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Testing for unit roots in short dynamic panels with serially correlated and heteroscedastic disturbance terms

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Abstract

In this paper we introduce fixed- T unit root tests for panel data models with serially correlated and heteroscedastic disturbance terms. The tests are based on pooled least squares estimators for the autoregressive coefficient of the AR(1) panel model adjusted for their inconsistency. The proposed test statistics have normal limiting distributions when the cross-section dimension of the panel grows large, provided a condition involving the $4 + \delta$ -th order moments of the first differences of the data is satisfied. Monte Carlo evidence suggests that the tests have empirical size close to the nominal level and considerable power, even for MA(1) disturbance terms which exhibit strong negative autocorrelation.

JEL classification: C22, C23

Keywords: Panel data; Unit Roots; Serial Correlation; Heteroscedasticity; Central Limit Theorem.

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1 Introduction

In the last decade growing interest in the econometrics literature has been focused on testing for unit roots in autoregressive panel data models. The tests that have been developed can be categorised according to the assumptions made about the time dimension of the panel, i.e. large T or fixed- T , and the estimation method employed, e.g. least squares (LS) or maximum likelihood (ML).¹ In each category, the cross-section dimension of the panel, N , is assumed to be large. The fixed- T tests are appropriate for panels with a small time dimension, which have traditionally been the object of micro-economic studies [see Chamberlain (1984), *inter alia*]. The LS based tests can be employed when the distributions of the initial conditions of the data and the disturbance terms are unknown.

In this paper, we introduce two fixed- T panel unit root tests for AR(1) panel models with and without individual drift parameters, respectively. Both tests allow for serially correlated and heteroscedastic disturbance terms and are based on pooled within LS estimators of the autoregressive coefficient of the AR(1) panel data model corrected for their inconsistency [see Kiviet (1995)]. The correction for the inconsistency of the estimators makes use of a consistent estimator of the autocovariance matrix of the disturbance terms. This correction still works with heterogeneity in the nuisance parameters. For example, it is valid for moving average disturbance terms with time-varying and heterogenous coefficients.² The proposed tests can be thought of as an extension of Harris and Tzavalis' (1999) tests which assume IID disturbance terms.

The limiting distributions of the proposed test statistics are normal. They are derived by applying standard asymptotic results treating the time-dimension of the panel as fixed. Like other LS based tests, our tests do not require any distributional assumptions about the disturbance terms, apart from that some conditions on the higher moments of the first differences of

¹See Baltagi and Kao (2000) and Hall and Mairesse (2001) for surveys. LS based tests have been suggested by Levin and Lee (1992), Quah (1994) and Im, Pesaran and Shin (1997), for large- T panels, and for fixed- T panels by Harris and Tzavalis (1999). ML based tests have been suggested by Kruiniger (2000) for the canonical autoregressive model, and by Binder, Hsiao and Pesaran (2001) for panel vector autoregression models.

²Efforts to extend Harris and Tzavalis' (1999) tests to the case of serially correlated disturbance terms have previously been made by Harris and Tzavalis (2000). However, their study is limited to the case of the canonical dynamic panel data model without individual effects and a moving average model for the disturbance terms of order one. The primary aim of this study was to investigate the effects of ignoring serial correlation on the properties of their panel data unit root tests for panels with large and small time dimensions.

the data are satisfied. These conditions are quite general and are often met in practice.

To evaluate the small- N sample properties of the tests, we run some Monte Carlo simulation experiments assuming that the disturbance terms follow MA(1) processes with possibly time-varying and heterogenous coefficients, and display heteroscedasticity in both the cross-section and time dimensions of the panel. The overall conclusion that can be drawn from the results of these experiments is that the tests have finite sample performance similar to that of fixed- T tests for analogous models with normally distributed white noise disturbance terms [see Harris and Tzavalis (1999), and Hall and Mairesse (2001)], and perform better than large- T panel unit root tests, which exhibit severe size distortions [see Karlsson and Lothgren (1999), and Wu and Yin (1999)]. The tests have empirical size close to the nominal level of significance at 5%. This is true even for large negative values of the MA parameters, where the single time series unit root tests are substantially oversized [see Schwert (1989)]. Furthermore, the power of the tests increases as N increases faster than T . The effect of an increase in T *per se* on the power of the tests is ambiguous since it not only increases the number of observations, but also the number of nuisance parameters of the covariance matrix that our tests allow for heterogeneity.

To illustrate the use of the tests, we re-examine whether the data of Summers and Heston (1991) are in agreement with one of the implications of the economic convergence hypothesis, namely that the real per capita income series is mean reverting for each country. To investigate this, we employ our tests to see whether the null hypothesis of a unit root in per capita income, which implies economic divergence, can be rejected in favour of the alternative hypothesis of mean reversion, which implies economic convergence. The results of the tests cannot reject the divergence hypothesis for the sample period 1960-1989. Apart from a unit root in the autoregressive component of the per capita income, we have also found that there is evidence of a significant MA(1) component in the disturbance terms.

The paper is organised as follows. Section 2 introduces the test statistics and derives their limiting distributions. In section 3 we conduct a Monte Carlo simulation study of the properties of the test statistics. Section 4 presents the results of an application of the tests to the empirical example. Section 5 concludes the paper.

2 The test statistics and their limiting distribution

Consider the following two autoregressive panel data models:

$$y_i = a_i(1 - \phi)e + \phi y_{i,-1} + u_i, \quad i = 1, 2, \dots, N, \quad (1)$$

and

$$y_i = a_i e + b_i(1 - \phi)\tau + \phi y_{i,-1} + u_i, \quad (2)$$

where $y_i = (y_{i1}, \dots, y_{iT})'$ is a T -vector of time-series observations on the dependent variable for cross-sectional unit i of the panel; $y_{i,-1} = (y_{i0}, \dots, y_{iT-1})'$ is the vector y_i lagged one time period; $e = (1, 1, \dots, 1)'$ and $\tau = (1, 2, \dots, T)'$ are constant T -vectors; and $u_i = (u_{i1}, \dots, u_{iT})'$ is a T -vector of unobserved disturbance terms. For $\phi = 1$, model (1) reduces to a non-stationary model with different initial conditions but without drift, while model (2) reduces to a non-stationary model with different initial conditions and drift parameters. Below, we will propose two test statistics for the unit root hypothesis, i.e. $\phi = 1$, which will be based on the LS estimators for ϕ in the regression models (1) and (2), respectively, corrected for their inconsistency [see Kiviet (1995)].

When $\phi = 1$, the LS estimators for ϕ in models (1) and (2), denoted by $\hat{\phi}$, can be rewritten as

$$\hat{\phi} - 1 = \left[\sum_{i=1}^N y'_{i,-1} G y_{i,-1} \right]^{-1} \left[\sum_{i=1}^N y'_{i,-1} G u_i \right], \quad (3)$$

where G is a $(T \times T)$ ‘within’ transformation matrix. For model (1), G will be defined as $G \equiv M = [I - \frac{1}{T}ee']$, where I is a $(T \times T)$ identity matrix, while for model (2), G will be defined as $G \equiv Q = [I - X(X'X)^{-1}X']$, where $X = (e, \tau)$. To distinguish between the LS estimators for ϕ in both models, we will hereafter denote the LS estimator for model (1) by $\hat{\phi}_{LSDV}$, and the estimator for model (2) by $\hat{\phi}_{LSDVT}$. The two acronyms LSDV and LSDVT stand for LS dummy variables, and LS dummy variables and trends, respectively.

It can be easily seen from equation (3) that the inconsistency of the LS estimators can stem from two different sources: the first is the within transformation of the data [see Nickell (1981)] and the second is the presence of serial correlation in the disturbance terms, u_{it} , which implies that $E(y_{it-1}u_{it}) \neq 0$, for all i and t .

The limiting distribution of the test statistics under the null hypothesis can be derived from (3) after specifying assumptions about the sequences of the disturbance terms, $\{u_{it}\}$, and the data, $\{y_{it}\}$. Note that we do not need to make any assumptions about the initial observations of the panel, y_{i0} , and, in the case of model (2), the individual effects, a_i . The test statistics that we propose are asymptotically invariant under the null hypothesis, $\phi = 1$, to y_{i0} and a_i . In what follows, we maintain the following assumption.

Assumption 1:

(a1) $\{u_i\}$ constitutes a sequence of independent random T -vectors with means $E(u_i) = \mathbf{0}$ and covariance matrices $E(u_i u_i') = \Gamma_i \equiv [\gamma_{i,st}]$ of unknown form apart from $\gamma_{i,1T} = \gamma_{i,T1} = 0$. (a2) The average population covariance matrix $\bar{\Gamma}_N \equiv \frac{1}{N} \sum_{i=1}^N \Gamma_i$ is bounded away from zero in large samples: $\bar{\gamma}_{N,tt} > \delta'$ for some $\delta' > 0$, and for all $N > N_0$, for some N_0 , and for at least one $t \in \{1, \dots, T\}$.

(b1) The $4+\delta$ -th population moments of Δy_i , $i = 1, 2, \dots, N$, are uniformly bounded, i.e. for every $l \in \Re^T$ such that $l'l = 1$, $E(|l'\Delta y_i|^{4+\delta}) < B < \infty$ for some B , where Δ is the difference operator. (b2) $\frac{1}{N} \sum_{i=1}^N l' \mathcal{V}(\text{vech}(\Delta y_i \Delta y_i')) l > \delta''$ for some $\delta'' > 0$, and for all $N > N_1$, for some N_1 , and for every $l \in \Re^{0.5T(T+1)}$ such that $l'l = 1$.

Assumption 1 specifies quite weak conditions, that are often met in practice, and enables us to derive the limiting distribution of $\hat{\phi} - 1$ by employing standard results from asymptotic theory under the assumption that N tends to infinity, while T remains fixed. Below, we discuss some of the implications of the assumption in more detail.

Condition (a1) allows for heterogeneity in the cross section dimension of the panel as well as variation in the values of the autocovariance parameters over time. Ignoring intertemporal differences among the elements of the Γ_i may lead to erroneous inference about ϕ . The condition $\bar{\gamma}_{i,1T} = \bar{\gamma}_{i,T1} = 0$ implies that the maximum order of serial correlation in the u_{it} 's should be less than $T - 1$. This assumption is required because our test statistics correct the LS estimator for the inconsistency due to the serial correlation in the disturbance terms by using a non-parametric estimator of the average population covariance matrix, $\bar{\Gamma}_N$, and because the time dimension of the panel is fixed.

Conditions (a2) and (b1)-(b2) allow us to apply the Markov law of large numbers (LLN) and the Lindeberg-Feller central limit theorem (CLT) in order to derive the limiting distributions of the test statistics. Condition (a2) also guarantees that the probability limits of the denominators of the LS estimators are different from zero.

The next theorem presents the limiting distribution of our unit root test

statistic for model (1), which is based on $\hat{\phi}_{LSDV}$ corrected for its inconsistency.

Theorem 1 *Let the sequence $\{y_{i,t}\}$ be generated according to model (1) and let Assumption 1 hold. Then under the null hypothesis $\phi = 1$, as $N \rightarrow \infty$*

$$Z_A \equiv V_{LSDV}^{-0.5} \hat{\delta}_{LSDV} \sqrt{N} \left(\hat{\phi}_{LSDV} - 1 - \frac{\hat{B}_{LSDV}}{\hat{\delta}_{LSDV}} \right) \xrightarrow{d} N(0, 1), \quad (4)$$

$$\hat{\delta}_{LSDV} - \text{tr}(\Lambda' M \Lambda \bar{\Gamma}_N) \xrightarrow{p} 0,$$

and

$$\hat{B}_{LSDV} - \text{tr}(\Lambda' M \bar{\Gamma}_N) \xrightarrow{p} 0,$$

where

$$V_{LSDV} = \text{vec}(M\Lambda)'(I - S) \left(\frac{1}{N} \sum_{i=1}^N \mathcal{V}(\text{vec}(\Delta y_i \Delta y_i')) \right) (I - S) \text{vec}(M\Lambda),$$

$$\hat{\delta}_{LSDV} = \frac{1}{N} \sum_{i=1}^N y'_{i,-1} M y_{i,-1} = \frac{\hat{\sigma}_u^2}{N \hat{\sigma}_{\hat{\phi}_{LSDV}}^2},$$

$$\hat{B}_{LSDV} = \text{vec}(M\Lambda)' S \left(\frac{1}{N} \sum_{i=1}^N \text{vec}(\Delta y_i \Delta y_i') \right),$$

$\hat{\sigma}_{\hat{\phi}_{LSDV}}^2$ is the pooled LS estimator of the variance of the LSDV estimator of ϕ , $\hat{\sigma}_u^2$ is the LS estimator of the variance of the disturbance terms of the auxiliary regression model (1), Λ is a $(T \times T)$ matrix defined as $\Lambda_{r,c} = 1$ if $r > c$ and $\Lambda_{r,c} = 0$ otherwise, S is a $(T^2 \times T^2)$ diagonal selection matrix defined as $S_{(c-1)T+r, (c-1)T+r} = 1 - d((\bar{\Gamma}_N)_{r,c} = 0)$, $\{r = 1, 2, \dots, T, c = 1, 2, \dots, T\}$, where $d(\cdot)$ denotes the Dirac function, $\mathcal{V}(\cdot)$ denotes the variance operator, “ $\xrightarrow{d}$ ” signifies convergence in distribution, and “ $\xrightarrow{p}$ ” signifies weak convergence in probability.

The proof of the theorem is given in the appendix.

In the test statistic suggested by Theorem 1, the term $\hat{B}_{LSDV}/\hat{\delta}_{LSDV}$ corrects the estimator $\hat{\phi}_{LSDV}$ for its inconsistency when $\phi = 1$, which is given

by $\frac{tr[\Lambda' M \bar{\Gamma}_N]}{tr[\Lambda' M \Lambda \bar{\Gamma}_N]}$, by using a non-parametric estimator for the average population covariance matrix, $\bar{\Gamma}_N$, which is based on $\frac{1}{N} \sum_{i=1}^N (\Delta y_i \Delta y_i')$. Under the null hypothesis the latter constitutes a consistent, semi-definite estimator of $\bar{\Gamma}_N$, since under the null hypothesis $u_i = \Delta y_i$.

The matrix S in $\hat{B}_{LSDV}$ plays an important role in the construction of our test statistic. Since it selects the elements of $\frac{1}{N} \sum_{i=1}^N (\Delta y_i \Delta y_i')$ whose means are different from zero, $\frac{1}{N} \sum_{i=1}^N Svec(\Delta y_i \Delta y_i')$ will be a consistent estimator of $vec(\bar{\Gamma}_N)$, too. Condition (a1) of Assumption 1 ensures that at least two elements of Γ_i are zero, and therefore by employing $\frac{1}{N} \sum_{i=1}^N Svec(\Delta y_i \Delta y_i')$ we correct for the inconsistency of $\hat{\phi}_{LSDV}$ due to the serial correlation in the disturbance terms, while preserving some sample variation in the resulting test statistic.

Given a consistent estimator for V_{LSDV} , the statistic proposed in Theorem 1 can be readily used in practice by employing the tables of the standard normal distribution to test for the null hypothesis of $\phi = 1$. A consistent estimator for V_{LSDV} is given by

$$\hat{V}_{LSDV} = vec(M\Lambda)'(I_{T^2} - S)\hat{\Omega}(I_{T^2} - S)vec(M\Lambda) \quad (5)$$

where

$$\begin{aligned} \hat{\Omega} &= \frac{1}{N} \sum_{i=1}^N (vec(\Delta y_i \Delta y_i') vec(\Delta y_i \Delta y_i'))' \\ &\quad - \left(\frac{1}{N} \sum_{i=1}^N vec(\Delta y_i \Delta y_i') \right) \left(\frac{1}{N} \sum_{i=1}^N vec(\Delta y_i \Delta y_i') \right)', \end{aligned}$$

where $\hat{\Omega}$ is a consistent, and positive semidefinite estimator of $\frac{1}{N} \sum_{i=1}^N \mathcal{V}(vec(\Delta y_i \Delta y_i'))$. It can be easily seen that $\hat{V}_{LSDV} \xrightarrow{p} V_{LSDV}$ when $N \rightarrow \infty$ by Markov's LLN and condition (b) of Assumption 1.

The limiting distribution of a unit root test statistic for model (2) that is based on the LSDVT estimator, $\hat{\phi}_{LSDVT}$, is given in the next theorem.

Theorem 2 *Let the sequence $\{y_{i,t}\}$ be generated according to model (2) and let Assumption 1 hold. Then under the null hypothesis $\phi = 1$, as $N \rightarrow \infty$*

$$Z_B \equiv V_{LSDVT}^{-0.5} \hat{\delta}_{LSDVT} \sqrt{N} \left(\hat{\phi}_{LSDVT} - 1 - \frac{\hat{B}_{LSDVT}}{\hat{\delta}_{LSDVT}} \right) \xrightarrow{d} N(0, 1), \quad (6)$$

$$\hat{\delta}_{LSDVT} - \text{tr}(\Lambda' Q \Lambda \bar{\Gamma}_N) \xrightarrow{p} 0,$$

and

$$\hat{B}_{LSDVT} - \text{tr}(\Lambda' Q \bar{\Gamma}_N) \xrightarrow{p} 0,$$

where

$$\begin{aligned} V_{LSDVT} &= \text{vec}(Q\Lambda)' (I_{T^2} - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1}e_{T^2}\tilde{e}')) \left(\nu \left(\frac{1}{N} \sum_{i=1}^N \text{vec}(\Delta y_i \Delta y_i') \right) \right) \\ &\quad \times (I_{T^2} - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1}e_{T^2}\tilde{e}'))' \text{vec}(Q\Lambda) \end{aligned}$$

$$\hat{\delta}_{LSDV} = \frac{1}{N} \sum_{i=1}^N y'_{i,-1} Q y_{i,-1} = \frac{\hat{\sigma}_u^2}{N \hat{\sigma}_{\hat{\phi}_{LSDVT}}^2},$$

$$\hat{B}_{LSDVT} = \text{vec}(Q\Lambda)' S (I_{T^2} - (\tilde{e}'\tilde{e})^{-1}e_{T^2}\tilde{e}') \left(\frac{1}{N} \sum_{i=1}^N \text{vec}(\Delta y_i \Delta y_i') \right),$$

$\hat{\sigma}_{\hat{\phi}_{LSDV}}^2$ is the LS estimator of the variance of the LSDVT estimator of ϕ , $\hat{\sigma}_u^2$ is the LS estimator of the variance of the residuals of the auxiliary regression model (2), and $\tilde{e}$ is the T^2 -vector $(I_{T^2} - S)e_{T^2}$, where e_{T^2} is a T^2 -vector of ones.

The proof of the theorem is given in the appendix.

The construction of the test statistic given by Theorem 2 is based on arguments that are largely similar to those underlying the test statistic presented in Theorem 1. The estimator $\hat{\phi}_{LSDVT}$ is corrected for its inconsistency under the unit root hypothesis, which stems from the within transformation of the data and the serial correlation in the disturbance terms. In theorem 1 the inconsistency correction of the LS estimator was achieved by applying the selection matrix S to the vec of the average sample covariance matrix, $\frac{1}{N} \sum_{i=1}^N \text{vec}(\Delta y_i \Delta y_i')$. However, under model (2) the limiting distribution of $\hat{\phi}_{LSDVT} - 1$ depends on the individual effects, a_i , because under the null hypothesis we now have $\Delta y_i = \alpha_i e + u_i$, which implies that $E(\Delta y_i \Delta y_i') = \Gamma_i + \alpha_i^2 I$. The simple correction used in Theorem 1 does not render the limiting distribution of $\hat{\phi}_{LSDVT} - 1$ free from the individual effects. To correct the limiting distribution for the inconsistency of $\hat{\phi}_{LSDVT}$ and to remove the average of the individual effects $\frac{1}{N} \sum_{i=1}^N \alpha_i^2$ which are present in this distribution, we post-multiply the selection matrix S by $(I_{T^2} - (\tilde{e}'\tilde{e})^{-1}e_{T^2}\tilde{e}')$.

The sample covariance matrix $\frac{1}{N} \sum_{i=1}^N (\Delta y_i \Delta y_i')$ has $\tilde{e}'\tilde{e}$ elements which can be used to eliminate $\frac{1}{N} \sum_{i=1}^N \alpha_i^2$ from the limiting distribution of $\hat{\phi}_{LSDVT} - 1$. These $\tilde{e}'\tilde{e}$ elements correspond to the zero elements of the average population covariance matrix of the disturbance terms, $\bar{\Gamma}_N$.

As in Theorem 1, the implementation of the unit root test statistic given by Theorem 2 requires a consistent estimator for V_{LSDVT} . A consistent estimator for V_{LSDVT} is given by

$$\begin{aligned} & \hat{V}_{LSDVT} \\ &= \text{vec}(Q\Lambda)' (I_{T^2} - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1}e_{T^2}\tilde{e}')) \\ & \quad \hat{\Omega} (I_{T^2} - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1}e_{T^2}\tilde{e}'))' \text{vec}(Q\Lambda). \end{aligned} \tag{7}$$

Again, it can be easily seen that $\hat{V}_{LSDVT} \xrightarrow{p} V_{LSDVT}$ by Markov's LLN and condition (b) of Assumption 1.

3 Simulation results

The test statistics given by Theorems 1 and 2 are based on asymptotic arguments. In this section we explore the finite sample performance of the tests for different combinations of N and T and different degrees of heterogeneity of the panel, across i or t . This is done based on 5000 Monte carlo simulation experiments and conducting two different sets of experiments. For each set, we report the size and power performance of the one-sided test statistics normalised by their standard deviation at a nominal level of significance 5%. For both sets of experiments, we assume that the fixed effects are generated as $a_i \sim NID(0, 1)$.

The first set of experiments investigates the performance of the test statistics under very general panel data conditions where the disturbance terms u_{it} follow a MA(1) process with time-varying parameters and heteroscedastic errors, across both dimensions of the panel, N and T . Specifically, we assume that u_{it} is given by

$$u_{it} = \varepsilon_{it} + \theta_{it}\varepsilon_{it-1},$$

with

$\theta_{it} = \theta + \omega_{it}$, $\theta = \{-0.50, 0.0, 0.50\}$, $\omega_{it} \sim U[-0.30, 0.30]$, and $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon_i}^2)$, where $U(\cdot)$ denotes the uniform distribution density function; the standard deviation of ε_{it} is drawn from the uniform distribution

$U[1.0, 2.0]$, for all i and t .³ The results of this set of simulation experiments can be found in Tables 1(a)-(b); Table 1(a) reports the results for the test statistic of Theorem 1, while Table 1(b) contains the results for the test statistic of Theorem 2.

The second set of experiments investigates the performance of the test statistics when u_{it} is homogenous, i.e. $\theta_{it} = \theta$ and $\sigma_{\varepsilon_i}^2 = \sigma_\varepsilon^2$, for all i and t . For the nuisance parameters, θ and σ_ε^2 , we consider the following set of values: $\theta = \{-0.80, -0.30, 0.0, 0.30, 0.80\}$ and $\sigma_\varepsilon^2 = 1$, for all i and t . This set of experiments can indicate whether our tests can be successfully applied to simpler covariance structures involving a smaller number of nuisance parameters of the covariance matrix of the disturbance terms than those estimated by implementing our tests. These results can be found in Tables 2(a)-(b).

To investigate the power of the test statistics, we generated the panel data according to the following two alternative hypotheses: $\phi = \{0.95, 0.90\}$. The initial conditions of the panel have been set as $y_{i0} = \varepsilon_{i0}$, for model (1), and $y_{i0} = a_i + \varepsilon_{i0}$, for model (2). These conditions imply that the panel data series have the same sample mean under both the null and alternative hypotheses, and thus will enable us to investigate whether our tests have the power to distinguish the null against local alternatives. For the second model, note that these initial conditions ensure that the rate of trend of the data is the same under both the null and alternative hypotheses. The results on the power performance are also reported in Tables 1(a)-(b) and 2(a)-(b), for each set of experiment. The tables report the size-adjusted power, using the actual 5% critical values. The nominal power at the 5% nominal level can be found in parentheses.

The results of the tables clearly indicate that both test statistics have size very close to the 5% nominal level, for all combinations of $\{N, T\}$ and θ considered. The findings are consistent across both sets of experiments conducted. The application of the tests to panels with homogenous nuisance parameters does not seem to affect the performance of the tests significantly. For both sets of experiments, the empirical size approaches its 5% nominal level as N increases faster relative to T and θ goes from negative to positive values. This size performance of the tests is clearly better than that of the large- T panel data unit roots tests, which are severely oversized when the disturbance term is serially correlated [see Karlsson and Lothgren (1999) and Wu and Yin (1999)]. The very good size performance of both test statistics, especially for negative large values of θ , may be considered as an advantage of the panel data based unit root tests, compared with those based on single

³Note that the above assumption of a MA(1) structure of the disturbance terms is consistent with many economic series [see Schwert (1989)].

time series which are critically oversized for negative values of θ [see Schwert (1989)]. This can be attributed to the fact that the panel data tests by exploiting cross-section information from the data can better distinguish the autoregressive models (1) and (2) with a unit root and values of θ closed to -1.0 .

The power of the tests increases as N increases faster than T , and is higher than the power of large- T tests [see Karlsson and Lothgren (1999)]. It also increases significantly as we move from large negative values of θ to positive ones. The effect of an increase in T *per se* on the power of the tests is ambiguous since it does not only increase the number of observations, but also the number of nuisance parameters of the covariance matrix that our tests allow for. As in the case of single time series and panel data unit root tests for IID disturbances, the power performance considerably reduces for the test statistic of Theorem 2, which allows for a deterministic trend under the alternative hypothesis of stationarity. For this test statistic, we found that one needs panels with much larger N and T , e.g. $N = 1000$, $T = 50$, to achieve the same performance as the test statistic for the case without the individual trends, given by Theorem 1.

4 Empirical application

As an illustration of the use of the fixed- T panel data unit roots tests, we re-examine one implication of the economic convergence hypothesis, namely that the level of real per capita income of each country is reverting to its steady state [see Mankiw, Romer and Weil (1992), *inter alia*]. This should be true for both versions of the convergence hypothesis: the conditional and unconditional versions [see Barro and Sala-i-Martin (1995)].

To investigate the validity of this hypothesis, we carry out the panel data unit root test given by Theorem 2, which allows for individual deterministic trends under the alternative hypothesis of stationarity. The null hypothesis of a unit root corresponds to economic divergence.⁴ Our tests allow for a MA(1) serial correlation component in the disturbance term. As Campbell (1994) has shown, a MA(1) component in the growth autoregression model can reflect a stochastic evolution of the technological process. Note that this component is disregarded in the cross-section tests of the economic conver-

⁴Panel data based tests for the economic convergence hypothesis have been recently suggested by various authors [see Islam (1995), and Caselli, Esquivel and Lefort (1996), *inter alia*]. These tests have become popular because they have more power to reject the divergence hypothesis than single time series based unit root tests [see Bernard and Durlauf (1994), *inter alia*] or cross section based tests [see Barro and Sala-i-Martin (1995)].

gence hypothesis [see Barro and Sala-i-Martin (1995), *inter alia*], which may have serious consequences for the inference.

We carry out our fixed- T unit root test for two different groups of countries of the Penn World Tables [see Summers and Heston (1991)]: (i) the Non-oil (89 countries) and (ii) the Intermediate (72 countries).⁵ We note that for both groups of countries, the cross section dimension of the panel is higher than the time-dimension. In order to mitigate possible effects of cross section correlation on the test results, time dummies have been included in the auxiliary regression models [see O'Connell (1998)].

Below, we report the values of the estimator $\hat{\phi}_{LSDVT}$, its inconsistency correction $\hat{B}_{LSDVT}/\hat{\delta}_{LSDVT}$, denoted by $\hat{B}_{LSDVT}^{adj}$, and the test statistic Z_B , as well as a pooled GMM estimate of the MA parameter θ obtained under the null hypothesis,⁶

$\hat{\phi}_{LSDVT}$	$\hat{B}_{LSDVT}^{adj}$	θ	Z_B
NONOIL (N = 89)			
0.86	-0.13	0.21 [4.44]	-0.70
INTER (N = 72)			
0.86	-0.13	0.22 [4.40]	-0.74

Notes: $\mathbf{Z}_B \equiv \hat{V}_{LSDVT}^{-1/2} \hat{\delta}_{LSDVT} \sqrt{N} \left(\hat{\phi}_{LSDVT} - 1 - \hat{B}_{LSDVT}^{adj} \right)$ and $\hat{B}_{LSDVT}^{adj} \equiv \frac{\hat{B}_{LSDVT}}{\hat{\delta}_{LSDVT}}$. t-ratios are in brackets.

⁵See Mankiw *et al* (1992), for this grouping of countries. The intermediate group of countries is a subset of the non-oil group which does not include countries with a population size less than one million.

⁶A GMM estimate of θ is obtained as follows. Let σ_ε be the average standard deviation of the MA disturbance terms and let $\xi = (\sigma_\varepsilon, \theta)'$. Then, the GMM estimator of ξ is given by

$$\hat{\xi}_{GMM} \equiv \xi \arg \min \tilde{m}(\xi)' W \tilde{m}(\xi),$$

where $\tilde{m}(\xi) = Dm(\xi)$, with

$$m(\xi) = \frac{1}{N} \sum_{i=1}^N vech(m_i(\xi)),$$

where $m_i(\xi) = \Delta y_i \Delta y_i' - (a^2 ee' + \bar{\Gamma}_N(\xi))$, D is the first-difference matrix, and W is the optimal weight matrix.

The above results indicate that the null hypothesis cannot be rejected for both groups of countries. They are therefore consistent with the results of Bernard and Durlauf (1995), which are based on single time series data. The results also show that there exists a significant MA(1) component in the stochastic growth model, with a small positive value for the MA coefficient, which lends support to Campbell's (1994) specification of the technological process.

5 Conclusions

This paper introduced unit root tests for short panels that allow for individual effects and trends under the alternative hypothesis, and disturbance terms which are heteroscedastic and serially correlated. The test statistics are based on pooled within LS estimators of the autoregressive coefficient of an AR(1) panel data model corrected for its inconsistency and have standard normal limiting distributions.

To assess the small sample performance of the tests, we carried out some Monte Carlo simulation experiments allowing for different degrees of heterogeneity of the data and assuming a MA(1) model for the disturbance term. The results of our simulation study indicate that the suggested test statistics have empirical size very close to the nominal level, at 5%, even for large negative values of the MA(1) parameter, where single time series unit root tests or large T panel unit root tests are severely oversized, especially in small samples. The power performance of the test statistics against local alternatives is found to be similar to those of Harris and Tzavalis (1999), for panel data models with white noise disturbance terms. The power of the tests increases as N increases faster than T , and is higher than the power of large- T tests.

To illustrate the use of the proposed tests, the paper re-examined the implication of the economic convergence hypothesis which implies that a unit root in the per capita output should be rejected against its stationary alternative. Based on an updated version of Summer's and Heston's data base, the results of the paper cannot reject divergence. They also indicate the existence of a significant MA(1) component in the stochastic growth model which can reflect a stochastic evolution of the technological process.

A Appendix

In this appendix we present the proofs of the main theoretical results of the paper.

Proof of Theorem 1: To derive the limiting distribution of the test statistic, we first show that the LSDV estimator, $\hat{\phi}_{LSDV}$, is inconsistent. We then construct a normalised statistic based on $\hat{\phi}_{LSDV}$ corrected for the inconsistency.

Decompose the vector $y_{i,-1}$ for model (1) under the null hypothesis $\phi = 1$ as

$$y_{i,-1} = ey_{i0} + \Lambda u_i, \quad (8)$$

where the matrix Λ is defined in Theorem 1.

Premultiplying (8) with the matrix M yields

$$My_{i,-1} = M\Lambda u_i, \quad (9)$$

since $Me = \mathbf{0} \equiv (0, 0, \dots, 0)'$. Under the null hypothesis $\phi = 1$, we also have $u_i = \Delta y_i$. It follows that

$$y'_{i,-1}Mu_i = u'_i\Lambda'Mu_i = \Delta y'_i\Lambda'M\Delta y_i. \quad (10)$$

It can now easily be seen that the numerator of $\hat{\phi}_{LSDV} - 1$ converges in probability by the Markov LLN as $N \rightarrow \infty$, i.e.

$$\frac{1}{N} \sum_{i=1}^N y'_{i,-1}Mu_i - \text{tr}(\Lambda'M\bar{\Gamma}_N) \xrightarrow{p} 0, \quad (11)$$

where $\bar{\Gamma}_N = \frac{1}{N} \sum_{i=1}^N \Gamma_i$. Since S picks up all elements of $\text{vec}(u_i u'_i)$ that have a mean which is different from zero, $\frac{1}{N} \sum_{i=1}^N S \text{vec}(\Delta y_i \Delta y'_i)$ is a consistent estimator of $\text{vec}(\bar{\Gamma}_N)$ under the null hypothesis.

Subtract the term $\hat{B}_{LSDV}/\hat{\delta}_{LSDV}$ from $\hat{\phi}_{LSDV} - 1$. This yields

$$\hat{\phi}_{LSDV} - 1 - \frac{N\hat{\sigma}_{\hat{\phi}_{LSDV}}^2}{\hat{\sigma}_u^2} \hat{B}_{LSDV} = \left[\sum_{i=1}^N D_{iT} \right]^{-1} \left[\sum_{i=1}^N W_{i,T} \right] \quad (12)$$

where

$$\begin{aligned} W_{i,T} &= y'_{i,-1}Mu_i - \text{vec}(M\Lambda)'S(\text{vec}(\Delta y_i \Delta y'_i)) \\ &= \Delta y'_i\Lambda'M\Delta y_i - \text{vec}(M\Lambda)'S(\text{vec}(\Delta y_i \Delta y'_i)) \\ &= \text{vec}(M\Lambda)'[\text{vec}(\Delta y_i \Delta y'_i) - S \text{vec}(\Delta y_i \Delta y'_i)] \\ &= \text{vec}(M\Lambda)'(I_{T^2} - S) \text{vec}(\Delta y_i \Delta y'_i) \end{aligned} \quad (13)$$

is a typical term in the numerator of the inconsistency corrected estimator $\hat{\phi}_{LSDV}$, and

$$\begin{aligned} D_{i,T} &= y'_{i,-1} M y_{i,-1} \\ &= u'_i \Lambda' M \Lambda u_i \\ &= \Delta y'_i \Lambda' M \Lambda \Delta y_i \end{aligned} \quad (14)$$

is a typical term in the denominator of $\hat{\phi}_{LSDV}$.

The equalities in (14) also follow from (9), and the equality $u_i = \Delta y_i$, which is only valid under the null hypothesis. The terms $W_{i,T}$ have expectation zero by construction, since the matrix $(I_{T^2} - S)$ selects those elements of $\text{vec}(u_i u'_i)$ that have mean zero. The variance of $W_{i,T}$, $V_{i,T}$, is given by $\text{vec}(M \Lambda)' (I_{T^2} - S) \mathcal{V}(\text{vec}(\Delta y_i \Delta y'_i)) (I_{T^2} - S) \text{vec}(M \Lambda)$. These variances are different across i . Notice that the variance of $W_{i,T}$ is well defined, when $\mathcal{V}(\text{vec}(\Delta y_i \Delta y'_i))$ exists. The latter follows by condition (b1) of Assumption 1, which requires the existence of the $4 + \delta$ -th population moments of Δy_i .

Application of the Markov LLN yields that $\hat{\delta}_{LSDV} - \text{tr}(\Lambda' M \Lambda \bar{\Gamma}_N) = N^{-1} \sum_{i=1}^N D_{i,T} - \text{tr}(\Lambda' M \Lambda \bar{\Gamma}_N) \xrightarrow{p} 0$ (under the null hypothesis) by condition (b1) of Assumption 1. Condition (a2) ensures that $\text{plim}_{N \rightarrow \infty} \hat{\delta}_{LSDV}$ is different from zero. If conditions (b1) and (b2) of Assumption 1 are satisfied, then $N^{-0.5} \sum_{i=1}^N V_{LSDV}^{-0.5} W_{i,T} \xrightarrow{d} N(0, 1)$ by the Lindeberg-Feller CLT, where $V_{LSDV} = N^{-1} \sum_{i=1}^N V_{i,T}$ is the cross-sectional average of the variances of the $W_{i,T}$, $i = 1, 2, \dots, N$. Finally, by applying the Cramér theorem, we obtain

$$\sqrt{N} \left(\frac{\hat{\delta}_{LSDV}}{V_{LSDV}^{0.5}} \right) (\hat{\phi}_{LSDV} - 1 - \frac{\hat{B}_{LSDV}}{\hat{\delta}_{LSDV}}) \xrightarrow{d} N(0, 1). \quad (15)$$

Proof of Theorem 2: To derive the limiting distribution of the test statistic, we will follow the same steps as we did for Theorem 1. Decompose the vector $y_{i,-1}$ for model (2) under the null hypothesis that $\phi = 1$ as

$$y_{i,-1} = e y_{i0} + \Lambda e a_i + \Lambda u_i. \quad (16)$$

Premultiplying (16) by Q yields

$$Q y_{i,-1} = Q \Lambda u_i, \quad (17)$$

since $Q(e, \Lambda e) = [\mathbf{0}, \mathbf{0}]$.

Notice that in the case of model (2) the vector of disturbances u_i is not observed net of the individual effects under the null hypothesis $\phi = 1$, since now $\Delta y_i = u_i + a_i e$. Nevertheless, since $Q \Lambda e = \mathbf{0}$, we have under the null hypothesis

$$y'_{i,-1} Q u_i = u'_i \Lambda' Q u_i = \Delta y'_i \Lambda' Q \Delta y_i. \quad (18)$$

It follows by the Markov LLN that the numerator of $\hat{\phi}_{LSDVT} - 1$ converges in probability as $N \rightarrow \infty$, i.e.

$$\frac{1}{N} \sum_{i=1}^N y'_{i,-1} Q u_i - \text{tr}(\Lambda' Q \bar{\Gamma}_N) \xrightarrow{p} 0. \quad (19)$$

The cross-sectional average of the sample covariances of the first-differences of the y_i no longer yields a consistent estimator for $\bar{\Gamma}_N$. Instead, we have

$$\begin{aligned} & \frac{1}{N} \sum_{i=1}^N Svec(\Delta y_i \Delta y'_i) - Se_{T^2} \alpha_N^2 - vec(\bar{\Gamma}_N) \\ &= \frac{1}{N} \sum_{i=1}^N Svec((u_i u'_i) + a_i(u_i e') + a_i(e u'_i)) - vec(\bar{\Gamma}_N) \xrightarrow{p} 0, \end{aligned} \quad (20)$$

where $\alpha_N^2 = N^{-1} \sum_{i=1}^N a_i^2$. It follows from condition (b2) that $\alpha_N^2 = O(1)$.

We notice that under the null hypothesis α_N^2 can be estimated consistently by those elements of the matrix $N^{-1} \sum_{i=1}^N \Delta y_i \Delta y'_i$ that correspond to the zero elements of the Γ_i , $i = 1, 2, \dots, N$. For instance, if the maximum order of serial correlation of the $\{u_{i,t}\}$ processes is $T - 2$, there will be two such elements of $N^{-1} \sum_{i=1}^N \Delta y_i \Delta y'_i$: the upper right element and the lower left element.

By construction $\tilde{e} = (I_{T^2} - S)e_{T^2}$ picks up all elements of $E(vec(\Delta y_i \Delta y'_i))$ which are equal to a_i^2 . Since the total number of such elements is given by $\tilde{e}'\tilde{e}$, we have

$$\frac{1}{\tilde{e}'\tilde{e}} \tilde{e}' \left(\frac{1}{N} \sum_{i=1}^N vec(\Delta y_i \Delta y'_i) \right) - \alpha_N^2 \xrightarrow{p} 0. \quad (21)$$

The last three results taken together imply that the probability limit of $\hat{B}_{LSDVT}$ is the same as that of the numerator of $\hat{\phi}_{LSDVT} - 1$, that is

$$\begin{aligned} & \frac{1}{N} \sum_{i=1}^N y'_{i,-1} Q u_i - \hat{B}_{LSDVT} \\ &= \frac{1}{N} \sum_{i=1}^N \Delta y'_i \Lambda' Q \Delta y_i - \hat{B}_{LSDVT} \\ &= \frac{1}{N} \sum_{i=1}^N \Delta y'_i \Lambda' Q \Delta y_i - \text{tr}(\Lambda' Q \bar{\Gamma}_N) + vec(Q \Lambda)' vec(\bar{\Gamma}_N) \\ & \quad - vec(Q \Lambda)' S (I_{T^2} - (\tilde{e}'\tilde{e})^{-1} e_{T^2} \tilde{e}') \left(\frac{1}{N} \sum_{i=1}^N vec(\Delta y_i \Delta y'_i) \right) \xrightarrow{p} 0 \end{aligned} \quad (22)$$

Relationship (22) indicates that $\hat{B}_{LSDVT}$ can be used to correct the limiting distribution of $\hat{\phi}_{LSDVT} - 1$ for the inconsistency of $\hat{\phi}_{LSDVT}$, which arises from the presence of the fixed effects and the serially correlated disturbance terms. This can be done as follows.

Subtract $\hat{B}_{LSDVT}/\hat{\delta}_{LSDVT}$ from $\hat{\phi}_{LSDVT}$. This yields

$$\hat{\phi}_{LSDVT} - 1 - \frac{N\hat{\sigma}_{\hat{\phi}_{LSDVT}}^2}{\hat{\sigma}_u^2} \hat{B}_{LSDVT} = \left[\sum_{i=1}^N \tilde{D}_{i,T} \right]^{-1} \left[\sum_{i=1}^N \tilde{W}_{i,T} \right], \quad (23)$$

where $\tilde{W}_{i,T}$ is given by

$$\begin{aligned} \tilde{W}_{i,T} &= y'_{i,-1} Q u_i - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1} e_{T^2} \tilde{e}') \text{vec}(\Delta y_i \Delta y'_i) \\ &= \Delta y'_i \Lambda' Q \Delta y_i - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1} e_{T^2} \tilde{e}') \text{vec}(\Delta y_i \Delta y'_i) \\ &= \text{vec}(Q\Lambda)' \text{vec}(\Delta y_i \Delta y'_i) - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1} e_{T^2} \tilde{e}') \text{vec}(\Delta y_i \Delta y'_i) \\ &= \text{vec}(Q\Lambda)' (I_{T^2} - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1} e_{T^2} \tilde{e}')) \text{vec}(\Delta y_i \Delta y'_i) \end{aligned} \quad (24)$$

and $\tilde{D}_{i,T}$ is given by

$$\begin{aligned} \tilde{D}_{i,T} &= y'_{i,-1} Q y_{i,-1} \\ &= u'_i \Lambda' Q \Lambda u_i \\ &= \Delta y'_i \Lambda' Q \Lambda \Delta y_i. \end{aligned} \quad (25)$$

The above relationships for $\tilde{W}_{i,T}$ and $\tilde{D}_{i,T}$ have been obtained by making use of (17), the equality $\Delta y_i = u_i + a_i e$, which only holds under the null hypothesis, and the equality $Q\Lambda e = \mathbf{0}$. Notice that $\tilde{W}_{i,T}$ has expectation zero by construction and variance, $\tilde{V}_{i,T}$, given by

$$\begin{aligned} &\text{vec}(Q\Lambda)' (I_{T^2} - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1} e_{T^2} \tilde{e}')) \mathcal{V}(\text{vec}(\Delta y_i \Delta y'_i)) \\ &\times (I_{T^2} - S(I_{T^2} - (\tilde{e}'\tilde{e})^{-1} e_{T^2} \tilde{e}'))' \text{vec}(Q\Lambda) \end{aligned} \quad (26)$$

for each $i \in \{1, \dots, N\}$. The variances of $\tilde{W}_{i,T}$ are well defined by condition (b1) of Assumption 1.

Finally, by applying the same arguments as in the proof of Theorem 1, we obtain $\hat{\delta}_{LSDVT} - \text{tr}(\Lambda' Q \Lambda \bar{\Gamma}_N) = N^{-1} \sum_{i=1}^N \tilde{D}_{i,T} - \text{tr}(\Lambda' Q \Lambda \bar{\Gamma}_N) \xrightarrow{p} 0$, and

$$\sqrt{N} \left(\frac{\hat{\delta}_{LSDVT}}{V_{LSDVT}^{0.5}} \right) (\hat{\phi}_{LSDVT} - 1 - \frac{\hat{B}_{LSDVT}}{\hat{\delta}_{LSDVT}}) \xrightarrow{d} N(0, 1), \quad (27)$$

where $V_{LSDVT} = N^{-1} \sum_{i=1}^N \tilde{V}_{i,T}$ is the cross-sectional average of the variances of the $\tilde{W}_{i,T}$, $i = 1, 2, \dots, N$.

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Table 1a Size and power of $\hat{V}_{LSDV}^{-1/2} \hat{\delta}_{LSDV} \sqrt{N} \left(\hat{\phi}_{LSDV} - 1 - \frac{\hat{B}_{LSDV}}{\hat{\delta}_{LSDV}} \right)$									
N, T	$\theta_{it} = -0.50 + \omega_{it}$			$\theta_{it} = 0.00 + \omega_{it}$			$\theta_{it} = 0.50 + \omega_{it}$		
	Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$	
		0.95	0.90	1.00	0.95	0.90	1.00	0.95	0.90
25, 5	0.061	0.06	0.14	0.055	0.09	0.18	0.053	0.12	0.21
50, 5	0.053	0.06	0.19	0.054	0.12	0.27	0.047	0.17	0.34
500, 5	0.046	0.10	0.36	0.051	0.40	0.67	0.050	0.68	0.66
25, 10	0.063	0.06	0.13	0.059	0.08	0.16	0.068	0.09	0.18
50, 10	0.056	0.06	0.17	0.06	0.09	0.22	0.06	0.12	0.24
500, 10	0.042	0.10	0.29	0.051	0.29	0.50	0.052	0.43	0.61

Notes: $\hat{C}_{LSDV} = \hat{V}_{LSDV} / \hat{\delta}_{LSDV}^2$

Table 1b Size and power of $\hat{V}_{LSDVT}^{-1/2} \hat{\delta}_{LSDVT} \sqrt{N} \left(\hat{\phi}_{LSDVT} - 1 - \frac{\hat{B}_{LSDVT}}{\hat{\delta}_{LSDVT}} \right)$									
N, T	$\theta_{it} = -0.50 + \omega_{it}$			$\theta_{it} = 0.00 + \omega_{it}$			$\theta_{it} = 0.30 + \omega_{it}$		
	Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$	
		0.95	0.90		0.95	0.90		0.95	0.90
25, 5	0.058	0.05	0.05	0.069	0.05	0.05	0.075	0.05	0.05
50, 5	0.059	0.05	0.05	0.06	0.05	0.05	0.066	0.05	0.05
500, 5	0.046	0.05	0.05	0.054	0.06	0.07	0.05	0.06	0.10
25, 10	0.061	0.05	0.06	0.070	0.06	0.06	0.070	0.06	0.07
50, 10	0.060	0.05	0.06	0.070	0.06	0.06	0.071	0.06	0.08
500, 10	0.049	0.05	0.07	0.051	0.07	0.13	0.054	0.08	0.21
500, 15	0.046	0.06	0.08	0.052	0.09	0.19	0.053	0.11	0.29

Notes: $\hat{C}_{LSDVT} = \hat{V}_{LSDVT} / \hat{\delta}_{LSDVT}^2$

Table 2a		Size and power of $\hat{V}_{LSDV}^{-1/2} \hat{\delta}_{LSDV} \sqrt{N} \left(\hat{\phi}_{LSDV} - 1 - \frac{\hat{B}_{LSDV}}{\hat{\delta}_{LSDV}} \right)$ for homogeneous panels													
N, T	$\theta = -0.80$			$\theta = -0.30$			$\theta = 0.00$			$\theta = 0.30$			$\theta = 0.80$		
	Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$	
		0.95	0.90		0.95	0.90	1.00	0.95	0.90	1.00	0.95	0.90	1.00	0.95	0.90
25, 5	0.056	0.05	0.12	0.055	0.07	0.16	0.054	0.09	0.19	0.054	0.10	0.22	0.06	0.11	0.22
50, 5	0.056	0.05	0.17	0.053	0.08	0.21	0.060	0.11	0.27	0.054	0.14	0.31	0.051	0.17	0.32
500, 5	0.055	0.05	0.21	0.053	0.18	0.47	0.055	0.38	0.65	0.055	0.58	0.68	0.054	0.70	0.67
25, 10	0.066	0.05	0.12	0.061	0.07	0.14	0.060	0.08	0.18	0.071	0.08	0.16	0.061	0.09	0.18
50, 10	0.060	0.05	0.16	0.049	0.08	0.20	0.052	0.09	0.20	0.063	0.10	0.23	0.056	0.11	0.23
500, 10	0.045	0.06	0.21	0.050	0.16	0.36	0.049	0.29	0.48	0.052	0.35	0.52	0.055	0.37	0.55

Table 2b		Size and power of $\hat{V}_{LSDV}^{-1/2} \hat{\delta}_{LSDV} \sqrt{N} \left(\hat{\phi}_{LSDV} - 1 - \frac{\hat{B}_{LSDV}}{\hat{\delta}_{LSDV}} \right)$ for homogeneous panels													
N, T	$\theta = -0.80$			$\theta = -0.30$			$\theta = 0.00$			$\theta = 0.30$			$\theta = 0.80$		
	Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$		Size	Power, $\phi =$	
		0.95	0.90		0.95	0.90	1.00	0.95	0.90	1.00	0.95	0.90	1.00	0.95	0.90
25, 5	0.06	0.05	0.05	0.065	0.05	0.05	0.076	0.05	0.05	0.069	0.05	0.05	0.072	0.05	0.06
50, 5	0.06	0.05	0.05	0.056	0.05	0.05	0.057	0.05	0.05	0.065	0.05	0.06	0.062	0.05	0.06
500, 5	0.046	0.05	0.05	0.055	0.05	0.05	0.055	0.06	0.07	0.051	0.06	0.08	0.052	0.07	0.10
25, 10	0.062	0.05	0.05	0.059	0.05	0.05	0.070	0.05	0.06	0.070	0.06	0.07	0.072	0.06	0.07
50, 10	0.054	0.05	0.05	0.059	0.05	0.06	0.062	0.06	0.07	0.062	0.06	0.07	0.065	0.06	0.08
500, 10	0.053	0.05	0.05	0.05	0.06	0.07	0.051	0.07	0.12	0.052	0.08	0.16	0.054	0.09	0.23
500, 15	0.046	0.05	0.05	0.043	0.06	0.08	0.045	0.07	0.13	0.045	0.09	0.19	0.053	0.10	0.27

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