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The Paradoxes of the Liberal Ethics of Non-interference

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Abstract

We analyse the liberal ethics of non-interference applied to social choice. Two liberal principles capturing non-interfering views of society, inspired by J.S. Mill's conception of liberty are examined, which capture the idea that society should not penalise agents after changes in their situation that do not affect others. Two paradoxes of liberal approaches are highlighted. First, it is shown that a restricted view of non-interference, as reflected in the Individual Damage Principle, together with some standard axioms in social choice leads straight to welfare egalitarianism. Second, it is proved that every weakly pareian social welfare ordering that satisfies a general principle of noninterference must be dictatorial. Both paradoxes raise important issues for liberal approaches in social choice and political philosophy.

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1 Introduction

Liberal approaches are prominent in political philosophy and in the social sciences. Liberal principles in philosophy and social choice tend to express some notion of *individual autonomy or freedom*. In Sen's ([22]) seminal work and the literature that it sparked, the core of liberalism is formalised as the requirement that an individual has a sphere of complete control on society's preferences over minimal subsets of social states. Despite its intuitive appeal, the latter axiom of minimal liberty conflicts with some other desirable requirements of social orderings. In particular, as argued by Riley ([20]), the essence of Sen's impossibility result can be interpreted as identifying a contradiction between the Pareto principle as a *democratic* rule (unanimity) and a respect for absolute *libertarian rights*, given unrestricted domain. Given the vast literature on liberal approaches originated from Sen's contribution, one would imagine that little more can be added that is significantly enlightening. Yet, the problems raised by Sen remain as puzzles for liberal approaches and although a number of solutions have been proposed, "There seems every reason to doubt the possibility of a comprehensive and coherent modern philosophy of liberalism" (Dunne, [6], p.54, quoted in [20], p.1135).

This paper provides various additional arguments and formal results that throw new light on liberal approaches in social choice theory and political philosophy, and it raises further doubts on liberalism by highlighting some new paradoxes and inconsistencies, building on our own work in [13] and [14]. To be specific, we analyse two liberal principles suited to Bergson-Samuelson Social Welfare Orderings (henceforth, SWOs), which are meant to incorporate a liberal ethics of noninterference, whose intellectual origin can be traced back to John Stuart Mill's famous essay '*On Liberty*'. Our analysis aims to capture some of the fundamental insights of the Harm Principle, which arguably lie at the core of all liberal approaches, namely the idea that society should not interfere with individual choices *whenever the latter have no effect on others*.

The Harm Principle is conceptually different from Sen's idea of minimal liberty in two respects. First, personal autonomy is explicitly related to the effects of individual choices on other people's welfare. Second, it is explicitly formulated as a negative freedom. Our axioms stipulate conditions under

which agents have a power to *veto* society from expressing some preferences that interfere with an agent's personal sphere when nobody else is affected. In its negative prescription, the Harm Principle arguably captures an even more fundamental aspect of liberalism, and one that is often endorsed even by people who do not subscribe to a liberal philosophy. For example, in the Brundtland report, sustainable development is defined as "development that meets the needs of the present without compromising the ability of future generations to meet their needs" (Brundtland et al., 1987: 43; Wolf, [31] provides a thorough analysis of intergenerational justice from a liberal perspective).

The two axioms analysed in this paper capture different aspects of non-interfering views, but both lead to results that are problematic for liberal theories. First, we consider the Individual Damage Principle, which embodies a more limited version of noninterference and obtain an arguably counterintuitive result: together with the standard axioms of Strong Pareto and Anonymity, the Individual Damage Principle leads straight to the strongly egalitarian leximin SWO. This result is interesting from both a formal and a theoretical viewpoint, because the leximin SWO is characterised by appealing to axioms without any clear egalitarian content.

Next, we extend our notion of noninterference and obtain a rather striking impossibility result. Any SWO that satisfies the standard axiom of Weak Pareto Optimality and our general Principle of Non-Interference must be dictatorial.

Thus, our analysis arguably identifies a deep and fundamental inconsistency in liberal approaches: if the general Principle of Non-Interference is adopted, an impossibility result emerges. However, even if the bite of noninterference is significantly limited, and a possibility result obtains, the social welfare ordering is not distinctly liberal. Indeed, as shown in [14], a similar conclusion is obtained if a number of alternative ways of constraining the Principle of Non-Interference are considered. Besides, it is unclear that a convincing theoretical justification for such restrictions can be provided *a priori, from a liberal perspective*.

We focus on societies with only two individuals but all results can be generalised to economies with $N > 2$ agents (see, [13] and [14]. See also [11] and [12], where economies with an infinite number of agents are also analysed). We present here some new results and conceptual arguments, and provide a number of new graphical proofs and examples.

The conclusions of the analysis may seem to lead to a dead end, and one

may ask what is the solution to the inconsistencies in liberal thinking highlighted in this paper. However, first, these results “can also be interpreted as guideposts to the creation of a more acceptable theory of liberal democracy” (Riley, [20], pp.1135-6). Second, following Sen ([23], p.28), one may argue that “there is nothing much to ‘resolve’ anyway. The impossibility ... just brings out a conflict of principles ... The really interesting issues relate to the implications of the conflict. There are implications both for the evaluation of outcomes and for choice of decision procedures”.

The rest of the paper is organised as follows. Section 2 discusses the Individual Damage Principle and Section 3 proves that, together with Anonymity and Strong Pareto, it characterises the leximin SWO. Section 4 analyses the Principle of Non-Interference and Section 5 proves that any weakly paretian SWO that satisfies the Principle of Non-Interference must be dictatorial. Section 6 concludes.

2 The Individual Damage Principle

To illustrate the basic idea behind the first liberal principle analysed in this paper, called the Individual Damage Principle, we use a simple example.

Example 1 *The Victorian library example*

Consider a society with two agents, me and you, and consider the social choice between two welfare profiles p and q . All entries in the table below, including α and β , represent welfare levels:

	Me	You
p	10	α
q	7	β

Suppose that (for whatever reason) p is socially preferred to q .

Now consider a change to two different profiles p' and q' . In both of these new profiles, I have lost some welfare compared to the corresponding profiles p and q .

	Me	You
p'	8	α
q'	6	β

Your welfare at p' and q' is identical to your welfare at p and q , respectively. What should the social preference between p' and q' be?

We suggest that in *any* non-interfering view of society, the following argument should apply. I suffered damage at p' and q' compared to p and q , while you are exactly in the same conditions. The cause of such damage is not clarified. It might, or might not, have been a result of my negligence. Perhaps it was just bad luck. Perhaps I did not work hard enough, or failed to insure myself. What is for sure is that you were not involved either by my bad luck or by my negligence. As far as you are concerned, the choice between p and q is exactly the same as the choice between p' and q' . So society should *not reverse* the strict preference between p and q to a strict preference for q' over p' (a switch to indifference is admitted). Switching the social preference to q' would imply a *further* damage for me in addition to the loss of welfare, a social punishment. Not only did I lose welfare, but the social choice moves against me in addition. But there is no information whatever to justify such a punishment. In the absence of such a justification, society should not interfere.

To see the implications of the Individual Damage principle, suppose that the numerical example depicts social choice concerning access to public libraries in the Victorian era. Suppose that I enjoy reading, but I am relatively poor and can only buy very few books: 10 is my welfare from situation p where, among other things, access to public libraries is free, while 7 is my welfare in situation q in which society takes an action that reduces my welfare, such as a restriction of access to public libraries - either in the form of a general policy (e.g. census-based membership, given that I am poor); or ad hominem (e.g., directly against me, for whatever reason). All other things, except access to libraries, are held constant at p and q .

Next, consider p' and q' , and the corresponding welfare profiles, which may arise in a number of possible ways: I may have lost my job, due to a general economic downturn, and without any fault of my own; or my house may have burnt down, and I had failed to insure myself; or perhaps I have found "Lady Chatterley's Lover" on a bench and read it. Then 8 is my welfare in situation p' in which I have access to libraries, while 6 is my welfare in situation q' , in which society sanctions me - for being unemployed, or for losing my house, or for reading the book - by restricting my access to libraries. Note that my welfare decreases at p' and q' : this is obvious if I have been laid off, or my house has burnt down, but it may also happen when I

find the book if, after reading it, I utterly dislike it.¹ It is also important to note that the book is not borrowed from the library and thus the decision concerning access bears no direct relation with my reading the book. In any case, the Individual Damage Principle stipulates that *if* society chose open access when I was employed, or my house was still intact, or I had not read the book, then my bad luck, or negligence, or even my action of reading the book should not lead to a change in social preferences: in any case, I should not be punished given that nobody's welfare is involved. It is important to note, though, that the principle is not liberal in the sense of prescribing that access to public libraries be open, that is, the axiom does *not* require that p be socially preferred to q .

Furthermore, the principle aims to capture only *some* aspects of noninterfering views of society, and it does not impose any constraints on social choice in a number of cases. If social preferences over p and q were different, then the principle would be silent. Maybe society should *compensate* me for the loss by switching from a preference for q over p to a preference for p' over q' . But maybe not. Similarly, if I was better off at q' than at p' , the principle would also be silent on whether I should be compensated by a switch in social preferences to coincide with my own. Further, the principle is silent in cases when *your* welfare changes, too: if you are affected, this may provide *prima facie* ground for interference. But maybe not. Because we do not aim to provide a complete liberal theory of government, the axiom is appropriately silent on these controversial cases.²

To be sure, from a non-liberal perspective, there may be many reasons to switch preferences against me: for instance, total or average utility may now be lower at p' than at q' , or maybe p' may be more egalitarian than q' . But all these statements incorporate nonliberal concerns, which we explicitly

¹It may even be the case that, even if I enjoy the book, some of my acquaintances decide to stop socialising with me, so that again the net effect of reading the book on my welfare may be negative. In Mill's analysis of the Harm Principle, generalised moral reprobation and enforcement via moral coercion following my reading the book are not justified, and the Harm Principle is meant to protect agents from the tyranny of public opinion as much as from legal coercion. Yet, people have no duty to interact with me and in reading the book I may incur "the inconveniences which are strictly inseparable from the unfavourable judgement of others" (Mill, *On Liberty*, chapter IV; in [30]). It is worth noting that people may decide to stop meeting me *even if* their welfare is not directly affected by knowing that I read it.

²For a discussion of the necessary and sufficient conditions for interference in classical liberalism, see, e.g., Berger ([2]) and Rees ([19]).

wish to set aside.

By imposing a restriction on society's choices when individual actions and events do not affect others, we believe that the Individual Damage Principle incorporates a concern for negative freedom (from interference) with a clear liberal content. Yet, given the relevance of nonwelfaristic concerns in liberal approaches, it may be worth defending our choice to focus on a welfaristic framework. The following arguments are also relevant for the discussion of the principle of Non-Interference in section 4 below.

First, it is important to emphasise again that we do not aim to provide a complete theory of liberalism, and our axioms capture only *some* implications of liberal views. The axiom is formulated as a purely negative prescription: it requires noninterference in a set of social settings, but it does not characterise all the situations in which noninterference is morally required, and thus it provides no guidance as to when interference is indeed legitimate. Nonwelfaristic concerns are typically central in liberal theories in the treatment of the latter problems, which involve issues such as, for example, the distinction between the so-called "private and public versions of the harm principle" (Danley, [4], p.420) or the difference between harm caused by action and harm caused by inaction (see, e.g., Berger, [2], p.255ff). By assuming that other agents' welfare is constant, the axiom assumes *all* types of harm away and makes the latter distinctions arguably less relevant *in our analysis*.

One of the main arguments against a purely welfaristic framework is that not all losses in welfare count as harms, both in Mill's approach and in plausible versions of liberalism. For example, losses incurred as the effect of the operation of competitive markets (with losers and winners) do not count as harms in Mill (see, e.g., the discussion in Arneson, [1], section 1). Although this point is important in general, it is unclear that it invalidates our analysis, because our axioms state that if the welfare of all other agents is unchanged - *whatever the source of the change* in one agent's welfare - then there is no ground for interference, in the sense of reversing social preferences. Of course, there may be changes in the welfare of other agents that a liberal would not count as harms and would not modify the conclusion that q' should not be strictly preferred to p' . However, given the purpose of our analysis, it is desirable to have weaker axioms that rule out *all* welfare changes.³

³It is worth noting that if agent i 's welfare losses were caused by some other agents' choices, there may be some room for nonwelfaristic concerns. However, one may argue that the existence of actions taken by agent j that affect agent i , but do not modify j 's welfare at all, is highly implausible. Perhaps more importantly, even in such case, it would

Second, even if liberal theories cannot be fully captured in a purely welfaristic framework, and typically involve issues of rights and entitlements, welfare considerations arguably play a central role in the major approaches, and the Harm Principle itself can be plausibly interpreted in welfaristic terms. Although a thorough interpretation of the classic texts in liberal philosophy is beyond the scope of this paper, some points are worth making here in support of our welfaristic approach.

In the opening pages of his essay on liberty, J.S. Mill explicitly notes that “It is proper to state that I forego any advantage which could be derived to my argument from the idea of abstract right as a thing independent of utility” (*On Liberty*, Chapter I; p.136 in [30]). Even more explicitly, Mill goes on to say that “I regard utility as the ultimate appeal on all ethical questions; but it must be utility in the largest sense, grounded on the permanent interests of man as a progressive being” (*On Liberty*, Chapter I; p.136 in [30]). And although the consistency between his analysis of liberal principles and his utilitarian approach has been questioned, a number of scholars have forcefully argued that Mill is best interpreted as a “utilitarian liberal”(see, e.g., Wallack, [29], and the contributions in Gray and Smith, [8]). Furthermore, in Mill’s own formulation, the Harm principle is concerned with actions that affect other people’s welfare, and not only their rights or freedoms, and “the formula ‘coercion to prevent coercion’ used by some liberals is distinct from Mill’s ‘coercion to prevent harm’” (McCloskey, [16], p.147).

Despite the emphasis on rights, welfaristic considerations are central also in entitlement theories of justice, and this is true in particular for those aspects that relate to noninterference. Consider, for example, the famous Lockean proviso, according to which the initial acquisition of unowned resources is justified as long as ‘enough and as good is left for others’ of the resources appropriated. Locke himself defends the relevance of the proviso by claiming that, given the abundance of natural resources at his time, acquisition could take place “without straitening anybody” (quoted in Wolf, [31], p.798), which naturally lends itself to a welfaristic interpretation. Miller ([15], p.406-7) actually argues that the relevant part of the proviso is ‘as good’, rather than ‘enough’, and that ‘as good’ should be interpreted in welfaristic terms. In Nozick’s theory, “the crucial point is whether an appropriation of an unowned object worsens the situation of others” (Nozick, [18], p.175) and many

seem all the more desirable to impose that q' be not strictly preferred to p' , as a *minimal* requirement of justice.

authors have argued that a welfaristic approach provides the most promising interpretation of the proviso, which solves a number of conceptual difficulties, such as those related to heterogeneous land or to the rights of future generations (see, e.g., Sanders, [21]; Wolf, [31]), which would make it virtually impossible to satisfy the proviso.⁴ In general terms, the rationale of the Lockean proviso may be stated as requiring that “no individual be made worse off (in some appropriate sense) by the appropriation (compared with the situation before appropriation)” (Vallentyne, [28], p.7).⁵

In sum, the point is not that a welfaristic framework captures all that is relevant in a liberal, or libertarian, approach: this is, of course, untrue. It seems, however, reasonable to say at least that a welfaristic interpretation of liberal noninterfering views does capture many of the relevant issues at stake and in the most plausible, and widespread, construals of liberal approaches, welfare information plays a pivotal role in the analysis of noninterference.

A different objection to our axiomatic treatment of noninterference is worth noting here. It might be argued that all meaningful individual acts have an effect on others, so that our axioms are empirically irrelevant (see, e.g., the discussion in Rees, [19], p.171ff). In Mill’s own words: “No person is an entirely isolated being; it is impossible for a person to do anything seriously or permanently hurtful to himself, without mischief reaching at least to his near connections, and often far beyond them” (quoted in McCloskey, [16], p.149). In the Victorian library example above, one may argue that other agents - for example, people holding strong views on morals, - would indeed be affected by my reading the book, or by my not having insured the house, or maybe even by my being unemployed, and therefore their welfare will not remain unchanged. This criticism is not specifically aimed at our axioms, though, and it is typically moved to all liberal views based on some version of the Harm Principle. If correct, this critique would indeed be quite damaging for liberal views but, if anything, from a theoretical viewpoint it would reinforce our critical analysis. Perhaps more importantly, it is im-

⁴See, e.g., Miller([15], p.406ff) and the discussion in Kymlicka([10], pp.308ff). Sanders actually argues that “Nozick suggests an interpretation that might make use of indifference curves” (Sanders, [21], p.380, fn.18).

⁵Interestingly, a welfaristic approach is even more cogent within the so-called *left-libertarian* approaches, which strengthen the proviso by requiring that compensatory payments be made. See the contributions in Steiner and Vallentyne ([25]). See in particular, Steiner ([24]) and Vallentyne’s ([28], pp.11ff) discussion of the ‘social fund’ to compensate for the depletion of natural assets and of welfarism in left-libertarian views.

portant to note that ours is a logical inquiry into the consistency of liberal approaches, whereas the criticism focuses mostly on an empirical issue. From a logical viewpoint, it is certainly possible to draw a meaningful distinction between the two types of actions (viz., self-regarding vs. other-regarding) and the idea that *all* conceivable acts have effects on others does not seem plausible. Actually, many liberals have defended the view that an empirically meaningful distinction exists.⁶

3 The first paradox: noninterference implies equality

Let N be a 2–agent society. Consider social welfare orderings \succcurlyeq on \mathcal{R}^2 , where \mathcal{R} is the set of real numbers and \mathcal{R}^2 denotes the set of welfare allocations in this society. That is, if $p \in \mathcal{R}^2$, then p_i represents the welfare level of agent $i \in N = \{1, 2\}$. The vector notation is: $x > y$ (resp. $x \gg y$) iff $x_i \geq y_i$ for all i and $x \neq y$ (resp. iff $x_i > y_i$ for all i).

Define the following two basic axioms for \succcurlyeq , for all $p, q \in \mathcal{R}^2$:

Strong Pareto Optimality (SPO): $p > q \Rightarrow p \succ q$.

Anonymity (A): $p = \pi q$ for some permutation $\pi \Rightarrow p \sim q$.

Next, we introduce the formal version of the Individual Damage Principle. Starting from two welfare allocations p and q for which p is socially preferred to q , consider two different welfare allocations p' and q' such that agent i is worse off at these than at the corresponding starting allocations, the other agents are equally well off, and agent i prefers p' to q' . The principle requires that society should *not reverse* the strict preference between p and q to a strict preference for q' over p' (a switch to indifference is admitted):

Individual Damage Principle (IDP): Let $p \succ q$, and let p' and q' be such

⁶See the discussion in Berger ([2], p.243ff). One context in which the distinction seems particularly meaningful concerns intergenerational justice since not *all* actions of the present generation have effects on removed generations (certainly not on predecessors, but possibly also on successors). For a discussion see McCloskey ([16]).

that, for some $i \in N$,

$$\begin{aligned} p'_i &< p_i \\ q'_i &< q_i \\ p_j &= p'_j \text{ for } j \neq i \\ q_j &= q'_j \text{ for } j \neq i \end{aligned}$$

Then $p' \succ q'$ whenever $p'_i > q'_i$.

For any $p \in \mathcal{R}^2$ let \bar{p} denote the permutation of p such that the components are ranked in ascending order, so that \bar{p}_1 is the welfare level of the worst-off agent and \bar{p}_2 of the second worst-off agent.

Define the *leximin ordering* \succ^{LM} by:

$$p \succ^{LM} q \Leftrightarrow \bar{p}_1 > \bar{q}_1 \text{ or } \bar{p}_1 = \bar{q}_1 \& \bar{p}_2 > \bar{q}_2$$

We can now show that the combination of Strong Pareto Optimality, Anonymity, and the Individual Damage Principle characterises the leximin ordering.⁷

Theorem 2 : *A social ordering \succ satisfies Strong Pareto Optimality, Anonymity and the Individual Damage Principle if and only if it is the leximin ordering.*

Proof: 1. (Necessity) As is well-known, the leximin satisfies Strong Pareto Optimality and Anonymity, so we only need to prove that it satisfies the Individual Damage Principle. Consider the following diagram, where any point in the plane represents a welfare allocation to agents 1 and 2:

[Add Figure 1 about here.]

Without loss of generality, consider points p and q : p is strictly leximin preferred to q , because the welfare of the worst off agent is higher at p (agent 2) than at q (agent 1). It is immediately clear that any perturbation of p and q such that the welfare of agent 1 (resp. agent 2) decreases in both allocations but is higher at p' than at q' (resp. at p'' than at q'') implies that p' is still

⁷We provide here a graphical proof of the Theorem. A full-fledged standard demonstration can be found in the appendix. A proof for societies with N agents was first derived in [13] and later generalised in [11].

strictly leximin preferred to q' (resp. p'' is still strictly leximin preferred to q'').

2. (Sufficiency) For any pairs of welfare allocations p and q , if p is leximin indifferent to q then p is a permutation of q and therefore the two vectors are indifferent by Anonymity. Therefore we only need to prove that if p is strictly leximin preferred to q , then p is strictly socially preferred to q . The result immediately follows by Strong Pareto (and Anonymity), if p Pareto-dominates (the permutation of) q . Therefore suppose that p and (the permutation of) q cannot be Pareto-ranked and suppose by contradiction that p is strictly leximin preferred to q but q is strictly socially preferred to p . By Anonymity, we can restrict attention to the case with: $q_i > p_j \geq p_i > q_j$, where $i, j \in \{1, 2\}, i \neq j$. Without loss of generality, suppose that p and q are as shown in Figure 2.

[Add Figure 2 about here.]

Then, starting from p and q , it is possible to construct vectors p' and q' such that $p'_1 = q_2 + \varepsilon$, where ε is an arbitrarily small number, and $q'_1 = p_2$, such that q' is weakly socially preferred to p' by the Individual Damage Principle, but p' is strictly socially preferred to q' by Strong Pareto Optimality and Anonymity. Therefore a contradiction is obtained.

Therefore it follows that if p is strictly leximin preferred to q , then q cannot be strictly socially preferred to p . The possibility that q and p are socially indifferent is ruled out by noting that in this case it is possible to slightly perturb p , to obtain a vector r such that r is strictly leximin preferred to q , but q is strictly socially preferred to r (by Strong Pareto Optimality), which we have ruled out. ■

The previous result is interesting for two reasons. Theoretically, from the viewpoint of liberal approaches, it proves that an anonymous constitution that satisfies unanimity and a liberal criterion of noninterference leads to egalitarian allocations. Formally, this result is surprising, because the Individual Damage Principle has no obvious egalitarian content. To show the latter point, our result should be compared with the classical characterisation of the leximin ordering obtained by Hammond ([9]).⁸ Beside Anonymity and Strong Pareto Optimality, his characterisation uses the axiom now usually called Hammond Equity. This is a strongly egalitarian condition that can be formally expressed as follows.

⁸See also the generalisation in Tungodden ([27]).

Hammond Equity: If $p_i < q_i < q_j < p_j$ for two profiles p and $q \in \mathcal{R}^2$, then $q \succcurlyeq p$.

So one additional interesting implication of our result is that Hammond Equity and the Individual Damage Principle are equivalent in the presence of Anonymity and Strong Pareto Optimality. It is possible to show, however, that, if Strong Pareto Optimality holds, then Hammond Equity implies the Individual Damage Principle, but the converse does not hold.

Proposition 3 : *Suppose that a social ordering \succ on \mathcal{R}^2 satisfies Strong Pareto Optimality. If it satisfies Hammond Equity, then it satisfies the Individual Damage Principle, but the converse is not true.*

Proof: See Appendix. ■

Proposition 3 has two interesting implications. First, from a technical viewpoint, it shows that our characterisation is not trivial: under Strong Pareto, Hammond Equity is logically stronger than the Individual Damage Principle.⁹ Second, as shown in the proof of the latter part of the statement, the Individual Damage Principle has indeed no obvious egalitarian content, as it is consistent (together with Strong Pareto Optimality) with some of the least equitable social welfare orderings, such as the serial dictatorships. In this sense, together with Theorem 2, Proposition 3 clearly demonstrates the ethical relevance of the axiom of Anonymity and the normative strength of conceptions of justice as impartiality.

4 The Principle of Non-Interference

The egalitarian implications of the Individual Damage Principle are counterintuitive and pose a challenge to liberal approaches. It may be argued, however, that the Individual Damage Principle captures only some of the intuitions behind the Harm principle, and the egalitarian implications would disappear in a more general framework. In this section, we address this issue and extend our analysis of the liberal notion of noninterference.

⁹It is worth noting that an analogous result can be proved for transitive social welfare relations, without assuming completeness. However, the proof of Proposition 3 does not hold for societies with more than two agents and thus we cannot claim that Hammond Equity is *in general* stronger than the Individual Damage Principle under Strong Pareto Optimality.

In our framework, the core of Non-Interference is the following idea: an individual has the right to make society *remain passive* in all circumstances of change in his welfare, *with the only exception* of those circumstances in which the welfare of other individuals is also affected. Non-Interference says in particular that changes in one individual's welfare that leave all other individuals unaffected should not constitute a motivation for penalising the individual in the social welfare judgement, *whether the change involves a damage or a benefit for him*. By 'penalising' we simply mean a switch in society's strict rankings of social alternatives against the individual, with respect to the ranking of the original alternatives (before the adverse or beneficial change for the individual under consideration occurred).

To illustrate concretely how Non-Interference works, consider again the two-person example discussed in section 2 above.

Example 4 *The Victorian library example reconsidered*

Again, social choice is between two welfare profiles p and q :

	Me	You
p	10	α
q	7	β

Suppose that p is socially preferred to q .

Now consider a change to two different profiles p' and q' which affects only me. In both of these new profiles, I have either suffered a damage or gained a benefit, compared to the corresponding profiles p and q . You, instead, enjoy exactly the same welfare levels at p' and q' as you enjoyed at p and q , respectively:

	Me	You
p'	$10 + \varepsilon$	α
q'	$7 + \varepsilon'$	β

In the table above, ε and ε' are positive or negative numbers such that $\text{sign}(\varepsilon) = \text{sign}(\varepsilon')$.

What should the social preference between p' and q' be? The ethical arguments provided in section 2 above to defend the Individual Damage Principle seem to extend quite naturally to the latter example: Non-Interference says that it should *not reverse* the strict preference between p and q to a strict

preference for q' over p' (a switch to indifference is admitted), possibly except when I prefer otherwise. Non-Interference is silent on what should actually happen if I prefer q' over p' .¹⁰ In other words, *I can veto society from switching its strict preferences after a change that affects only me and nobody else.* The fact that society is allowed to switch from a strict preference for p over q to an indifference between p' and q' even when $10 + \varepsilon > 7 + \varepsilon'$ (i.e. I strictly prefer p' over q') makes my veto power, and hence Non-Interference, relatively mild.

To illustrate the axiom, suppose again that the above example describes social choice over library access in the Victorian era. If $\text{sign}(\varepsilon) = \text{sign}(\varepsilon') < 0$, then we are back to the situation described in section 2 above, where the change in welfare arises from losing a job, without any fault of my own; or from failing to insure myself against my house burning down; or from finding on a bench and reading “Lady Chatterley’s Lover”. (Observe how we allow for a different disutility from punishment in the SWO depending on whether I suffer a damage or not, a feature that stands to reason.) The axiom, however, now encompasses the possibility that my welfare increases at p' and q' compared with p and q , so that $\text{sign}(\varepsilon) = \text{sign}(\varepsilon') > 0$: maybe I won a lottery; or I have finally been able to solve a difficult mathematical problem; or I have found on a bench “Lady Chatterley’s Lover” and thoroughly enjoyed reading it. Then $10 + \varepsilon$ is my welfare in situation p' in which I have access to libraries, while $7 + \varepsilon'$ is my welfare in situation q' , in which society sanctions me - for winning the lottery, or for solving the mathematical problem, or for reading the book - by restricting my access to libraries.¹¹ In any case, the Principle of Non-Interference stipulates that if society chose open access when I had not won the lottery, or the problem was still unsolved, or I had not read the book, my good luck, or effort, or even my action of reading the book should not lead to a change in social preferences: in any case, I should not be punished given that nobody’s welfare is involved. Again, the principle is not liberal in the sense of prescribing that access to public libraries be open, that is, the axiom does *not* require that p be socially preferred to q .

As already noted, the principle of noninterference does not aim to provide a complete description of liberal, noninterfering approaches to social choice,

¹⁰A hard-minded application of the underlying philosophy might suggest that p' should still be preferred to q' : but for our purposes (an impossibility result), as we shall see, it would be pointless, while possible, to strengthen the principle in this way.

¹¹Again, we assume that the book is not borrowed from the library and thus the decision concerning access bears no direct relation with my reading the book.

yet it is clearly more general than the Individual Damage Principle analysed in sections 2 and 3 above. In particular, it arguably captures some relevant liberal ethical intuitions more thoroughly and, although it seems interesting enough on its own merits to warrant investigation, we shall highlight its theoretical relations with some prominent liberal approaches.

First of all, in our opinion, the principle of Non-Interference translates faithfully the substantial aspects of J.S. Mill's 'Harm Principle', notably when he says:

"The only purpose for which power can be rightfully exercised over any member of a civilized community, against his will, is to prevent harm to others...The only part of the conduct of anyone, for which he is amenable to society, is that which concerns others. In the part which merely concerns himself, his independence is, of right, absolute." (*On Liberty*, Chapter I; p.135 in [30]).

In our framework, 'exercising power...against his will' corresponds to society switching its ranking in a way contrary to the individual's welfare. It is hard to deny that any exercise of power against one's own will has precisely this welfare consequence.

Mill's description of the Harm Principle is couched in agency terms, e.g.

"The sole end for which mankind are warranted, individually or collectively, in interfering with the *liberty of action* of any of their number, is self-protection." (*On Liberty*, Chapter I, our italics; p.135 in [30]).¹²

The agency aspect is obviously lost in a welfaristic framework, but, as argued in section 2 above, this is not essential to our analysis of the Harm Principle, especially if one notes that all relevant forms of interference will have some welfare implications. Besides, as already noted, we only aim to capture *some* aspects of the Harm Principle, and our axioms rule out interference only in a core of social situations, namely when interference decreases agents' welfare. As shown in the Victorian library example, a switch in social preferences over library access certainly represents a relevant form of interference, which would diminish my welfare. The Principle of Non-Interference does capture an essential aspect of Mill's view by requiring that *if* society adopted a liberal policy at p against q , then no switch in social preferences should occur following my decisions or acts, given that the latter

¹²In the general context of Mill's theory, self-protection is not to be interpreted literally, and may be extensively understood to refer to well-being and not strictly to physical integrity only.

do not influence anyone else. According to Mill, “he already bears, or will bear, the whole penalty of his error; if he spoils his life by mismanagement, we shall not, for that reason, desire to spoil it further” (*On Liberty*, Chapter IV; p.210 in [30]). And a similar principle applies, in a liberal perspective, to welfare-enhancing decisions, acts, or events that do not affect others.

Secondly, the Principle of Non-Interference is relevant for libertarian approaches, too. In general, protection from interference is central for such approaches, for it is the essential ingredient of (legitimately acquired) private property, and negative rights (such as those incorporated in our axiom) are foundational in libertarian notions of justice (Wolf, [31] p.793; Arneson, [1], p.322). Indeed, in Locke’s theory, noninterference on property follows from noninterference on people’s individual attributes and choices, as it derives from the rights correlated to self-ownership. Non-Interference is also implicated in the transfer of property: voluntary transactions (without externalities) do not cause harm and therefore the state should not interfere.

Furthermore, and perhaps more importantly for our paper, the principle of Non-Interference touches on the very moral legitimacy of the institution of private property in a libertarian perspective, because of its relevance for the just initial acquisition of resources. In Locke’s ‘labour theory of property’, the initial appropriation of natural resources in the state of nature, which gives rise to the institution of private property, is legitimate if (i) one mixes one’s labour with unowned goods; (ii) one does not appropriate an amount of things in excess of those one can use, so that they do not rot or get wasted; and, as already noted, (iii) a proviso is satisfied, namely that ‘there is enough and as good left for others’ of the resources appropriated. Recent discussions on Locke and on neo-Lockean theories, however, suggest that that the force of entitlement theories of justice stems from the proviso (iii), suitably modified, rather than the other conditions.¹³ In particular, as concerns (ii), libertarians consider it an unjustified limitation of the rights associated with ownership (which normally include the possibility of disposing of the good, consuming it, and even destroying it), and this seems especially unsatisfactory if the proviso holds, so that destroying the good cannot harm anybody.¹⁴

¹³See, e.g., Nozick ([18]), Kymlicka ([10]), and Arneson ([1]). As for (i), Nozick forcefully argues that mixing one’s labour with natural resources has not much moral force in terms of justifying private ownership especially if the only ‘labour’ performed is the acquisition itself. For the opposite view, see Sanders ([21]).

¹⁴Interestingly, when Locke argues that condition (ii) does not apply to monetary economies he states that “a man may fairly possess more land than he himself can use

The force of the proviso itself derives from its being an application of the Harm Principle: if there is enough and as good left for others, then appropriation cannot possibly harm them, and thus nobody can complain or demand ‘interference’ on property claims.¹⁵ As persuasively stated by Wolf ([31], p.795), “The proviso functions to stipulate conditions in which this presumptive claim [a claim to property] will be undefeated, or overriding, and will therefore impose *duties of noninterference* on others”. Therefore, although we do not aim to capture all relevant aspects of entitlement theories of justice, it is possible to provide one plausible interpretation of the above conditions in our welfaristic framework that highlights the relevance of our Principle of Non-Interference for some core issues in libertarian approaches.

In a welfaristic framework, the proviso may be reformulated as requiring that nobody’s welfare is (negatively) affected by appropriation. Thus the *strong libertarian proviso* might seem to be a variant of the *strong pareto principle*: in the Lockean state of nature, let p be the welfare profile of all agents when private acquisition of some resource is allowed and let q be the welfare profile of all agents when it is not. Then, the welfaristic lockean proviso could be interpreted as stating that private acquisition is legitimate if $p > q$ (this could be imagined as the outcome of a sequential process in which one analyses private acquisition by one agent at a time and a series of pareto-improving acquisitions take place; if some acquisition is not pareto-improving - there is not ‘enough and as good’ - left for others, then the process stops and private property is not legitimate). The parallel is imperfect, though, because what is required is that no *other* agent is harmed, but the appropriating agent may as well be worse off: this would not decrease the legitimacy of the acquisition.¹⁶ At any rate, the strength of our Principle of Non-Interference is that we need not be involved into this discussion: whatever the libertarian criteria that may lead to $p \succ q$, the Principle of Non-Interference (as applied in the Lockean state of nature) can be seen as a *weak libertarian claim (or proviso)*: suppose the choice between p and q in the numerical examples above, is a choice between allowing me to appropriate a natural resource (p), or not (q), in Locke’s state of nature, in

the product of, by receiving in exchange for the overplus, Gold and Silver, which may be hoarded up *without injury to anyone*” (quoted in Miller, [15], p.407, italics added).

¹⁵See, e.g., Gibbard ([7], pp.28-30); Cohen ([3], p.256); Kymlicka ([10], p.308); and Arneson ([1], pp.329-330).

¹⁶This raises the issue of why the agent appropriates the resource, given that she is worse off, but this has nothing to do with legitimacy.

which economic interaction is presumably limited. For example, I grab a lot of land and then subsist on it. If, consistently with Locke's view, $p \succ q$, then it follows that whatever I get from the resource, or whatever luck (misfortune) I have with it, if the Lockean proviso holds, and thus nobody else is affected by my decisions (due to the limited interactions in the state of nature), and if $p'_i > q'_i$ it should certainly be $p' \succsim q'$, i.e., if I still prefer to appropriate the resource, society should not actively prevent me from doing it. Given the arguments concerning the Lockean criterion (ii) above, the legitimacy of my acquisition would not be diminished by my spoiling or depleting the resource, and thus no assumptions need be made concerning the source of change in welfare from p, q to p', q' .¹⁷

In our opinion, the latter arguments support the idea that the Principle of Non-interference captures some essential aspects of liberal views. This view is not uncontroversial, though, and some important criticisms of the social choice interpretation of rights have been put forward. According to one of the most prominent critics, liberal views require a purely procedural formulation of rights and thus cannot be captured in a social choice theoretic framework, and especially in a welfaristic context (Sugden, [26]). A thorough discussion of Sugden's interesting arguments goes beyond the scope of this paper, but we note here that they are arguably not conclusive. On the one hand, his interpretation of Mill as developing a theory of rights, and in particular of the right to a personal sphere (see Sugden, [26], p.128) is disputable. As argued above, it is doubtful that a close reading of Mill's texts would support this interpretation and there are many passages in which Mill explicitly rejects this view.¹⁸

On the other hand, the defence of a purely procedural formulation of rights is arguably weak and incomplete. It is incomplete because, even in purely deontological approaches such as Nozick's, considerations about outcomes are kicked out of the door just to come back from the window: the

¹⁷Given its welfarist structure and given its silence on the reasons why $p \succ q$ in the antecedent, the Principle of Non-Interference allows us to eschew all discussions of the baseline model against which to analyse appropriation in the state of nature. Various authors criticise Nozick for illegitimately restricting the proviso by comparing private ownership only to the alternative of *no-ownership* (instead of, say, *common* ownership and productive use) of resources (e.g., Cohen, [3]). Our Principle of Non-Interference is silent on the 'underlying structure' of q , instead.

¹⁸The main quote from Mill used by Sugden to support his argument, for example, is arguably taken out of the context (see Sugden, [26], p.131).

Lockean proviso is the best example (see the discussion above for more examples). It is weak because Sugden ([26], p.136) grants that Sen's objections to procedural approaches are relevant and in his reply to Sen's 'Donna, Charles, and Ali' famous example, Sugden does not defend the absolute importance of constraints (against Sen's advocacy of trade-offs) with a direct ethical argument, but only by invoking second-order prudential arguments, such as the possibility that people are self-serving or self-deluding, or that they overestimate dangers, etc. leading to violations of rights based on welfare considerations that turn out to be false ex post (see, for instance, Sugden, [26] pp.142 and 148). Even if correct, this does not provide a first-order ethical argument and if the second-order prudential factors are not relevant (which is only an empirical matter), there remains no ethical defence of rights.

5 The second paradox: the impossibility of non-interference

We shall now substantially weaken the axioms of Strong Pareto and Anonymity, and replace them with the following basic properties for \succcurlyeq , for all $p, q \in \mathcal{R}^2$:

Weak Pareto Optimality (WPO): $p \gg q \Rightarrow p \succ q$.

Non-Dictatorship (ND): For all $i \in N$, there are $p, q \in \mathcal{R}^2$ such that $p_i > q_i$ and $q \succcurlyeq p$.

Next, we introduce the formal version of the Principle of Non-Interference. Starting from two welfare allocations p and q for which p is socially preferred to q , consider two different welfare allocations p' and q' such that agent i is either worse off, or better off, at these than at the corresponding starting allocations, the other agents are equally well off, and agent i prefers p' to q' . The principle requires that society's preference over p' and q' should agree with person i 's preferences.

Non-Interference (NI): Let $p \succ q$, and let p' and q' be such that, for some $i \in N$,

$$\begin{aligned} (p_i - p'_i)(q_i - q'_i) &> 0 \\ p_j &= p'_j \text{ for } j \neq i \\ q_j &= q'_j \text{ for } j \neq i \end{aligned}$$

Then $p' \succcurlyeq q'$ whenever $p'_i > q'_i$.

In other words, *I can veto society from switching its strict preferences after a change that affects only me and nobody else.* The fact that society is allowed to switch from a strict preference for p over q to an indifference between p' and q' even when I strictly prefer p' over q' makes my veto power, and hence Non-Interference, relatively mild.

We can now show that the conditions defined so far are incompatible.¹⁹

Theorem 5 : *There is no social ordering \succcurlyeq that satisfies Weak Pareto Optimality, Non-Dictatorship, and Non-Interference.*

Proof: Consider the following diagram, where any point in the plane represents a welfare allocation to agents 1 and 2. By Nondictatorship, there are at least two pairs of allocations (x, y) and (p, q) such that if agent 1 strictly prefers y to x , x is weakly socially preferred to y ; and if agent 2 strictly prefers p to q , q is weakly socially preferred to p . By Weak Pareto Optimality, social preferences among the two pairs of alternatives can be taken to be strict, and it can be assumed that agent 2 strictly prefers x to y , and agent 1 strictly prefers q to p , without loss of generality. Suppose that the two pairs of allocations are as depicted in Figure 3.

[Add Figure 3 about here.]

It is possible to perturb p and q as shown in Figure 3 so that q' is weakly socially preferred to p' by Non-Interference (only agent 1's welfare is affected and she strictly prefers q' to p'). Similarly, it is possible to perturb x and y as shown in Figure 3 so that x' is weakly socially preferred to y' by Non-Interference (only agent 2's welfare is affected and she strictly prefers x' to y'). However, p' strictly pareto dominates x' and y' strictly pareto dominates q' . Therefore a contradiction is obtained because q' is weakly socially preferred to p' which is strictly socially preferred to x' (by Weak Pareto Optimality), and x' is weakly socially preferred to y' which is strictly socially preferred to q' (by Weak Pareto Optimality).

It is easy to show that starting from any initial configuration of vectors p, q, x, y , a contradiction is obtained with a similar reasoning. ■

¹⁹We provide here a graphical proof of the Theorem. A standard demonstration can be found in the appendix.

Remark 1 It is worth noting that completeness is not necessary to prove Theorem 5, which can be strengthened to apply to all transitive social welfare relations.

In order to illustrate the logic of the impossibility result, consider the following two-person example.

Example 6 *The neighbouring desert islands*

Suppose that Andrea is young, vegetarian, and loves meeting new people (but she does not wish to live with anyone), whereas Bob is middle-aged and does not enjoy company, but has a passion for fishing. They live in two different islands and each of them is the only inhabitant of their island. The two islands are sufficiently far apart that neither of them can see what the other is doing, but they are sufficiently close that a number of decisions must be taken collectively. Consider first social choice between two welfare profiles p and q , where the only difference between the two allocations is that in q a boat with a large number of young people partying is allowed to sail along the canal separating the two islands, whereas in p it is barred from the canal. Given their preferences, and all other things equal, we assume that the following welfare profiles are associated with the two alternatives.

	Andrea	Bob
p	10	15
q	20	8

Suppose that q is (strictly) socially preferred to p .

Consider next social choice between two welfare profiles x and y , where the only difference between the two allocations is that in x empty beer bottles cannot be thrown in the open sea, whereas in y all sorts of rubbish can be freely disposed of in the canal. Given their preferences, and all other things equal, we assume that the following welfare profiles are associated with the two alternatives.

	Andrea	Bob
x	12	13
y	23	6

Suppose that x is (strictly) socially preferred to y .

Note that the assumption that q is socially preferred to p and that x is socially preferred to y entail no loss of generality because by Nondictatorship (and Weak Pareto) two such pairs will always exist. Now consider a change from welfare profiles p and q to two different profiles p' and q' which affects only Andrea: maybe she has casually found a hidden cave to explore in her island, or - after a long series of experiments - she has found a way of producing an alcoholic drink from a plant growing in her island, so that her welfare has increased compared to the corresponding profiles p and q . Bob, instead, enjoys exactly the same welfare levels at p' and q' which he enjoyed at p and q , respectively:

	Andrea	Bob
p'	16	15
q'	21	8

Similarly, consider a change from welfare profiles x and y to two different profiles x' and y' which affects only Bob: maybe he has found a hidden mountain lake with a large stock of fish, or maybe he has invented a new portable chair that allows him to fish more comfortably. In both of these new profiles, his welfare has increased, compared to the corresponding profiles x and y . Andrea, instead, enjoys exactly the same welfare levels at x' and y' which she enjoyed at x and y , respectively:

	Andrea	Bob
x'	12	14
y'	23	9

The impossibility result can now be illustrated. Since q is (strictly) socially preferred to p , then by Non-Interference, q' is (weakly) socially preferred to p' . Then by Weak Pareto Optimality, p' is (strictly) socially preferred to x' . Finally, since x is (strictly) socially preferred to y , then by Non-Interference, x' is (weakly) socially preferred to y' . But then, since by Weak Pareto Optimality y' is (strictly) socially preferred to q' , the desired contradiction obtains.

In order to appreciate the strength of our result, it is worth comparing it with Sen's ([22]) classic contribution. In Sen's ([22]) Minimal Liberty axiom, the core of liberalism is formalised as the requirement that an individual has a sphere of complete control on society's preferences over minimal subsets

of social states. The characteristics of the social states are not specified, however, and in this sense the Minimal Liberty axiom cannot capture Mill’s Harm Principle; for no mention is made of other agents’ welfare and the notion of personal autonomy is not explicitly related to the idea of no harm to others.

Conceptually, Sen’s ML captures individual autonomy by defining a sphere of ‘positive’ freedom, overriding in principle concerns about other people’s welfare. This is an important difference from our Principle of NonInterference, which embodies a notion of personal autonomy in terms of negative freedom, namely as the protection from interference under certain circumstances not involving others. This characteristic of NI is even more transparent if one notes that the consequent of NI might equivalently be written as requiring $q' \succ p'$ whenever $p'_i > q'_i$. Similarly, if Sen’s Minimal Liberty axiom is understood as a “social choice formulation of rights” (Sugden, [26], p.128ff), then it can be interpreted as assuming the existence of a personal sphere over which individuals have a right to have their preferences respected. In the language of rights, the Principle of NonInterference would capture a different, and arguably weaker type of right, namely the right to be protected from adverse changes in strict social preferences.

Formally, one way to read NI is as ‘local decisiveness’ condition. Sen’s ‘minimal liberalism’, for example, asserts that an individual i must be decisive *at least* on one pair of alternatives. NI asserts that an individual i must be decisive *on every* pair of alternatives that is reached through certain ‘precedents’ (society has already demonstrated a strict preference between p and q and the only change involved concerns i alone). The individual is decisive not because of the nature of the alternatives but on the basis of the ranking of society itself in another situation which is *identical* to the one under consideration when seen from the viewpoint of all other members of society. Once again, observe how the required individual’s decisiveness is weak since she cannot prevent society’s indifference in the face of her strict preference.

6 Concluding remarks

We formalise and study principles of noninterference applied to social welfare orderings. The Principle of Non-Interference captures aspects of liberal approaches to social decision making. Broadly speaking, it requires that an

individual has the right to make society remain passive in all circumstances of change in his welfare, with the only exception of those circumstances in which the welfare of other individuals is also affected. In its full generality, NI produces an impossibility result: in fact, together with Pareto Optimality, it implies that a Social Welfare Ordering must be dictatorial. However, by restricting the direction of the allowed changes in circumstance, we can characterise the Leximin Social Welfare Ordering which satisfies, in addition to Pareto Optimality, also Anonymity. Actually, as shown in Mariotti and Veneziani ([14]), different restrictions on the form of these changes allow us to characterise the leximax, the Utilitarian and the Nash orderings.

This highlights a deep inconsistency in liberal approaches. On the one hand, if a general view of noninterference is held, then an impossibility result emerges. We take this result as a demonstration that it is impossible to make social welfare judgements without considering the distributional consequences of changes in welfare, even when those changes affect only one individual. There cannot be any ‘protected sphere’ for the individual in this sense. Of the appeals of the individuals to be left alone because ‘nobody but me has been affected’, at least some will necessarily have to be overruled. On the other hand, even if the bite of noninterference is restricted, and possibility results are derived, then social welfare orderings obtained are not recognisably, or distinctly, liberal.

7 Appendix: Formal Proofs

Proof of Theorem 2: *If.* We verify only that the leximin ordering satisfies the IDP. Suppose $p \succ^{LM} q$ and consider p', q' with $q'_i < p'_i$, $p'_i < p_i$, $q'_i < q_i$, some $i \in N$, and $p_j = p'_j$ and $q_j = q'_j$, $j \neq i$. We prove that $p' \succ^{LM} q'$.

Note first that if it was $q'_j \leq p'_j$, then p' would Pareto dominate q' and the conclusion would be reached immediately. So in what follows assume $q'_j > p'_j$. Therefore we have $q'_j = q_j > p'_j = p_j$. Furthermore, if $q_i > p'_j = p_j$, then $\min\{q_1, q_2\} > \min\{p_1, p_2\}$ contradicting $p \succ^{LM} q$, so we must have

$$q'_j > p'_j \geq q_i$$

Given that $q_i > q'_i$, it follows from these inequalities that $q'_j > p'_j > q'_i$. Since by assumption $p'_i > q'_i$ we can conclude that $\min\{p'_1, p'_2\} > \min\{q'_1, q'_2\}$

Only if. Let a social ordering \succ satisfy the axioms and let $p, q \in \mathcal{R}^2$ with $p \neq q$. If $p \sim^{LM} q$ then $p_i = q_j$, $i \neq j$, and it follows immediately by A that

$p \sim q$. So we show that $p \succ^{LM} q$ implies that $p \succ q$. If p Pareto dominates q we are done by SPO. If p Pareto dominates the permutation of q then we are done by A and SPO. So let $p_i \neq q_j$, $i \neq j$, and suppose by contradiction that $p \succ^{LM} q$ but $q \succ p$. Without loss of generality, let

$$\begin{aligned} p_1 &> q_2 \\ q_1 &> p_2 \end{aligned}$$

Let $\varepsilon \in \mathcal{R}_+$. Define $p' = (p'_1, p'_2) = (q_2 + \varepsilon, p_2)$ and $q' = (q'_1, q'_2) = (p_2, q_2)$. Consider three cases:

- a) $q_2 \geq q_1$. In view of the displayed inequalities it follows that $p_1 > q_2 \geq q_1 > p_2$, contradicting the assumption $p \succ^{LM} q$.
- b) $p_1 \leq p_2$ and $q_1 > q_2$. As by assumption $p_1 > q_2$ it follows that $p_2 \geq p_1 > q_2$. For a sufficiently small ε , by the IDP $q \succ p$ implies $q' \succ p'$, whereas by A and SPO it must be $p' \succ q'$, contradiction.
- c) $p_1 > p_2$ and $q_1 > q_2$. Then $p \succ^{LM} q$ implies $p_2 \geq q_2$. If it was $p_2 = q_2$ then $p \succ^{LM} q$ would imply $p_1 > q_1$. This yields $p_1 > q_1 > p_2 \geq q_2$ and p would Pareto dominate q , contradicting $q \succ p$. So let $p_2 > q_2$: if $p_1 \geq q_1$ a contradiction would immediately obtain. Therefore let $q_1 > p_1$ and, by A, it is possible to repeat the reasoning of the previous case.

This shows that it must be $p \succ q$ whenever $p \succ^{LM} q$. If $p \sim q$, it is possible to slightly reduce the welfare of one of two agents to obtain r close enough to p such that $r \succ^{LM} q$ but (by SPO) $q \succ r$, and the argument above can be applied to r and q , which concludes the proof. ■

Proof of Proposition 3:²⁰ 1. Consider two vectors $p, q \in \mathcal{R}^2$. Without loss of generality, assume $q_2 > q_1$.

Case (i). Suppose $p > q$. By SPO $p \succ q$ and it is easy to check that for any $p', q' \in \mathcal{R}^2$ such that $p'_i < p_i$, $q'_i < q_i$, and $p_j = p'_j$, $q_j = q'_j$ for $j \neq i$, if $p'_i > q'_i$ then $p' \succ q'$ by SPO.

Case (ii). Suppose $q_1 < p_1 \leq p_2 < q_2$. By HE and SPO, it is immediate to prove that it must be $p \succ q$. Consider $p', q' \in \mathcal{R}^2$ such that $p'_2 < p_2$, $q'_2 < q_2$, and $p_1 = p'_1$, $q_1 = q'_1$: if $p'_2 > q'_2$ then $p' \succ q'$ by SPO. Consider $p', q' \in \mathcal{R}^2$ such that $p'_1 < p_1$, $q'_1 < q_1$, and $p_2 = p'_2$, $q_2 = q'_2$: if $p'_1 > q'_1$ then by HE $p' \succ q'$.

²⁰We thank Michele Lombardi for various suggestions that have greatly improved the proof.

Case (iii). Suppose $q_1 < p_2 < p_1 \leq q_2$. Again, by HE and SPO, it must be $p \succ q$ and the rest of the proof follows as in the previous case except for the following combination: $q'_1 < p_2 \leq p'_1 < q_2$. Yet, by HE and SPO it is immediate to prove that in the latter case, too, it must be $p' \succ q'$.

Case (iv). Suppose $q_1 < p_2 < q_2 < p_1$. By HE and SPO, it is immediate to prove that it must be $p \succ q$. Consider $p', q' \in \mathcal{R}^2$ such that $p'_2 < p_2$, $q'_2 < q_2$, and $p_1 = p'_1$, $q_1 = q'_1$: if $p'_2 > q'_2$ then again $p' \succ q'$ by SPO. Consider $p', q' \in \mathcal{R}^2$ such that $p'_1 < p_1$, $q'_1 < q_1$, and $p_2 = p'_2$, $q_2 = q'_2$, and $p'_1 > q'_1$. If $q'_1 < p_2 < q_2 < p'_1$ the proof that $p' \succ q'$ follows as at the beginning of this case. If $q'_1 < p_2 < p'_1 \leq q_2$ the proof that $p' \succ q'$ follows as at the beginning of case (iii). If $q'_1 < p'_1 \leq p_2 < q_2$ then by SPO and HE, it must be $p' \succ q'$.

Cases (v) and (vi). If $q_1 = p_2 < p_1 \leq q_2$ or $q_1 = p_2 < q_2 < p_1$, it is easily checked that IDP holds in these cases, too.

Finally, all other configurations of $p, q \in \mathcal{R}^2$ are dealt with using symmetrical arguments.

2. We want to show that there exists a SWO \succcurlyeq on \mathcal{R}^2 that satisfies SPO and IDP but not HE. Consider the following lexicographically dictatorial SWO \succcurlyeq^{LD} by $p \sim^{LD} q \Leftrightarrow p = q$ and $p \succ^{LD} q \Leftrightarrow p_1 > q_1$ or $p_1 = q_1 \& p_2 > q_2$. \succcurlyeq^{LD} satisfies IDP and SPO but not HE. ■

Proof of Theorem 5: 1. Consider $p, q \in \mathcal{R}^2$, $p \neq q$, such that $p_i > q_i$. If $q \succcurlyeq p$, then by WPO it is immediate to prove that there exist $p', q' \in \mathcal{R}^2$, $p' \neq q'$, such that $p'_i > q'_i$, $q'_j > p'_j$, and $q' \succ p'$.

2. By ND and step 1, there exist $p, q \in \mathcal{R}^2$ such that $p_1 < q_1$, $p_2 > q_2$, and $q \succ p$. By ND and step 1 above, there also exist $x, y \in \mathcal{R}^2$ such that $x_1 < y_1$, $x_2 > y_2$, and $x \succ y$. We show that there exist vectors p', q', x', y' such that $q' \succcurlyeq p'$, $x' \succcurlyeq y'$, $p' \succcurlyeq x'$, and $y' \succcurlyeq q'$.

3. There are in principle a number of cases to consider. Case (i): suppose that $x_1 > p_1$, $x_2 < p_2$, $y_1 > q_1$, and $y_2 < q_2$. Then it is possible to construct two vectors x', y' such that $x_2 < x'_2 < p_2$, $y_2 < q_2 < y'_2$, $x_1 = x'_1$, $y_1 = y'_1$, and $y'_2 < x'_2$: by NI, it follows that $x' \succcurlyeq y'$. Then, construct two vectors p', q' , such that $p'_1 > x'_1 = x_1 > p_1$, $y'_1 = y_1 > q'_1 > q_1$, $p'_2 = p_2$, $q'_2 = q_2$, and $q'_1 > p'_1$: by NI, it follows that $q' \succcurlyeq p'$. Furthermore, by WPO, it follows that $p' \succ x'$ and $y' \succ q'$, yielding the desired contradiction.

4. It is easily checked that starting from any initial configuration of the vectors p, q, x, y , a contradiction is obtained with a similar reasoning in a finite number of steps using NI and WPO. ■

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Figure 1: Leximin - Necessity

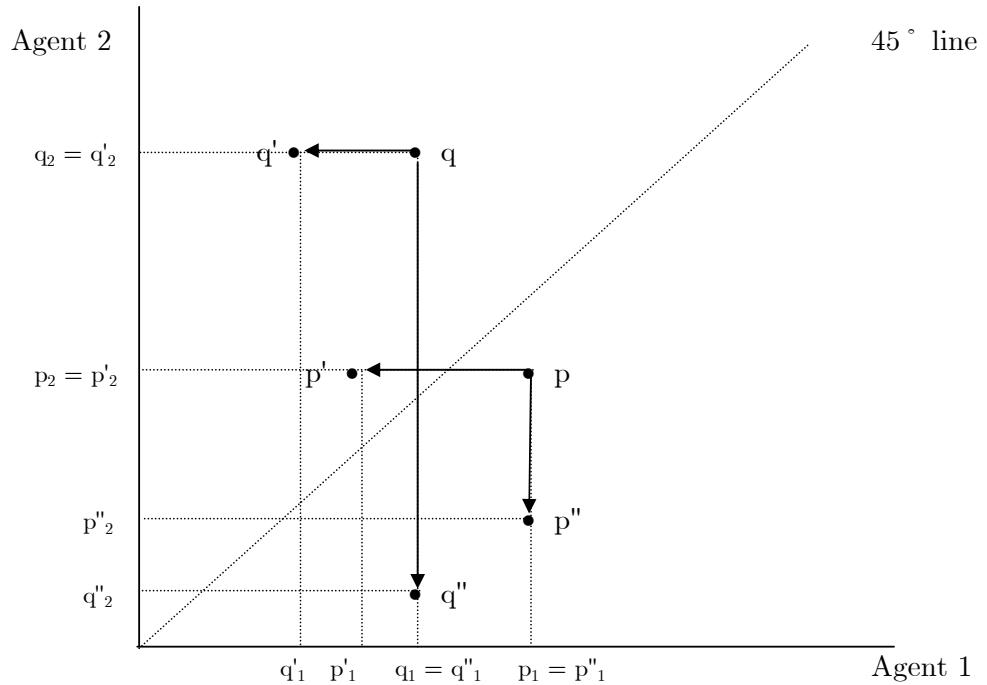


Figure 2: Leximin - Sufficiency

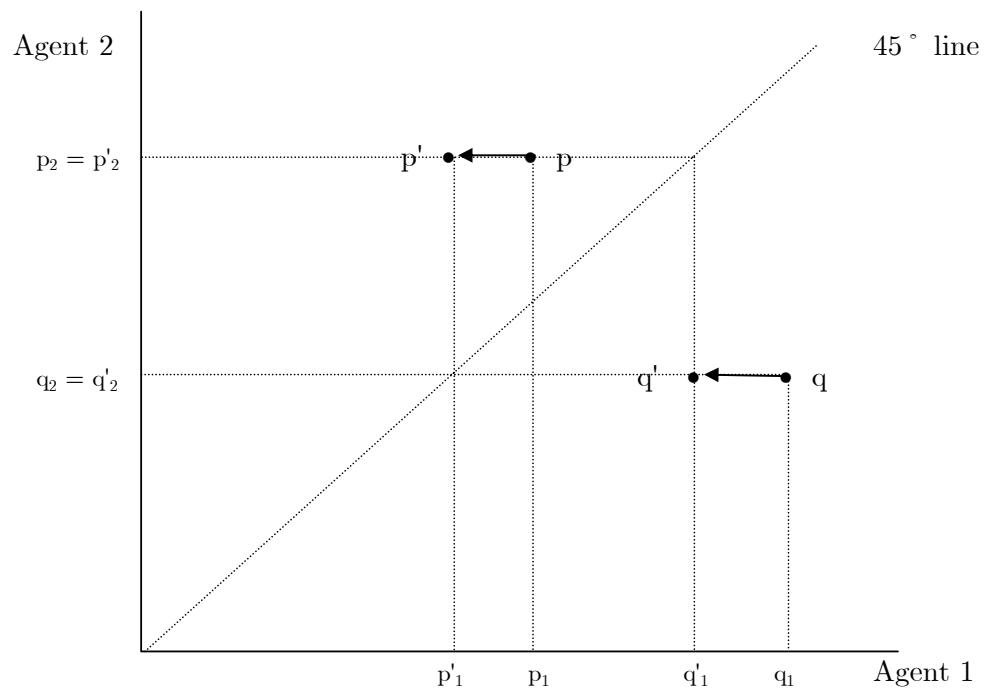
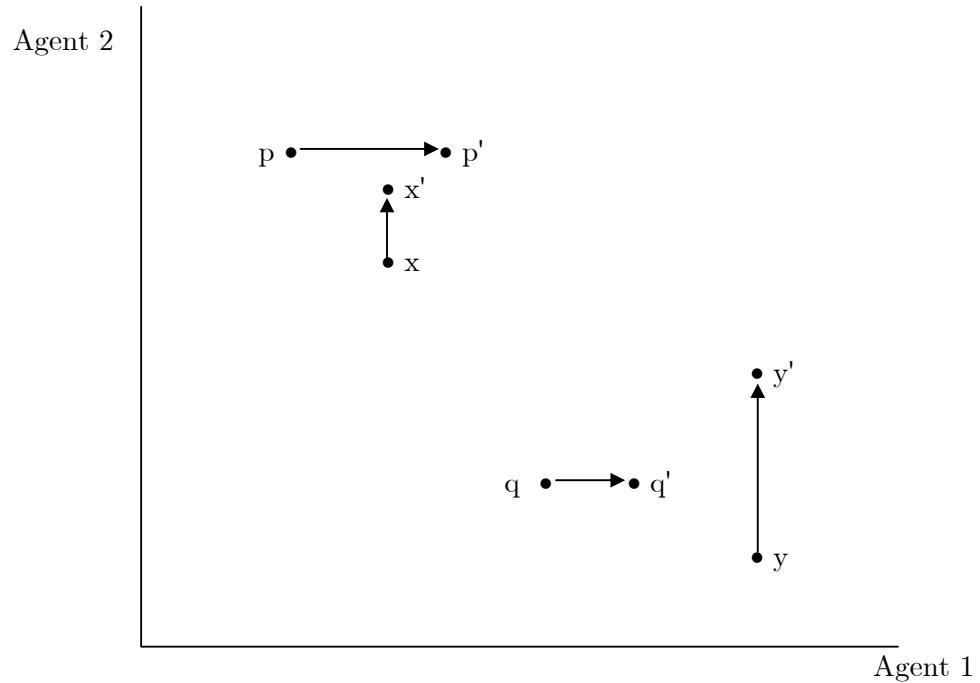


Figure 3: The impossibility of NonInterference



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