Department of Economics

Testing for Exogeneity in Nonlinear Threshold Models

George Kapetanios

Working Paper No. 515 July 2004 ISSN 1473-0278

Queen Mary
University of London

Testing for Exogeneity in Nonlinear Threshold Models

George Kapetanios*
Queen Mary, University of London

April 2004

Abstract

Most work in the area of nonlinear econometric modelling is based on a single equation and assumes exogeneity of the explanatory variables. Recently, work by Caner and Hansen (2003) and Psaradakis, Sola, and Spagnolo (2004) has considered the possibility of estimating nonlinear models by methods that take into account endogeneity but provided no tests for exogeneity. This paper examines the problem of testing for exogeneity in nonlinear threshold models. We suggest new Hausman-type tests and discuss the use of the bootstrap to improve the properties of asymptotic tests. The theoretical properties of the tests are discussed and an extensive Monte Carlo study is undertaken.

Key Words: Threshold Models, Endogeneity, Bootstrap.
JEL Classification: C12, C15, C22.

1 Introduction

Nonlinear models have been used extensively in recent years to investigate macroeconomic phenomena. A number of classes of models have been popularised in the literature. Two of the main classes considered are threshold models and smooth transition models. The main characteristic of both classes is the non-constancy of the response of the dependent variables to the

*Department of Economics, Queen Mary, University of London, Mile End Road, London E1 4NS. email: G.Kapetanios@qmul.ac.uk

explanatory variables. This response which is, in linear regression models, simply the coefficient of the explanatory variable, is allowed to vary depending on the occurrence of given trigger events. The main difference between threshold models and smooth transition models is that the first class assumes that coefficients change abruptly (discontinuously) with respect to the trigger events whereas for smooth transition models the responses change gradually (continuously).

Most work in the area of nonlinear econometric modelling is based on a single equation and assumes exogeneity of the explanatory variables. However, just like in linear models, this assumption is mostly suspect in economics. Explanatory variable endogeneity can cause at least as much trouble in nonlinear models as in linear ones. However, endogeneity has not been taken into account in most of the literature. Recently work by Caner and Hansen (2003) and Psaradakis, Sola, and Spagnolo (2004) has considered the possibility of estimating nonlinear models by methods that take into account endogeneity. Hence, two-stage least squares and GMM estimators have appeared in the literature. However, the crucial problem remains. Should one use these estimators to alleviate bias in case of endogeneity or use standard estimators which, although biased under endogeneity, are likely to be more efficient if there is no endogeneity? Clearly this problem can be addressed using Hausman type tests, which compare standard estimators with those that account for endogeneity. This topic is addressed in this paper.

Unlike linear models where estimators such as OLS are known to be efficient under relatively mild conditions this is not necessarily the case for nonlinear models. Hence, the standard results of Hausman (1978) cannot be invoked automatically to derive asymptotic distributions for test statistics. The problem can be alleviated using bootstrap methods. The problem is then seen to be easily tackled. We thus propose new estimators which are a mixture of standard and endogeneity robust ones where a Hausman type test is used to decide which estimator to use for a given sample. It is seen, via simulations, that the standard asymptotic theory of the Hausman test appears not to be a good guide in small samples. On the other hand the bootstrap appears much more useful. In this paper we focus on threshold models which have been more extensively analysed in the case of endogeneity by Caner and Hansen (2003). Of course, all methods are readily adapted for any class of nonlinear models. We note that a specific way to test for endogeneity in Markov-Switching models has been recently suggested by Kim (2004).

The paper is organised as follows: Section 2 discusses the model and the problem addressed. Section 3 discusses the bootstrap in this context. Section 4 provides Monte Carlo evidence in favour of the new methods. Finally, Section 5 concludes.

2 The Model

The model we consider is that of Caner and Hansen (2003). In the simple form analysed there the model is given by

$$y_i = \beta_1' z_i I_{q_i \leq \gamma} + \beta_2' z_i I_{q_i > \gamma} + \epsilon_i \tag{1}$$

and

$$z_i = \Pi_1 x_i I_{q_i \leq \rho} + \Pi_2 x_i I_{q_i > \rho} + u_i \tag{2}$$

where y_i is a scalar variable, z_i is an $m \times 1$ vector of (possibly) endogenous variables, x_i is a $p \times 1$ vector of exogenous variables, q_i is an exogenous scalar variable controlling the regime switches, ϵ_i is an i.i.d. sequence with variance σ_ϵ^2, β_j, $j = 1, 2$ are $m \times 1$ vectors of parameters, Π_j, $j = 1, 2$ are $m \times p$ matrices of parameters, γ and ρ are threshold parameters and ϵ_t is a martingale difference sequence with respect to an information set timed at $t-1$ containing the past of x_i and z_i. We also define $\beta = (\beta_1', \beta_2')'$.

For this model Caner and Hansen (2003) define 2SLS and GMM estimators that allow for the endogeneity of z_i. More specifically, let $\hat{z}_i$ be the fitted value from the estimation of (2). Define

$$Y = (y_1, \ldots, y_N)'$$

$$Z_\gamma = (\hat{z}_1 I(q_1 < \gamma), \ldots, \hat{z}_N I(q_N < \gamma))'$$

and

$$\tilde{Z}_\gamma = (\hat{z}_1 I(q_1 \geq \gamma), \ldots, \hat{z}_N I(q_N \geq \gamma))'$$

Let $S_N(\gamma)$ denote the sum of squared residual of a regression of Y on Z_γ and $\tilde{Z}_\gamma$ for given γ. Then, the threshold is estimated as the value of γ which minimises $S_N(\gamma)$ and denoted $\hat{\gamma}$. Then, assuming that γ is known and given by $\hat{\gamma}$ one can estimate the coefficients via 2SLS or GMM for the two subsamples implicitly defined by $\hat{\gamma}$. The problem is thus reduced to a linear estimation for the two subsamples. Caner and Hansen (2003) prove consistency of the estimators and derive the asymptotic distribution of $\hat{\gamma}$ under the assumption of a 'small threshold' asymptotic framework, i.e. assuming that $\beta_1 - \beta_2 = \beta_{1N} - \beta_{2N} = o(1)$. It is also shown that the asymptotic distribution of the estimates of β_1 and β_2 are $\sqrt{N}$-consistent and asymptotically normal.

Theoretical results that go beyond this point are not available. Disregarding the presence of endogeneity and estimating a threshold model such as (1) by the standard method, which involves applying OLS for every point on a threshold parameter grid, is likely to lead to inconsistency (both for coefficients and threshold parameter estimates), although the asymptotic bias is not available in closed form and is likely to depend in complicated ways on the true model parameters. On the other hand, it is not clear to what extent estimation by IV methods introduces inefficiency in the estimation. Nevertheless, it is clear that in either case use of an inappropriate estimation will lead to suboptimal outcomes. It is obvious that a Hausman (1978) type test can provide guidance on which estimator to use.

A formalisation of that idea is the topic of the paper. The null hypothesis of the test is that

$$H_0 : E(z_i \epsilon_i) = 0, \forall i \tag{3}$$

The test statistic is of the form

$$S = (\hat{\beta} - \tilde{\beta})' \hat{V}^{-1} (\hat{\beta} - \tilde{\beta}) \tag{4}$$

where $\hat{\beta}$ is the IV-type estimator (such as the 2SLS or GMM estimator) and $\tilde{\beta}$ is the OLS-type estimator. V is the variance of $(\hat{\beta}_j - \tilde{\beta})$ and $\hat{V}$ denotes its estimate. This statistic is asymptotically distributed as χ^2_m. This result follows easily from the asymptotic normality of both $\hat{\beta}$ and $\tilde{\beta}$ under the null hypothesis.

If $\hat{\beta}$ is efficient (i.e. achieves the Cramer-Rao lower variance bound) then by Hausman (1978) it is known that

$$V = Var(\tilde{\beta}) - Var(\hat{\beta}) \tag{5}$$

and $\hat{V}$ can be obtained by plugging in estimates of $Var(\tilde{\beta})$ and $Var(\hat{\beta})$ in the above expression. Asymptotically, estimation of the threshold is irrelevant for the estimation of the coefficients due to the superconsistency of the threshold parameter estimate. As Caner and Hansen (2003) state, under homoscedasticity, both 2SLS and GMM estimators are semi-parametrically efficient. Hoewever, full parametric efficiency would require distributional assumptions on ϵ_i. Further, this result is asymptotic. As Kapetanios (2000) has shown the asymptotic irrelevance of the threshold parameter estimation is not relevant in small samples where the adjective small applies even to samples much larger that usually encountered in, say, macroeconomics. Hence, using this estimate of V is likely to be misleading. For that reason we address the estimation of V in the next section using the bootstrap. For the purposes of this section we assume

Assumption 1 *There exists a consistent estimate of V, denoted $\hat{V}$.*

For simplicity we will concentrate on $\tilde{\beta}$ being the 2SLS estimator. Let c_α denote the critical value for the 100*(1-α)% significance level of the Hausman-type test. In what follows α may be allowed to depend on the sample size N. Then, we define a new estimator given by

$$\hat{\beta}_\alpha^V = \hat{\beta} I_{S>c_\alpha} + \tilde{\beta} I_{S\leq c_\alpha} \tag{6}$$

We assume assumptions 1 and 2 of Caner and Hansen (2003) and conditions 1-4 of Chan (1993). We also make the following assumptions

Assumption 2 *If $E(z_i\epsilon) \neq 0$ then $plim_{N\to\infty}\tilde{\beta} = \beta^* \neq \beta$ and $plim_{N\to\infty}\tilde{\gamma} = \gamma^*$ for some constants β^* and γ^*.*

It is easy to see that under assumption 2 if one estimates the model by OLS for $\gamma = \gamma^*$ then $\tilde{\beta} - \beta^* = O_p(N^{-1/2})$.

Assumption 3 $\lim_{N\to\infty} \alpha_N = \infty$ *and* $\lim_{N\to\infty} \ln \alpha_N / N = 0$*, where* $\alpha_N \equiv \alpha$

Assumption 2 is essentially the cause of the problem we address. It is reasonable to assume it, in the absence of a closed form expression for the asymptotic bias, as otherwise the problem of inconsistency of $\tilde{\beta}$ does not arise. Also define the asymptotic mean square error of an estimator $\hat{\beta}$ as

$$MSE(\hat{\beta}) = lim_{N\to\infty} E(T(\hat{\beta} - \beta)'(\hat{\beta} - \beta)) \tag{7}$$

Then, we have the following theorem

Theorem 1 *Under assumptions 1-2 of Caner and Hansen (2003), conditions 1-4 of Chan (1993) and assumptions 1-3 above, β_α^V attains the semi-parametrically efficient MSE bound both in the case of endogeneity and no endogeneity.*

Proof

To prove the theorem we need to show that β_α^V attains the same MSE as $\hat{\beta}$ under the alternative and $\tilde{\beta}$ under the null. In other words we need to show that

$$lim_{N\to\infty} Pr(I_{S>c_\alpha} = 1) = 1 \tag{8}$$

under the alternative and

$$lim_{N\to\infty} Pr(I_{S>c_\alpha} = 1) = 0 \tag{9}$$

under the null hypothesis. To show (8) we note that under assumption 2 $(\hat{\beta} - \tilde{\beta}) = O_p(1)$. Further, by assumption 1, and assumption 2 (convergence in probability of $\tilde{\beta}$ to some constant) $\hat{V} = O_p(N^{-1})$ implying that $S = O_p(N)$. Since we have introduced dependence of α and hence c_α on N, we require $c_{\alpha_N}/N \to 0$ for (8) to hold. For this we mirror the analysis of Hosoya (1989). We have that

$$\alpha_N \leq \int_{c_{\alpha_N}}^{\infty} c(x|m)dx \leq exp(-c_{\alpha_N}/d)$$

for some $d > 0$, where $c(x|m)$ is the pdf of a χ_m^2. Thus,

$$\frac{-log(\alpha_N)}{N} \geq c_{\alpha_N}/Nd$$

So if the second part of assumption 3 holds then $c_{\alpha_N}/N \to 0$ holds and (8) holds. To show (9), we note that under the null, $S = O_p(1)$. Hence by the first part of assumption 3, (9) holds.

Q.E.D.

A similar estimator can, of course, be defined for the threshold parameter, which we consider in the Monte Carlo experiments. However, in this case the extent of the inefficiency arising out of using IV methods in the case of no endogeneity is less clear. Monte Carlo simulations will be used in Section 4 to investigate this issue.

3 The Bootstrap

As we have seen in the last section, a new estimator for β can be obtained as long as Assumption 1 is satisfied. In other words we need a consistent variance estimator. The asymptotic analysis of Hausman (1978) requires full parametric efficiency, which may not be achievable via the standard estimators in nonlinear models in a number of cases such as, e.g., heteroscedasticity. An alternative estimator maybe provided by the bootstrap. There are a number of issues that need to be addressed. The first issue concerns the validity of the bootstrap for threshold models. The bootstrap relies on parametrically or non-parametrically resampling from the available data. The standard bootstrap procedure is in general able to provide an estimate of the exact distribution of an estimator and hence of the variance of the estimator. Under mild assumptions this estimator is consistent. Further, under the assumption of asymptotic pivotalness (independence of the asymptotic distribution from nuisance parameters) the bootstrap estimator may converge more quickly to the true distribution compared to the asymptotic approximation. However, in the case of the standard estimator of the threshold parameter, the asymptotic distribution is not asymptotically pivotal. Further, it is not even clear whether the standard bootstrap estimator is consistent in this case, as Coakley and Fuertes (2002) claim.

To explain in detail why consistency is in doubt, we denote the distribution of the threshold parameter estimate, $\hat{\gamma}_N$, obtained by minimising the conditional sum of squares, by $\mathcal{L}_N(\hat{\gamma}, F_N)$ where F_N denotes the joint distribution function of the sample $((y_1, z_1')', \ldots, (y_N, z_N')')'$. We denote a generic parameter of $\mathcal{L}_N(\hat{\gamma}, F_N)$ by $\theta_N(\hat{\gamma}, F_N)$. For example, θ_N could be the variance of $\hat{r}_T$ or the 95% quantile of its distribution. The crucial regularity condition for consistency of the standard bootstrap approach is the continuity of the mapping $F_N \to \theta_N$ (see e.g. Beran and Ducharme (1991)). To appreciate the difficulty of showing this for the threshold models we note that the asymptotic distribution of $N(\hat{\gamma}_N - \gamma)$ is given by that of the lower bound, M_-, of a random interval where a functional of two independent compound Poisson processes is minimised almost surely. This continuity condition is not necessary for consistency of the bootstrap but most theorems available on this subject assume it. In this context we note the work of Inoue and Kilian (2003) where it is shown that for an unrelated problem (unit root inference) continuity of that form is not needed for bootstrap validity. Nevertheless, all the above relate to the threshold parameter estimate. However, our test statistic S depends only on the coefficient estimates. These satisfy the continuity assumption since their limit distribution is normal. Hence, the bootstrap may or may not be valid for full inference on threshold models but it clearly is valid for bootstrapping S.

Incidentally, the question of bootstrap validity in threshold models can be addressed using alternative bootstrap approaches. Thus, whereas the standard bootstrap where inference is based on resampling samples of size N maybe invalid, using subsampling can provide valid inference. Subsampling is essentially carrying out the bootstrap but resampling smaller samples to carry out inference. More specifically, rather that resampling samples of size N, samples of size B where $B \to \infty$, but $B/N \to 0$ are resampled. Subsampling has been suggested and discussed by, among other, Politis and Romano (1994a) and Bickel, Gotze, and van Zwet (1997). Unlike the continuity assumption needed for the standard bootstrap, the only assumptions needed for subsampling validity are the existence of a limit distribution for the statistic of interest and, in the case of nonparametric resampling, strong mixing for the process being resampled. We will examine the performance of parametric subsampling in our context in the next section.

Another important issue is the mode of resampling so that the null hypothesis is imposed of the bootstrap samples. This is not as important when only the variance is bootstraped as in our case but is crucial when the whole distribution is bootstrapped and used to carry out the test as an alternative to retaining the asymptotic χ^2_m approximation. In that case, not imposing the null distribution on the bootstrap samples will lead to an inconsistent test. To see this note that if the null hypothesis is not imposed then endogeneity will prevail in the bootstrap samples. Hence, the bootstrap test statistics will tend to infinity asymptotically rather than be $O_p(1)$. Note that it is the fact that $(\hat{\beta}_j - \tilde{\beta}) = O_p(1)$ that makes S tend to infinity as both under the null and under the alternative $Var(\hat{\beta}_j - \tilde{\beta}) = O(N^{-1})$. Hence, imposing the null is crucial for bootstrapping the whole distribution but less crucial for bootstrapping the variance only. To investigate this further we examine in detail a number of possible bootstrap implementations.

The first step in the bootstrap implementation is the resampling of z_i, x_i

and q_i. As we do not assume any parametric model for these, we need to use the nonparametric bootstrap. Depending on the presence of temporal dependence in these variables, one may want to use the block bootstrap or the stationary bootstrap of Politis and Romano (1994b). In that case the block size needs to be determined. If subsampling, is undertaken, then one needs to choose the subsample size too. Note that if one wishes to assume a parametric model for z_i is terms of x_i then a parametric bootstrap may be used. In the case of the nonparametric bootstrap it is important that one resamples rows (or blocks) from the matrix $W = ((z_1', x_1', q_1)', \ldots, (z_N', x_N', q_n)')'$ rather than resample independently x_i, z_i and q_i. This is so as to retain the contemporaneous dependence between these variables. However, this stage is relatively straightforward.

Then, one needs to resample parametrically y_i given the bootstrap sample $W^* = ((z_1^{'*}, x_1^{'*}, q_1^*)', \ldots, (z_N^{'*}, x_N^{'*}, q_n^*)')'$ where stars denote a generic bootstrap sample. To parametrically resample y_i one can either use the parameter estimates obtained via 2SLS or via OLS. Since the test is constructed under the null hypothesis, the best choice is to use the more efficient OLS estimates and the OLS residuals for resampling ϵ_i. Care needs to be applied in the resampling of ϵ_i. Under the alternative $E(z_i\epsilon_i) \neq 0$. But for the bootstrap sample we must impose $E^*(z_i^*\epsilon_i^*) = 0$, where E^* denotes bootstrap expectation which is conditional on the sample realisation. We suggest two alternatives for that. Denote the OLS residual by $\tilde{\epsilon}_i$. The first alternative regresses $\tilde{\epsilon}_i$ on z_i and resamples from the residual of that regression, denoted $\tilde{\epsilon}_i^z$. $\tilde{\epsilon}_i^z$ is normalised to have variance $\hat{\sigma}_\epsilon^2$. By construction then $E^*(z_i^*\epsilon_i^*) = 0$. The second alternative is to use the wild bootstrap. This involves constructing the bootstrap error terms as $\epsilon_i^* = \eta_i\tilde{\epsilon}_i$, where η_i is an i.i.d. zero mean sequence independent of all other variables whose variance is unity. Again $E^*(z_i^*\epsilon_i^*) = E^*(z_i^*\tilde{\epsilon}_i\eta_i^*) = 0$. Thus, the null hypothesis is imposed on the bootstrap sample. By the (i) consistency of the parameter estimates both of the OLS and 2SLS estimators proved, under the null by Chan (1993) and Caner and Hansen (2003), (ii) the fact that, via the nonparametric bootstrap the joint distributions of $W^* = ((z_1^{'*}, x_1^{'*}, q_1^*)', \ldots, (z_N^{'*}, x_N^{'*}, q_n^*)')'$ are the same as those of $W = ((z_1^{'}, x_1^{'}, q_1)', \ldots, (z_N^{'}, x_N^{'}, q_n)')'$ it follows that the bootstrap samples satisfy assumptions 1-2 of Caner and Hansen (2003), if z_i follow a linear or threshold model, and conditions 1-4 of Chan (1993). The validity of the wild bootstrap approach then easily follows since asymptotically $S^* \sim \chi_m^2$. This is summarised in the theorem below.

Theorem 2 *The wild bootstrap approach is asymptotically valid since* $S^* \xrightarrow{d} \chi^2_m$.

Similar results for the other bootstrap implementations can be obtained.

4 Monte Carlo

In this section we carry out a detailed Monte Carlo simulation of the new methods. We use the following model

$$y_i = z_i I_{q_i \leq 0} + 2 z_i I_{q_i > 0} + \epsilon_i \tag{10}$$

and

$$z_i = \sum_{i=1}^{p} x_i I_{q_i \leq 0} + \sum_{i=1}^{p} 2 x_i I_{q_i > 0} + u_i \tag{11}$$

where $p = 1, 2$. We introduce endogeneity by specifying $E(\epsilon_i u_i) = \sigma_{\epsilon u}$ and setting $\sigma_{\epsilon u} = 0, 0.5, 0.95$. Throughout, $\epsilon_i, u_i \sim N(0,1)$. We set $N = 100, 200$. $x_i, q_i \sim i.i.d.N(0,1)$.

The IV estimator used is 2SLS. Under the data generation process considered it is as efficient as GMM. Both for 2SLS and OLS estimation the threshold parameter grid is made up of 20 equally spaced points between the 10% and 90% quantiles. To minimise computational cost we assume that $\gamma = \rho$ and impose that restriction in estimation.

We try out the plain 2SLS and OLS estimators denoted in the tables, as before, by $\hat{\beta}$ and $\tilde{\beta}$. We also try out $\hat{\beta}^{\hat{V}^h}_{0.05}$ where $\hat{V}^h$ is the estimated asymptotic variance covariance matrix as discussed in Hausman (1978). As it appears from the notation we use a significance level of 95%.

We now discuss the bootstrap implementation in detail. We consider 5 different bootstrap implementations. The first is the standard parametric bootstrap where the null hypothesis is imposed by regressing the OLS residuals on z_i. The bootstrap variance estimator, denoted $\hat{V}^z$, from this bootstrap is used to construct S. The second implementation uses the wild bootstrap to get the variance, denoted by $\hat{V}^w$. The third implementation uses a fully nonparametric bootstrap which resamples nonparametrically y_i

as well as x_i, z_i and q_i. This is not a block bootstrap as the data are i.i.d. Obviously this bootstrap does not impose the null hypothesis on the bootstrap samples. This variance estimate is denoted $\hat{V}^n$. The fourth implementation uses subsampling where the subsample size is set to 80% of the sample size. The variance obtained is denoted by $\hat{V}^s$. Finally, we consider a bootstrap test which bootstraps the whole distribution of S rather that only the variance. This estimator is denoted $\hat{\beta}^b_{0.05}$. The wild bootstrap is used for that. Other bootstrap implementations were tried but had minimal power indicating that only the wild bootstrap can adequately impose the null on the bootstrap samples. For all experiments we used 99 bootstrap replications and 1000 Monte Carlo replications.

To evaluate the performance of the estimators we report the rejection probabilities of the Hausman type test. These are reported in boldface font in all tables. For the coefficients we report the bias and RMSE in the first and second of the relevant columns for each estimator. For the threshold parameter we report the 5%, 50% and 95% quantiles of its empirical distribution in the three relevant columns in the tables. Results are presented in tables 1-8.

Results make very interesting reading. A first interesting point is that the threshold parameter estimate does not seem to be adversely influenced by endogeneity. Of course, this conclusion needs to be qualified by the specificity of the Monte Carlo design. Nevertheless, the coefficients are substantially influenced as expected leading to the tentative conclusion that endogeneity may not be more problematic for nonlinear models than for linear ones.

The asymptotic version of the S test performs badly. It underrejects very substantially, as it never rejects under the null hypothesis. Of course, this has the beneficial byproduct of very good performance of the estimator under the null since then the estimator is equivalent to the OLS estimator. The downside is of course low power and bad performance under the alternative hypothesis.

Moving to the bootstrap tests we see that the best performer by far is the one using the wild bootstrap variance estimator. This test is correctly sized and has very good power. Even for 100 observations the power is at least 95%. The wild bootstrap works well even when the whole bootstrap distribution is being used, even though its power is considerably less than that of the wild bootstrap variance test. The second best bootstrap variance test is based on V^z. The nonparametric bootstrap comes next and the worst performer is the subsample bootstrap which has quite low power. Overall, all the bootstrap tests work better that the asymptotic test.

5 Conclusion

Endogeneity of the explanatory variables is an issue that has received little attention in the analysis of nonlinear econometric models. Recently work by Caner and Hansen (2003) and Psaradakis, Sola, and Spagnolo (2004) have looked at ways of estimating nonlinear models under the presence of endogeneity. However, the question of how to determine the presence of endogeneity is open. This paper addresses this issue.

We propose a Hausman-type test to test for endogeneity. The asymptotic analysis of Hausman (1978) provides a solution to the construction of the test but only under the prohibitively restrictive assumption of having an efficient estimator for the parameters under the null hypothesis. In most nonlinear models such an assumption may be difficult to satisfy. Furthermore, asymptotic analysis is less likely to be of relevance in small samples for nonlinear model than for linear ones.

This paper therefore proposes bootstrap based tests to solve the problem. Monte Carlo analysis clearly shows the importance of imposing the null hypothesis of no endogeneity in the bootstrap samples for the construction of a test with desirable properties. It turns out that imposing exogeneity is not straightforward. We propose a version of the wild bootstrap which seems to perform very well under all circumstances considered.

Threshold models have been used as a vehicle for the analysis for the simple reason that via the work of Caner and Hansen (2003) they have been analysed in the context of endogeneity. But the techniques discussed here are straightforwardly applicable to many alternative nonlinear models. More generally, the use of the wild bootstrap to impose exogeneity in the bootstrap samples is of more general interest in the context of bootstrap testing of exogeneity.

References

Beran, R., and G. R. Ducharme (1991): *Asymptotic Theory for Bootstrap Methods in Statistics.* Les Publications CRM, Centre de Recherches Mathematiques, Universite de Montreal, Montreal, Canada.

Bickel, P. J., F. Gotze, and W. R. van Zwet (1997): "Resampling Fewer than n observations: Gains, Losses and Remedies for Losses," *Statistica Sinica*, 7(1), 1–31.

Caner, M., and B. Hansen (2003): "Instrumental Variables Estimation of a Threshold Model," *Mimeo.*

Chan, K. S. (1993): "Consistency and Limiting Distribution of the Least Squares Estimator of a Threshold Autoregressive Model," *The Annals of Statistics*, 21(1), 520–533.

Coakley, J., and A. Fuertes (2002): "Bootstrap LR Tests of Sign and Amplitude Asymmetry," *Birkbeck College Discussion Paper 4/2000.*

Hausman, J. A. (1978): "Specification Tests in Econometrics," *Econometrica*, 46, 1251–1271.

Hosoya, Y. (1989): "Hierarchical Statistical Models and a Generalised Likelihood Ratio Test," *Journal of the Royal Statistical Society B*, 51(3), 435–448.

Inoue, A., and L. Kilian (2003): "The continuity of the Limit Distribution in the Parameter of Interest Is Not Essential for the Validity of the Bootstrap," *Econometric Theory*, 19, 944–961.

Kapetanios, G. (2000): "Small Sample Properties of the Conditional Least Squares Estimator in SETAR Models," *Economics Letters*, 69(3), 267–276.

Kim, C. J. (2004): "Markov-Switching Models with Endogenous Explanatory Variables," *Journal of Econometrics*, Forthcoming.

Politis, D. N., and J. P. Romano (1994a): "Large Sample Confidence Regions Based on Subsamples under Minimal Assumptions," *Annals of Statistics*, 22(4), 2031–2050.

Table 1: $\tilde{\beta}$

N	$\sigma_{\epsilon u}$		No. of instruments					
			1			2		
		β_1	0.000	1.098	*	0.005	0.797	*
	0	β_2	0.000	0.432	*	-0.004	0.241	*
		r	-0.176	-0.000	0.122	-0.146	-0.003	0.107
		β_1	0.246	7.035	*	0.168	3.512	*
100	0.5	β_2	0.098	1.341	*	0.054	0.527	*
		r	-0.196	0.000	0.133	-0.147	-0.005	0.111
		β_1	0.482	23.892	*	0.320	10.779	*
	0.95	β_2	0.191	4.005	*	0.102	1.262	*
		r	-0.174	0.002	0.154	-0.148	-0.005	0.104
		β_1	0.000	0.616	*	0.003	0.379	*
	0	β_2	-0.004	0.218	*	-0.003	0.123	*
		r	-0.096	-0.011	0.067	-0.091	-0.013	0.059
		β_1	0.258	7.164	*	0.173	3.343	*
200	0.5	β_2	0.095	1.096	*	0.053	0.394	*
		r	-0.102	-0.009	0.076	-0.093	-0.013	0.065
		β_1	0.481	23.472	*	0.322	10.618	*
	0.95	β_2	0.186	3.640	*	0.101	1.126	*
		r	-0.101	-0.009	0.073	-0.102	-0.011	0.069

——— (1994b): "The Stationary Bootstrap," *Journal of the American Statistical Association*, 89, 1303–1313.

PSARADAKIS, Z., M. SOLA, AND F. SPAGNOLO (2004): "Testing the Unbiased Forward Exchange Rate Hypothesis using a Markov-Switching Model and Instrumental Variables," *Journal of Applied Econometrics*, Forthcoming.

Table 2: $\hat{\beta}$								
N	$\sigma_{\epsilon u}$		No. of instruments					
			1			2		
		β_1	-0.002	2.237	*	0.001	1.114	*
	0	β_2	-0.001	0.599	*	-0.005	0.283	*
		r	-0.173	0.000	0.124	-0.146	-0.003	0.107
		β_1	-0.005	2.533	*	0.001	1.115	*
100	0.5	β_2	-0.011	0.622	*	-0.004	0.308	*
		r	-0.270	-0.003	0.153	-0.154	-0.007	0.111
		β_1	0.050	3.170	*	0.012	1.221	*
	0.95	β_2	-0.021	0.932	*	-0.010	0.328	*
		r	-0.592	-0.010	0.295	-0.231	-0.011	0.107
		β_1	-0.000	1.206	*	0.001	0.552	*
	0	β_2	-0.006	0.284	*	-0.005	0.149	*
		r	-0.099	-0.011	0.068	-0.091	-0.013	0.059
		β_1	0.004	1.110	*	0.004	0.587	*
200	0.5	β_2	-0.010	0.298	*	-0.007	0.155	*
		r	-0.157	-0.012	0.080	-0.099	-0.015	0.063
		β_1	0.027	1.469	*	0.008	0.559	*
	0.95	β_2	-0.019	0.449	*	-0.009	0.150	*
		r	-0.427	-0.022	0.103	-0.128	-0.014	0.066

Table 3: $\tilde{\beta}^{Vh}_{0.05}$								
N	$\sigma_{\epsilon u}$		No. of instruments					
				1			2	
		β_1	0.000	1.098	*	0.005	0.797	*
	0	β_2	0.000	0.432	**0.000**	-0.004	0.241	**0.000**
		r	-0.176	-0.000	0.122	-0.146	-0.003	0.107
		β_1	0.236	7.108	*	0.167	3.504	*
100	0.5	β_2	0.093	1.382	**0.022**	0.054	0.531	**0.002**
		r	-0.202	0.000	0.132	-0.149	-0.005	0.111
		β_1	0.361	19.311	*	0.306	10.554	*
	0.95	β_2	0.125	3.662	**0.215**	0.096	1.293	**0.029**
		r	-0.591	-0.008	0.150	-0.191	-0.006	0.104
		β_1	0.000	0.616	*	0.003	0.379	*
	0	β_2	-0.004	0.218	**0.000**	-0.003	0.123	**0.000**
		r	-0.096	-0.011	0.067	-0.091	-0.013	0.059
		β_1	0.242	6.925	*	0.172	3.349	*
200	0.5	β_2	0.086	1.094	**0.052**	0.052	0.396	**0.003**
		r	-0.114	-0.011	0.075	-0.094	-0.013	0.065
		β_1	0.097	5.644	*	0.263	9.114	*
	0.95	β_2	0.011	1.056	**0.809**	0.077	0.968	**0.168**
		r	-0.427	-0.024	0.072	-0.116	-0.014	0.061

Table 4: $\tilde{\beta}^{Vz}_{0.05}$								
N	$\sigma_{\epsilon u}$		No. of instruments					
				1			2	
		β_1	0.001	1.205	*	0.004	0.842	*
	0	β_2	-0.000	0.446	**0.032**	-0.004	0.248	**0.034**
		r	-0.176	-0.000	0.122	-0.146	-0.003	0.107
		β_1	0.080	4.277	*	0.018	1.428	*
100	0.5	β_2	0.020	0.785	**0.673**	-0.000	0.330	**0.889**
		r	-0.235	-0.004	0.131	-0.155	-0.007	0.111
		β_1	0.102	6.126	*	0.023	1.579	*
	0.95	β_2	0.002	1.283	**0.859**	-0.007	0.339	**0.970**
		r	-0.569	-0.016	0.159	-0.230	-0.011	0.104
		β_1	-0.000	0.675	*	0.003	0.408	*
	0	β_2	-0.004	0.230	**0.038**	-0.003	0.129	**0.042**
		r	-0.100	-0.011	0.068	-0.091	-0.013	0.059
		β_1	0.009	1.271	*	0.004	0.591	*
200	0.5	β_2	-0.009	0.314	**0.980**	-0.007	0.155	**0.998**
		r	-0.157	-0.012	0.077	-0.099	-0.015	0.063
		β_1	0.032	1.823	*	0.008	0.559	*
	0.95	β_2	-0.015	0.497	**0.980**	-0.009	0.150	**1.000**
		r	-0.427	-0.023	0.091	-0.128	-0.014	0.066

Table 3: $\tilde{\beta}^{Vw}_{0.05}$								
N	$\sigma_{\epsilon u}$		No. of instruments					
			1			2		
		β_1	-0.000	1.306	*	0.003	0.863	*
	0	β_2	-0.000	0.448	**0.047**	-0.004	0.248	**0.049**
		r	-0.176	-0.000	0.121	-0.146	-0.003	0.107
		β_1	0.003	2.741	*	0.002	1.160	*
100	0.5	β_2	-0.008	0.646	**0.942**	-0.004	0.312	**0.977**
		r	-0.271	-0.005	0.134	-0.157	-0.007	0.111
		β_1	0.050	3.195	*	0.012	1.221	*
	0.95	β_2	-0.021	0.934	**0.996**	-0.010	0.328	**1.000**
		r	-0.592	-0.010	0.282	-0.231	-0.011	0.107
		β_1	-0.000	0.687	*	0.002	0.416	*
	0	β_2	-0.005	0.229	**0.042**	-0.004	0.131	**0.069**
		r	-0.100	-0.011	0.068	-0.091	-0.013	0.059
		β_1	0.004	1.114	*	0.004	0.589	*
200	0.5	β_2	-0.010	0.297	**0.999**	-0.007	0.155	**0.999**
		r	-0.157	-0.012	0.080	-0.099	-0.015	0.063
		β_1	0.027	1.469	*	0.008	0.559	*
	0.95	β_2	-0.019	0.449	**1.000**	-0.009	0.150	**1.000**
		r	-0.427	-0.022	0.103	-0.128	-0.014	0.066

Table 6: $\tilde{\beta}_{0.05}^{Vn}$									
N	$\sigma_{\epsilon u}$		No. of instruments						
			1			2			
		β_1	0.000	1.143	*	0.004	0.839	*
	0	β_2	-0.000	0.435	**0.020**	-0.004	0.246	**0.040**
		r	-0.176	-0.000	0.122	-0.146	-0.003	0.107
		β_1	0.093	4.396	*	0.024	1.546	*
100	0.5	β_2	0.025	0.887	**0.622**	0.002	0.340	**0.845**
		r	-0.231	-0.004	0.124	-0.155	-0.007	0.111
		β_1	0.110	6.362	*	0.026	1.636	*
	0.95	β_2	0.005	1.277	**0.845**	-0.006	0.340	**0.963**
		r	-0.554	-0.015	0.160	-0.222	-0.011	0.104
		β_1	-0.001	0.685	*	0.003	0.408	*
	0	β_2	-0.005	0.223	**0.033**	-0.004	0.128	**0.039**
		r	-0.100	-0.011	0.068	-0.091	-0.013	0.059
		β_1	0.010	1.333	*	0.005	0.593	*
200	0.5	β_2	-0.008	0.316	**0.976**	-0.007	0.156	**0.996**
		r	-0.157	-0.012	0.077	-0.099	-0.015	0.063
		β_1	0.037	2.047	*	0.008	0.559	*
	0.95	β_2	-0.014	0.525	**0.972**	-0.009	0.150	**1.000**
		r	-0.427	-0.023	0.090	-0.128	-0.014	0.066

Table 7: $\tilde{\beta}^{Vs}_{0.05}$								
N	$\sigma_{\epsilon u}$		No. of instruments					
			1			2		
		β_1	0.001	1.129	*	0.004	0.806	*
	0	β_2	0.000	0.435	**0.007**	-0.004	0.242	**0.016**
		r	-0.176	-0.000	0.122	-0.146	-0.003	0.107
		β_1	0.125	5.009	*	0.038	1.780	*
100	0.5	β_2	0.039	0.925	**0.491**	0.006	0.352	**0.759**
		r	-0.223	-0.003	0.128	-0.153	-0.007	0.111
		β_1	0.132	7.253	*	0.031	1.891	*
	0.95	β_2	0.014	1.458	**0.802**	-0.005	0.359	**0.951**
		r	-0.554	-0.015	0.159	-0.230	-0.011	0.104
		β_1	-0.000	0.637	*	0.003	0.398	*
	0	β_2	-0.005	0.222	**0.013**	-0.003	0.126	**0.021**
		r	-0.100	-0.011	0.067	-0.091	-0.013	0.059
		β_1	0.017	1.538	*	0.005	0.604	*
200	0.5	β_2	-0.006	0.335	**0.951**	-0.007	0.155	**0.995**
		r	-0.157	-0.012	0.077	-0.099	-0.015	0.063
		β_1	0.038	2.082	*	0.009	0.565	*
	0.95	β_2	-0.014	0.532	**0.969**	-0.009	0.150	**0.999**
		r	-0.427	-0.023	0.088	-0.128	-0.014	0.066

Table 8: $\tilde{\beta}^{b}_{0.05}$								
N	$\sigma_{\epsilon u}$		No. of instruments					
			1			2		
		β_1	-0.001	1.374	*	0.003	0.872	*
	0	β_2	0.000	0.438	**0.034**	-0.004	0.244	**0.043**
		r	-0.176	-0.000	0.122	-0.146	-0.003	0.107
		β_1	0.036	3.707	*	0.018	1.483	*
100	0.5	β_2	0.015	0.856	**0.733**	0.004	0.348	**0.827**
		r	-0.269	-0.005	0.133	-0.155	-0.008	0.111
		β_1	0.053	3.458	*	0.012	1.236	*
	0.95	β_2	-0.017	0.998	**0.966**	-0.009	0.329	**0.996**
		r	-0.592	-0.014	0.196	-0.231	-0.011	0.106
		β_1	-0.001	0.691	*	0.003	0.428	*
	0	β_2	-0.004	0.220	**0.034**	-0.003	0.125	**0.037**
		r	-0.100	-0.011	0.067	-0.091	-0.013	0.059
		β_1	0.006	1.205	*	0.004	0.596	*
200	0.5	β_2	-0.008	0.314	**0.976**	-0.007	0.155	**0.995**
		r	-0.157	-0.012	0.078	-0.099	-0.015	0.063
		β_1	0.027	1.469	*	0.008	0.559	*
	0.95	β_2	-0.018	0.461	**0.995**	-0.009	0.150	**1.000**
		r	-0.427	-0.023	0.099	-0.128	-0.014	0.066

Queen Mary
University of London

This working paper has been produced by
the Department of Economics at
Queen Mary, University of London

Department of Economics
Queen Mary, University of London
Mile End Road
London E1 4NS
Tel: +44 (0)20 7882 5096
Fax: +44 (0)20 8983 3580
Website: www.econ.qmul.ac.uk/papers/wp.htm