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Deconstructing the Consumption Function: New Tools and Old Problems

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DECONSTRUCTING THE CONSUMPTION FUNCTION: NEW TOOLS AND OLD PROBLEMS

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In this paper, we analyse anew the relationship between aggregate income and consumption in the United Kingdom. Our analysis entails a close examination of the structure of the data, for which we employ a variety of spectral methods which depend on the concepts of Fourier analysis. We discover that fluctuations in the rate of growth of consumption tend to precede similar fluctuations in income, which contradicts a common supposition. We also highlight the difficulty of uncovering from the aggregate data a structural equation representing the behaviour of consumers.

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JEL Classification: C22

1. Introduction: The Evolution of the Consumption Function

Over many years, the aggregate consumption function has provided a context in which problems of econometric modelling have been debated and from which significant innovations in methodology have often emerged.

For almost two decades, beginning in the mid 1950's, successes in modelling the income–consumption relationship were seen as grounds for congratulating the econometricians. They had managed to reconcile the formulations of Keynesian macroeconomic theory with some empirical findings which had seemed, at first, to contradict the theory.

The so-called "static" Keynesian consumption function, which remains a feature of macroeconomic textbooks, proposes that the marginal propensity to consume is a constant or a declining function of income and that the average propensity to consume is a declining function of income. Such a formulation is in accordance with the supposition of Keynes (1936) himself that, as aggregate income increases, the economy will be beset by worsening problems of underconsumption.

Some simple linear forms of the Keynesian consumption function were fitted by least-squares regression in the late 1940's and early 1950's. When

these were used for forecasting post-war consumption in the United States, they greatly under-predicted its values—see Davis (1952), for example. Doubt was also cast on the estimated consumption functions by data produced by Kuznets (1942), which demonstrated that income and consumption had maintained a rough proportionality over many years.

At the same time, it was recognised that there is a double relationship between consumption and income. There is the relationship that is summarised in the consumption function and there is a relationship which follows from the fact that consumption expenditures are a major factor in determining the level of aggregate income—see, for example, the analysis of Haavelmo (1947) which was partly recapitulated in the textbook of Ackley (1961). This truism has been both ignored and called to mind many times in subsequent years. To disentagle the two relationships from the macreconomic data is a more difficult task than many optimistic analysts have assumed.

The Keynesian formulation was reconciled with the findings of Kuznets in a variety of dynamic models in which the relationship of consumption to income was subject to time lags. Models of this nature were provided by Duesenberry (1949), who propounded the relative-income hypothesis, by Modigliani and Brumberg (1954), who propounded the life-cycle hypothesis—see Modigliani (1975), also—and by Friedman (1957), who propounded the permanent-income hypothesis. According to these models, rapid increases in income will give rise, in the short run, to less-that-proportional increases in consumption. Over longer periods, consumption will gradually regain its long-run relationship with income.

The observations of Granger and Newbold (1974) and others on the spurious nature of regression relationships between trended economic variables led many to suspect that the apparent success in modelling the relationship between income and consumption might be illusory. Whereas such regressions account remarkably well for the level of consumption, they often perform poorly in the far more stringent task of predicting the changes in the level of consumption for one period to another.

A dynamic regression model in levels can always be expressed, via a linear reparametrisation, as a so-called error-correction model which comprises the differences of the variables together with a term expressing the current disproportion of income and consumption. The fitted error-correction model will have the same residual sum of squares as the model in levels.

The residual sum of squares will often look small in comparison with the total sum of squares of a trended dependent variable. This will result in a high value for the R^2 coefficient of determination for the model in levels, which might be seen as indicative of its success. The same residual sum of squares may seem large in comparison with the sum of squares of the differences of the dependent variable. Therefore, the error-correction model, which is just a reparametrised version of a model in levels, often appears to fit the data poorly.

The recognition of the fact that, in the case of trended variables, the error-correction formulation was the appropriate context in which to conduct

statistical tests of significance, whilst the model in levels was, clearly, an inappropriate context, led to a thorough reappraisal of the supposed successes in modelling the income–consumption relationship.

In the late 1970's, there were considerable doubts about the validity of many of the contemporary exercises in macroeconometric modelling. These doubts were somewhat relieved by a number of successful modelling exercises which adopted the error-correction formulation as their starting point. Amongst the most celebrated of these was the paper by Davidson *et al.* (1978) which proposed a model of the income–consumption relationship in the U.K. that produced a remarkably high value for the coefficient of determination.

In effect, the paper of Davidson *et al.* succeeding in reestablishing the traditional consumption function within a viable econometric framework. The model proposed in this paper was revisited by Hendry, Muellbauer and Murphy, (1990) who also surveyed some of the contemporaneous developments in econometric theory which were associated with the error-correction model and with the concept of cointegration.

An influential paper by Hall (1978), which was published in the same year as that of Davidson *et al.*, appeared to draw opposite conclusions. Hall considered the problem of the intertemporal maximisation of the utility of a representative consumer; and he adopted the premise of rational expectations. He demonstrated that, in theory, the best forecast of the current consumption of an optimising individual is their consumption in the previous period. This implies that the evolution of consumption follows a random walk; and Hall supported this proposition with an econometric analysis of the aggregate data. Economists who accepted Hall's analysis were encouraged to forsake further investigation of the aggregate consumption function in favour microeconomic investigations of consumer behaviour. Such endeavours have come to dominate the field of consumption theory.

Enough time has elapsed since the publication of the article by Davidson *et al.* for the data series to have more than doubled in length. In spite of the various economic vicissitudes that are reflected in the extended data set, their model continues to fit remarkably well, with newly estimated parameters that are not vastly different from the original ones. One of the purposes of the present paper is to examine the basis of this apparent success. Another purpose is to determine whether it is possible to extract from the aggregate data an autonomous structural equation representing the supposed behaviour of a representative consumer.

2. The Data and the Four-Period Difference Filter

In evaluating any model, we should begin by inspecting the data. The data series of consumption and income have two prominent characteristics. The first characteristic is their non-stationarity. Over the extended data period, the logarithms of the data show an upward trend that is linear in it basis. The second characteristic of the data series is that they both show evident patterns of seasonal variation which play on the backs of the rising trends.

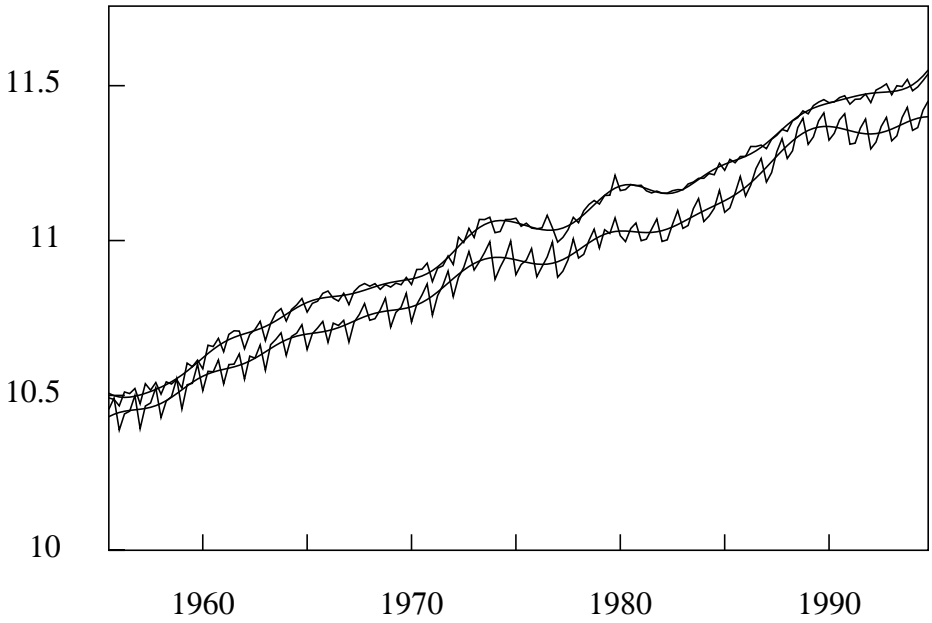

Figure 1. The quarterly series of the logarithms of income (upper) and consumption (lower) in the U.K., for the years 1955 to 1994, together with their interpolated trends.

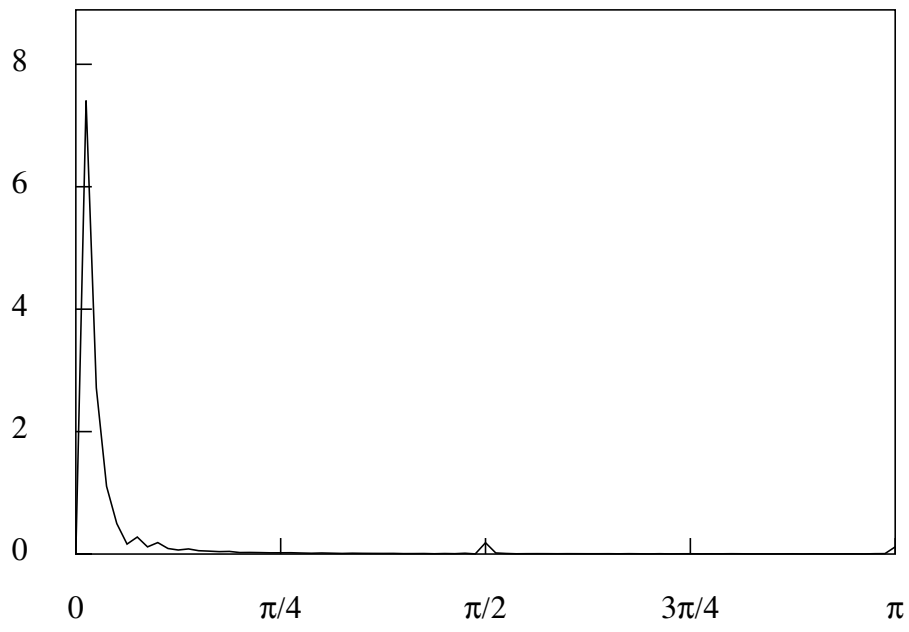

Figure 2. The periodogram of the logarithms of consumption in the U.K., for the years 1955 to 1994.

The seasonal pattern is more evident in the consumption series than it is in the income series. Therefore, the question arises of whether seasonal fluctuations in consumption are induced by those in income or whether they have their origin in an independent influence which impinges on both income and consumption.

If we were to take the view that the seasonal fluctuations in consumption originate in the income stream, then we should proceed to construct a transfer function relationship between income and consumption which is based on the raw, non-deseasonalised data. If we were to take the view that the seasonality in both series is due to exogenous influences, then we should base our estimate of the transfer function on some deseasonalised data.

Models like that of Davidson *et al.* seek to explain an annual growth rate in consumption which is derived from quarterly data. The dependent variable of the model is obtained by passing the logarithm of the consumption series, which we shall denote by $y(t)$, through a four-period difference filter of the form $\nabla_4 = 1 - L^4 = (1 - L)(1 + L + L^2 + L^3)$. Here L is the lag operator which has the effect that $Ly(t) = y(t - 1)$, where $y(t) = \{y_t; t = 0 \pm 1, \pm 2, \ldots\}$ is a series of observations taken at three-monthly intervals. The filter removes from $y(t)$ both the trend and the seasonal fluctuations; and it removes much else besides.

The gain of the filter is depicted in Figure 3. The operator nullifies the component at zero frequency and it diminishes the power of the trend components whose frequencies are in the neighbourhood of zero. This is the effect of $\nabla = (1 - L)$, which is a factor of ∇_4. The filter also removes the components at the seasonal frequency of $\pi/2$ and at its harmonic frequency of π, and it attenuates the components in the neighbourhoods of these frequencies. This is the effect of the four-point summation operator $S_4 = 1 + L + L^2 + L^3$ which is the other factor of ∇_4. It is also apparent that the filter amplifies the cyclical components of the data that have frequencies in the vicinities of $\pi/4$ and $3\pi/4$; and, as we shall discover later, this is a distortion that can have a marked effect upon some of the estimates that are derived from the filtered data.

The effect of the filter upon the consumption series can be discerned in the periodogram of Figure 4. The periodogram is the sequence of the coefficients ρ_j^2 from the Fourier expression

$$y(t) = \sum_{j=0}^{[T/2]} \rho_j \cos(\omega_j t - \theta_j), \tag{1}$$

where T is the sample size and $[T/2]$ in the integral part of $T/2$.

The most striking aspect of this periodogram, in comparison with that of the unfiltered data, shown in Figure 2, is the diminution of the power at the frequencies in the vicinity of zero, which is where the trend components are to be found, and in the vicinities of $\pi/2$ and π, where the seasonal components and their harmonics are to be found. The degree of the amplification of the components in the vicinities of $\pi/4$ and $3\pi/4$ can be judged in comparison with

a periodogram of the detrended data, presented in Figure 7, which has been obtained by the fitting of a linear trend.

In this paper, we shall propose new methods for detrending the data and for deseasonalising it, which are designed to remove the minimum quantity of information from the processed series. We shall use these alternative methods mainly because we are mindful of the distortions induced by the differencing operator.

The method of deseasonalising that we use allows us to vary our definition of the seasonal component quite widely in view of what we find in the data. At its simplest, the method is equivalent to using simple seasonal dummy variables to capture an invariable pattern of fluctuations which is the same in every year. However, the method also allows us to capture a changing pattern of seasonal fluctuations by including in the definition various components at non-harmonic frequencies which are adjacent to the seasonal frequencies.

3. The Error-Correction Model and its Implications

The consumption function of Davidson *et al.* (1978) was calculated originally on a data set from the U.K. running from 1958 to 1970, which was a period of relative economic quiescence. When the function is estimated for an extended data period, running from 1956 to 1994, it yields the following results:

(2)

$$\nabla_4 y(t) = \underset{(0.040)}{0.70}\nabla_4 x(t) - \underset{(0.060)}{0.156}\nabla\nabla_4 x(t) + \underset{(0.015)}{0.068}\{x(t-4) - y(t-4)\} + e(t)$$

$$R^2 = 0.77 \qquad \text{D–W} = 0.920.$$

Here $y(t)$ and $x(t)$ represent, respectively, the logarithms of the consumption sequence and the income sequence, without seasonal adjustment. The operators $\nabla = 1 - L$ and $\nabla_4 = 1 - L^4$ are, respectively, the one-period and the four-period difference operator. Therefore $\nabla_4 y(t)$ and $\nabla_4 x(t)$ represent the annual growth rates of consumption and income, whilst $\nabla_1 \nabla_4 x(t)$ represents the acceleration or deceleration in the growth of income.

This specification reflects an awareness of the difficulty of drawing meaningful inferences from a regression equation that incorporates nonstationary variables. The difference operators are effective in reducing the sequences $x(t)$ and $y(t)$ to stationarity. The synthetic sequence $x(t-4) - y(t-4)$ is also presumed to be stationary by virtue of the cointegration of $x(t)$ and $y(t)$; and its role within the equation is to provide an error-correction mechanism which tends to eliminate any disproportion that might arise between consumption and income.

A feature of the error-correction term in equation (2) is that it is expected always to be positive, since income will invariably exceed consumption. In an alternative formulation, there would be a positive coefficient γ associated with $x(t-4)$, representing the income-elasticity of consumption. In that case, one would expect the term $\gamma x(t-4) - y(t-4)$ to have an average over the sample period close to zero. Also, the estimated coefficient of $\nabla_4 x(t)$ would increase,

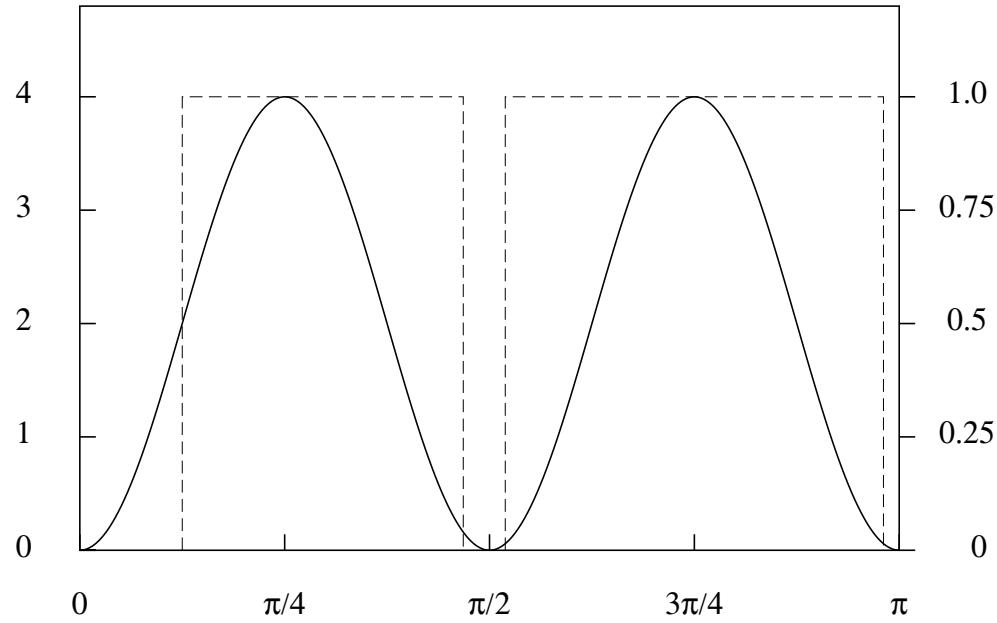

Figure 3. The gain of the four-period difference filter $\nabla_4 = 1 - L^4$ (continuous line and left scale) and the frequency selection of the deseasonalised detrended data (broken line and right scale).

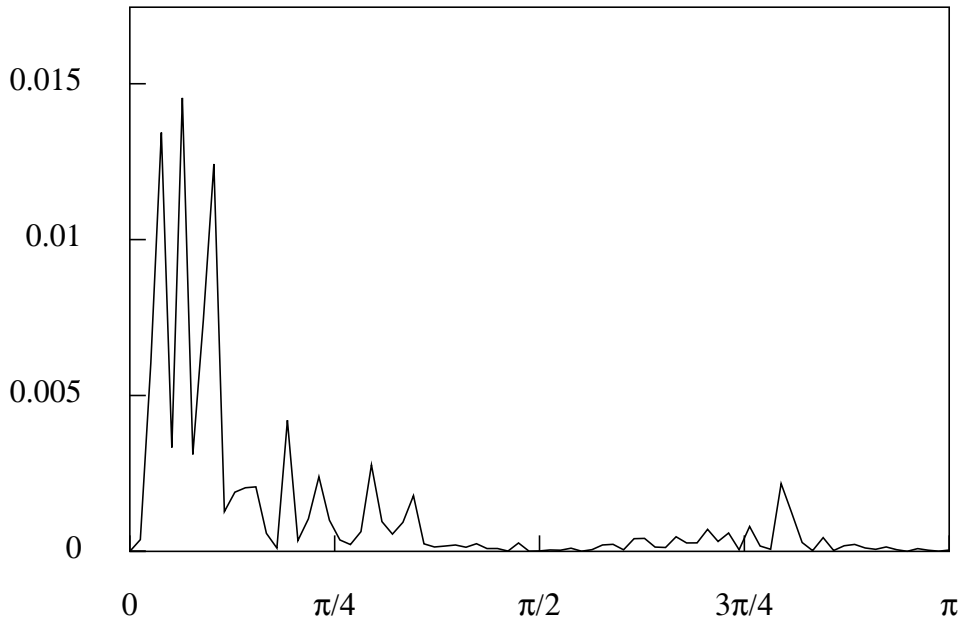

Figure 4. The periodogram of the filtered series $\nabla_4 y(t)$ representing the annual growth rate of consumption.

unless an intercept term were included in the equation to compensate for the closure of the error-correction gap. The logic of the specification, as it stands, is that the gap between $x(t)$ and $y(t)$ is related to the rate of growth of income. Only if the data were derived from an economy in a state of zero growth would the gap be close to zero on average.

The specification also bears the impress of some of the earlier experiences in modelling the consumption function which we have described in the introduction. The variable $\nabla_1\nabla_4 x(t)$ with its associated negative-valued coefficient allows the growth of consumption to lag behind the growth of income when the latter is accelerating. This is the sort of response that the analysts of the late 1940's and 1950's, who were intent on reconciling the Keynesian formulations with the empirical findings, were at such pains to model.

We can evaluate the roles played by the terms of the RHS of equation (2) by modifying the specification and by observing how the coefficients of the fitted regression are affected and how the goodness of fit is affected.

The first modification is to replace $x(t-4)-y(t-4)$ by a constant dummy variable. The result is a slight change in the estimates of the remaining parameters of the model and a negligible loss in the goodness of fit. This suggests that we can dispense with the error-correction term at very little cost:

$$\begin{aligned} \nabla_4 y(t) &= 0.006 + 0.682\nabla_4 x(t) - 0.160\nabla\nabla_4 x(t) + e(t) \\ &\quad\ (0.001)\ \ (0.053) \qquad\qquad (0.066) \\ R^2 &= 0.76 \qquad \text{D–W} = 0.93. \end{aligned} \tag{3}$$

The second modification is to eliminate both the error-correction term and the acceleration term $\nabla_1\nabla_4 x(t)$ and to observe how well the annual growth in consumption is explained by the annual growth of income. In this case, we observe that the coefficient of determination of the fitted regression is 0.72, compared with 0.77 for the fully specified model, while the error sum of square increases to 0.053 from 0.044. We conclude from this that the acceleration term does have some effect:

$$\begin{aligned} \nabla_4 y(t) &= 0.769\nabla_4 x(t) + e(t) \\ &\quad\ (0.028) \\ R^2 &= 0.72 \qquad \text{D–W} = 1.15. \end{aligned} \tag{4}$$

The fact that the acceleration term enters the consumption function with a negative coefficient seem to suggest that the response of consumption to rapid changes in income is laggardly more often that not. This would fit well with the various hypotheses regarding consumer behaviour that have been mentioned in the introduction. However, the significance of the estimated coefficient is not very great and it is considerably reduced when the coefficient is estimated using only the first third of the data. We shall reconsider the acceleration term at the end of the paper where we shall discover that its effect is reversed when we analyse the relationship between the trends depicted in Figure 1.

A final issue that should be considered in connection with the consumption function of Davidson *et al.* is the problem of simultaneous-equations bias. This is an issue that was raised by Haavelmo (1947) in the early days of macro econometric modelling. According to a familiar textbook demonstration—see, for example, Johnston (1984, p. 440)—the estimate of the marginal propensity to consume is liable to be biased upwards in a static linear consumption function that relates current consumption to current income. The bias will be positively related to the variance of the disturbances or 'innovations' affecting the consumption function and inversely related to the variance of those affecting income.

This result, which is easily understood in reference to the diagram known as the Keynesian cross, is also true for more complicated dynamic versions of the consumption function, such as that of Davidson *et al.*, that include current income amongst their explanatory variables. The matter has been analysed carefully by Urbain (1992) who uses the concepts of exogeneity expounded by Engle *et al.* (1983). It important, therefore, to discover the relative sizes of the innovation variances in the consumption function and in the income equation—and we shall attend to this matter in a later section of the paper.

If it transpires that the consumption innovations are relatively large, then doubt will be cast on the ordinary least-squares estimates of the parameters of an equation such as (2). In the face of a severe problem of simultaneous-equations bias, it may be wise to forsake the ambition of uncovering a structural equation, reflecting the behaviour of the typical consumer, and to be content with a reduced-form equation of the sort that expresses current consumption in terms of exogenous and predetermined variables.

In the section that follows, we shall conduct an analysis of the income and consumption data which entails making a clear separation of the trends of the two series from their remaining components. We shall proceed to separate the seasonal components from the detrended series in a manner which is designed to have the minimum effect upon the remaining information. However, we shall discover that there is very little information remaining in either series when they have been both detrended and deseasonalised. Therefore our analysis of the income–consumption relationship is bound to rest heavily upon an analysis of the trends.

4. A Fourier Method for Detrending the Data

In the previous section, we have seen how the difference operator $1-L$ and the four-point summation operator $S_4 = 1+L+L^2+L^3$ are liable to remove a substantial part of the information that is contained in the data of the consumption series. In this section, we shall propose alternative devices for detrending and for deseasonalising the data that leave much of the information intact.

Our basic objective is to remove from the data only those Fourier components that contribute to the trend or to the seasonality, and to leave the other components of the data unaffected. The idea of discriminating amongst the Fourier components of the data according to their frequency values is an old

one. In particular, Engle (1974) has proposed a method of "band-spectrum regression" in which selected subsets of the spectral components of the data series are subjected to an ordinary regression analysis. However, is seems that this technique has not been used much in econometric analysis; and, when it is applied to nonstationary data sequences, it is beset by some problems which need to be resolved.

A normal requirement for the use of the standard methods of statistical Fourier analysis is that the data in question should be generated by stationary processes, and this requirement is a hardly ever satisfied in econometric analysis.

To understand the problems that can arise in applying Fourier methods to trended data, one must recognise that, in analysing a finite data sequence, one is making the implicit assumption that it represents a single cycle of a periodic function which is defined over the entire set of positive and negative integers. This function may be described as the periodic extension of the data sequence.

In the case of a trended sequence, there are bound to be radical disjunctions in the periodic function where one replication of the data sequence ends and another begins. Thus, for example, if the data follow a linear trend, then the function which is the subject of the Fourier analysis will have the appearance of the serrated edge of a saw blade.

In Figure 5, we plot of a single cycle of a discrete periodic the saw-tooth function

$$y(t) = \{(t + T/2) \bmod T\} - T/2; \quad t = 0, \ldots, 63 = T - 1 \tag{5}$$

and, in Figure 6, we show the periodogram of the function. This periodogram reveals the structure that underlies the periodogram of the trended consumption data which is shown in Figure 2.

It is clear, from Figure 6, that the synthesis of the disjunction which arises from the periodic extension of a data sequence will involve all of the Fourier frequencies. Therefore, it is incorrect to identify the trend component of a data sequence with the low-frequency components alone; and it appears, at first, that there will be difficulties in extracting the trend from a data sequence via Fourier methods.

The problem is resolved by using an approach which is familiar from the forecasting of ARIMA processes. We begin by differencing the data sequence as many times as may be necessary to reduce it to a state of stationarity. We proceed to eliminate the low-frequency components from the differenced data. Then, by cumulating or 'integrating' the resulting sequence, we will obtain the detrended version of the data. The trend of the data can be obtained, likewise, by cumulating the low-frequency components that have been extracted from the differenced data.

The effect of differencing the data should be to remove or, at least, greatly to diminish the disjunctions in the periodic extension of the data sequence. Some further steps can be taken to minimise any remaining effect. To begin with, if the data show a high degree of seasonality, then it makes sense to apply

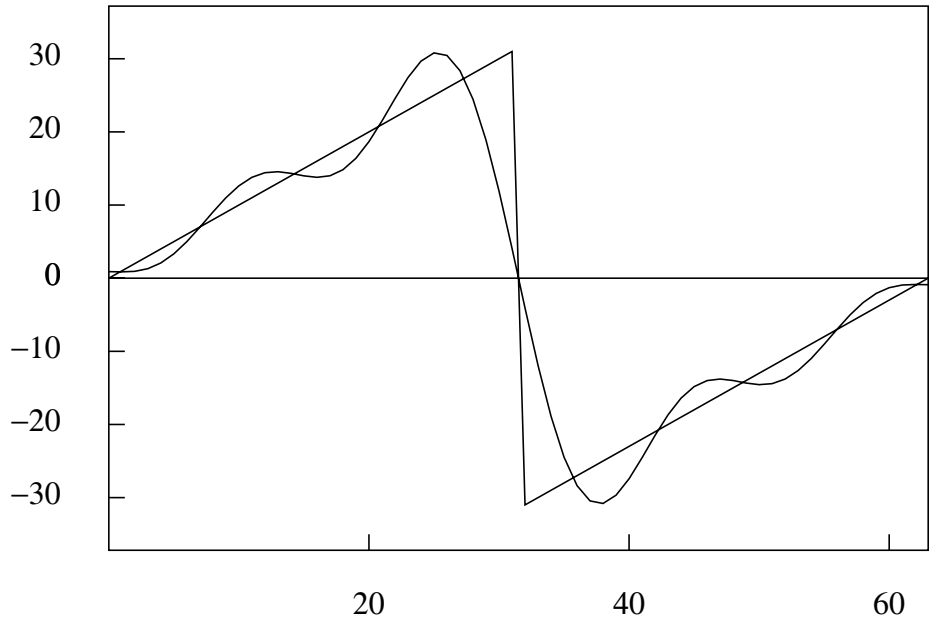

Figure 5. The sawtooth function and it Fourier approximation based on four cosine functions.

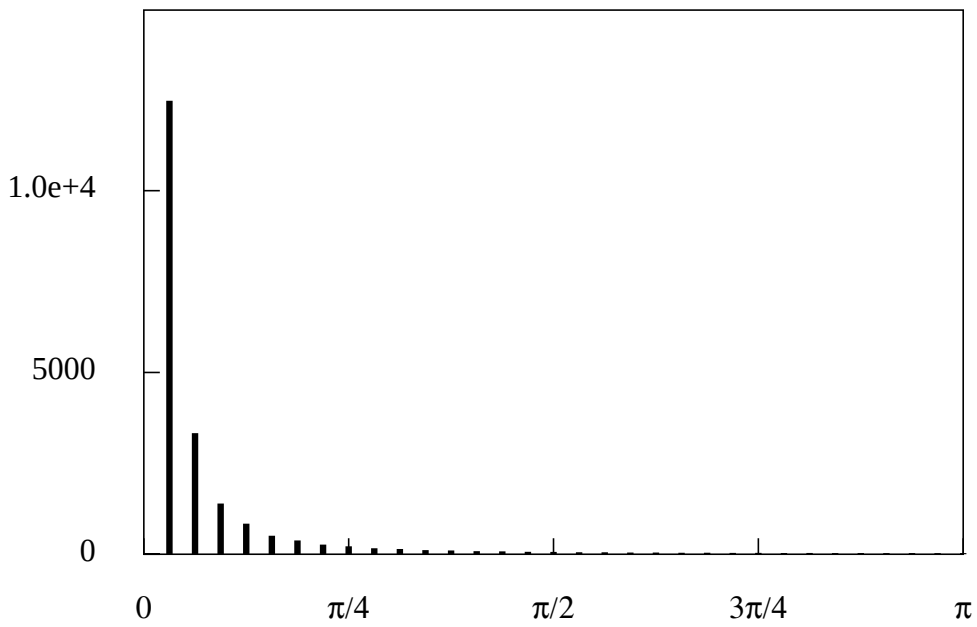

Figure 6. The periodogram of the sawtooth function.

the Fourier analysis to a differenced sequence that contains an integral number of seasonal cycles. For this purpose, one can discard a few points from the start of the data sequence where the oldest observations are to be found.

A second step that can be taken to reduce or to eliminate the disjunctions from the periodic function is to taper the differenced data so that its end points are both reduced to zero. The tapering is achieved by multiplying each of the data points by a scalar factor. The factors should tend to zeros as the ends of the sample are approached. A common tapering sequence or "data window" is the split cosine bell—see Bloomfield (1976, p. 85), for example. Tapering can be accompanied by a process of reverse tapering which may be applied to the differenced data sequence after its trend component, or some other component, has been removed. In practice, we have not found it necessary to use this device.

The process by which the trend components are cumulated after they have been extracted from the differenced data sequence calls for some initial conditions or starting values. To provide expressions for these values, we need to describe the matrix versions of the difference operator and of the summation or cumulation operator, which is its inverse.

Let the identity matrix of order T be denoted by

$$I_T = [e_0, e_1, \ldots, e_{T-1}], \tag{6}$$

where e_j represents a column vector which contains a single unit preceded by j zeros and followed by $T - j - 1$ zeros. Then the finite-sample lag operator is the matrix

$$L_T = [e_1, \ldots, e_{T-1}, 0] \tag{7}$$

which has units on the first subdiagonal and zeros elsewhere. The matrix which takes the d-th difference of a vector of order T is given by $\Delta = (I - L_T)^d$.

Taking differences within a vector entails a loss of information. Therefore, if $\Delta = [Q_*, Q]'$, where Q_*' has d rows, then the d-th differences of a vector $y = [y_0, \ldots, y_{T-1}]'$ are the elements of the vector $g = [g_d, \ldots, g_{T-1}]'$ which is found in the equation

$$\begin{bmatrix} g_* \\ g \end{bmatrix} = \begin{bmatrix} Q_*' \\ Q' \end{bmatrix} y. \tag{8}$$

The vector $g_* = Q_*'y$ in this equation, which is a transform of the vector $[y_0, \ldots, y_{d-1}]$ of the leading elements of y, is liable to be discarded.

The inverse of the difference matrix is the matrix $\Delta^{-1} = \Sigma = [S_*, S]$. This has the effect that

$$S_* g_* + Sg = y. \tag{9}$$

The vector y can be recovered from the differenced vector g only if the vector g_* of initial conditions is provided.

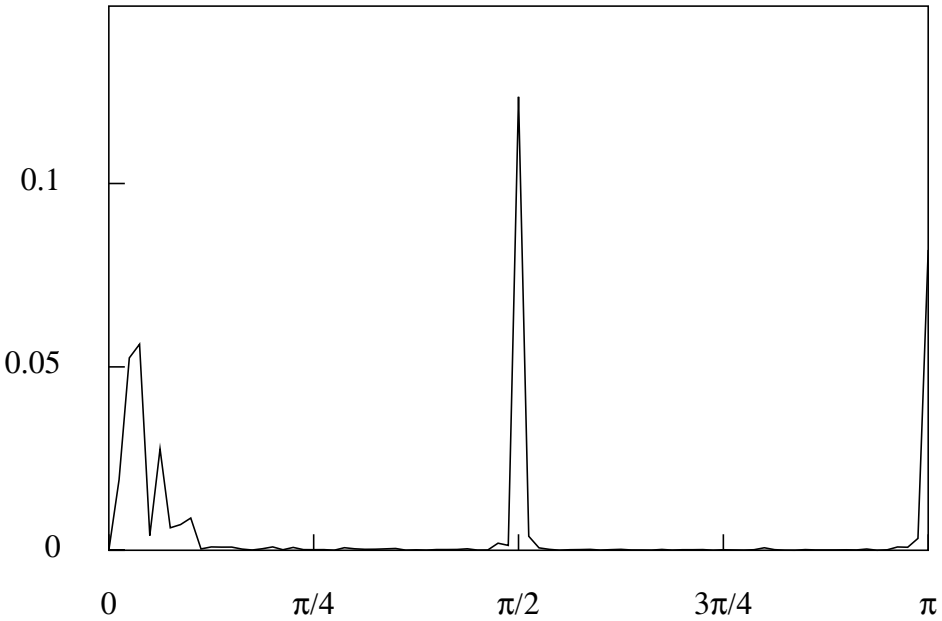

Figure 7. The periodogram of the residuals obtained by fitting a linear trend through the logarithmic consumption data of Figure 1.

Now let z represent the differenced version of the trend component which requires to be cumulated to form $x = S_* z_* + Sz$. Then the initial conditions in z_* should be chosen so as to ensure that the trend is aligned with the data as closely as possible. The criterion is

$$\text{(10)} \qquad \text{Minimise} \quad (y - S_* z_* - Sz)'(y - S_* z_* - Sz) \quad \text{with respect to} \quad z_*.$$

The solution for the starting values is

$$\text{(11)} \qquad z_* = (S_*' S_*)^{-1} S_*'(y - Sz).$$

The facility that we have constructed for removing the trend from the data allows us to select a cut-off point which marks the highest frequency amongst the Fourier components which constitute the trend. The decision of where to place the cut-off point should be guided by an appraisal of the spectral structure of the data. Figure 7 shows the periodogram of the residual sequence obtained by fitting a linear trend through the logarithms of the consumption series. Fitting a linear trend overcomes the problems of non-stationarity without destroying the information relating to the trend components. In effect, it removes from the periodogram of the logarithmic data a component whose form is illustrated by Figure 7.

We choose to place the cut-off point at $\pi/8$ radians which is in a dead space of the periodogram where there are no ordinates of any significant size. Given that the observations are at quarterly intervals, this implies that the trend includes all cycles of four years duration of more. The detrended consumption

series is show in Figure 8. A similar analysis of the income data suggests that the same cut-off point is appropriate. The trends in the consumption and income series that have been calculated on this basis are depicted in Figure 1.

5. A Fourier Method for Deseasonalising the Data

As well as removing the trend from the data, we also wish to remove the seasonal fluctuations. This can be done in much the same way. At its simplest, we can define the seasonal component to consist only of those Fourier components, extracted from the differenced data $\{g_d, \ldots, g_{T-1}\}$, which are at the seasonal frequency and at the harmonically related frequencies. In the case of quarterly data, the component would be described by the equation

$$w(t) = \alpha_1 \cos\left(\frac{\pi t}{2}\right) + \beta_1 \sin\left(\frac{\pi t}{2}\right) + \alpha_2(-1)^t, \tag{12}$$

wherein

$$\begin{aligned} \alpha_1 &= \frac{2}{T}\sum_t g_t \cos\left(\frac{\pi t}{2}\right), \\ \beta_1 &= \frac{2}{T}\sum_t g_t \sin\left(\frac{\pi t}{2}\right), \\ \alpha_2 &= \frac{1}{T}\sum_t g_t(-1)^t. \end{aligned} \tag{13}$$

In fact, this scheme is equivalent to one which uses seasonal dummy variables with the constraint that their associated coefficients must sum to zero. It will generate a pattern of seasonal variation which is the same for every year.

A more sophisticated pattern of seasonality, which might vary gradually from year to year, can be obtained by comprising within the Fourier sum a set of components whose frequencies are adjacent to the seasonal frequency and its harmonics.

The combined effect of two components at adjacent frequencies depends upon whether their sinusoids are in phase, in which case they reinforce each other, or out of phase, in which case they tend to interfere with each other destructively. Two sinusoids whose frequencies are separated by θ radians will take a take a total of $\tau = 2\pi/\theta$ periods to move from constructive interference to destructive interference and back again.

It remains to describe how the seasonal components that have been extracted from the differenced data are to be cumulated to provide an estimate of the seasonal component. Where the seasonal component is concerned, it seems reasonable to chose the starting values so as to minimise the sum of squares of the seasonal fluctuations. Let $w = S_*u_* + Su$ be the cumulated seasonal component, where u_* is a vector of s starting values and u is the vector of the seasonal component that has been extracted from the differenced data. Then the criterion is

$$\text{Minimise} \quad (S_*u_* + Su)'(S_*u_* + Su) \quad \text{with respect to} \quad u_*. \tag{14}$$

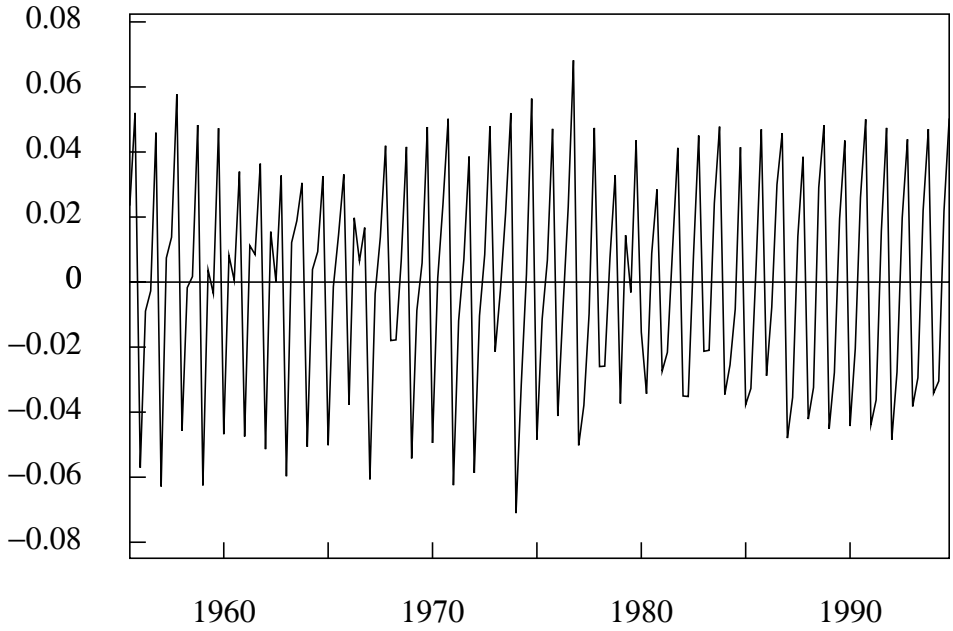

Figure 8. The detrended consumption series.

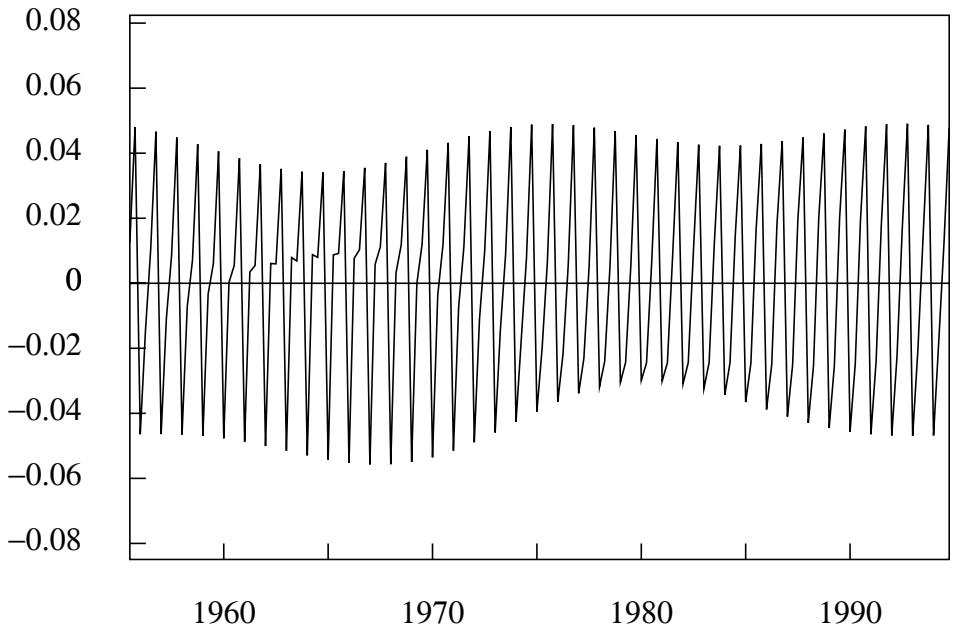

Figure 9. The estimated seasonal component of the consumption series.

The solution for the starting values is

$$u_* = -(S_*'S_*)^{-1}S_*'Su. \tag{15}$$

In Figure 9, we show the estimated seasonal component of the consumption series. The seasonal series is synthesised from the trigonometric functions at the seasonal frequency of $\pi/2$ and at its harmonic frequency of π, together with a handful of components at the adjacent non-seasonal frequencies. It comprises two non-seasonal components below $\pi/2$ and one above, and it also comprises one non-seasonal component below π. These choices have resulted from an analysis of the periodogram of Figure 7. Figure 3 indicates, via the dotted lines, the frequencies that are present in the detrended and deseasonalised data.

The seasonal component of consumption accounts for the 93 percent of the variation of the detrended consumption series. When the seasonal component is estimated for the income series using the same set of frequencies, it accounts for only 46 percent of the variance of the corresponding detrended series.

6. An Appraisal of the Income–Consumption Relationship

In the previous section, we have described some new techniques for detrending the data and for extracting the seasonal components. We have discovered that the seasonal fluctuations in consumption are of a greater amplitude than those of the income series. They also appear to be more regular. These findings persuade us to reject the notion that the fluctuations have been transferred from income to consumption. It seems more reasonable to treat the seasonal fluctuations in both series as if they derive from external influences. Therefore, in seeking to establish a relationship between the detrended series, it is best to work with the deseasonalised versions.

When we turn to the deseasonalised and detrended consumption series, we find that its variance amounts to only 7 percent of the variance of the detrended series. It is hardly worthwhile to attempt to model this series. Instead, we conduct a cursory examination of its relationship to the detrended and deseasonalised income series. In Figure 11, we plot the empirical cross-correlation function of the two series.

The plot indicates that the elements of the consumption series are more strongly correlated with the elements of the income series that follow them in time than with those that precede them. This finding also deters us from estimating a causal transfer-function relationship mapping from income to consumption. Indeed, according to the basic causality test of Granger (1969) and Sims (1972), we can accept the hypothesis that the income series does not cause the consumption series, whereas we must reject the hypothesis that the consumption series does not cause the income series.

The periodogram of Figure 7 also makes it clear that there is very little information in the data of the consumption sequence which is not attributable either to the trend or to the seasonal component. If it is accepted that the seasonal component needs no further explanation, then attention may be confined

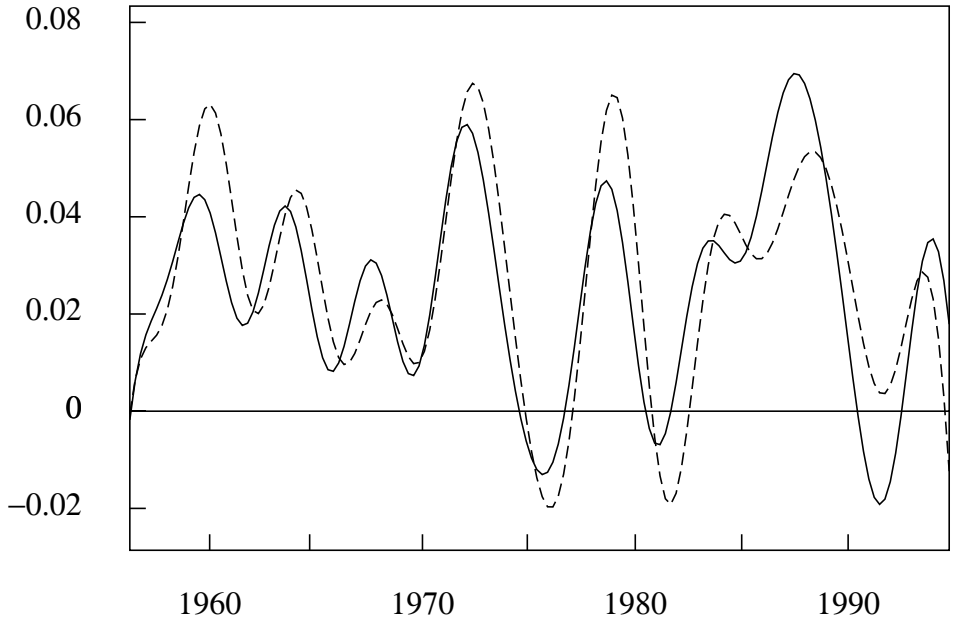

Figure 10. The annual differences of the trend of the logarithmic consumption series (solid line) and of the trend of the logarithmic income series (broken line).

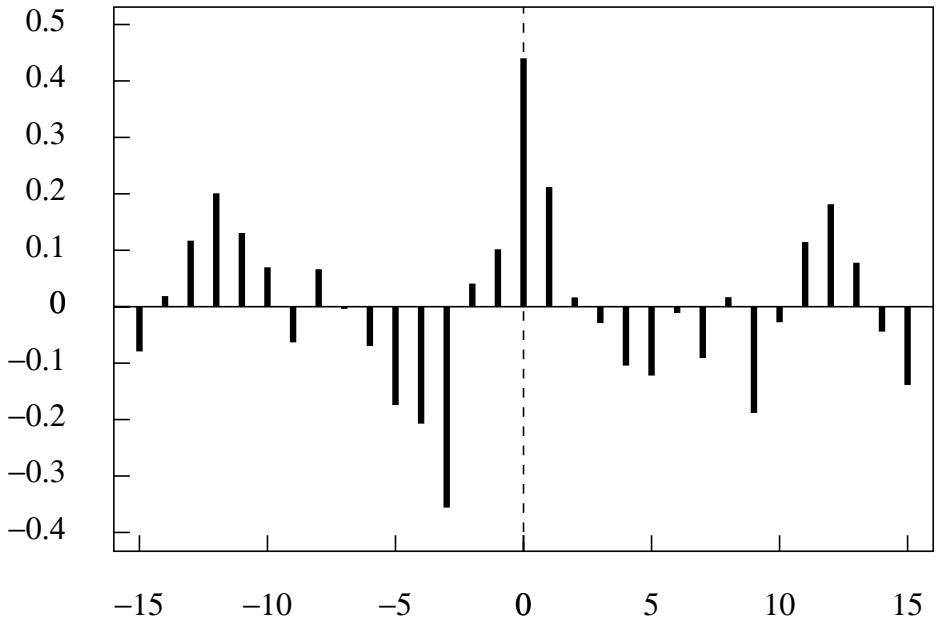

Figure 11. The cross-correlation function $\mathrm{Corr}\{y(t), x(t-\tau)\}; \tau = -15, \ldots, 15$, between the detrended and deseasonalised series of consumption and income.

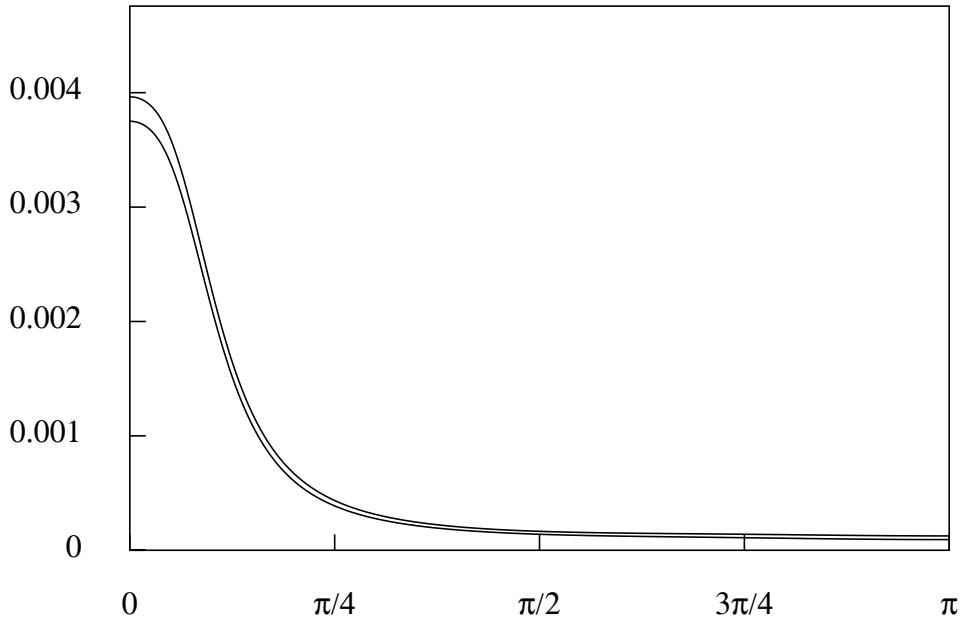

Figure 12. The spectrum of the consumption growth sequence $\nabla_4 y(t)$ (the outer envelope) and that of its innovation component $\{\alpha(L)/\pi(L)\}\varepsilon(t)$ (the inner envelope).

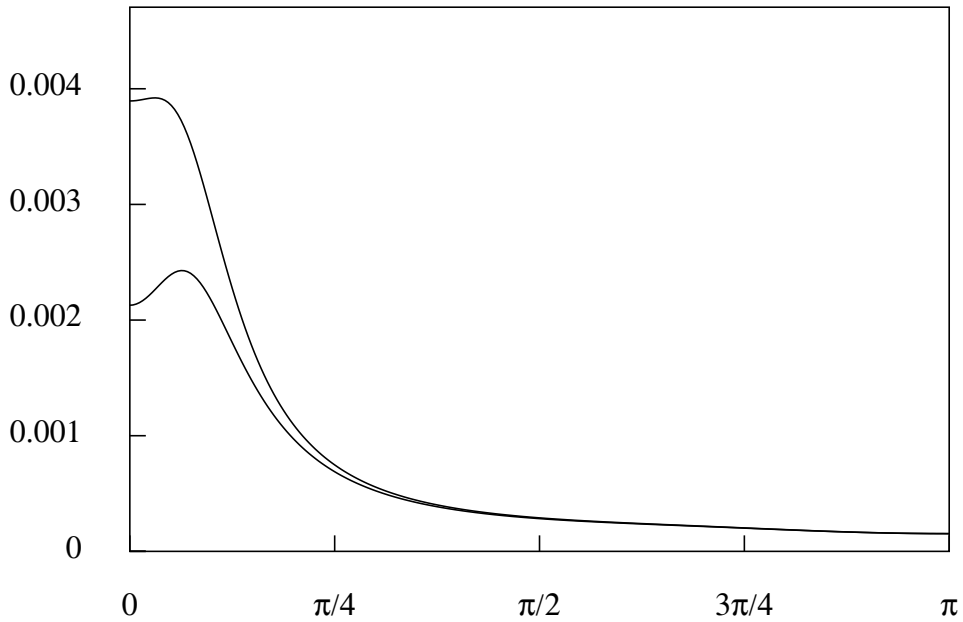

Figure 13. The spectrum of the income growth sequence $\nabla_4 x(t)$ (the outer envelope) and that of its innovation component $\{\gamma(L)/\pi(L)\}\eta(t)$ (the inner envelope).

to the trend. The use of ordinary linear statistical methods dictates that any explanation of the consumption trend is bound to be in terms of data components whose frequencies are bounded by zero and by the cut-off point of $\pi/8$ radians. That is to say, the trend in consumption can only be explained by similar trends in other variables.

We therefore turn to the essential parts of the income and the consumption series, which are their trends. We take the annual differences of the logarithmic trends by applying the operator $\nabla_4 = I - L^4$; and the results are a pair of smooth series which represent the annual growth rates of income and consumption. By combining the two series in one graph, which is Figure 10, we are able to see that, in the main, the fluctuations in the growth in consumption *precede* similar fluctuations in the growth of income.

It may be recalled the income-acceleration term $\nabla\nabla_4 x(t)$ enters the consumption functions of equation (1) and (2) with a negative coefficient. This is in spite of the clear indication of Figure 10 that the consumption-growth series leads the income-growth series. However, when the smoothed growth series $\nabla_4\hat{y}(t)$ and $\nabla_4\hat{x}(t)$ of Figure 10 are used in these equations in place of $\nabla_4 x(t)$ and $\nabla_4 y(t)$, the sign on the coefficient of the acceleration term is reversed:

$$\begin{aligned} \nabla_4\hat{y}(t) &= 0.006 + 0.689\nabla_4\hat{x}(t) + 1.055\nabla\nabla_4\hat{x}(t) + e(t) \\ &\qquad (0.001)\ \ (0.044) \qquad\qquad (0.170) \\ R^2 &= 0.87. \end{aligned} \tag{16}$$

The explanation of this anomaly must lie in the nature of the gain of the four-period difference filter $\nabla_4 = I - L_4$ which is represented in Figure 3. The effect of the filter is to amplify some of the minor components of the data which lie in the dead spaces of the periodogram of Figure 7 on either side of the frequencies $\pi/4$ and $3\pi/4$. Thus it can be concluded that, notwithstanding its specious justification, the negative acceleration term is an artifact of the differencing filter. This finding conflicts with the belief that consumption responds in a laggardly fashion to rapid changes in income.

The perception that the series of the annual growth rate in consumption is leading the corresponding series in income can be reaffirmed within the context of a bivariate vector autoregressive model. The model must be applied to the unsmoothed growth rates obtained by taking the four-period differences of the logarithms of the two series. It cannot be applied to the smoothed growth-rate series of Figure 10, which have have band-limited spectra. The reason is that an autoregressive model presupposes a spectral density function which is nonzero everywhere in the frequeuncy range except on a set of measure zero. (The consequence of fitting the model to the smoothed growth rates would be the virtual singularity of the empirical moment matrix and a likely violation, by the estimated parameters, of the conditions of stability.)

The basic model takes the form of

$$\nabla_4 y(t) = c_y + \sum_{i=1}^{p} \phi_i \nabla_4 y(t-i) + \sum_{i=1}^{p} \beta_i \nabla_4 x(t-i) + \varepsilon(t), \tag{17}$$

$$\nabla_4 x(t) = c_x + \sum_{i=1}^{p} \psi_i \nabla_4 x(t-i) + \sum_{i=1}^{p} \delta_i \nabla_4 y(t-i) + \eta(t). \tag{18}$$

The terms c_y and c_y stand for small constants which are eliminated from the model when the differenced series are replaced by deviations about their mean values. The deviations may be denoted by $\tilde{y}(t) = \nabla_4 y(t) - E\{\nabla_4 y(t)\}$ and $\tilde{x}(t) = \nabla_4 x(t) - E\{\nabla_4 x(t)\}$. The expected values can be represented by the corresponding sample means.

In the case of $p = 2$, the estimated equations are

$$\begin{aligned} \tilde{y}(t) &= 0.51\tilde{y}(t-1) + 0.34\tilde{y}(t-2) + 0.27\tilde{x}(t-1) - 0.38\tilde{x}(t-2) + e(t), \\ &\quad (0.086) \qquad (0.087) \qquad (0.073) \qquad (0.072) \end{aligned} \tag{19}$$

$$\begin{aligned} \tilde{x}(t) &= 0.52\tilde{x}(t-1) - 0.10\tilde{x}(t-2) + 0.16\tilde{y}(t-1) + 0.25\tilde{y}(t-2) + h(t). \\ &\quad (0.093) \qquad (0.092) \qquad (0.11) \qquad (0.11) \end{aligned} \tag{20}$$

To facilitate the analysis of the model, it is helpful to write the equations (17) and (18) in a more summary notation which uses polynomials in the lag operator to represent the various sums. Thus

$$\phi(L)\tilde{y}(t) - \beta(L)\tilde{x}(t) = \varepsilon(t), \tag{21}$$

$$-\delta(L)\tilde{y}(t) + \psi(L)\tilde{x}(t) = \eta(t), \tag{22}$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\beta(L) = \beta_1 L + \cdots + \beta_p L^p$, $\psi(L) = 1 - \psi_1 L - \cdots - \psi_p L^p$ and $\delta(L) = \delta_1 L + \cdots + \delta_p L^p$.

The notion that the sequence $\tilde{y}(t)$ is driving the sequence $\tilde{x}(t)$ would be substantiated if the influence of the innovations sequence $\varepsilon(t)$ upon $\tilde{y}(t)$ were found to be stronger that the influence of $\eta(t)$ upon the corresponding sequence $\tilde{x}(t)$. The matter can be investigated via the moving-average forms of the equations which express $\tilde{x}(t)$ and $\tilde{y}(t)$ as functions only of the innovations sequences $\varepsilon(t)$ and $\eta(t)$. The moving-average equations, which are obtained by inverting equations (21) and (22) jointly, are

$$\tilde{y}(t) = \frac{\psi(L)}{\pi(L)}\varepsilon(t) + \frac{\beta(L)}{\pi(L)}\eta(t), \tag{23}$$

$$\tilde{x}(t) = \frac{\delta(L)}{\pi(L)}\varepsilon(t) + \frac{\phi(L)}{\pi(L)}\eta(t), \tag{24}$$

where $\pi(L) = \phi(L)\psi(L) - \beta(L)\delta(L)$.

Since there is liable to be a degree of contemporaneous correlation between innovations sequences, the variance of the observable sequences $\tilde{y}(t)$ and $\tilde{x}(t)$ will not equal the sum of the variances of the components in $\varepsilon(t)$ and $\eta(t)$ on the RHS. The problem can be overcome by reparametrising the two equations so that each is expressed in terms of a pair of uncorrelated innovations. Such a procedure has been adopted by Geweke (1982), for example.

Consider the innovation sequence $\eta(t)$ within the context of equation (23) which is for $\tilde{y}(t)$. We may decompose $\eta(t)$ into a component which lies in the space spanned by $\varepsilon(t)$ and a component $\zeta(t)$ which is in the orthogonal complement of the space. Thus

$$\begin{aligned}\eta(t) &= \frac{\sigma_{\eta\varepsilon}}{\sigma_\varepsilon^2}\varepsilon(t) + \left\{\eta(t) - \frac{\sigma_{\eta\varepsilon}}{\sigma_\varepsilon^2}\varepsilon(t)\right\} \\ &= \frac{\sigma_{\eta\varepsilon}}{\sigma_\varepsilon^2}\varepsilon(t) + \zeta(t),\end{aligned} \tag{25}$$

where $\sigma_\varepsilon^2 = V\{\varepsilon(t)\}$ is the variance of the consumption innovations and $\sigma_{\varepsilon\eta}^2 = C\{\varepsilon(t), \eta(t)\}$ is the covariance of the consumption and income innovations. Substituting (25) in equation (23) and combining the terms in $\varepsilon(t)$ gives

$$\tilde{y}(t) = \frac{\alpha(L)}{\pi(L)}\varepsilon(t) + \frac{\beta(L)}{\pi(L)}\zeta(t), \tag{26}$$

where

$$\alpha(L) = \psi(L) + \frac{\sigma_{\eta\varepsilon}}{\sigma_\varepsilon^2}\beta(L). \tag{27}$$

By a similar reparametrisation, the equation (24) in $\tilde{x}(t)$ becomes

$$\tilde{x}(t) = \frac{\gamma(L)}{\pi(L)}\eta(t) + \frac{\delta(L)}{\pi(L)}\xi(t), \tag{28}$$

where

$$\begin{aligned}\gamma(L) &= \phi(L) + \frac{\sigma_{\eta\varepsilon}}{\sigma_\eta^2}\delta(L), \\ \xi(t) &= \varepsilon(t) - \frac{\sigma_{\eta\varepsilon}}{\sigma_\eta^2}\eta(t),\end{aligned} \tag{29}$$

and where $\eta(t)$ and $\xi(t)$ are mutually uncorrelated.

The relative influences of $\varepsilon(t)$ on $\tilde{y}(t)$ and of $\eta(t)$ on $\tilde{x}(t)$ can now be assessed by an analysis of the corresponding spectral density functions. Figure 12 shows the spectrum of $\tilde{y}(t)$ together with that of its own innovation component $\{\alpha(L)/\pi(L)\}\varepsilon(t)$ which is the lower envelope. Figure 13 shows the spectrum of $\tilde{x}(t)$ together with that of $\{\gamma(L)/\pi(L)\}\eta(t)$.

From a comparison of the figures, it is clear that the innovation sequence $\varepsilon(t)$ accounts for a much larger proportion of $\tilde{y}(t)$ than $\eta(t)$ does for $\tilde{x}(t)$. Thus the consumption growth series appears to be driven largely by its own innovations. These innovations also enter the income growth series to the extent that the latter is not accounted for by its own innovations. Figure 13 shows that the extent is considerable.

The fact the consumption innovations play a large part in driving the bivariate system implies that the consumption function of Davidson *et al*, which

is equation (2), cannot be properly construed as a structural econometric relationship. For it implies that the estimates are bound to suffer from a considerable simultaneous-equations bias. Nevertheless, in so far as the mechanisms generating the data remain unchanged, the equation will retain its status as an excellent predictor of the growth rate of consumption that is based on a parsimonious information set.

7. Conclusions

The traditional macroeconomic consumption function depicts a delayed response of consumption spending to changes in income; and many analysts would expect this relationship to be readily discernible in the macroeconomic data. Instead, the data seem to reflect a delayed response of aggregate income to changes in consumption. Although the two responses can easily coexist, it is the dominant response which is liable to discerned in the data at first sight.

A crucial question is whether both responses can be successfully disentangled from the macroeconomics data. The construction of a bivariate autoregressive model is the first step in the process of their disentanglement. However, given the paucity of the information contained in the data, one is inclined to doubt whether the process can be carried much further. Indeed, the efforts which have been devoted the microeconomic analysis of consumer behaviour in the last twenty years can be construed as a reaction to limited prospects facing macroeconomic investigations.

Much has already been accomplished in the microeconomic analysis of consumer behaviour; and an excellent account of some of the numerous influences which affect consumer behaviour directly has been provided recently by Muellbauer and Latimore (1995). However, what is lacking is a methodology which would enable the consumption behaviour of identifiable social and economic groups to be aggregated into a macroeconomic consumption function.

We have found that, within a bivariate autoregressive system designed to explain the growth rates on income and consumption, the innovations sequence of the consumption equation dominates the corresponding innovations sequence of the income equation. Thus the fluctuations in the growth rate of consumption have been depicted mainly as the result of autonomous influences.

Although the innovations sequences are an artifact of the statistical analysis, they are not entirely devoid of worldly connotations. By a detailed study of the historical circumstances, we should be able to relate the consumption innovations to the fiscal policies of the central governments, the state of the financial markets, the rate of inflation, the political and social climate, and to much else besides. Although some of these influences have been included in macroeconomic consumption functions, it seems that, in the main, there has been a remarkable oversight of the circumstantial details in most attempts at explaining the aggregate level of consumption. The present analysis is, regrettably, no exception.

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