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Andrea Carriero, Haroon Mumtaz, Konstantinos Theodoridis and Angeliki Theophilopoulou

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Queen Mary University of London

The Impact of Uncertainty Shocks under Measurement Error. A Proxy SVAR approach

Andrea Carriero
Queen Mary, University of London

Haroon Mumtaz
Bank of England

Konstantinos Theodoridis
Bank of England

Angeliki Theophilopoulou
University of Westminister

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Abstract

A growing empirical literature has considered the impact of uncertainty using SVAR models that include proxies for uncertainty shocks as endogenous variables. In this paper we consider the possible impact of measurement error in the uncertainty shock proxies on the estimated impulse responses from these SVAR models. We show via a Monte Carlo experiment that measurement error can result in attenuation bias in the SVAR impulse responses. In contrast, the proxy SVAR that uses the uncertainty shock proxy as an instrument to identify the underlying shock does not suffer from this bias. Applying this proxy SVAR method to the Bloom (2009) data set results in estimated impulse responses to uncertainty shocks that are larger in magnitude and persistence than those obtained from a standard recursive SVAR.

JEL Classification: C15, C32, E32.

Keywords: Uncertainty Shocks, Proxy SVAR, Non-linear DSGE models.

1 Introduction

The recent financial crisis and ensuing recession has spurred a growing literature on the impact of uncertainty shocks on the economy. While a number of theoretical papers focus on modelling the channels of transmission of these shocks (see for example Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011) and Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011)), a large strand of this literature is empirical and focusses on estimating the percentage change in real activity following a shock to a measure of uncertainty via empirical models such as structural VARs (SVARs).

A seminal paper that applies a SVAR model to this issue is Bloom (2009). The author builds a dummy variable indicator of volatility shocks for the US. The indicator takes a value of one when a measure of options implied stock market volatility (VIX) significantly exceeds its mean. This indicator is then added as an endogenous variable in a SVAR model containing standard macroeconomic variables. The author finds that a shock to the volatility indicator leads to a 1% decline in industrial production. Baker et al. (2012) build an index of US economic policy uncertainty by using a combination of textual analysis, data on tax code expiration and dispersion of economic forecasts. In an SVAR model, a 102 point increase in this uncertainty index reduces industrial production by 2.5% while aggregate employment declines by 2.3 million. Leduc and Liu (2012) use survey based measures of uncertainty in an SVAR model and find that an increase in uncertainty depresses economic activity.

This strand of the literature on uncertainty has two common elements. First, these studies necessarily use proxies as measures of uncertainty as the true value is not directly observed. Second these proxies are entered directly into the VAR systems as endogenous variables.

In this paper we explore the consequences of these features for estimates of the impact of uncertainty. First, we use a simulation experiment to show that when the proxy for uncertainty differs from the true underlying measure, estimates of the impulse response from VARs that include the uncertainty measure are biased downwards. In contrast, structural VARs that use this measure as an 'external instrument' (this proxy SVAR approach was proposed in Stock and Watson (2008) and Mertens and Ravn (2012)) to identify the uncertainty shock are less susceptible to this bias. Second we re-visit the empirical work in Bloom (2009) using this proxy SVAR approach and find important differences in the estimates of the impact of uncertainty shocks and their importance over the business cycle. Using the proxy SVAR, the estimated impact of these shocks is larger and more persistent.

2 The SVAR approach to estimating the impact of uncertainty shocks

The existing empirical papers on the impact of uncertainty mentioned above consider the following SVAR models

$$Y_t = c + \sum_{j=1}^{P} B_j Y_{t-p} + A_0 \varepsilon_t \tag{1}$$

where Y_t is a matrix of endogenous variables that include a measure of uncertainty $\hat{\sigma}_t$ and a set of macroeconomic variables of interest. The structural shocks ε_t are related to the VAR residuals u_t via the relation $A_0 \varepsilon_t = u_t$ where A_0 is a matrix such that $VAR(u_t) = \Omega = A_0 A_0'$. In applications to uncertainty A_0 is typically chosen to be the Cholesky decomposition of Ω with $\hat{\sigma}_t$ usually ordered before the macroeconomic variables. For example the benchmark VAR in Bloom (2009) includes a stock price index, the dummy variable measure of uncertainty shocks, the federal funds rate, wages, CPI, hours, employment and industrial production.

Given that $\hat{\sigma}_t$ is a proxy for true underlying value for uncertainty, it is reasonable to assume a degree of measurement error. For example, the relationship between the constructed measure of uncertainty and its underlying value may be defined as $\hat{\sigma}_t = \sigma_t + \sigma_v v_t$ where v_t is standard normal. It is easy to see that the presence of measurement error would bias the estimate of the structural shock of interest. In addition, it is well known that OLS estimates of the VAR coefficients would suffer from attenuation bias due to the correlation between the RHS variables and the residuals introduced by the measurement error.

In contrast, the proxy SVAR approach is less susceptible to the measurement error problem. The underlying VAR model is given by the following equation:

$$\tilde{Y}_t = c + \sum_{j=1}^{P} B_j \tilde{Y}_{t-p} + \tilde{A}_0 \tilde{\varepsilon}_t \tag{2}$$

The matrix of endogenous variables $\tilde{Y}_t$ does not contain the constructed measure of uncertainty shocks directly but, instead, this is used as an instrument to estimate the structural shock of interest. Denoting the structural shock related to uncertainty by $\tilde{\varepsilon}_t^{\sigma}$ and the remaining shocks by $\tilde{\varepsilon}_t^{\bullet}$, this approach requires the proxy for uncertainty $\hat{\sigma}_t$ to satisfy the following conditions

$$\begin{aligned} E\left(\hat{\sigma}_t, \tilde{\varepsilon}_t^{\sigma}\right) &= \alpha \neq 0 \\ E\left(\hat{\sigma}_t, \tilde{\varepsilon}_t^{\bullet}\right) &= 0 \end{aligned} \tag{3}$$

The first expression in equation 3 states the instrument $\hat{\sigma}_t$ is correlated with the structural shock to be estimated, while the second expression rules out a correlation between $\hat{\sigma}_t$ and the remaining structural shocks and establishes exogeneity of the instrument. As shown in Stock and Watson (2008), Mertens and Ravn (2012) and Mertens and Ravn (2011), these conditions along with the requirement that the structural shocks $\tilde{\varepsilon}_t$ are contemporaneously uncorrelated can be used to derive a GMM estimator for the column of $\tilde{A}_0$ that corresponds to $\tilde{\varepsilon}_t^{\sigma}$.Mertens and Ravn (2011) also provide a measure of reliability of the instrument. The reliability statistic is a measure of the correlation between the instrument and the shock of interest and can be used to gauge the quality of the instrument.

Equation 3 imposes less stringent conditions on the quality of $\hat{\sigma}_t$. In particular, the only requirements are that $\hat{\sigma}_t$ is correlated with the shock of interest and uncorrelated with other shocks. These conditions can be satisfied even if $\hat{\sigma}_t$ is measured with error.

2.1 A simple Monte Carlo experiment

To gauge the possible impact of measurement error on VAR estimates of responses to uncertainty shocks we conduct a simple simulation experiment. In particular we generate data from a simple non-linear DSGE model where the variance of a structural shock of interest is characterised by stochastic volatility. We use the generated data to estimate the standard recursive VAR and the proxy SVAR. Using these VAR estimates, we check if the DSGE responses can be recovered using the empirical models.

2.1.1 The data generating process

The data is generated from a standard model of the monetary transmission mechanism. The model is derived in detail in appendix 1. Here we present an overview of the key characteristics.

The household side of the model consists of a continuum of households that consume, save in bonds, work and pay taxes. On the firms side, there is continuum of intermediate good producers that sell differentiated goods to final output producers. Intermediate good producers face a quadratic cost of adjusting prices (see Rotemberg (1982))–there is full indexation to either steady state value added inflation or to a lagged measure of inflation.

The government purchases units of final output and finances its expenditure using lump-sum taxes.

Finally, the monetary policy authority follows a rule for the nominal interest rate (R_t) that responds to deviations of CPI inflation (π_t) from its target (Π), and to deviations of output (y_t) from its steady-state value. This gives the following rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left(\frac{\pi_t}{\pi}\right)^{\frac{(1-\phi_R)\phi_\Pi}{4}} \left(\frac{y_t}{y}\right)^{(1-\phi_R)\phi_Y} \varepsilon_t^R \tag{4}$$

R is the steady state nominal interest rate that ensures that CPI inflation is at target in the long run. We assume that ε_t^R is a *heteroscedastic* interest rate shock, given by

$$\log \varepsilon_t^R = \rho_{\varepsilon^R} \log \varepsilon_{t-1}^R + \sigma_t^R \eta_t^R \tag{5}$$

The evolution of policy uncertainty is given by

$$\log \sigma_t^R = (1 - \rho_{\sigma^R})\, \sigma_{\varepsilon^R} + \rho_{\sigma^R} \log \sigma_{t-1}^R + \sigma_{\sigma^R} \eta_t^{\sigma^R} \tag{6}$$

The model, therefore, incorporates uncertainty in the monetary policy rule and this is the focus of the estimation on the generated data described below.

The model is solved using third-order perturbation methods (see Judd (1998)) since for any order below three, stochastic volatility shocks that we are interested in do not enter into the decision rule as independent components. The calibration of the parameters is standard and is described in Table 2.

We use artificial data for $\sigma_t^R, y_t, \pi_t, R_t$ and the structural shock to volatility $M_t = \sigma_{\sigma^R} \eta_t^{\sigma^R} + v_t$, with $v_t \sim N(0, \sigma_v^2)$. Note that v_t is assumed to be a measurement error, and when this equals zero, the structural shock is measured perfectly. In the experiment below we assume that σ_v^2 varies between 0 and 5. Note that the calibrated value σ_{σ^R} equals 1 and therefore these values for the variance of the measurement error cover a large range.

The data is generated for 2200 periods with the first 2000 observations discarded to remove the impact of starting values. The final 200 observations are used to estimate the following VAR models:

First we estimate the standard recursive SVAR:

$$Y_t^{(1)} = c + \sum_{j=1}^{P} B_j Y_{t-j}^{(1)} + A_0^{(1)} \varepsilon_t^{(1)} \tag{7}$$

where $Y_t^{(1)} = \{M_t, y_t, \pi_t, R_t\}$ and $A_0^{(1)}$ is obtained via a Cholesky decomposition with the ordering of the variables as in $Y_t^{(1)}$. This mimics the kind of SVAR models considered for example in Bloom (2009) where a measure of the uncertainty shock enters the VAR system directly.

The second empirical model is the proxy SVAR that takes the following form

$$Y_t^{(2)} = d + \sum_{j=1}^{P} D_j Y_{t-j}^{(2)} + A_0^{(2)} \varepsilon_t^{(2)} \tag{8}$$

where $Y_t^{(2)} = \{\sigma_t^R, y_t, \pi_t, R_t\}$. The first shock in $\varepsilon_t^{(2)}$ is the volatility shock and is identified by using the following moment restrictions:

$$E\left(M_t, \varepsilon_{1,t}^{(2)}\right) = \alpha \neq 0 \tag{9}$$

$$E\left(M_t, \varepsilon_{i,t}^{(2)}\right) = 0, i = 2, 3, 4 \tag{10}$$

In figure 1 we consider the scenario where measurement error equals zero and $M_t = \sigma_{\sigma^R} \eta_t^{\sigma^R}$. The dotted lines in the figure present the response of the macroeconomic variables to a one unit increase in policy uncertainty in the DSGE model. The blue line and the shaded area present the median and the 90% error band (based on 1000 replications) of the same response estimated using the proxy and recursive SVARs. When the uncertainty shock is observed without error, the two SVAR models deliver a similar performance. The median response of Y and R from the SVAR models tracks the true response closely. While the contemporaneous SVAR response of π is close to the DSGE response, there appears to be a slight downward bias at medium horizons. This probably reflects the fact that the linear VAR models abstract from the non-linear dynamics present in the reduced form of the DSGE model obtained via third order perturbation.

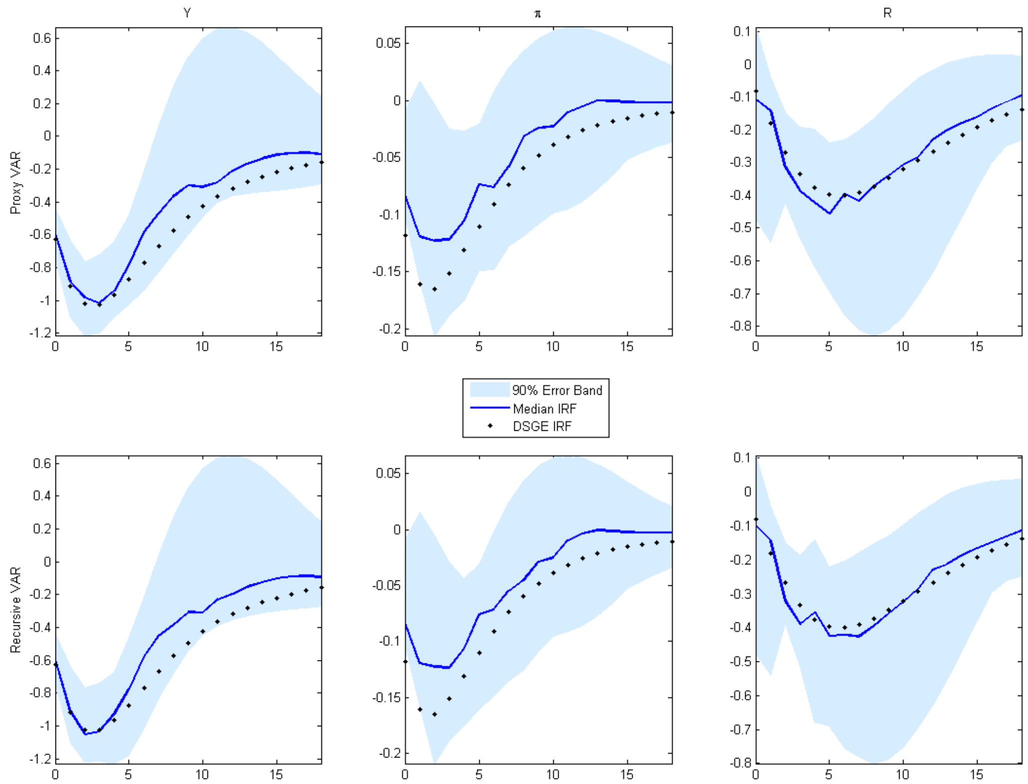

Figure 1: A comparison of SVAR and DSGE impulse responses to policy uncertainty shocks in the absence of measurement error.

Figure 2 presents the results of the simulation when measurement error is present. Each panel of the figure reports the median bias in the SVAR impulse responses (Z-axis) as the variance of the measurement error increases in importance relative to σ_{σ^R}. The impact of the measurement error on the estimated responses from the proxy SVAR is muted. As discussed above, this is because the mis-measured uncertainty shock does not enter the VAR system directly but is used as an instrument. In contrast to these results, there is a clear attenuation bias evident in the responses estimated using the recursive SVAR model. Even for relatively small values of $\sigma_v/\sigma_{\sigma^R}$ the estimated impulse response is less negative than the DSGE response, with this difference much more pronounced than the proxy SVAR and the recursive VAR that

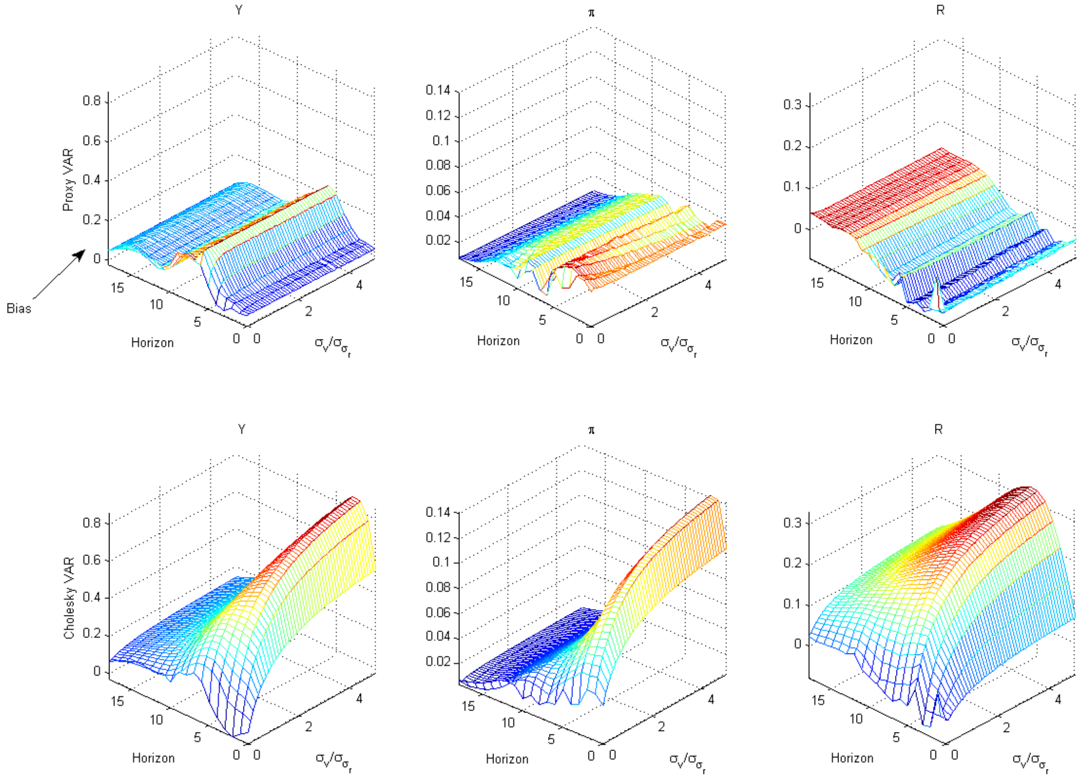

Figure 2: Bias in SVAR impulse responses to policy uncertainty shocks under measurement error.

allows for measurement error. Note that this bias is present both at horizon 0 and beyond indicating that the estimates of the contemporaneous impact matrix and the VAR coefficients are affected by the measurement error problem.

3 Empirical results: The Bloom (2009) VAR model re-visited

In this section, we re-estimate the VAR model in Bloom (2009) and consider the possible role of measurement error. Bloom (2009) estimate a variety of VAR models that contain the following variables (in this order): (1) log S&P500 stock market index ,

(2) an indicator of shocks to stock-market volatility, (3) Federal Funds Rate, (4) log average hourly earnings , (5) log consumer price index , (6) hours, (7) log employment , and (8) log industrial production. The benchmark volatility shock indicator is constructed by the author to correspond to periods when stock market volatility is above a given threshold. As shown in figure 1 of Bloom (2009), the constructed shocks correspond closely to periods of economic and/or political turbulence. The different VAR specifications in Bloom (2009) correspond to different measures of stock market volatility shocks constructed by the author. The author shows, however, that the key results remain unchanged across the different measures. In particular, all the VARs that include the different measures result in very similar responses for industrial production.

In the left panel of figure 3 we produce the results in Bloom (2009) using the benchmark volatility shock measure employed in Bloom (2009)[1]. The panel shows the response to 1 unit volatility shock.[2]. As in Bloom (2009), both industrial production and employment decline. Both variables increase subsequently, with the rise in industrial production statistically significant. Note that the response of the stock price index shows a similar pattern–there is an initial decline and a subsequent bounceback. The shock also results in a fall in hours, the federal funds rate, wages and CPI, with the decline in these variables lasting for less than a year.

The right panel of figure 3 shows the impulse responses to a one unit volatility shock from a version of the VAR that uses the benchmark Bloom volatility shock measure as an instrument. In particular, we estimate the following VAR(12) model:

$$Z_t = d + \sum_{j=1}^{12} D_j Z_{t-j} + A_0 \varepsilon_t \tag{11}$$

where Z_t contains the VIX stock market volatility index and the 7 macroeconomic and financial variables included in the Bloom (2009) VAR model above. For convenience,

[1] We use the data and data transformations employed by Bloom (2009). The data can be downloaded at http://www.stanford.edu/~nbloom/replication.zip. The data is monthly and available from 1962M7 to 2008M6. Following Bloom (2009) we employ a lag length of 12.

[2] Note that we scale the underlying volatility shock measure to have the same units as the VIX. Therefore the 1 unit volatility shock is comparable across the two VAR models presented in this section.

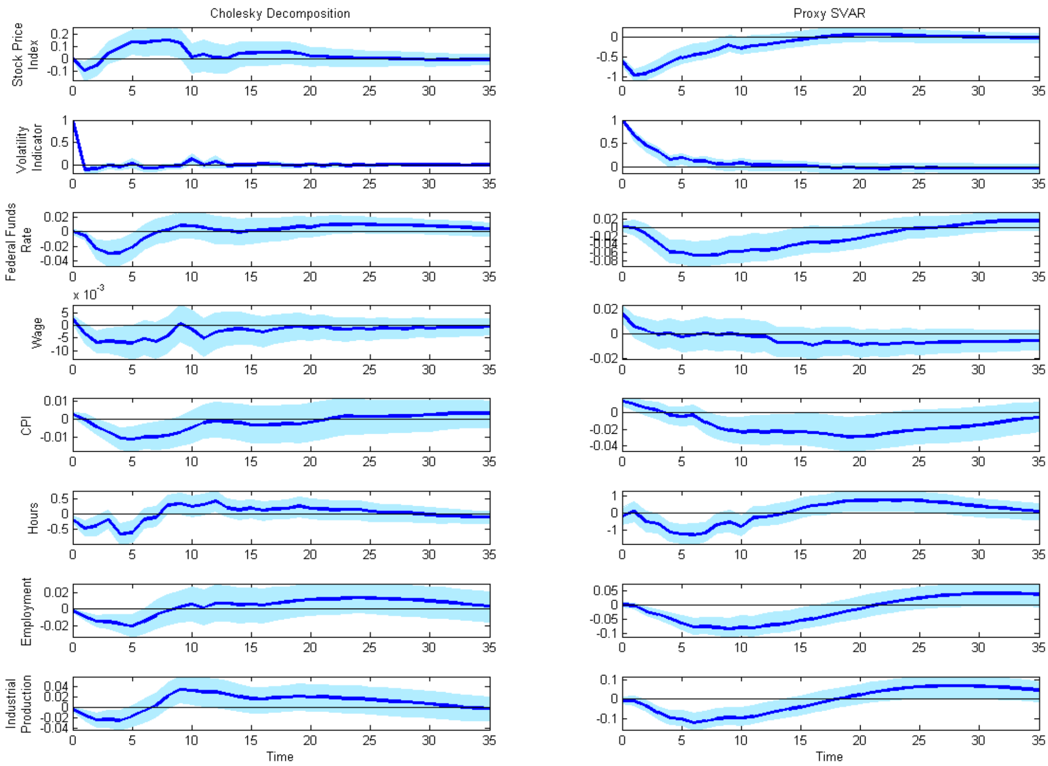

Figure 3: Impulse response to a volatility shock using a recursive VAR and the Proxy SVAR. The shaded area represents the 90% confidence interval estimated using a wild bootstrap with 10,000 replications. The shock is scaled to the units of the VIX index.

the VIX is ordered first in the VAR model. The shock to volatility is identified using the following moment conditions

$$E\left(M_t, \varepsilon_{1,t}\right) = \alpha \neq 0 \tag{12}$$

$$E\left(M_t, \varepsilon_{i,t}\right) = 0, i = 2, 3, ..8 \tag{13}$$

where M_t is the benchmark volatility shock measure employed by Bloom (2009) in their VAR model.[3] Thus unlike the VAR model in Bloom (2009), M_t does not enter directly into the VAR, but is used as an instrument to estimate the first column of the

[3] We show in the sensitivity analysis that similar results are obtained using the alternative definitions of the volatility shock employed by Bloom (2009).

A_0 matrix and the volatility shock $\varepsilon_{1,t}$. The reliability statistic is estimated to be 0.6, suggesting a high correlation between M_t and the shock of interest. The estimated impulse responses suggest a larger response to the volatility shock. For instance, while the stock market index declines by 0.1% in Bloom's SVAR, the estimated response is around 1% in the proxy SVAR. Note also that this response in the proxy SVAR is persistent and lasts for about one year. Similarly, the responses of employment, CPI, Federal Funds rate and industrial production from the proxy SVAR are estimated to be larger and more persistent. Note that the bounceback in industrial production occurs at around the 20 month horizon, rather than after about 6 months in the Cholesky case.

These results match those obtained in the Monte-Carlo simulation described above. In particular, the responses to the volatility shock appear to be smaller in size when the shock measure is included directly into the VAR system. In contrast, when the shock measure is used as an instrument to identify the volatility shock in the proxy SVAR, the estimated impulse responses are larger in size and persistence. This is consistent with the attenuation bias revealed by the Monte-Carlo experiment.

Figure 4 plots the contribution of the estimated volatility shock to the main variables using the two VAR models. The black lines in the figure represent the de-trended data for each variable. The blue and the red lines are the counterfactual estimates of these series assuming the presence of only the volatility shock, where the two VAR models are used, respectively, to identify the volatility shock. The volatility shock estimated using the proxy SVAR appears to be more important. For instance, the contribution of this shock in the benchmark VAR model to fluctuations in the stock market index is relatively small. In contrast, the results from the proxy SVAR imply that this shock accounts for a large proportion of the movement in this variable. This feature is especially apparent during the large troughs in the stock market index in the early and mid-1970s, the early 1980s and during the recession in the early 2000's. Similarly, the proxy VAR suggests that the volatility shock makes a more important contribution to employment and industrial production, especially during the recession in the early years of the last decade. This estimated contribution is smaller when the benchmark VAR model is used.

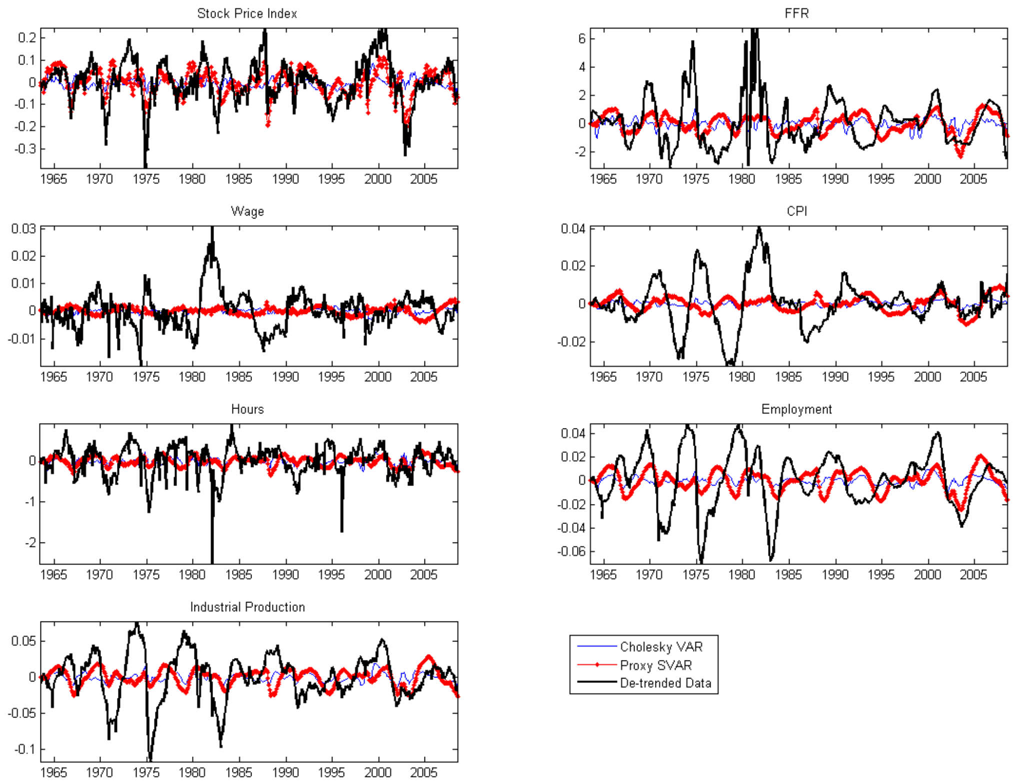

Figure 4: Historical Decomposition: The contribution of the volatility shock using the Cholesky VAR and the Proxy SVAR.

3.1 Sensitivity Analysis

We test the robustness of the empirical results along two dimensions. First, we re-estimate the proxy SVAR using the additional volatility shock measures considered by Bloom (2009). Second, we consider alternatives to the VIX index included in the proxy SVAR.

Figure 5 shows the impact of a 1 unit volatility shock from the benchmark VAR model and using the additional volatility shock measures constructed by Bloom (2009) as instruments. The figure shows that the impulse response of the key variables are similar in magnitude and persistence across the different instruments.

Figure 6 shows the impulse response to a volatility shock using two alternatives

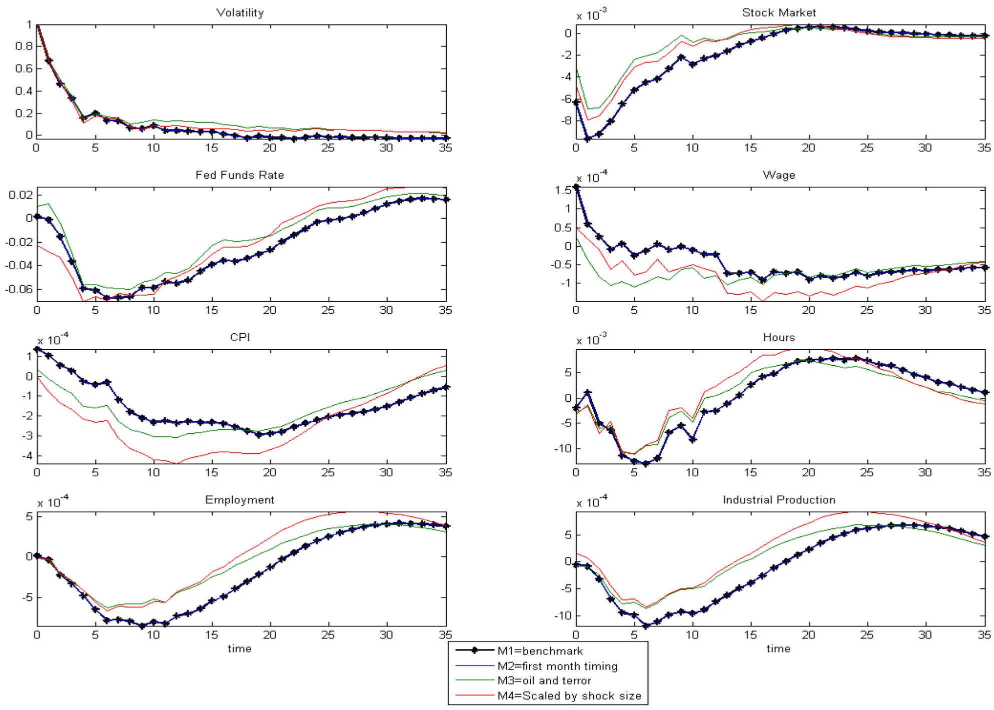

Figure 5: The impact of a volatility shock from the proxy SVAR using different shock measures. 'First month timing' refers to the indicator where shocks are dated by first month rather than the highest month. 'Oil and Terror' refers to shocks associated with war, terrorism and oil. The final measure is the benchmark shock scaled by the size of movement in stock market volatility. These different shock measures are described in appendix A1 of Bloom (2009).

to the VIX measure of volatility.[4] First, we employ a non-parametric estimate of stock market volatility where the monthly standard deviation is estimated as the sample standard deviation of the the daily observations within that month. Second, we use a stochastic volatility model to estimate the volatility. This model is defined as $\Delta S_t = h_t^{1/2} e_t$ where $e_t \tilde{} N(0,1)$, $h_t = \alpha + \vartheta h_{t-1} + g^{1/2} v_t$, $v_t \tilde{} N(0,1)$ and S_t denotes the monthly S&P500 stock price index.[5] Figure 6 shows that the impulse responses using the alternative measures are similar to the benchmark case.

[4]The benchmark shock measure is used as an instrument for each model.

[5]The model is estimated using the MCMC algorithm described in Jacquier et al. (2004).

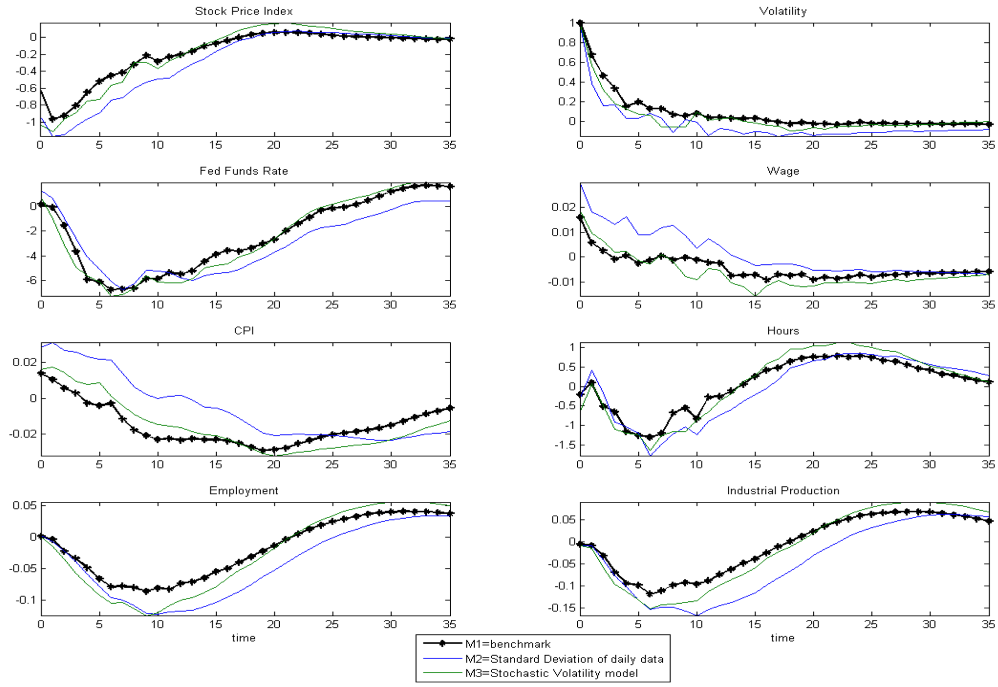

Figure 6: Impulse responses to the volatility shock from the proxy SVAR using different measures of stock market volatility. Model M2 uses a monthly volatility measure based on standard deviation of daily stock returns. Model M3 uses a measure estimated via a stochastic volatility model.

4 Conclusions

This paper re-considers the SVAR approach to estimating the impact of volatility shocks and investigates the role played by measurement error. First by estimating VAR models on data simulated from a DSGE model with stochastic volatility, we show that estimates of impulse responses to volatility shocks from a recursive SVAR suffer from a downward bias in the presence of measurement error. In contrast, the proxy SVAR produces impulse responses close to the underlying DSGE responses. This is because the proxy SVAR uses the volatility shock as an instrument rather than an endogenous variable, thus ameliorating the effect of measurement error.

An application of the proxy SVAR to the Bloom (2009) data-set results in responses to the volatility shock that are larger in magnitude to those obtained using the recursive SVAR employed in Bloom (2009). Similarly, a historical decomposition exercise using the volatility shock estimated from the proxy SVAR suggests a larger role for this shock than implied by the recursive SVAR. These results suggest that it may be important to account for measurement error when considering the impact of volatility using VAR models that include a proxy for volatility shocks.

Appendix 1: the nonlinear DSGE model

This appendix describes the model used in the Monte Carlo simulation exercise discussed in section 2.1.

4.1 Households

There is a continuum of households defined on the zero one interval $j \in [0,1]$. Households consume, save in bonds, work and pay taxes. The functional utility function:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \varepsilon_t^C \left\{ \frac{\left(c_{j,t} - hc_{j,t-1}\right)^{1-\sigma_C}}{1-\sigma_C} - \psi \frac{{l_{j,t}}^{1+\sigma_L}}{1+\sigma_L} \right\} \tag{14}$$

where $c_{j,t}$ denotes consumption, $l_{j,t}$ denotes the household's labour supply, σ_C is the inverse of elasticity of intertemporal substitution, σ_L is the elasticity of labour supply with respect to the real wage, h is the habit formation parameter and $\mathrm{E}_t[\cdot]$ is the expectations operator. The period utility function is affected by a consumption shock $\left(\varepsilon_t^C\right)$ that increases the utility of current consumption relative to future consumption:

$$\log \varepsilon_t^C = (1-\rho_{\varepsilon^C}) \log \varepsilon^C + \rho_{\varepsilon^C} \log \varepsilon_{t-1}^C + \sigma_{\varepsilon^C} \eta_t^C \tag{15}$$

Utility is maximised with respect to the budget constraint:

$$P_t w_{j,t} l_{j,t} + R_{t-1} P_{t-1} B_{j,t-1} + P_t \xi_{j,t} = P_t c_{j,t} + P_t B_{j,t} + P_t \tau_{j,t}$$

where $w_{j,t}$ denotes real wages, $B_{j,t}$ is the value of real government debt, $\xi_{j,t}$ stands for real profits, $\tau_{j,t}$ is lump-sum taxes, P_t is the consumer price index and R_t is the gross value of the nominal interest rate. In real terms the budget constraint becomes:

$$w_{j,t} l_{j,t} + \frac{R_{t-1}}{\Pi_t} B_{j,t-1} + \xi_{j,t} = c_{j,t} + B_{j,t} + \tau_{j,t} \tag{16}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is consumer price inflation. If we focus on a symetric equilibrium then the maximisation of (14) subject to (16) with respect to $c_{j,t}$, $B_{j,t}$ and l_{jt} deliver the marginal utility function, the consumption Euler equation and the labour supply, respectively:

$$\frac{\varepsilon_t^C}{(c_t - hc_{t-1})^{\sigma_C}} - E_t \frac{\beta h \varepsilon_{t+1}^C}{(c_{t+1} - hc_t)^{\sigma_C}} = \lambda_t \tag{17}$$

where is the Lagrange multiplier associated with the budget constraint

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right\} \tag{18}$$

$$w_t \lambda_t = \varepsilon_t^C \psi l_t^{\sigma_L} \tag{19}$$

4.2 Firms

Intermediate Good Producers There is a continuum of intermediate good producers defined on the interval 0 to 1 and subindexed by n. The production function of firm n is given by

$$y_{n,t} = A_t l_{n,t} \tag{20}$$

where A_t is a temporary total factor productivity shock common to all intermediate good producers:

$$\log A_t = (1 - \rho_A) \log A + \rho_A \log TFP_{t-1} + \sigma_A \eta_t^A \tag{21}$$

Final Good Producers Intermediate firms sell a differentiated goods to final output producers with firm j facing demand

$$y_{n,t} = \left(\frac{P_{n,t}}{P_t} \right)^{-\varepsilon_p} y_t \tag{22}$$

from the producers of final goods, where $P_{n,t}$ is the price set by firm n, y_t and P_t are the aggregate quantity and price of final good and ε_p is the elasticity of substitution. The aggregate quantity of final good is given by the CES aggregator

$$y_t = \left[\int_0^1 \left(y_{n,t}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \right) dn \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{23}$$

and the final good price index is given by

$$P_t = \left[\int_0^1 (P_{n,t})^{1-\varepsilon_p} dn \right]^{\frac{1}{1-\varepsilon_p}} \tag{24}$$

Intermediate good producers face a quadratic cost of adjusting prices, where ϕ_p is the parameter that determines the degree of price stickiness in this sector and where there is full indexation to either steady state value added inflation (Π^*), or to a lagged measure of inflation. Each firm n solves the following problem

$$\max_{P_{n,t},l_{n,t}} \mathrm{E}_t \left[\sum_{i=0}^{\infty} \beta^i \frac{\lambda_{t+i}}{\lambda_t} \left(P_{n,t} y_{n,t} - P_{n,t} w_t l_{jt} - \Phi_p \left(\frac{P_{n,t}}{P_{n,t-1}}, P_{n,t} y_{n,t} \right) \right) \right] \tag{25}$$

subject to the production function (20) and demand equation (22). The first order with respect to labour delivers the labour demand equation and again focusing only on a symmetric equilibrium

$$mc_t = \frac{w_t}{A_t} \tag{26}$$

where mc_t is the shadow cost of one additional unit of output for the firm, which equals the real marginal cost. While the inflation Philips curve equation is derived from the maximisation of the profit function with respect to $P_{n,t}$

$$1 = \frac{\varepsilon_p}{\varepsilon_p - 1} mc_t - \phi_p \left\{ \left(\zeta_t^p - 1 \right) \zeta_t^p - \beta \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{t+1} y_{t+1}}{y_t} \left(\zeta_{t+1}^p - 1 \right) \zeta_{t+1}^p \right] \right\} \tag{27}$$

where

$$\zeta_t^p = \frac{\pi_t}{\pi^{1-\iota_p} \pi_{t-1}^{\iota_p}} \tag{28}$$

4.3 The government sector

The government purchases g units of final output and finances its expenditure using lump-sum taxes. The government's budget constraint is

$$g y_t + \frac{R_{t-1}}{\pi_t} B_{t-1} = \tau_t + B_t \tag{29}$$

4.4 Monetary policy

The monetary policy maker follows a rule for the nominal interest rate that responds to deviations of CPI inflation from its target (Π), and to deviations of output from its steady-state value. This gives the following rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\phi_R} \left(\frac{\pi_t}{\pi} \right)^{\frac{(1-\phi_R)\phi_\Pi}{4}} \left(\frac{y_t}{y} \right)^{(1-\phi_R)\phi_Y} \varepsilon_t^R \tag{30}$$

R is the steady state nominal interest rate that ensures that CPI inflation is at target in the long run, and ε_t^R is a conditional heteroscedastic interest rate shock, given by

$$\log \varepsilon_t^R = \rho_{\varepsilon^R} \log \varepsilon_{t-1}^R + \sigma_t^R \eta_t^R \tag{31}$$

The evolution of policy uncertainty is given by

$$\log \sigma_t^R = (1 - \rho_{\sigma^R})\, \sigma_{\varepsilon^R} + \rho_{\sigma^R} \log \sigma_{t-1}^R + \sigma_{\sigma^R} \eta_t^{\sigma^R} \tag{32}$$

4.5 Aggregation and market-clearing conditions

After some algebra the market clearing condition is

$$y_t = c_t + g y_t + \frac{\phi_p}{2} \left(\Pi_t - \Pi\right)^2 y_t \tag{33}$$

For simplicity and without loss of generality we set g equal to zero implying that

$$y_t = c_t + \frac{\phi_p}{2} \left(\Pi_t - \Pi\right)^2 y_t \tag{34}$$

Table 1 collects all the equation required for the solution of the model

Table 1: DSGE Nonlinear Model Equations

Equations	Mnemonics
Marginal Utility of Cosumption	$\frac{\varepsilon_t^C}{(c_t - hc_{t-1})^{\sigma_C}} - E_t \frac{\beta h \varepsilon_{t+1}^C}{(c_{t+1} - hc_t)^{\sigma_C}} = \lambda_t$
Consumption Euler Equation	$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right\}$
Labour Supply	$w_t \lambda_t = \varepsilon_t^C \psi l_t^{\sigma_L}$
Production Function	$y_t = A_t l_t$
Labour Demand	$mc_t = \frac{w_t}{A_t}$
Price Phillips Curve	$1 = \frac{\varepsilon_p}{\varepsilon_p - 1} mc_t - \phi_p \left\{ \left(\zeta_t^p - 1\right) \zeta_t^p - \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{t+1} y_{t+1}}{y_t} \left(\zeta_{t+1}^p - 1\right) \zeta_{t+1}^p \right] \right\}$
Policy Rule	$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left(\frac{\pi_t}{\pi}\right)^{\frac{(1-\phi_R)\phi_\Pi}{4}} \left(\frac{y_t}{y}\right)^{(1-\phi_R)\phi_Y} \varepsilon_t^R$
Market Clearing Condition	$y_t = c_t + \frac{\phi_p}{2} \left(\Pi_t - \Pi\right)^2 y_t$
Consumption Preference Shock	$\log \varepsilon_t^C = (1 - \rho_{\varepsilon^C}) \log \varepsilon^C + \rho_{\varepsilon^C} \log \varepsilon_{t-1}^C + \sigma_{\varepsilon^C} \eta_t^C$
Temporary TFP Shock	$\log A_t = (1 - \rho_A) \log A + \rho_A \log TFP_{t-1} + \sigma_A \eta_t^A$
Policy Shock	$\log \varepsilon_t^R = \rho_{\varepsilon^R} \log \varepsilon_{t-1}^R + \sigma_t^R \eta_t^R$
Policy Uncertainty	$\log \sigma_t^R = (1 - \rho_{\sigma^R})\, \sigma_{\varepsilon^R} + \rho_{\sigma^R} \log \sigma_{t-1}^R + \sigma_{\sigma^R} \eta_t^{\sigma^R}$

4.6 Steady-States

The steady-states of this model are readily derived. From (27) we obtain the solution for marginal cost

$$mc = \frac{\varepsilon_p - 1}{\varepsilon_p} \tag{35}$$

The steady-state of hours has been calibrated to 1/3 and this delivers the steady-state value of output using the intermediate goods production function (20)

$$y = Al \tag{36}$$

From the Euler equation (18) we get the value of the nominal interest rate

$$R = \frac{\pi}{\beta} \tag{37}$$

The steady state value of consumption is given by using the market clearing condition (34)

$$c = y$$

The marginal utility expression (17) is used to derive the steady-state value of Lagrange multiplier

$$\lambda = \frac{\varepsilon^C (1 - \beta h)}{((1 - h)\, c)^{\sigma_C}} \tag{38}$$

We use the labour demand equation (26) to obtain wages

$$w = Amc \tag{39}$$

Finally, we solve for ψ that ensures that $l = 1/3$ at the steady-state

$$\psi = \frac{w\lambda}{\varepsilon^C l^{\sigma_L}} \tag{40}$$

4.7 Solution and Calibration

The model is solved using third-order perturbation methods (see Judd (1998)) since for any order below three stochastic volatility shocks that we are interested in do not enter into the decision rule as independent components. One difficulty of using these higher-order solution techniques is that paths simulated by the approximated policy function often explode. As it is explained by Kim et al. (2008) regular perturbation approximations are polynomials that have multiple steady state and could yield unbounded solutions. In other words, this approximation is valid only locally and along the simulation path we may enter into a region where its validity is not preserved anymore.

To avoid this problem Kim et al. (2008) suggest to 'prune' all those terms that have an order that is higher than the approximation order, while Andreasen et al. (2013) show how this logic can be applied to any order. Although there are studies that question the legitimacy of this approach (see den Haan and de Wind (2010)), it has by now been widely accepted as the only reliable way to get the solution of n – where $n > 1$ – order approximated DSGE model. Finally, due to model's nonlinearity we employ the procedure introduced to Koop et al. (1996) (known as generalised impulse responses) to study the agents' dynamic responses to structural disturbances.

The calibration of the model is fairly standard, similar to Justiniano et al. (2010) we set $\sigma_C = 1$ (log utility), while following Christiano et al. (2005) and Adolfson et al. (2007) we set $\sigma_L = 1$. The values of $\epsilon_p = 21$, $\phi_p = 236.1$, $\beta = 0.9945$, $\pi = 1.0045$ ($R = \frac{\pi}{\beta} = 4\%$) are taken from the study of Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011), the habit formation parameter value ($h = 0.75$) is the one estimated by Christiano et al. (2005) and it is also used by Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011). The policy reaction function parameters ($\phi_R = 0.83$, $\phi_\pi = 2.03$ and $\phi_y = 0.3$) and the coefficients of the stochastic process ($\rho_{\varepsilon^C} = 0.18$, $100\sigma_{\varepsilon^C} = 0.23$, $\rho_A = 0.95$, $100\sigma_A = 0.45$, $\rho_{\varepsilon^R} = 0.15$, $100\sigma_{\varepsilon^R} = 0.24$) are those estimated by Smets and Wouters (2007). Finally, the parameters of the policy uncertainty process have been set $\rho_{\sigma^R} = 0.9$ and $100\sigma_{\sigma^R} = 1$. Table 2 provides a summary of the calibration.

Table 2: DSGE Parameters

Mnemonics	Values	Source
β	0.9945	Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011)
σ_C	1	Justiniano et al. (2010)
σ_L	1	Christiano et al. (2005)
ϵ_p	21	Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011)
ϕ_p	236.1	Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011)
ι_p	0.75	Benati (2008)
π	1.0045	Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011)
ϕ_R	0.81	Smets and Wouters (2007)
ϕ_π	2.03	Smets and Wouters (2007)
ϕ_y	0.3	Smets and Wouters (2007)
h	0.75	Christiano et al. (2005)
ρ_{ϵ^C}	0.18	Christiano et al. (2005)
ρ_A	0.95	Christiano et al. (2005)
ρ_{ϵ^R}	0.15	Christiano et al. (2005)
ρ_{σ^R}	0.9	Mumtaz and Theodoridis (2012)
$100\sigma_{\epsilon^C}$	0.23	Christiano et al. (2005)
$100\sigma_A$	0.45	Christiano et al. (2005)
$100\sigma_{\epsilon^R}$	0.24	Christiano et al. (2005)
$100\sigma_{\sigma^R}$	1	Mumtaz and Theodoridis (2012)

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Queen Mary
University of London

This working paper has been produced by
the School of Economics and Finance at
Queen Mary, University of London

School of Economics and Finance
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 7356
Fax: +44 (0)20 8983 3580
Web: www.econ.qmul.ac.uk/papers/wp.htm