer to the version without capital. Further, inflation and real variables are now interconnected. As before, the NKPC (41) provides one such connection. The other connection comes from equations (39) and (40), which relate inflation to capital gains as in equation (44); recall that in the version without adjustment costs, under r equal to zero, inflation depends, negatively, only on the monetary policy shock. Here, the monetary policy shock again contributes negatively to current inflation, but the capital gains term contributes positively. From (44):

$$\pi_t = \frac{1}{\nu}\left(-\xi_t + E_t\pi_{t+1} + E_t\widehat{G}_{t+1}\right).$$

The model can again be solved by the method of undetermined coefficients, guessing $\widehat{c}_t$, π_t, $\widehat{y}_t$, and $\widehat{k}_{t+1}$ as linear functions of $\widehat{k}_t$ and ξ_t. Relative to the system of restrictions in the previous section, only those resulting from equation (40) are different

$$-\overline{\kappa} - a_0 + (1-r)a_0 f_0 - \frac{r(1+\eta)}{1-\alpha}d_0 f_0 + \frac{r(1+\alpha\eta)}{1-\alpha}f_0 + 2\overline{\kappa}f_0 - \overline{\kappa}f_0 = 0,$$

$$-a_1+(1-r)a_0f_1+(1-r)a_1\rho-\frac{r(1+\eta)}{1-\alpha}d_0f_1-\frac{r(1+\eta)}{1-\alpha}d_1\rho+\frac{r(1+\alpha\eta)}{1-\alpha}f_1+2\overline{\kappa}f_1-\overline{\kappa}f_0f_1-\overline{\kappa}f_1\rho=0.$$

Observe that the system preserves the recursive structure of the previous section.

The equilibrium function for the ex-ante real rate can again be computed from the equilibrium functions for inflation and capital, using the formula in (38). Capital adjustment costs reduce, in absolute value, both b_1 and the fourth (capital) effect in equation (38), making the response of the real interest rate more likely to be positive.

Figures 7-9 show the responses of the model under three different parameterizations of the capital adjustment cost, $\kappa \in \{0.1, 0.2, 0.5\}$. The rest of the parameterization is as usual; specifically, $\theta = 0.7$ and $\rho = 0.5$. Observe that as κ increases, the model starts to produce responses consistent with the real rate channel. Specifically, at $\kappa = 0.1$ the model still suffers from producing a decline in the nominal interest rate and only a gradual increase in the ex-ante real rate. At $\kappa = 0.2$, the ex-ante real rate increases on impact, but the nominal interest rate still falls. At $\kappa = 0.5$, finally, both the ex-ante real rate and the nominal interest rate increase on impact. Throughout these experiments, the increase in the ex-ante real rate occurs exclusively due to expected capital gains.

Sufficiently high capital adjustment costs thus make the model consistent, at least qualitatively, with the real rate channel and the empirical impulse-responses noted it the Introduction. Capital adjustment costs in this model work in the same way as in asset-pricing business cycle models (e.g., Jermann, 1998; Tallarini, 2000). They prevent consumption smoothing. Here, consumers want to smooth consumption when income declines, as producers cut output in face of a drop in inflation, as dictated by the NKPC. To prevent consumption smoothing in equilibrium, capital gains has to be sufficiently high. As a result, through the no-arbitrage principle, the ex-ante real interest rate—the real yield on a one-period bond—has to also be high. This outcome, while consistent with the real rate channel, is only a by product of the decline in output, not its cause. It reflects the desire of households to smooth consumption in the face of the drop in output.

4 Additional observations

Here we make two additional points, related to the real rate channel. First, the usual description of the real rate channel relies on concepts such as aggregate demand and aggregate demand shocks. We demonstrate that even the basic model does not allow a representation as an AS-AD system. Second, we argue that the well-known Taylor principle of monetary policy reaction function has little to do with the real rate channel.

4.1 AS-AD representation?

Recall that the basic model collapses into a system of two equations, (13) and (14). Equation (13) is downward sloping in the $(\widehat{y}_t, \pi_t)$ space, whereas equation (14) is upward sloping in the $(\widehat{y}_t, \pi_t)$ space. For this reason, the first equation is often referred to as the 'aggregate demand curve' and the second equation as the 'aggregate supply curve'. Furthermore, as the monetary policy shock shows up in equation (13), it is sometimes referred to as an 'aggregate demand shock'. These interpretations of the system and the shock, however, are misleading. Supply and demand functions are supposed to determine the market clearing quantity and price. Here, however, the goods market clears along both curves, not just in their intersection—recall that the goods market clearing condition (7) has been used to derive equations (13) and (14). Strictly speaking, equation (13) characterizes the combinations of the inflation rate and *market clearing output* that are consistent with intertemporal optimality of consumers and the monetary policy rule. Equation (14), on the other hand, characterizes, strictly speaking, the combinations of the inflation rate and *market clearing output* that are consistent with the labor market equilibrium and optimal price setting. The intersection satisfies both types of optimality.

What if the market clearing condition is *not* used when deriving the equilibrium system? The system then becomes

$$-\widehat{c}_t = \nu\pi_t - E_t\widehat{c}_{t+1} + \xi_t - E_t\pi_{t+1}, \tag{45}$$

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}(\eta\widehat{y}_t + \widehat{c}_t) + \beta E_t\pi_{t+1},$$

$$\widehat{c}_t = \widehat{y}_t. \tag{46}$$

Here, in period t, $\widehat{c}_t$ can be interpreted as the amount of goods demanded, while $\widehat{y}_t$ can be interpreted as the amount of goods supplied, given the values of all other variables. Equation (46) clears the market. Eliminating $\widehat{c}_t$ from the NKPC, so as to express the equation only in terms of goods supplied, yields

$$\pi_t = -\frac{1}{\phi(\varepsilon - 1)}(\eta\widehat{y}_t - \nu\pi_t + E_t\widehat{c}_{t+1} - \xi_t + E_t\pi_{t+1}) + \beta E_t\pi_{t+1}. \tag{47}$$

Equation (45) is downward sloping in the $(\widehat{c}_t, \pi_t)$ space, while equation (47) is upward sloping in the $(\widehat{y}_t, \pi_t)$ space. The monetary policy shock, however, ends up shifting both curves. The Appendix shows that when the model is cast into an output-real interest rate space, another common 'AS-AD' representation, the monetary policy shock ends up shifting only the AS curve. Again, the interpretation as a demand shock is misleading.

4.2 The Taylor Principle

The Taylor Principle is a theorem according to which $\nu > 1$ is a necessary condition for the existence of a unique stationary equilibrium of the New-Keynesian model (e.g., Bullard and Mitra, 2002). A common interpretation of this result, however, is based on the real interest rate channel of monetary policy transmission and goes like this: The central bank has to move the nominal interest rate more than one-for-one with the inflation rate, so as to affect the real interest rate and thus aggregate demand. Through its effect on aggregate demand, monetary policy controls by how much producers adjust prices and thus keeps inflation in check. This interpretation, however, is misleading as the New-Keynesian model does not contain the real rate channel as the transmission mechanism of monetary policy. Indeed, as the upper panels of Figure 1 show, the unique stationary equilibrium derived for the basic model, under $\nu > 1$, often features the nominal interest rate responding much less than the inflation rate (for instance, for $\theta = 0.5$, the nominal interest rate does not respond at all while the inflation rate declines by about 0.7 percentage points).

Rather than to do with suppressing aggregate demand, the Taylor principle in the New-Keynesian model works just like in a flexible-price model: ensuring that a bubble term in the model's asset pricing equation can be excluded (a point made by Cochrane, 2011). In both models, combining equations (10) and (12), the inflation rate has to satisfy

$$\pi_t = \frac{1}{\nu} E_t \pi_{t+1} + \frac{1}{\nu} \left(E_t \widehat{y}_{t+1} - \widehat{y}_t \right) - \frac{\xi_t}{\nu},$$

where the expression in brackets is a deviation of the real interest rate from steady state. Under fully-flexible prices, $\widehat{y}_t = 0$ for all t and the deviation of the real interest rate is equal to zero. Clearly, by forward substitution, $\nu > 1$ is a necessary condition for a unique stationary equilibrium, as $1/\nu < 1$ works as a discount factor of future values of the shock ξ_t, the 'dividends' in the model, and of $E_t \pi_{t+\iota}$, $\iota \to \infty$, the bubble term. When prices are sticky, output (and thus the real interest rate) is endogenous, given by the NKPC

$$\widehat{y}_t = \frac{1}{\Omega} (\pi_t - \beta E_t \pi_{t+1}).$$

But as long as output is bounded (for instance by a time constraint on labor), $\nu > 1$ again ensures that future (stationary) dividend terms, which now also include $E_t \widehat{y}_{t+1} - \widehat{y}_t$, are discounted and the bubble term can be eliminated.[13]

5 Conclusion

How does monetary policy affect inflation and output in the economy? A widely accepted view is that through its effect on the ex-ante real interest rate. In this paradigm, a justification for the transmission from the nominal interest rate—the monetary policy instrument—to the real interest rate—a price that ultimately affects decisions of the private sector—rests on

[13] A formal proof of the Taylor principle is contained in the Appendix.

nominal price rigidities. Introducing this channel into a modern dynamic stochastic general equilibrium environment was one of the motivations for the development of New-Keynesian models. These models, both in their basic and extended (medium-scale DSGE) forms are routinely used at central banks around the world to guide monetary policy. This paper scrutinizes the transmission mechanism in New-Keynesian models and argues that the presence of capital is important for understanding of how the transmission mechanism in these models works. Capital and investment are integral parts of real business cycle models, as well as the medium-scale New-Keynesian DSGE models. These models, however, contain many other features that, while helpful in matching the data, obscure the working of the transmission mechanism.

We demonstrate that the transmission mechanism in New-Keynesian models does not operate through the real interest rate channel. Instead, as a first pass, inflation is determined by a Fisherian principle, through current and expected future monetary policy shocks, while output is then pinned down through a Keynesian principle, the New-Keynesian Phillips curve. The real interest rate largely only reflects the desire and ability of households to smooth consumption in response to movements in output (income). A decline, increase, or no change in the ex-ante real interest rate is consistent with declines in output and inflation in response to a positive monetary policy shock. High enough capital adjustment costs make the model appear as if it operated through the real interest rate channel—the real interest rate has to adjust so as to prevent consumption smoothing in equilibrium; a decline in output therefore coincides with an increase in the real interest rate. While observationally equivalent to the real rate channel, this is not how monetary policy transmits in New-Keynesian models. The critique applies equally to the basic framework without capital, which is a special case of the model with capital when capital adjustment costs are infinite.

The policy implication of our analysis is that (i) either monetary policy in actual economies does transmit through a real interest rate channel, but then the New-Keynesian model is not suitable for its analysis or (ii) the New-Keynesian model—for its micro-foundations of the price-setting behavior and internal consistency—is a useful description of actual economies, but then policy makers relying on this framework need to rethink the way monetary policy transmits into inflation and real activity.

Appendix

A.1. An alternative 'AS-AD' representation of the basic model

Sometimes, the basic model is interpreted in light of another popular 'AS-AD' representation, a representation in the real interest rate-output space. Here, the real interest rate plays the role of a price. Starting with the system

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \widehat{i}_t - E_t\pi_{t+1},$$

$$\pi_t = \Omega y_t + \beta E_t\pi_{t+1},$$

$$\widehat{i}_t = \nu\pi_t + \xi_t,$$

write the first equation as

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + E_t\widehat{r}_{t+1},$$

where

$$E_t\widehat{r}_{t+1} = \widehat{i}_t - E_t\pi_{t+1}.$$

Then eliminate π_t from the NKPC by substitution from the Taylor rule and rearrange terms to get

$$\widehat{i}_t - \nu\beta E_t\pi_{t+1} = \nu\Omega\widehat{y}_t + \xi_t$$

or

$$\widehat{i}_t - E_t\pi_{t+1} + (1-\beta\nu)E_t\pi_{t+1} = \nu\Omega\widehat{y}_t + \xi_t.$$

If $(1-\beta\nu) \approx 0$, we have a system

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + E_t\widehat{r}_{t+1}$$

$$E_t\widehat{r}_{t+1} = \nu\Omega\widehat{y}_t + \xi_t.$$

The first equation is downward sloping in the $(E_t\widehat{r}_{t+1}, \widehat{y}_t)$ space, giving rise to an interpretation as an AD curve, whereas the second equation is upward sloping, giving rise to an interpretation as an AS curve. The monetary policy shock, however, shows up in the second equation.

A.2. Proof of the Taylor Principle under sticky prices

Two equations characterize the equilibrium

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu\pi_t + \xi_t - E_t\pi_{t+1},$$

$$\pi_t = \Omega\widehat{y}_t + \beta E_t\pi_{t+1}.$$

Substituting out for $\widehat{y}_t$ gives

$$-\frac{1}{\Omega}\pi_t + \frac{\beta}{\Omega}E_t\pi_{t+1} = -E_t\left(\frac{1}{\Omega}\pi_{t+1} - \frac{\beta}{\Omega}E_{t+1}\pi_{t+2}\right) + \nu\pi_t + \xi_t - E_t\pi_{t+1}.$$

To determine the stability of this second-order stochastic differences equation, it is sufficient to study the dynamic properties of its deterministic homogenous part, which can be written as

$$\pi_t - \frac{1+\beta+\Omega}{1+\Omega\nu}\pi_{t+1} + \frac{\beta}{1+\Omega\nu}\pi_{t+2} = 0.$$

Using $\pi_t = L\pi_{t+1}$ and $\pi_t = L^2\pi_{t+2}$, this can be rewritten as

$$L^2 - \frac{1+\beta+\Omega}{1+\Omega\nu}L + \frac{\beta}{1+\Omega\nu} = 0.$$

The roots of this equation satisfy

$$\lambda_1 + \lambda_2 = \frac{1+\beta+\Omega}{1+\Omega\nu} \quad \text{and} \quad \lambda_1\lambda_2 = \frac{\beta}{1+\Omega\nu} > 0 \text{ and} < 1.$$

Based on the second equation, the two roots have the same sign and at least one is less than one. In order to have a unique stationary solution, both roots have to be less than one, so that inflation far in the future matters less and less for current inflation. Thus the sum of the two roots has to be less than two. This requires

$$\frac{1+\Omega}{1+\Omega\nu} < 1,$$

which holds when $\nu > 1$.

A.3. Forward-looking Taylor rule

The monetary policy rule is $\widehat{i}_t = \nu E_t \pi_{t+1} + \xi_t$. Under this rule, the equilibrium is characterized by

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu E_t\pi_{t+1} + \xi_t - E_t\pi_{t+1},$$

$$\pi_t = \Omega\widehat{y}_t + \beta E_t\pi_{t+1}.$$

Guessing $y_t = a\xi_t$ and $\pi_t = b\xi_t$, yields

$$b = -\frac{1}{(1-\rho)\frac{1-\beta\rho}{\Omega} + (\nu-1)\rho} < 0,$$

which differs only slightly from the solution for the contemporaneous Taylor rule by the second term in the denominator: $(\nu-1)\rho$ in the case of the forward-looking rule vs $(\nu-\rho)$ in the case of the contemporaneous rule.

A.4. Backward-looking Taylor rule

The monetary policy rule is $\widehat{\imath}_t = \nu\pi_{t-1} + \xi_t$. Under this rule, the equilibrium is characterized by

$$-\widehat{y}_t = -E_t\widehat{y}_{t+1} + \nu\pi_{t-1} + \xi_t - E_t\pi_{t+1},$$

$$\pi_t = \Omega\widehat{y}_t + \beta E_t\pi_{t+1}.$$

In this version of the model, there are two state variables: ξ_t and π_{t-1}. Suppose that equilibrium output and the inflation rate are linear functions of the state

$$y_t = a_0\pi_{t-1} + a_1\xi_t \quad \text{and} \quad \pi_t = b_0\pi_{t-1} + b_1\xi_t.$$

Using again the method of undetermined coefficients yields the following system of four equations in four unknowns

$$-a_0 = -a_0b_0 + \nu - b_0^2,$$

$$-a_1 = -a_0b_1 - a_1\rho + 1 - b_0b_1 - b_1\rho,$$

$$b_0 = \Omega a_0 + \beta b_0^2,$$

$$b_1 = \Omega a_1 + \beta b_0b_1 + \beta b_1\rho.$$

This system has a recursive structure: the first and third equations determine a_0 and b_0, independently of a_1 and b_1. Combining these two equations yields a third-order polynomial

$$\beta b_0^3 + (1 + \beta + \Omega)b_0^2 + b_0 + \Omega\nu = 0,$$

which has up to three distinct roots. For stability, we require either a unique root $|b_0| < 1$ or one root less than one in absolute value, to load on past inflation, and two roots greater than one in absolute value, to eliminate bubble terms. Once b_0 is determined, the other coefficients are determined uniquely as

$$a_0 = \frac{1}{\Omega}(b_0 - \beta b_0^2)$$

and

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 1-\rho & -(a_0 + b_0 + \rho) \\ -\Omega & (1 - \beta b_0 - \beta\rho) \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

Notice that neither b_0 nor a_0 depend on the persistence of the shock. Thus, the higher the persistence of the shock, the more are the dynamics of the endogenous variables governed by the exogenous shock and the less by past inflation.

For the baseline parameterization $\beta = 0.99$, $\eta = 1$, $\nu = 1.5$, $\theta = 0.7$ (and $\rho = 0.5$), the polynomial has a unique root, equal to -0.1478. Figure A.1 compares the impulse-responses under the backward-looking Taylor rule with those under the contemporaneous Taylor rule. The differences are small.

References

Bernanke, B. S., Gertler, M., 1995. Inside the black box: The credit channel of monetary transmission. Journal of Economic Perspectives 9, 27–48.

Bullard, J., 2015. Permazero as a possible medium-term outcome for the U.S. and the G-7. Speech at the FRB of Philadelphia Policy Forum, The New Normal for the U.S. Economy.

Bullard, J., Mitra, K., 2002. Learning about monetary policy rules. Journal of Monetary Economics 49, 1105–29.

Calvo, G., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 38398.

Christiano, L. J., Eichenbaum, M., Evans, C. L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1–45.

Clarida, R., Gali, J., Gertler, M., 1999. The science of monetary policy: A new-Keynesian perspective. Journal of Economic Literature XXXVII, 1661–1707.

Cochrane, J., 2011. Determinacy and identification with Taylor rules. Journal of Political Economy 119, 565–615.

Cochrane, J., 2016. Do higher interest rates raise or lower inflation? Mimeo, Hoover Institution.

Gali, J., 2015. Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications, 3rd Edition. Princeton University Press, Princeton.

Ireland, P., 2015. The monetary transmission mechanism. In: Blume, L., Durlauf, S. (Eds.), The New Pelgrave Dictionary of Economics, 2nd Edition. Pelgrave Macmillan, Hampshire.

Jermann, U. J., 1998. Asset pricing in production economies. Journal of Monetary Economics 41, 257–75.

Kocharlakota, N., 2016. Fragility of purely real macroeconomic models. NBER Working Paper 21866.

Kydland, F. E., Prescott, E. C., 1982. Time to build and aggregate fluctuations. Econometrica November, 1345–70.

Linde, J., Smets, F., Wouters, R., 2015. Challenges for macro models used at central bank. Mimeo.

Long, J. B., Plosser, C. I., 1983. Real business cycles. Journal of Political Economy 91, 39–69.

McCandless, G., 2008. The ABCs of RBCs: An Introduction to Dynamic Macroeconomic Models. Harvard University Press, Cambridge.

Mishkin, F. S., 1996. The channels of monetary transmission: Lessons for monetary policy. NBER Working paper No. 5464.

Rotemberg, J., 1982. Monopolistic price adjustment and aggregate output. Review of Economic Studies 49, 517–31.

Smets, F., Wouters, R., 2007. Shocks and frictions in U.S. business cycles: A Bayesian DSGE approach. American Economic Review 97, 586–606.

Tallarini, T. D., 2000. Risk-sensitive real business cycles. Journal of Monetary Economics 45, 507–532.

Walsh, C. E., 2010. Monetary Theory and Policy, 3rd Edition. MIT Press, Cambridge.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

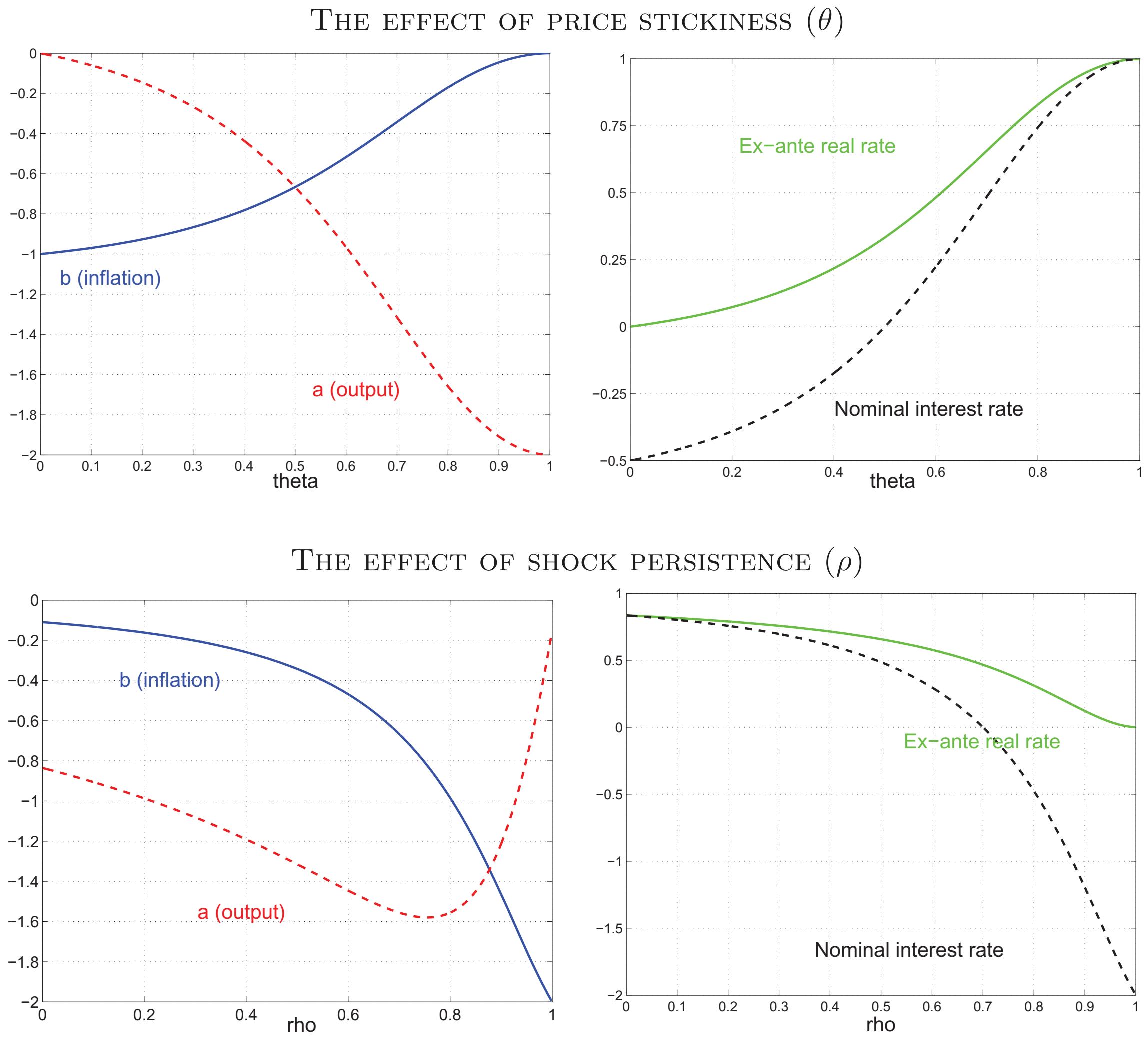

Figure 1: The effect of price stickiness (Calvo parameter θ) and of the persistence of the monetary policy shock (ρ) on the equilibrium coefficients (loading onto ξ_t) in the decision rule for output and the pricing functions for inflation, the nominal interest rate, and the ex-ante real rate. The baseline parameters are $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, and $\rho = 0.5$.

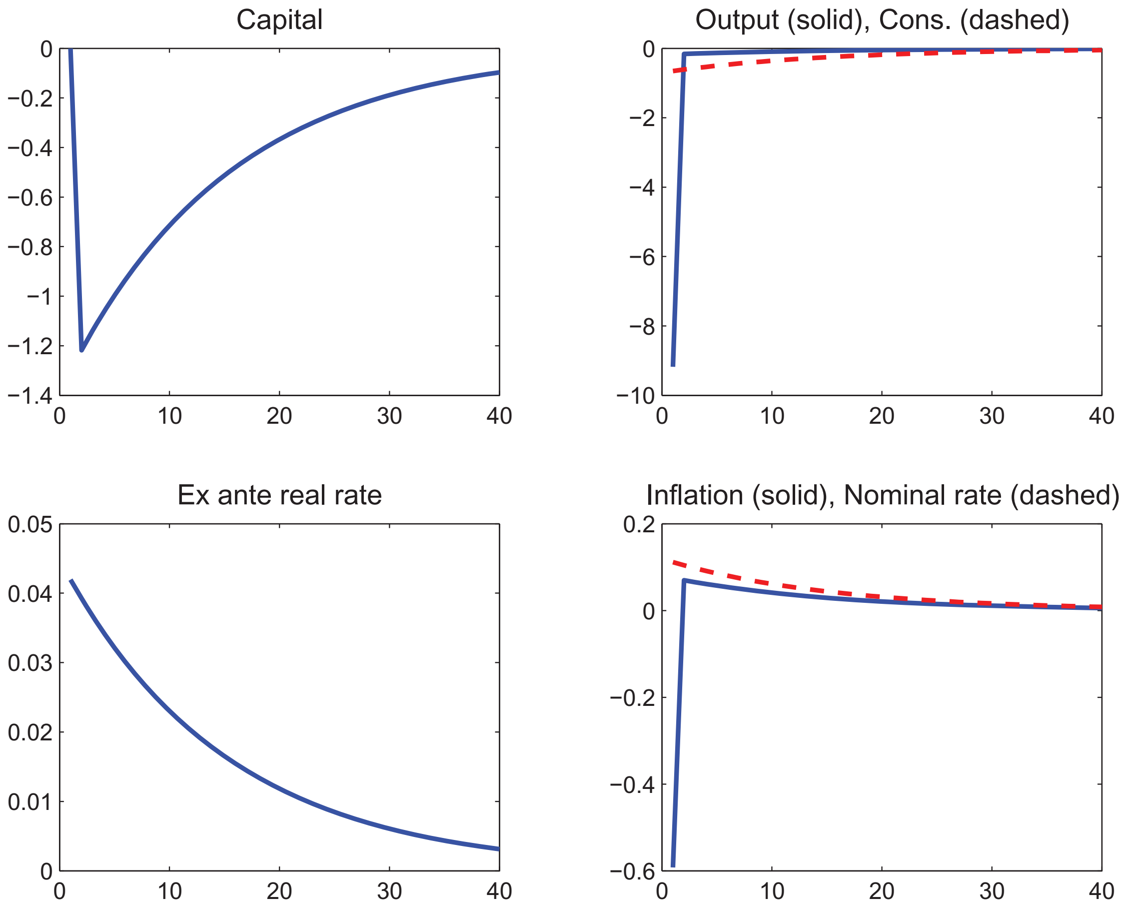

Figure 2: The model with capital, $\rho = 0$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

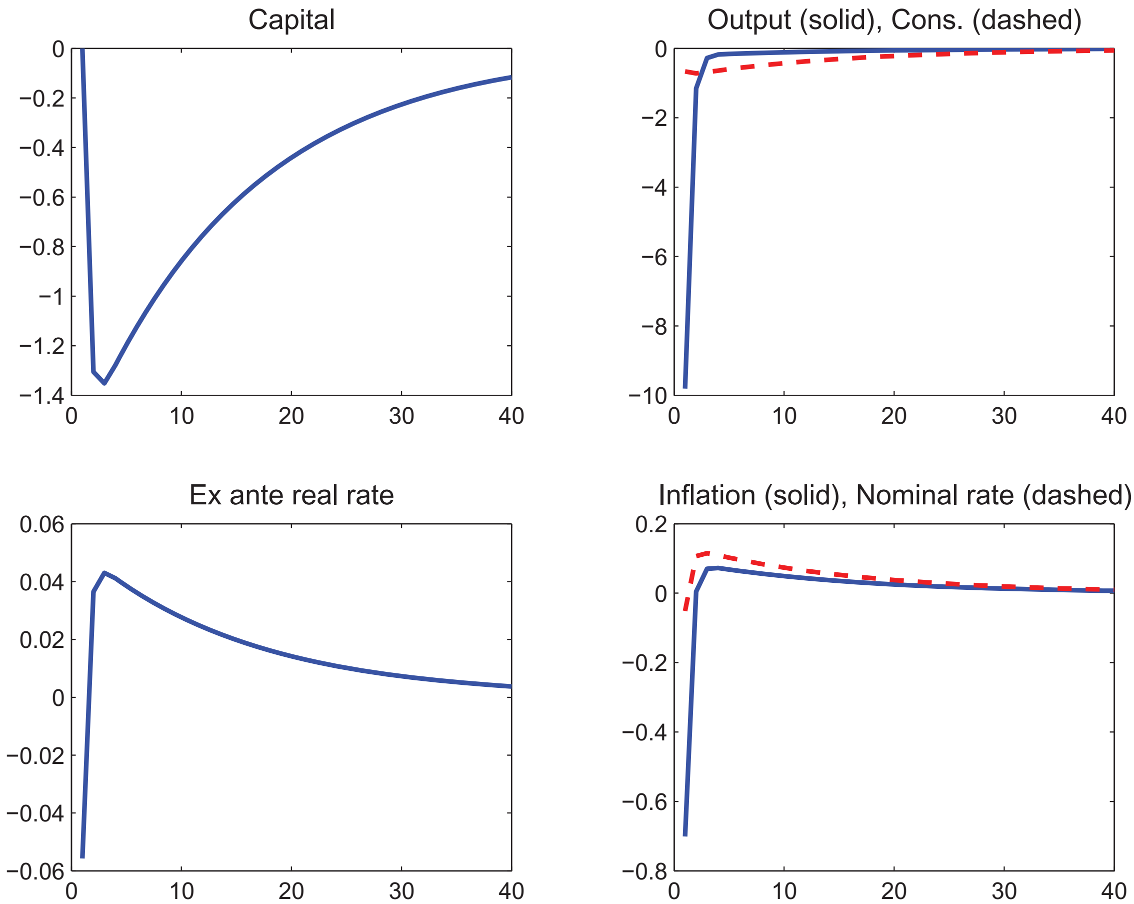

Figure 3: The model with capital, $\rho = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

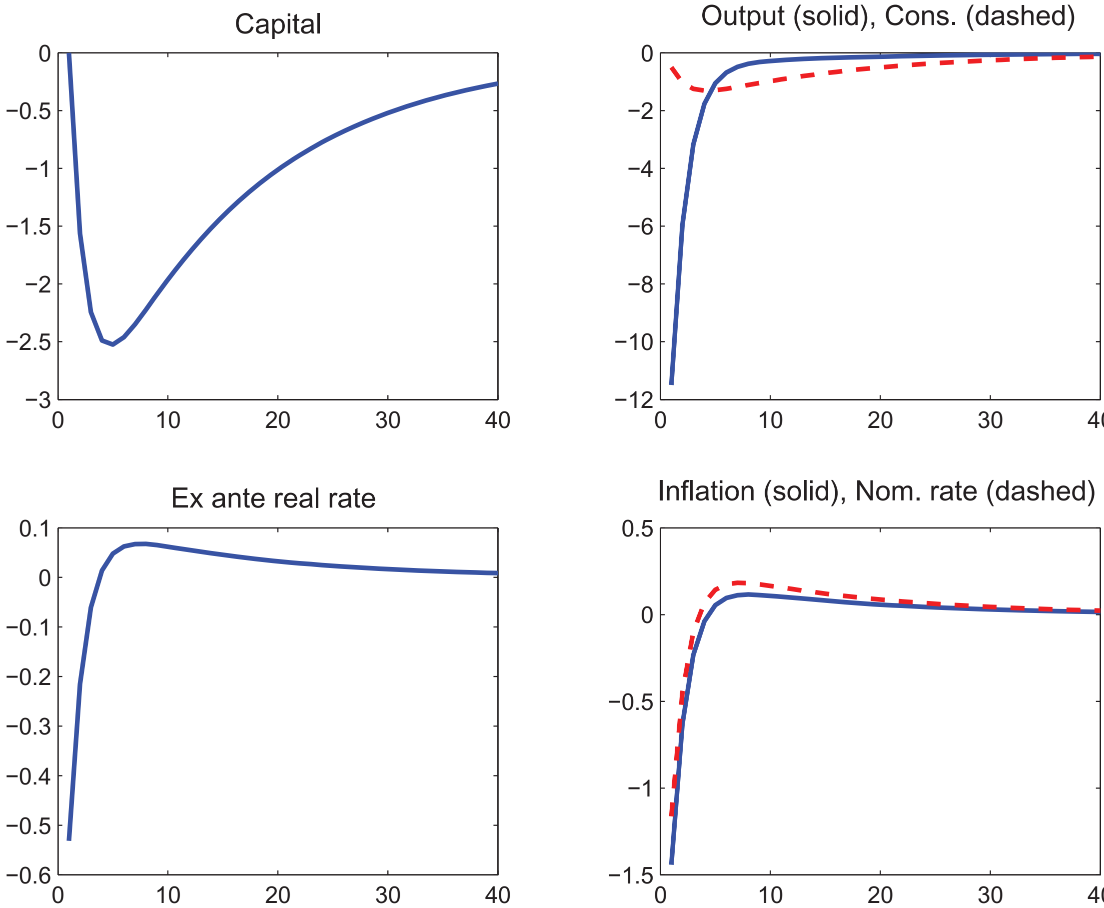

Figure 4: The model with capital, $\rho = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

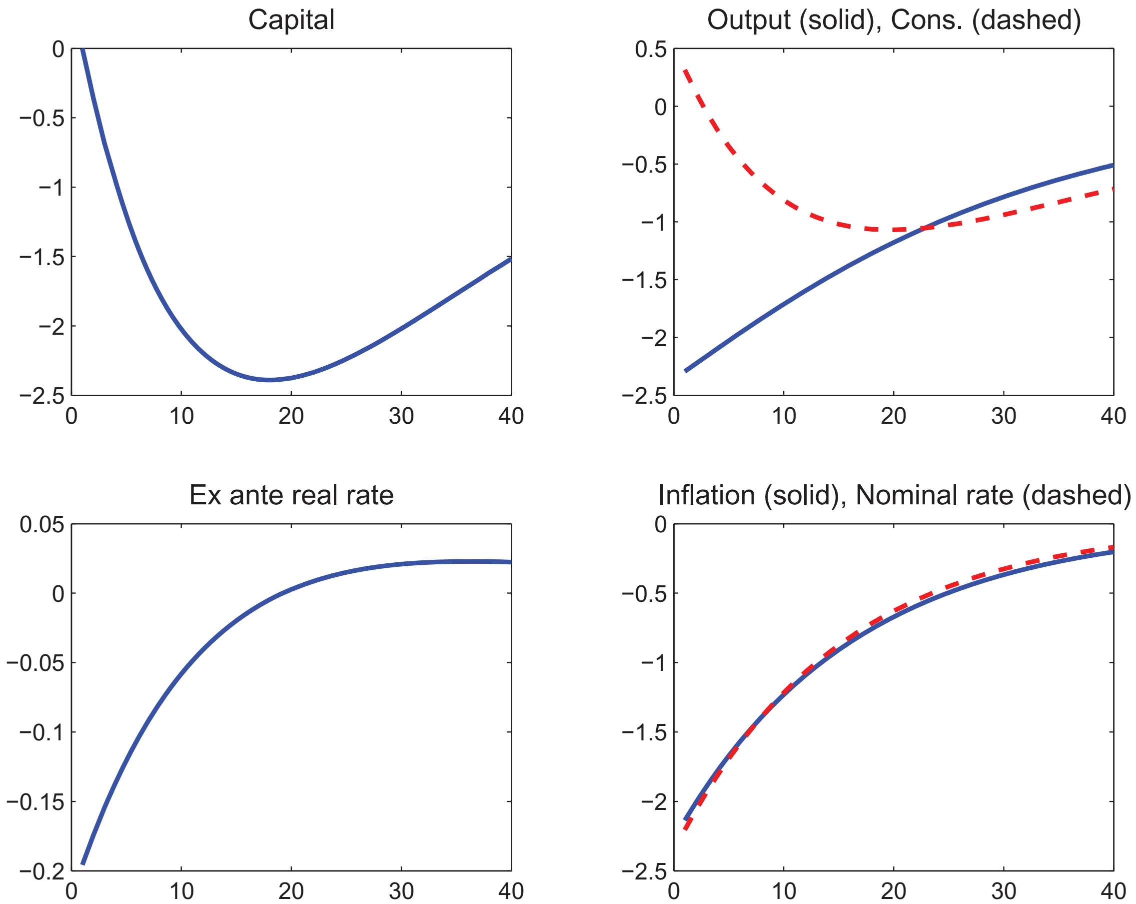

Figure 5: The model with capital, $\rho = 0.95$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

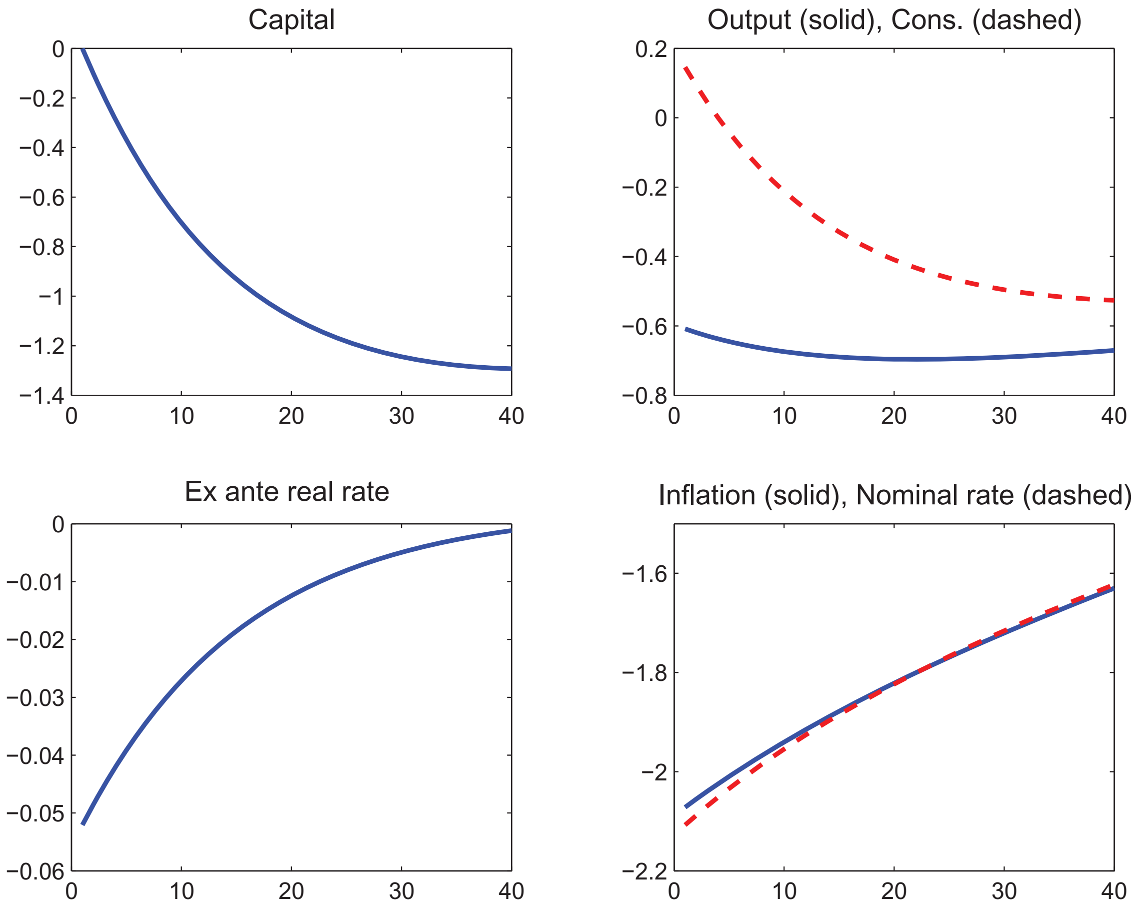

Figure 6: The model with capital, $\rho = 0.995$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, and $\delta = 0.025$.

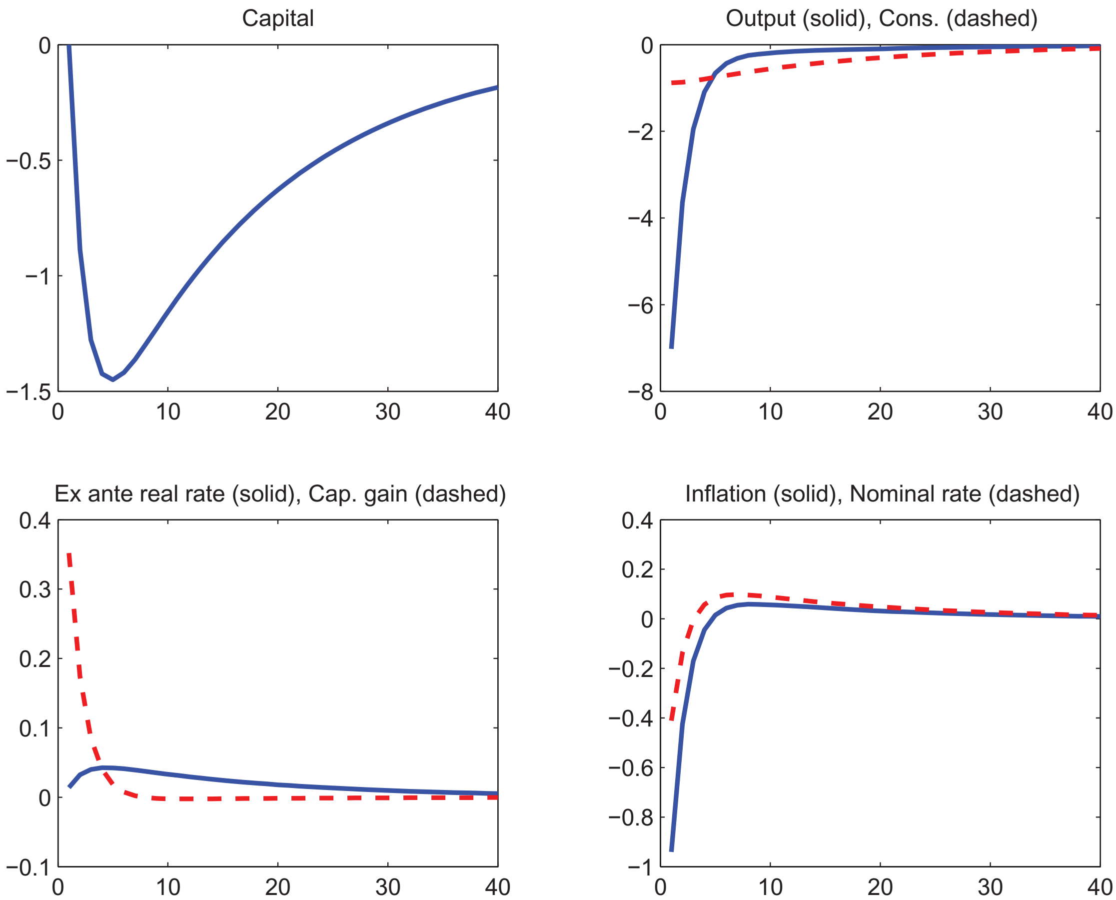

Figure 7: The model with capital adjustment costs, $\kappa = 0.1$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

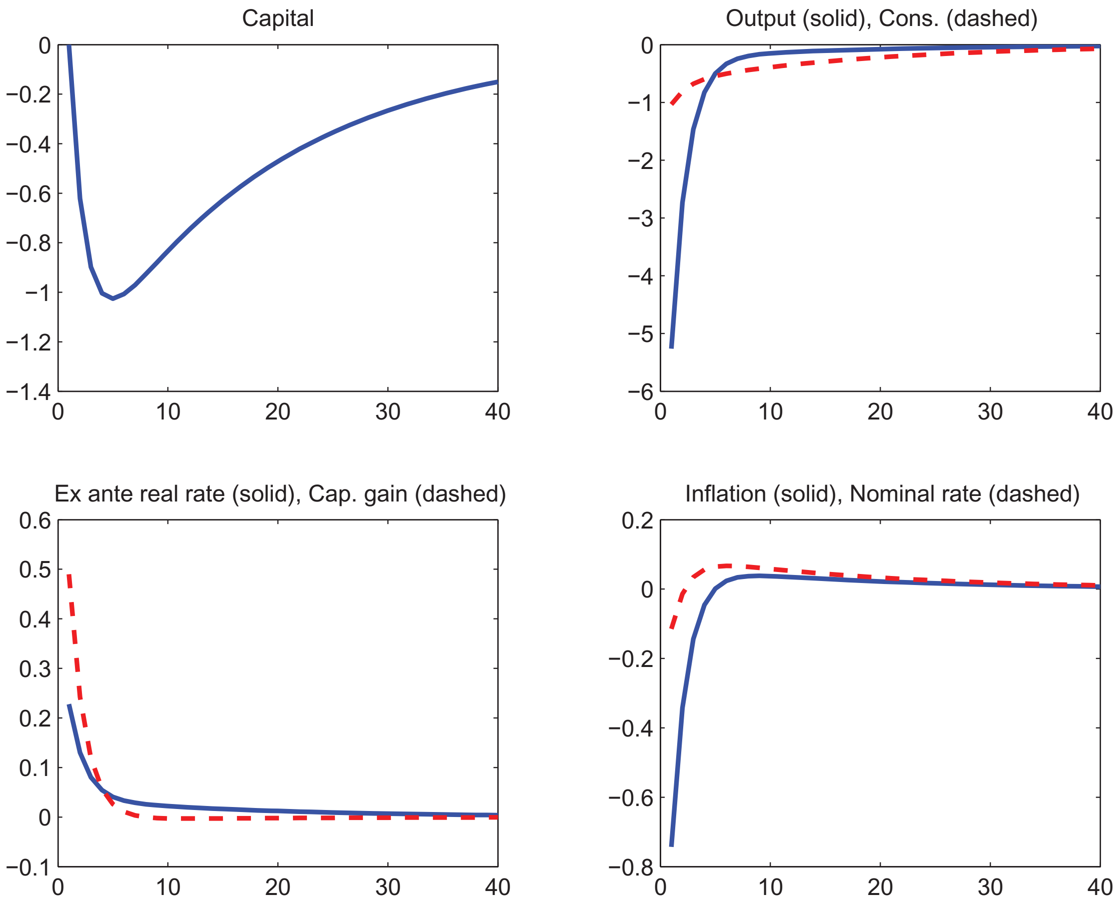

Figure 8: The model with capital adjustment costs, $\kappa = 0.2$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

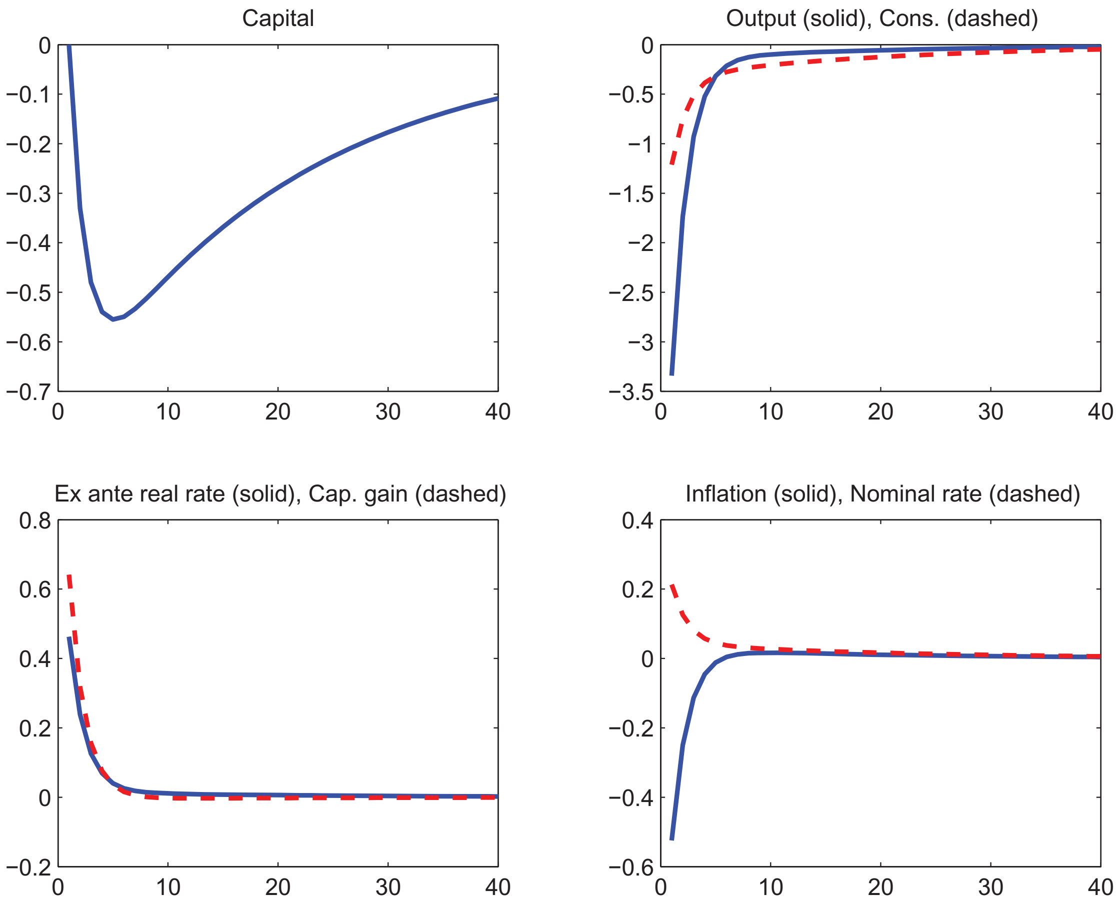

Figure 9: The model with capital adjustment costs, $\kappa = 0.5$. Responses to 1 percentage point increase in ξ_t. The remaining parameterization is: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\varepsilon = 0.83$, $\nu = 1.5$, $\alpha = 0.3$, $\delta = 0.025$, and $\rho = 0.5$.

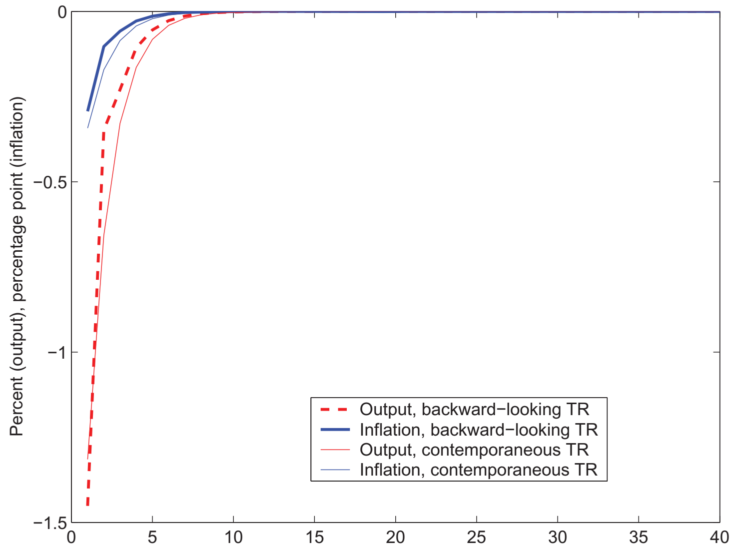

Fig. A.1. Backward-looking vs contemporaneous Taylor rule. Responses to a 1 percentage point increase in the monetary policy shock ξ_t. The parameterization is $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, and $\rho = 0.5$.

Queen Mary
University of London

This working paper has been produced by
the School of Economics and Finance at
Queen Mary University of London

School of Economics and Finance
Queen Mary University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 7356
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/research/workingpapers/