Department of Economics

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Working Paper No. 550 December 2005 ISSN 1473-0278

Queen Mary
University of London

Panel Data Unit Roots tests: The role of Serial Correlation and the Time Dimension*

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October 2005

Abstract

We investigate the influence of residual serial correlation and of the time dimension on statistical inference for a unit root in dynamic longitudinal data, known as panel data in econometrics. To this end, we introduce two test statistics based on method of moments estimators. The first is based on the generalised method of moments estimators, while the second is based on the instrumental variables estimator. Analytical results for the IV based test in a simplified setting show that (i) large time dimension panel unit root tests will suffer from serious size distortions in finite samples, even for samples that would normally be considered large in practice, and (ii) negative serial correlation in the error terms of the panel reduces the power of the unit root tests, possibly up to a point where the test becomes biased. However, near the unit root the test is shown to have power against a wide range of alternatives. These findings are confirmed in a more general set-up through a series of Monte Carlo experiments.

Keywords: Dynamic longitudinal (panel) data; Generalized method of moments; Instrumental variables; Unit roots; Moving average errors.
JEL: C22, C23.

1 Introduction

There has been much recent interest – both theoretical and applied – in testing for unit roots in longitudinal data, known in econometrics as panel data. Existing panel data unit root tests can be classified into two categories: the first treats the time dimension of the panel, T, as large (see Levin, Lin, and Chu (2002), Im, Pesaran, and Shin (1995) and Hadri (2000), inter alia) while the second treats T as fixed (short) (see Harris and Tzavalis (1999)). Asymptotic theories for both categories of tests assume that the cross-section dimension of

*The authors thank Subir Ghosh (the editor), an associate editor and two anonymous referees of the Journal of Statistical Planning and Inference, and Karim Abadir, Valentina Corradi, Garry Phillips, Zacharias Psaradakis and Ron Smith for useful comments and Steve Bond and Frank Windmeijer for helpful discussions. This research was funded by the ESRC under grant R000239139.

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the panel, N, goes to infinity; for large-T tests, also T is assumed to increase without bound, either jointly with N or sequentially. The fixed-T tests can be thought of as more appropriate for panels where the time dimension is small, while the large T tests are naturally suited to those panels where the time dimension can be considered large (see Chamberlain (1984), inter alia). From the point of view of statistical inference, however, there is no rule that allows one to classify the time dimension of a panel as small or large.

In this paper, we assess the influence of the time dimension of a panel on statistical inference for a unit root in the presence of serially correlated errors, a set-up which has proven to be challenging for single time series tests (see Schwert (1989) and Wu and Yin (1999)). Our aims are twofold. Firstly, the paper intends to characterize the specific problems for unit root testing that are introduced by allowing for serial correlation in the errors. We will show that tests lose power as serial correlation grows large and negative, up to a point where they may become biased. Near the unit root, however, the tests are powerful against a wide range of alternatives. Secondly, our study will help to investigate the minimum number of time series observations that are appropriate in order for short or large panel data unit root tests to be applicable. This will shed light on existing evidence that, for both single time series and panel data, unit root test statistics that assume that T is large seem to be critically oversized in small samples, especially when the panel disturbance (error) terms are negatively serially correlated.

We start our study by introducing a Generalized Method of Moments (GMM) based unit root test statistic. This is primarily designed for short T panels, as it is based on cross-sectional averaging only and allows the nuisance parameters to be heterogeneous across both the N and T dimensions of the panel. The paper then introduces an Instrumental Variables (IV) based test statistic under the additional assumption that the nuisance parameters of the panel are homogeneous across both dimensions of the panel. This test can be applied to panels where the T dimension is short, or large. The IV based test statistic will help us to analytically examine the influence of the time dimension of the panel and the serial correlation nuisance parameters on panel data unit root tests for the class of the method of moments based test statistics to which the IV based test belongs. Moreover, it allows us to examine power properties in a tractable setting.

To address these issues in a simple framework, consider the first order autoregressive panel data model,

$$z_{i,t} = \eta_i(1-\rho) + \rho z_{i,t-1} + u_{i,t} \qquad i = 1, ..., N; \; t = 1, ..., T \tag{1}$$

where the error terms $u_{i,t}$ are zero mean p-dependent processes[1] which are independent across i, with $E|u_{i,t}|^{4+\delta}$ uniformly bounded over i and t, for some $\delta > 0$ and $p < T$. η_i are individual-specific long-run means[2] of the processes when $\rho < 1$ - in the limit as $\rho \to 1$ the processes become driftless random walks; in this way individual-specific trends are ruled out

[1] That is $u_{i,t}$ and $u_{i,t-p-1}$ are independent random variables, but $u_{i,t}$ and $u_{i,\tau}$, $t \geq \tau \geq t-p$ may be dependent. Note that the dependence structure is allowed to be heterogeneous across i.

[2] $\eta_i(1-\rho)$ are often referred to as "fixed effects" in the econometrics literature.

for all values of ρ. The assumption that $p < T$ means that the order of serial correlation is smaller than the T dimension of the panel. It is required to derive unit root test statistics where T is fixed. The above assumption on the disturbance terms $u_{i,t}$ is quite general yet enables us to apply standard asymptotic results across the N dimension of the panel. When discussing the IV statistic, we will focus on the case where $u_{i,t}$ follows a homogeneous MA(1) process. More specifically

$$u_{i,t} = v_{i,t} + \theta v_{i,t-1} \tag{2}$$

where the error terms $v_{i,t}$ are independent zero mean random variables with $E|v_{i,t}|^{4+\delta} < \infty$.

At this point we do not make any assumption on the initial conditions of the panel $z_{i,0}$. The test statistics we derive are invariant to $z_{i,0}$ under the null hypothesis $\rho = 1$. This is achieved by subtracting $z_{i,0}$ from each observation $z_{i,t}$ as in Breitung and Meyer (1994): we define the new series

$$y_{i,t} = z_{i,t} - z_{i,0} \qquad i = 1, ..., N;\ t = 1, ..., T \tag{3}$$

and employ $y_{i,t}$ rather than $z_{i,t}$ in deriving the limiting distributions of the test-statistic of the hypothesis $\rho = 1$ in model (1). Model (1) is written in terms of y as

$$y_{i,t} = (\eta_i - z_{i,0})(1 - \rho) + \rho y_{i,t-1} + u_{i,t} \qquad i = 1, ..., N;\ t = 2, ..., T \tag{4}$$

The paper is organized as follows. Section 2 introduces the test statistics and derives their limiting distribution. Section 3 conducts a Monte Carlo study to appraise the small sample performance of our test statistics and to confirm some of the theoretical results derived in Section 2. Section 4 concludes the paper. All proofs are relegated to the Appendix.

2 The Test Statistic and its Limiting Distribution

Under the assumptions made in Section 1, ρ can be consistently estimated under $H_0 : \rho = 1$ by a GMM estimator based on orthogonality moment conditions of the form

$$E(y_{i,s}u_{i,t}(\rho)) = E(y_{i,s}(y_{i,t} - \rho y_{i,t-1})) = 0 \qquad t = p+2, ..., T;\ s = 1, ..., t-p-1 \tag{5}$$

A GMM estimator based on the above moment conditions takes the general form

$$\widehat{\rho}_{GMM} = \left[\left(\sum_{i=1}^{N} y_{i,-1}' W_i\right)\widehat{\Omega}^{-1}\left(\sum_{i=1}^{N} W_i' y_{i,-1}\right)\right]^{-1}\left[\left(\sum_{i=1}^{N} y_{i,-1}' W_i\right)\widehat{\Omega}^{-1}\left(\sum_{i=1}^{N} W_i' y_i\right)\right] \tag{6}$$

where

$$W_i = \begin{bmatrix} y_{i,1} & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & y_{i,1} & y_{i,2} & & 0 & & 0 \\ \vdots & & & \ddots & & & \\ 0 & 0 & 0 & \cdots & y_{i,1} & \cdots & y_{i,T-p-1} \end{bmatrix}$$

is the $(T-p-1)\times((T-p-1)(T-p)/2)$ matrix of instruments, $y_i = (y_{i,p+2}, ..., y_{i,T})'$ is a $(T-p-1)$-vector of observations, $y_{i,-1}$ is its one-period lagged value and $\widehat{\Omega} = \frac{1}{N}\sum_{i=1}^{N} W_i'\Delta y_i \Delta y_i' W_i$ is a consistent (as $N \to \infty$) estimator of the optimal GMM weight matrix $\Omega = E(\frac{1}{N}\sum_{i=1}^{N} W_i' u_i u_i' W_i)$ under the null hypothesis that $\rho = 1$ (since then $u_{i,t} = \Delta y_{i,t}$).

Under the conditions given below expression (1), it can be shown that $\widehat{\rho}_{GMM}$ is a consistent estimator of ρ under the hypothesis that $\rho = 1$. Appropriately normalized, this estimator can be used to construct a test-statistic for the null that $\rho = 1$:

Theorem 1 *Let $E|u_{i,t}|^{4+\delta}$ be uniformly bounded over i and t, for some $\delta > 0$. Then, for fixed T, under the null hypothesis that $\rho = 1$,*

$$\tau_1 = \widehat{V}_0^{-1/2}\sqrt{N}\left(\widehat{\rho}_{GMM} - 1\right) \xrightarrow{L} N(0,1) \tag{7}$$

as $N \to \infty$, where $\widehat{V}_0 = \left[\left(\frac{1}{N}\sum_{i=1}^{N} y_{i,-1}' W_i\right)\widehat{\Omega}^{-1}\left(\frac{1}{N}\sum_{i=1}^{N} W_i' y_{i,-1}\right)\right]^{-1}$ is a consistent estimator of the variance of $\widehat{\rho}_{GMM}$ under the null.

The test statistic given by Theorem 1 enables us to test for the null hypothesis of a unit root using the tables of the normal distribution under quite general assumptions on the distribution of the error terms $u_{i,t}$. It allows for $u_{i,t}$ to be non-normally distributed with non-constant variance across both dimensions of the panel data and serially correlated across the time dimension with a cross-sectionally heterogeneous time-dependence structure.

Under the alternative hypothesis, $\widehat{\rho}_{GMM}$ is not a consistent estimator[3] of ρ. As a consequence, the test may lack power or even be biased against some specific alternatives. It is difficult to characterize these cases within the general set-up used here, but below we show in the Monte Carlo experiments as well as in a simplified formal set-up (see Theorem 3) that problems will likely occur when residual serial correlation is strongly negative. As mentioned in the introduction, negative residual serial correlation is known to be a difficult scenario for unit root tests in a pure time-series set-up. Our finding indicates that this remains true in a fixed-T panel data setting.

A possible reaction to this problem is to try and base the test-statistic on an estimator that is consistent under both the null and the alternative. It is easy to construct a consistent GMM estimator for ρ when $\rho < 1$ based on moment conditions of the form $E(y_{i,s}(\Delta y_{i,t} - \rho\Delta y_{i,t-1}))$, $s < t-p-2$ instead of those in (5) (see e.g. Arellano and Bond (1991)). However, this estimator cannot identify ρ when $\rho = 1$: the instrument $y_{i,t-s}$ will not be correlated with $\Delta y_{i,t-1}$. If one is, on the other hand, willing to sacrifice generality by imposing additional assumptions on the process under the alternative, it is possible to construct GMM estimators that are consistent under both the null and the alternative[4] (see e.g. Arellano and Bover (1995), Ahn and Schmidt (1995) and Blundell and Bond (1998)). Unit root tests based on

[3] This is an instance of the Neyman and Scott (1948) incidental parameters problem: the individual-specific parameters $\eta_i - z_{i,0}$ are not consistently estimable as $N \to \infty$.

[4] Examples of such assumptions are mean or covariance stationarity. Note that the estimators of this kind in the literature are based on untransformed data, i.e. on equation (1) rather than (4). They would therefore not directly apply to the set-up in this paper.

such estimators have recently been examined in a set-up without serial dependence in $u_{i,t}$ (see Bond, Nauges, and Windmeijer (2005)) and were found to perform poorly.

When choosing the instrument matrix W_i as $diag(y_{i,1}, ..., y_{i,T-p-1})$ and summing up the resulting moment conditions over t into the single condition $E\left[\sum_{t=1}^{T-p-1} y_{i,t}u_{i,t+p+1}(\rho)\right] = 0$, the GMM estimator reduces to the IV estimator, defined as

$$\widehat{\rho}_{IV} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T-p-1} y_{i,t}y_{i,t+p}\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T-p-1} y_{i,t}y_{i,t+p+1}\right). \tag{8}$$

Although the IV estimator $\widehat{\rho}_{IV}$ is asymptotically less efficient than the GMM estimator $\widehat{\rho}_{GMM}$, it is well-defined and consistent independently of whether T or N or both tend to infinity (see Arellano (2003))[5]. This makes the IV estimator a convenient vehicle for studying the influence of the time dimension on panel unit root tests under serial correlation in the error terms $u_{i,t}$ more rigorously. To this end, in the next theorem we give the limiting distribution of the test statistic based on the IV estimator. In order to obtain interpretable analytic results, we assume that the error processes are homogeneous MA(1) processes, i.e. $u_{i,t} = v_{i,t} + \theta v_{i,t-1}$ for all i and t, where the MA innovations $v_{i,t}$ are IID with zero mean and constant variance[6]. The extension to higher-order residual dependence is conceptually similar but less tractable.

Theorem 2 *Let* $u_{i,t} = v_{i,t} + \theta v_{i,t-1}$ *with*[7] $\theta \neq -1$ *and* $v_{i,t} \sim IID(0, \sigma_v^2)$, $\forall i$ *and* t. *Then, under the null hypothesis that* $\rho = 1$,

$$\tau_2 = [C(\theta, T)]^{-\frac{1}{2}} \sqrt{N}\left(\widehat{\rho}_{IV} - 1\right) \xrightarrow{L} N(0, 1) \tag{9}$$

as $N \to \infty$, *where* $C(\theta, T) = \frac{R(\theta,T)}{D(\theta,T)^2}$ *and* $R(\theta, T)$ *and* $D(\theta, T)$ *are polynomial functions of* T *and* θ *defined in the Appendix.*

Since the variance of the limiting distribution of $\sqrt{N}\left(\widehat{\rho}_{IV} - 1\right)$, given by $C(\theta, T)$, depends on the moving average nuisance parameter θ, implementation of the test statistic τ_2 in Theorem 2 requires an estimator of θ that is consistent under the null hypothesis that $\rho = 1$. A convenient estimator of θ can be obtained in two stages, following MacDonald and MacKinnon (1985). First, the correlation coefficient γ between $u_{i,t}$ and $u_{i,t-1}$ is estimated using the fact that $u_{i,t} = \Delta y_{i,t}$ is observed under the null hypothesis. The implied estimate

[5] In contrast, for the GMM estimator an increase in T implies a proliferation of the number of moment conditions as well as the need to estimate $O(T^2)$ nuisance parameters (the weighting matrix); one may expect that this will result in inferior behaviour if T is large relative to N.

[6] One may instead construct a unit root test based on the estimator in (8) and a scaling factor estimated by the square root of

$$\widehat{V}_{IV} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T-p-1} y_{i,t}y_{i,t+p}\right)^{-2}\left(\sum_{i=1}^{N} y_i' u_{i,+p} u_{i,+p}' y_i'\right).$$

This test will be valid under the original general assumptions, but more difficult to analyze analytically.

[7] When $\theta \neq -1$, $y_{i,t} = v_{i,t} - v_{i,0}$ and the process is indistinguishable from a pure white noise process, that is, the case $\rho = 1$, $\theta = -1$ is observationally equivalent to $\rho = 0$, $\theta = 0$. Otherwise stated, at these values the parameters are not identified.

of θ can then be retrieved by inverting the correlation coefficient function of the MA(1) process, given by $\gamma = \theta/(1+\theta^2)$.

[Figure 1]

The dependence of the test statistic τ_2 on θ and T, through the variance function $C(\theta, T)$, enables us to investigate the behaviour of large T panel unit root tests in finite T samples. Note that, for sufficiently large T, $C(\theta, T)$ approaches $C(T) = \frac{2}{T^2}$ if $\theta \neq -1$. This no longer depends on the MA parameter θ. Analogously to Hahn and Kuersteiner (2002, Theorem 4) and Harris and Tzavalis (1999), when both N and $T \to \infty$, one can scale the statistic τ_2 by T and replace $C(\theta, T)$ by $C(T)$, leading to the panel data unit root test statistic τ_2'

$$\tau_2' = \frac{T\sqrt{N}}{\sqrt{2}}\left(\widehat{\rho}_{IV} - 1\right) \xrightarrow{L} N(0,1) \tag{10}$$

which is suitable for use in large T and N panels. Comparing the variance function $C(\theta, T)$ with $C(T)$, for large T, one sees that the two variances differ by a factor of $\frac{T^2 C(\theta,T)}{2}$, which approaches unity as T becomes large. Plotting $\frac{T^2 C(\theta,T)}{2}$ against T, for various values of θ (see Figure 1) can help explain the serious size distortions of the large T panel unit root tests mentioned in the literature (see Wu and Yin (1999), inter alia).

Inspection of Figure 1 leads to the following conclusions. First, the large T test statistic τ_2' will be oversized in small T panels, compared to τ_2. This happens because $\frac{T^2 C(\theta,T)}{2} > 1$ for all θ, which implies that τ_2' is scaled with smaller variance than τ_2 in finite T panels. Hence, the tails of the distribution of the statistic τ_2' are drawn out. Moreover, Figure 1 shows that, when θ is negative, $\frac{T^2 C(\theta,T)}{2}$ converges slower to its asymptote than when $\theta > 0$. Hence, in the presence of negative residual autocorrelation the statistic τ_2' is oversized even when T is relatively large. Finally note that, if θ takes the large negative value -0.8, the pattern of convergence of $\frac{T^2 C(\theta,T)}{2}$ to its asymptote is non-monotonic.

As for the GMM based test, a potential problem with the IV tests comes from the fact that $\widehat{\rho}_{IV}$ is an inconsistent estimator of ρ when $\rho < 1$. In particular, it is not in general true that $plim(\widehat{\rho}_{IV}) < 1$ when $\rho < 1$. This may lead to a biased test. We now characterize the region of values of θ for which the test is consistent. To do so, we assume that the distribution of the initial conditions $z_{i,0}$ under the alternative hypothesis has mean and variance equal to that of the stationary distribution of z. Some investigations of the case $T = 3$ under more general initial conditions assumptions confirmed the finding that negative residual serial correlation renders incorrect acceptance of the unit root hypothesis more likely (calculations not included).

Theorem 3 *Under a covariance stationary alternative with $\rho > 0$ one has that*

$$plim(\widehat{\rho}_{IV}) < 1 \quad \textit{if and only if} \quad \theta > -\rho \quad \textit{or} \quad \theta < -1/\rho$$

Moreover, the bias is independent of $Var(\eta)$.

This result has the somewhat counterintuitive implication that negative residual autocorrelation is more likely to lead to misleading test results the further one moves away from the unit root. For example, when $\rho = 0.5$ even mild residual correlation of $\theta = -0.6$ will in large samples lead to the conclusion that there is a unit root. On the other hand, near the unit root there will only be a small θ-region in which the test is biased. While purely "mechanical" application of our tests may therefore lead to rather odd conclusions, most empirical researchers will resort to unit root tests only when there is a strong ex-ante suspicion of a high value of ρ. Theorem 3 suggests that our tests will be a useful tool in such situations.

To further investigate the test's behaviour near the unit root, Theorem 4 establishes the asymptotic distribution of the IV estimator based on a sequence of mean-stationary local alternatives. Because full covariance stationarity would imply that the variance of the initial conditions is $O(\sqrt{N})$ under this sequence, we find it more natural to keep the variance of the initial conditions fixed when computing the limiting distribution.

Theorem 4 *Under a sequence of mean-stationary local alternatives with fixed variance of the initial conditions where $\rho_N = 1 - c/\sqrt{N}$ and T is kept fixed one has that*

$$\sqrt{N}\left(\widehat{\rho}_{N,IV} - 1\right) \xrightarrow{L} N\left[-c\left(1 + \frac{2\theta}{T(1+\theta)^2 - (1+4\theta+\theta^2)}\right), \frac{R(\theta,T)}{D^2(\theta,T)}\right]$$

This result shows that while the IV estimator is locally asymptotically unbiased when $\theta = 0$ (note that $\sqrt{N}\left(\rho_N - 1\right) = -c$), test power will disappear as θ approaches -1: in this case $\frac{2\theta}{T(1+\theta)^2-(1+4\theta+\theta^2)} = -1$. On the other hand, the test will gain power for positive values of θ. However, since the denominator in the bias term increases with T if $\theta \neq -1$, these effects will be attenuated as T increases, except when $\theta = -1$.

3 Simulation Experiments

In this section we present the results of Monte Carlo experiments to judge the finite sample performance of the test statistics introduced in the previous section. In particular, our analysis is focused on examining the behaviour of the tests in relation to the sign and degree of serial correlation of the error terms $u_{i,t}$, as T increases. For all experiments, we report results on both the size and the size-adjusted power of the test statistics for different combinations of N and T, using 5000 replications. In order to evaluate the power of the tests, we consider the three alternative hypotheses that $\rho = 0.8$, $\rho = 0.9$ and $\rho = 0.95$. The size-adjusted power of the tests is calculated by the empirical frequency with which the null hypothesis is rejected using the actual one sided 5% critical value of the empirical distribution of the test statistic under the null hypothesis.

The analysis of this section proceeds as follows. First, we evaluate the performance of the test statistic τ_1, given by Theorem 1, which allows for a general specification of the autocovariance function of the error terms $u_{i,t}$. Second, we assess the performance of the test statistics τ_2 and τ_2' based on the IV estimator of ρ. In all experiments, we assume that the order of MA serial correlation is $p = 1$ and $v_{i,t} \sim NIID(0,1)$. For the statistic τ_1, we assume that $u_{i,t} = v_{i,t} + \theta_{i,t} v_{i,t-1}$ where the MA parameter $\theta_{i,t}$ is assumed to be uniformly $(-\frac{1}{2}, \frac{1}{2})$ distributed around its mean θ, which takes values in the set $\{-0.8, -0.6, ..., 0.8\}$. We use only 1 instrument per moment time period: increasing the number of instruments led to further performance deterioration. In the analysis of the statistics τ_2 and τ_2', we use the same set of values for θ, but choosing $\theta_{i,t} = \theta$ for all i and t. Throughout, we set $\eta_i = 0$ $\forall i$.

[Table 1]

Table 1 reports the size (at the 5% nominal significance level) and power of the test statistic τ_1. The results indicate that this statistic has the correct size at the 5% nominal level when the time dimension is small relative to the number of cross-sectional observations, e.g. $(T, N) = \{(5, 50), (5, 100), (10, 100)\}$. For this case, the power of the test is also satisfactory. This is true even for values of $\theta \leq -0.4$, where single time series tests seem to be substantially oversized and biased (see Stock (1994)). However, when $\theta = -0.8$, one observes that test power hardly responds to decreases in ρ: this can be attributed to the phenomenon documented in Theorem 3.

When T increases relative to N, the GMM statistic becomes critically oversized. This happens because this test statistic is designed for short T and large N panel data sets. When T increases, the number of moment conditions becomes large relative to the number of cross-sectional observations. This renders the asymptotics, which are designed for the fixed-T case, less reliable[8]. Additionally, the number of nuisance parameters involved in the GMM weighting matrix Ω increases considerably. As a consequence, the scaling factor $\widehat{V}_0^{-1/2}$ in Theorem 1 is less precisely estimated[9].

[Table 2]

Tables 2 and 3 present the results for the test statistics τ_2 and τ_2', respectively. The results of Table 2 indicate that, when T is small relatively to N, both the size and power performance of the statistic are similar to those of the statistic reported in Table 1. When T increases relative to N, the statistic τ_2 performs much better than τ_1, and has power which increases faster with T than with N. To put this finding into perspective one should keep in mind that, in contrast to the statistic τ_1, the number of nuisance parameters involved in the statistic τ_2 remains the same as T increases, due to the homogeneity assumption on $\theta_{i,t}$. These results suggest that, although the GMM based statistic τ_1 is asymptotically more efficient than the IV based test statistic τ_2, the latter has better finite sample performance regardless of the time dimension of the panel when the nuisance parameters of the autocovariance function of the error terms $u_{i,t}$ are homogeneous across i and t. Finally, the results of Table 3 confirm our theoretical predictions made in Section 2. They clearly show that the statistic τ_2', which is based on a large T approximation of the variance function $C(\theta, T)$ in finite T samples, can

[8]For this reason, Newey and Windmeijer (2005) proposes a different kind of asymptotic approximation for moment condition models with many moments.

[9]Inference in GMM panel models is more generally problematic in small samples - see Bond and Windmeijer (2002).

lead to a seriously oversized test, especially for large negative values of the moving average parameter θ. These results suggest the need to use fixed T panel data tests in practice, especially when the error terms $u_{i,t}$ are negatively correlated.

[Table 3]

4 Conclusions

This paper has introduced two panel unit root test statistics with the aim to study the influence of the time dimension and the impact of serial correlation in the error terms. The first statistic is based on the GMM estimator and is appropriate for short panel data sets with large cross-section dimension and heterogeneous autocovariance function of the error terms, across both dimensions of the panel. The second statistic is based on the IV estimator. It is appropriate for panel data with homogenous autocovariance function of the error terms and can be implemented to panel data sets regardless of the time dimension.

Our results show that both test statistics have the correct size if the time dimension of the panel is short relative to the cross section dimension. However, when the time dimension increases, the GMM based test can lead to critical size distortions. Using the IV based test, the paper shows that strongly negative serial correlation in the error terms can lead to substantial loss of power, especially - and somewhat counterintuitively - far from the unit root. In these cases, using large time dimension approximations of the variance of the limiting distributions of panel unit root tests can lead to serious size distortions in finite samples.

Appendix

PROOF OF THEOREM 1

The reasoning is similar to that of Theorem 2 below. Therefore, we only provide the asymptotic normality part of the proof. From (6), one has that under H_0:

$$\begin{aligned}\sqrt{N}\left(\widehat{\rho}_{GMM}-1\right) &= \left[\left(\frac{1}{N}\sum_{i=1}^{N} y_{i,-1}' W_i\right)\widehat{\Omega}^{-1}\left(\frac{1}{N}\sum_{i=1}^{N} W_i' y_{i,-1}\right)\right]^{-1} \textit{times} \\ & \left[\left(\frac{1}{N}\sum_{i=1}^{N} y_{i,-1}' W_i\right)\widehat{\Omega}^{-1}\left(\frac{\sqrt{N}}{N}\sum_{i=1}^{N} W_i' u_i\right)\right]\end{aligned}$$

Asymptotic normality is demonstrated by applying Liapounov's CLT to $\sqrt{N}\frac{1}{N}\sum_{i=1}^{N} W_i' u_i$ and a LLN to all other factors. Starting with the former, one needs to show - applying the Cramer-Wold device - that for any vector λ with $\|\lambda\| = 1$ and $\Omega_N = Var(\sqrt{N}\frac{1}{N}\sum_{i=1}^{N} W_i' u_i)$

one has $\sqrt{N}\frac{1}{N}\sum_{i=1}^{N}\lambda'\Omega_N^{-1/2}W_i'u_i \rightarrow N(0,I)$. To do so, one observes that a typical component of the vector $\sqrt{N}\frac{1}{N}\sum_{i=1}^{N}W_i'u_i$ is $\sqrt{N}\frac{1}{N}\sum_{i=1}^{N}y_{i,t-p-s}u_{i,t},\ s \geq 1$. This is rewritten, under H_0, as $\sqrt{N}\frac{1}{N}\sum_{i=1}^{N}\sum_{\tau=1}^{t-p-s}u_{i,\tau}u_{i,t}$. The above result will therefore follow if the sequence $u_{i,\tau}u_{i,t}$ satisfies the Lindeberg condition. Note here that the time dimension is kept fixed when taking limits and hence that boundedness of the terms of a sum over t implies boundedness of the sum. The Lindeberg condition is satisfied since by the Cauchy-Schwartz inequality

$$\begin{aligned} E\left|u_{i,\tau}u_{i,t}\right|^{2+\delta} &\leq \left(E\left|u_{i,\tau}^{2+\delta}\right|^2\right)^{1/2}\left(E\left|u_{i,t}^{2+\delta}\right|^2\right)^{1/2} \\ &< \infty \quad if \quad E\left|u_{i,t}\right|^{4+2\delta} < \infty \ \ for\ some\ \delta > 0,\ \forall t. \end{aligned}$$

The latter condition is satisfied by assumption. Boundedness of $E\left|u_{i,\tau}u_{i,t}\right|^{2+\delta}$ implies boundedness of $E\left|y_{i,t-p-s}u_{i,t}\right|^{2+\varepsilon}$ by Minkowski's inequality.
Defining $m_N = E\left(\frac{1}{N}\sum_{i=1}^{N}W_i'y_{i,-1}\right)$, $m = \lim m_N$, $\Omega = \lim \Omega_N$ we now have

$$\left[m_N'\Omega_N^{-1}m_N\right]^{-1}m_N'\Omega_N^{-1}\sqrt{N}\frac{1}{N}\sum_{i=1}^{N}W_i'u_i \rightarrow N(0,\left(m'\Omega^{-1}m\right)^{-1})$$

Finally, apply Markov's LLN to $\frac{1}{N}\sum_{i=1}^{N}W_i'y_{i,-1}$ and $\widehat{\Omega}$. A typical component of the former is $\frac{1}{N}\sum_{i=1}^{N}y_{i,t-p-s}u_{i,t},\ s \geq 0$. Markov's LLN will apply, by the same reasoning as above, if $E\left|u_{i,\tau}u_{i,t}\right|^{1+\delta}$ is bounded for all τ, t. By the Cauchy-Schwartz inequality, this is the case under the assumptions of the Theorem. Thus $\frac{1}{N}\sum_{i=1}^{N}W_i'y_{i,-1} - m_N \rightarrow 0\ a.s.$ and hence in probability. Similarly, a typical component of $\widehat{\Omega}$ is $\frac{1}{N}\sum_{i=1}^{N}y_{i,t-p-s}u_{i,t}u_{i,\tau}y_{i,\tau-p-r}$ with $r, s \geq 0$. Decomposing $y_{i,t-p-s}$ and $y_{i,\tau-p-r}$ as above and using Minkowski's inequality, one sees that Markov's LLN will apply to terms of this type if $E\left|u_{i,k}u_{i,\tau}u_{i,t}u_{i,r}\right|^{1+\delta} < \infty$ for some $\delta > 0$. Repeated application of the Cauchy-Schwartz inequality shows that this is the case if $E\left|u_{i,t}\right|^{4+\varepsilon} < \infty$ for some $\varepsilon > 0$. Hence we have $\widehat{\Omega} - \Omega_N \rightarrow 0$, completing the proof.

PROOF OF THEOREM 2

The derivations of the results are based on the Lindeberg-Levy Central Limit Theorem (CLT) for independent, identical innovation processes (see White (1984)) and the Law of Large Numbers (LLN). We proceed in two stages. We first show that $\widehat{\rho}_{IV}$ is a consistent estimator of ρ under the hypothesis that $\rho = 1$. Then we derive the limiting distribution of $\widehat{\rho}_{IV} - 1$.
The consistency (as $N \rightarrow \infty$) of $\widehat{\rho}_{IV}$ can be shown by writing

$$\widehat{\rho}_{IV} - 1 = \frac{\sum_{i=1}^{N}Z_{i,T}}{\sum_{i=1}^{N}D_{i,T}} \tag{A1}$$

where $Z_{i,T} = \sum_{t=3}^{T} y_{i,t-2}u_{i,t}$ and $D_{i,T} = \sum_{t=3}^{T} y_{i,t-1}y_{i,t-2}$. Then

$$\begin{aligned}
plim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} Z_{i,T} &= \sum_{t=1}^{T-2}\sum_{s=t+2}^{T}\left(plim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} u_{i,t}u_{i,s}\right) \\
&= \sum_{t=1}^{T-2}\left[plim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\left(v_{i,t}+\theta v_{i,t-1}\right)\left(v_{i,t+2}+\theta v_{i,t+1}\right)\right] = 0 \qquad \text{(A2)}
\end{aligned}$$

The limiting distribution of $\widehat{\rho}_{IV} - 1$ can be obtained by calculating the variance of $Z_{i,T}$ and the probability limit of the denominator of (A1), and then applying standard large sample theory. In particular:

$$\begin{aligned}
E(Z_{i,T}^2) &= \sum_{t=1}^{T-2}\sum_{s=t+2}^{T} E(u_{i,t}^2 u_{i,s}^2) + 2\sum_{t=1}^{T-3}\sum_{x=t+2}^{T-1} E(u_{i,t}^2 u_{i,s}u_{i,s+1}) \\
&+2\sum_{t=1}^{T-3}\sum_{s=t+2}^{T-1} E(u_{i,t}u_{i,s}u_{i,t+1}u_{i,s+1}) + 2\sum_{t=1}^{T-3}\sum_{s=t+3}^{T} E(u_{i,t}u_{i,t+1}u_{i,s}^2) \\
&+2\sum_{t=1}^{T-4}\sum_{s=t+3}^{T-1} E(u_{i,t}u_{i,s}u_{i,t+1}u_{i,s+1})
\end{aligned}$$

which, after substituting $u_{i,t} = v_{i,t} + \theta v_{i,t-1}$ and simplifying, gives

$$E(Z_{i,T}^2) = \sigma_v^4 R(\theta, T) \qquad \text{(A3)}$$

where

$$R(\theta, T) = R_4\theta^4 + R_3\theta^3 + R_2\theta^2 + R_1\theta + R_0$$

with

$$\begin{aligned}
R_4 &= R_0 = \frac{1}{2}T(T-3) + 1 \\
R_3 &= R_1 = 2T(T-5) + 12 \\
R_2 &= 3T(T-5) + 20.
\end{aligned}$$

By a standard Law of Large Numbers, $plim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N} D_{i,T} = E(D_{i,T})$ and

$$\begin{aligned}
E(D_{i,T}) &= \sum_{t=3}^{T} E(y_{i,t-2}y_{i,t-1}) \\
&= \sum_{t=1}^{T-2}\sum_{s=1}^{t} E\left(u_{i,s}^2\right) + 2\sum_{t=1}^{T-2}\sum_{s=1}^{t} E\left(u_{i,s}u_{i,s+1}\right) - \sum_{t=1}^{T-2} E\left(u_{i,t}u_{i,t+1}\right) \\
&= \sigma_v^2 D(\theta, T)
\end{aligned}$$

with $D(\theta, T) = \frac{1}{2}(T-2)\left(T(1+\theta)^2 - (1 + 4\theta + \theta^2)\right)$.
Theorem 2 now follows by applying the CLT to the sums of $Z_{i,T}$, implying $\frac{1}{\sqrt{N}}\sum_{i=1}^{N} Z_{i,T} \to N\left(0, \sigma_v^2 R(\theta, T)\right)$ as $N \to \infty$.

PROOF OF THEOREM 3

For the analysis of the behaviour of the IV based test under the alternative, we restrict ourselves for the sake of analytical tractability to the case where $z_{i,t}$ is mean and variance stationary. This means that $z_{i,0} = \eta_i + v_{i,0} + \xi_i$ where $E(\xi_i) = 0$, $Var(\xi_i) = \sigma_v^2 (\rho + \theta)^2 / (1 - \rho^2)$ and ξ_i independent of all other random elements. The following is only a sketch of the reasoning - full details are available upon request from the authors.

By recursive substitution, $y_{i,t}$ can be written as

$$y_{i,t} = \sum_{j=0}^{t-1} \rho^j u_{i,t-j} + (\eta_i - z_{i,0})(1 - \rho^t) \tag{A4}$$

We now obtain the range of values of θ as a function of ρ for which $plim(\widehat{\rho}_{IV}) < 1$.To do so, first note that

$$\begin{aligned} plim(\widehat{\rho}_{IV}) &= \rho + plim \frac{\sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2}(\eta_i - z_{i,0})(1-\rho)}{\sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2} y_{i,t-1}} + plim \frac{\sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2} u_{i,t}}{\sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2} y_{i,t-1}} \\ &= \rho + (1-\rho) \frac{Num(\theta, T, \sigma_v^2, \sigma_\xi^2)}{Denom(\theta, T, \sigma_v^2, \sigma_\xi^2)} + 0 \end{aligned} \tag{A5}$$

Using standard algebra based on expression (A4) one finds that

$$\begin{aligned} Num(\theta, T, \sigma_v^2, \sigma_\xi^2) &= \sum_{t=3}^{T} \left[\left(\sigma_v^2 + \sigma_\xi^2\right)\left(1 - \rho^{t-2}\right) - \theta \sigma_v^2 \rho^{t-3} \right] \\ &= (T - 2 - \rho \frac{1 - \rho^{T-2}}{1 - \rho})(\sigma_v^2 + \sigma_\xi^2) - \theta \sigma_v^2 \frac{1 - \rho^{T-2}}{1 - \rho} \end{aligned}$$

A similar, but more cumbersome expression can be obtained in the same way for the factor $Denom(\theta, T, \sigma_v^2, \sigma_\xi^2)$ in (11). Like $Num(\theta, T, \sigma_v^2, \sigma_\xi^2)$, this expression does not depend on $Var(\eta)$. It turns out that the equation $plim(\widehat{\rho}_{IV}) = 1$, rewritten as $\rho Denom + (1-\rho) Num = Denom$, is quadratic in θ. The roots of this equation are $-\rho$ and $-1/\rho$. Note that this does not depend on the value of T or σ_v^2.

PROOF OF THEOREM 4

Starting with the usual decomposition

$$\begin{aligned} \sqrt{N}\left(\widehat{\rho}_{N,IV} - 1\right) &= \sqrt{N}\left(\rho_N - 1\right) + \sqrt{N} \frac{\frac{1}{N} \sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2}(\eta_i - z_{i,0})}{\frac{1}{N} \sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2} y_{i,t-1}} (1 - \rho_N) \\ &\quad + \sqrt{N} \frac{\frac{1}{N} \sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2} u_{i,t}}{\frac{1}{N} \sum_{t=3}^{T} \sum_{i=1}^{N} y_{i,t-2} y_{i,t-1}} \\ &=: I + II + III \end{aligned}$$

we examine each term separately. Since $\rho_N = 1 - c/\sqrt{N}$, we have that I equals $-c$. For II

and III, first examine the denominator:

$$\begin{aligned}\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}y_{i,t-1} &= \frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N}\rho_N y_{i,t-2}^2 + \frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}(\eta_i - z_{i,0})(1-\rho_N) \\ &\quad + \frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}u_{i,t-1} \\ &= \left(\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}^2 - O_p(n^{-1/2})\right) + \left(O_p(n^{-1/2})\right) \\ &\quad + \left(\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}u_{i,t-1}\right)\end{aligned}$$

where use is made of the fact that $1-\rho_N$ is $O(n^{-1/2})$ and that the variance of ξ_i is now kept fixed, hence $\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}^2$ is $O_p(1)$. By expression (A4) and the fact that $\rho_N^k = 1 - k\frac{c}{\sqrt{N}} + o(n^{-1/2})$ we have

$$\begin{aligned}y_{i,t-2} &= \sum_{j=0}^{t-3}\rho_N^j u_{i,t-2-j} + (\eta_i - z_{i,0})(1-\rho_N^{t-2}) \\ &= \sum_{j=0}^{t-3} u_{i,t-2-j} - \sum_{j=0}^{t-3} k\frac{c}{\sqrt{N}}u_{i,t-2-j} + (\eta_i - z_{i,0})k\frac{c}{\sqrt{N}} + o_p(n^{-1/2}). \qquad \text{(A.6)}\end{aligned}$$

Hence we have $y_{i,t-2}^2 = \left(\sum_{j=1}^{t-2} u_{i,j}\right)^2 + o_p(1)$. Applying a similar reasoning to the term $\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}u_{i,t-1}$ and using the calculations in the proof of Theorem 2 one obtains that

$$\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}y_{i,t-1} \xrightarrow{p} \sigma_v^2 D(\theta, T)$$

under the sequence of local alternatives. Now turning to the numerator of II one has that

$$\begin{aligned}\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}(\eta_i - z_{i,0}) &= -\sum_{t=3}^{T}\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j=0}^{t-3}\rho_N^j u_{i,t-2-j} + (\eta_i - z_{i,0})(1-\rho_N^{t-2})\right)(v_{i,0}+\xi_i) \\ &= -\sum_{t=3}^{T}\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j=0}^{t-3}(1 - j\frac{c}{\sqrt{N}} + o(n^{-1/2}))u_{i,t-2-j}(v_{i,0}+\xi_i)\right) + O_p(n^{-1/2}) \\ &= -\sum_{t=3}^{T}\frac{1}{N}\sum_{i=1}^{N} u_{i,1}(v_{i,0}+\xi_i) + o_p(1) \\ &\xrightarrow{p} -(T-2)\theta\sigma_v^2.\end{aligned}$$

and $\sqrt{N}(1-\rho_N) \to c$. Finally, for the numerator of III one has

$$\frac{1}{N}\sum_{t=3}^{T}\sum_{i=1}^{N} y_{i,t-2}u_{i,t} = \sum_{t=3}^{T}\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\sum_{j=0}^{t-3} u_{i,t-2-j}u_{i,t}\right)$$

$$
-\sum_{t=3}^{T}\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\sum_{j=0}^{t-3}k\frac{c}{\sqrt{N}}u_{i,t-2-j}+(\eta_i-z_{i,0})k\frac{c}{\sqrt{N}}+o_p(n^{-1/2})\right)u_{i,t}
$$
$$
= \sum_{t=3}^{T}\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left(\sum_{j=0}^{t-3}u_{i,t-2-j}u_{i,t}\right)+o_p(1)
$$

Repeating the calculations of the proof of Theorem (2) (the section calculating the variance of the term "$Z_{i,T}$") to this term and reassembling all terms completes the proof.

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N	10	25	25	50	50	100	100	100
T	5	5	10	5	10	5	10	25
				θ=-0.8				
size (ρ=1)	0.08	0.07	0.11	0.06	0.11	0.06	0.08	0.14
ρ=0.95	0.05	0.06	0.07	0.06	0.07	0.06	0.11	0.26
ρ=0.9	0.05	0.06	0.08	0.07	0.09	0.08	0.14	0.34
ρ=0.8	0.07	0.06	0.09	0.07	0.09	0.08	0.14	0.26
				θ=-0.6				
size (ρ=1)	0.07	0.08	0.11	0.06	0.10	0.06	0.08	0.18
ρ=0.95	0.06	0.06	0.09	0.07	0.11	0.09	0.18	0.60
ρ=0.9	0.07	0.07	0.13	0.10	0.18	0.10	0.30	0.77
ρ=0.8	0.07	0.08	0.15	0.11	0.22	0.12	0.36	0.73
				θ=-0.4				
size (ρ=1)	0.07	0.07	0.10	0.06	0.10	0.06	0.09	0.23
ρ=0.95	0.06	0.07	0.11	0.09	0.19	0.12	0.36	0.91
ρ=0.9	0.07	0.10	0.19	0.14	0.36	0.20	0.65	0.99
ρ=0.8	0.08	0.13	0.25	0.19	0.49	0.29	0.80	0.99
				θ=-0.2				
size (ρ=1)	0.07	0.06	0.11	0.06	0.10	0.06	0.10	0.27
ρ=0.95	0.06	0.10	0.19	0.13	0.31	0.17	0.55	0.99
ρ=0.9	0.08	0.14	0.31	0.22	0.60	0.35	0.91	1.00
ρ=0.8	0.11	0.22	0.46	0.37	0.82	0.58	0.99	1.00
				θ=0				
size (ρ=1)	0.08	0.07	0.13	0.07	0.11	0.06	0.10	0.28
ρ=0.95	0.07	0.10	0.19	0.16	0.41	0.27	0.71	1.00
ρ=0.9	0.09	0.18	0.37	0.30	0.78	0.56	0.98	1.00
ρ=0.8	0.15	0.32	0.59	0.56	0.96	0.88	1.00	1.00
				θ=0.2				
size (ρ=1)	0.07	0.06	0.13	0.07	0.11	0.06	0.10	0.31
ρ=0.95	0.08	0.13	0.23	0.19	0.47	0.32	0.78	1.00
ρ=0.9	0.11	0.23	0.46	0.41	0.86	0.69	1.00	1.00
ρ=0.8	0.18	0.46	0.71	0.76	0.99	0.97	1.00	1.00
				θ=0.4				
size (ρ=1)	0.08	0.07	0.13	0.06	0.12	0.06	0.10	0.30
ρ=0.95	0.08	0.14	0.24	0.23	0.50	0.35	0.81	1.00
ρ=0.9	0.13	0.28	0.52	0.50	0.89	0.77	1.00	1.00
ρ=0.8	0.21	0.57	0.79	0.87	1.00	0.99	1.00	1.00
				θ=0.6				
size (ρ=1)	0.07	0.07	0.13	0.06	0.12	0.06	0.10	0.29
ρ=0.95	0.10	0.16	0.27	0.25	0.55	0.38	0.83	1.00
ρ=0.9	0.16	0.32	0.56	0.56	0.92	0.82	1.00	1.00
ρ=0.8	0.26	0.63	0.83	0.92	1.00	1.00	1.00	1.00
				θ=0.8				
size (ρ=1)	0.07	0.07	0.12	0.06	0.11	0.06	0.09	0.28
ρ=0.95	0.10	0.15	0.27	0.24	0.55	0.42	0.86	1.00
ρ=0.9	0.15	0.31	0.57	0.58	0.94	0.84	1.00	1.00
ρ=0.8	0.28	0.65	0.85	0.94	1.00	1.00	1.00	1.00

Table 1: Empirical size and size adjusted power of τ_1

N	10	25	25	50	50	100	100	100
T	5	5	10	5	10	5	10	25
				θ=-0.80				
size (ρ=1)	0.13	0.09	0.13	0.09	0.12	0.07	0.10	0.13
ρ=0.95	0.09	0.11	0.17	0.13	0.21	0.17	0.28	0.49
ρ=0.9	0.14	0.19	0.29	0.24	0.37	0.32	0.49	0.65
ρ=0.8	0.25	0.35	0.45	0.42	0.50	0.51	0.56	0.55
				θ=-0.60				
size (ρ=1)	0.12	0.09	0.11	0.08	0.09	0.08	0.08	0.08
ρ=0.95	0.09	0.11	0.20	0.14	0.27	0.18	0.40	0.90
ρ=0.9	0.14	0.19	0.36	0.26	0.50	0.35	0.69	0.98
ρ=0.8	0.25	0.36	0.55	0.47	0.68	0.61	0.84	0.98
				θ=-0.40				
size (ρ=1)	0.10	0.08	0.08	0.07	0.07	0.06	0.07	0.07
ρ=0.95	0.09	0.11	0.25	0.14	0.38	0.20	0.61	1.00
ρ=0.9	0.14	0.20	0.52	0.28	0.74	0.43	0.94	1.00
ρ=0.8	0.25	0.40	0.77	0.55	0.93	0.76	0.99	1.00
				θ=-0.20				
size (ρ=1)	0.10	0.08	0.08	0.07	0.07	0.06	0.06	0.06
ρ=0.95	0.09	0.11	0.29	0.17	0.51	0.25	0.81	1.00
ρ=0.9	0.14	0.22	0.67	0.37	0.92	0.56	0.99	1.00
ρ=0.8	0.27	0.48	0.95	0.71	0.99	0.91	1.00	1.00
				θ=0.00				
size (ρ=1)	0.09	0.07	0.08	0.07	0.07	0.05	0.06	0.06
ρ=0.95	0.09	0.13	0.35	0.18	0.60	0.31	0.85	1.00
ρ=0.9	0.15	0.28	0.78	0.43	0.97	0.71	1.00	1.00
ρ=0.8	0.30	0.58	0.99	0.83	1.00	0.98	1.00	1.00
				θ=0.20				
size (ρ=1)	0.09	0.07	0.08	0.07	0.07	0.06	0.06	0.06
ρ=0.95	0.09	0.14	0.37	0.21	0.61	0.33	0.87	1.00
ρ=0.9	0.15	0.30	0.83	0.49	0.98	0.76	1.00	1.00
ρ=0.8	0.34	0.65	0.99	0.91	1.00	0.99	1.00	1.00
				θ=0.40				
size (ρ=1)	0.10	0.07	0.08	0.07	0.07	0.06	0.06	0.06
ρ=0.95	0.10	0.15	0.38	0.22	0.62	0.35	0.88	1.00
ρ=0.9	0.16	0.32	0.83	0.53	0.98	0.80	1.00	1.00
ρ=0.8	0.35	0.73	0.99	0.94	1.00	0.99	1.00	1.00
				θ=0.60				
size (ρ=1)	0.09	0.08	0.08	0.07	0.07	0.06	0.06	0.06
ρ=0.95	0.10	0.15	0.38	0.22	0.63	0.37	0.90	1.00
ρ=0.9	0.17	0.31	0.84	0.54	0.99	0.82	1.00	1.00
ρ=0.8	0.37	0.74	1.00	0.95	1.00	1.00	1.00	1.00
				θ=0.80				
size (ρ=1)	0.08	0.07	0.07	0.06	0.06	0.06	0.06	0.06
ρ=0.95	0.10	0.15	0.38	0.23	0.64	0.38	0.90	1.00
ρ=0.9	0.17	0.32	0.84	0.56	0.99	0.83	1.00	1.00
ρ=0.8	0.38	0.76	1.00	0.96	1.00	1.00	1.00	1.00

Table 2: Empirical size and size adjusted power of τ_2

N	10	25	25	50	50	100	100	100
T	5	5	10	5	10	5	10	25
				θ=-0.80				
size (ρ=1)	0.30	0.29	0.30	0.29	0.31	0.30	0.31	0.33
ρ=0.95	0.10	0.13	0.21	0.15	0.30	0.19	0.37	0.65
ρ=0.9	0.16	0.24	0.40	0.30	0.50	0.38	0.60	0.76
ρ=0.8	0.36	0.45	0.57	0.51	0.61	0.61	0.68	0.65
				θ=-0.60				
size (ρ=1)	0.27	0.26	0.24	0.27	0.23	0.27	0.23	0.18
ρ=0.95	0.10	0.13	0.25	0.16	0.34	0.22	0.49	0.96
ρ=0.9	0.16	0.23	0.48	0.31	0.62	0.43	0.80	1.00
ρ=0.8	0.33	0.45	0.66	0.55	0.78	0.70	0.90	1.00
				θ=-0.40				
size (ρ=1)	0.22	0.21	0.16	0.21	0.16	0.21	0.15	0.11
ρ=0.95	0.09	0.12	0.28	0.16	0.43	0.23	0.66	1.00
ρ=0.9	0.15	0.25	0.60	0.36	0.82	0.51	0.96	1.00
ρ=0.8	0.31	0.49	0.86	0.66	0.97	0.83	0.99	1.00
				θ=-0.20				
size (ρ=1)	0.20	0.17	0.14	0.17	0.11	0.17	0.11	0.09
ρ=0.95	0.10	0.11	0.31	0.18	0.53	0.28	0.82	1.00
ρ=0.9	0.16	0.12	0.70	0.41	0.94	0.62	0.99	1.00
ρ=0.8	0.32	0.56	0.97	0.79	1.00	0.95	1.00	1.00
				θ=0.00				
size (ρ=1)	0.18	0.16	0.11	0.15	0.10	0.13	0.10	0.08
ρ=0.95	0.09	0.14	0.35	0.19	0.60	0.32	0.86	1.00
ρ=0.9	0.16	0.30	0.79	0.46	0.97	0.73	1.00	1.00
ρ=0.8	0.34	0.65	0.99	0.87	1.00	0.99	1.00	1.00
				θ=0.20				
size (ρ=1)	0.15	0.14	0.10	0.13	0.10	0.12	0.09	0.08
ρ=0.95	0.09	0.15	0.37	0.22	0.61	0.33	0.87	1.00
ρ=0.9	0.16	0.31	0.83	0.50	0.98	0.77	1.00	1.00
ρ=0.8	0.36	0.72	0.99	0.92	1.00	0.99	1.00	1.00
				θ=0.40				
size (ρ=1)	0.16	0.14	0.10	0.12	0.10	0.11	0.09	0.07
ρ=0.95	0.09	0.15	0.39	0.23	0.62	0.36	0.88	1.00
ρ=0.9	0.15	0.32	0.83	0.54	0.98	0.81	1.00	1.00
ρ=0.8	0.34	0.73	0.99	0.95	1.00	0.99	1.00	1.00
				θ=0.60				
size (ρ=1)	0.15	0.13	0.11	0.12	0.09	0.11	0.08	0.07
ρ=0.95	0.09	0.15	0.38	0.22	0.63	0.37	0.90	1.00
ρ=0.9	0.16	0.32	0.83	0.52	0.99	0.83	1.00	1.00
ρ=0.8	0.38	0.76	1.00	0.95	1.00	1.00	1.00	1.00
				θ=0.80				
size (ρ=1)	0.15	0.13	0.10	0.12	0.10	0.11	0.09	0.08
ρ=0.95	0.10	0.14	0.39	0.24	0.65	0.38	0.88	1.00
ρ=0.9	0.17	0.33	0.85	0.56	0.99	0.84	1.00	1.00
ρ=0.8	0.39	0.75	1.00	0.97	1.00	1.00	1.00	1.00

Table 3: Empirical size and size adjusted power of τ_2'

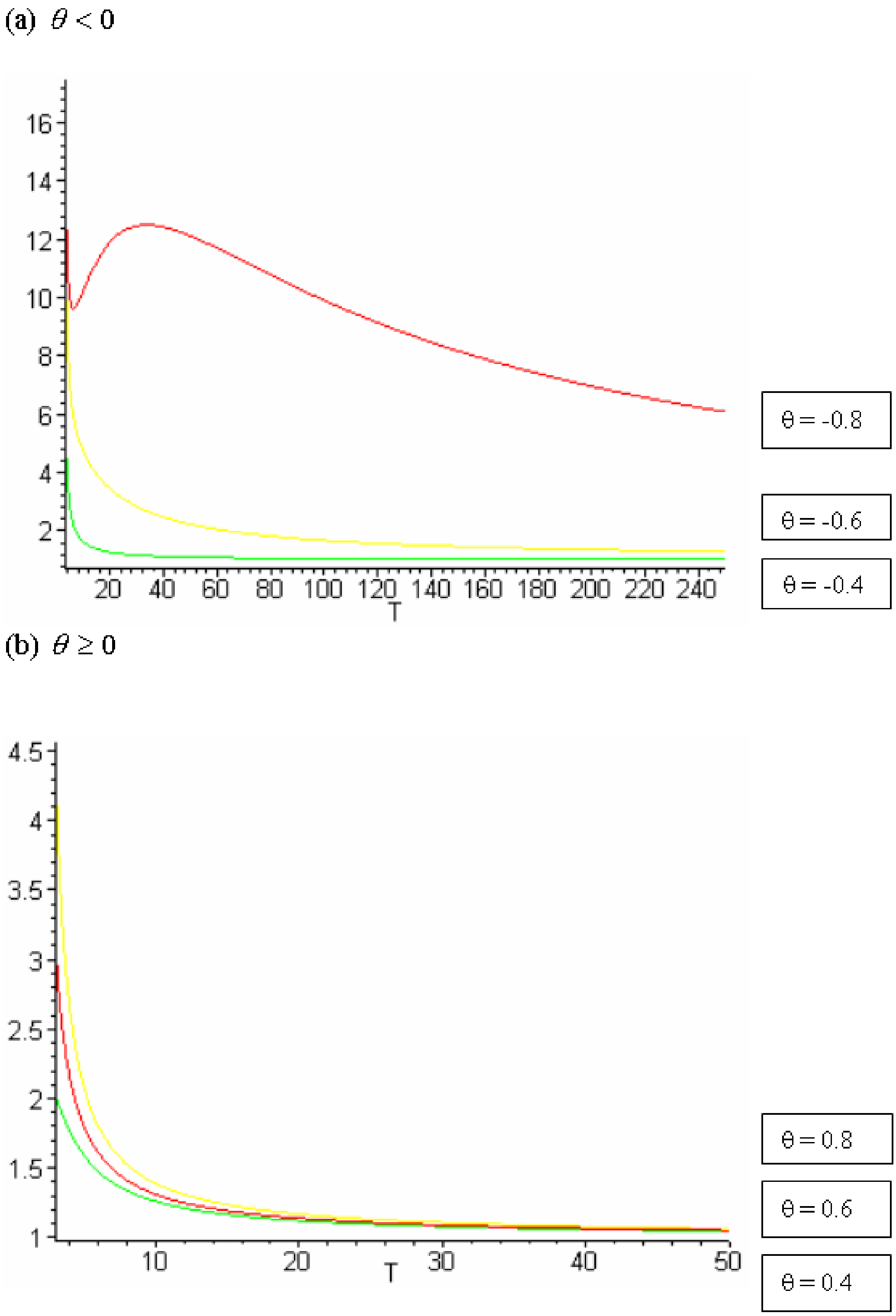

Figure 1: Plot of $\frac{T^2 C(\theta,T)}{2}$ against T.

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