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ABSTRACT. It is shown that altruism does not affect the equilibrium provision of public goods although altruism takes the form of unconditional commitment to contribute. The reason is that altruistic contributions completely crowd out selfish voluntary contributions. That is, egoists free ride on altruism. It is also shown that public goods are less likely to be provided in larger groups.

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1. Introduction

A recurrent issue in the private provision of public good is how group size affects the chance of voluntary provision of public goods. A large number of experiments have tried to test the Olson (1968) conjecture that public goods are less likely to be provided in larger groups as everyone has greater incentive to free ride on others' contributions. This prediction has found game-theoretic ground in Palfrey and Rosenthal (1986) for the provision of discrete public goods where it is shown that individual contribution rates fall to zero as group size increases to infinity (see Proposition 7, p. 182).[1] However it is possible that due to a sheer size effect, aggregate contribution rate may increase in spite of the decrease in individual contribution rates.[2] Taking the special case of Palfrey and Rosenthal model where only one contributor is necessary to secure provision, Dixit and Skeath (1999) show that individual contribution rates will decrease sufficiently to offset the increase in the size of the group, so that (discrete) public goods are less likely to be provided in a larger group (see chapter 11, pp. 388-392.). This result is obtained by assuming that everybody is self-interested. Given the apparent importance of altruism, the question is then how the presence of altruism may influence the impact of group size on the provision of public goods.[3] If we identify altruism with the unconditional commitment of contributing,[4] the natural expectation would be that altruism could overturn the result because a larger group has greater chance of comprising altruistic individuals thereby making the provision of public good more likely. However we shall see that the chance of any self-interested individual contributing will decrease sufficiently to offset exactly the increase in the chance of having some altruistic individuals among a larger group. The bottom line is that allowing for altruism does not increase the voluntary provision of public good. Changes in the probability of altruism are neutralized in equilibrium by offsetting

[1]For continuous public goods, a similar limit result has been obtained by Chamberlin (1974) when individuals are identical and then extended by Andreoni (1988) to the case where individuals have different endowments.

[2]Indeed, as shown by Andreoni (1988) for continuous public goods and group size increasing to infinity, individual contribution rate decreases to zero but total contribution rate increases to a finite value.

[3]A number of authors have obtained strong evidence of altruism in public good provision experiments: for recent evidence seeAndreoni and Miller (2001) and for a survey, see Ledyard (1993).

[4]This is consistent with the "warm glow" approach to altruism in which some agents may derive utility from the sheer fact of giving (see Andreoni, 1988). This form of altruism may be motivated either by a Kantian principle of unconditional commitment or by the fact that something is expected in return (see Sugden, 1986).

changes in selfish voluntary contributions. In contrast to this view, with continuous public good, altruism increases the level of provision (see Palfrey and Rosenthal, 1988). Intuitively, when agents care about the utilities of others, they may contribute beyond the point where their own marginal benefit equals marginal cost.

The model we use is an extension of Palfrey and Rosenthal (1984) to accomodate for the possibility of altruism. The model is tractable enough to allow us to derive sharp results on the effects of altruism and group size on the equilibrium contribution rate.[5]

The rest of the paper is organized as follows. Section 2 analyses the special case where it needs only one to contribute to secure provision. Section 3 extends the analysis for the case where several contributors are needed. Section 4 investigates the case of large groups using Poisson approximation. Section 5 concludes.

2. The model

We use a binary model similar to Palfrey and Rosenthal (1984) and allow for altruism. There is a finite set of individuals $N = \{1, ..., i, ..., n\}$ each with binary pure strategies s_i of either making a fixed contribution ($s_i = 1$) or not contributing at all ($s_i = 0$) to the provision of public good. This group is a random draw from a large population that comprises egoistic agents (in proportion $e \in (0, 1)$) and altruistic agents in proportion $(1 - e)$. The probability that there is no altruistic agents in a group of size n is $e^n > 0$ and the probability that there is at least one altruistic agent among this group is $1 - e^n < 1$.[6] We assume that for every altruistic player contributing ($s_i = 1$) is a dominant strategy. The public good is discrete: it is either provided or not. To take the case were altruism is more likely to facilitate the provision of public good we assume that it needs only one to contribute to secure the provision of the public good. (Note that Palfrey and Rosenthal consider the more general case in which the public good is provided if w out of n individuals in the group contribute (with $1 \leq w < n$). We study this case in the next section.)

[5]In Palfrey and Rosenthal (1984) the focus is rather on comparing how different institutional structures (like the refund and no-refund rules) affect the incentive to contribute.

[6]The usual trick for finding the probability that something happens is to calculate the chance it does not happen. Thus find the chance that everybody in the group is an egoistic person, and then substract this from 1 to obtain the probability to have at least one altruistic person in the group.

Every egoistic player i derives a benefit B from the provision of the public good and pays the cost C if contributing ($s_i = 1$). We assume that the public good is strongly desirable in the sense that $C < B$, so that any egoistic player would contribute if he was sure that no one else is going to contribute. However egoistic agents have an incentive to free ride getting the payoff B if someone else contributes instead of the lower payoff $B-C$ in contributing themselves. Given the size of the group $n \geq 2$, what is the equilibrium strategy for the egoistic players? Every single egoistic player considers that given the group size $n \geq 2$ there is at least one altruistic players among the $n-1$ other players with probability $1 - e^{n-1} < 1$. Hence given $n \geq 2$ (and ignoring risk aversion), it is a dominant strategy of not contributing ($s_i = 0$) for any egoistic player if

$$B - C \leq (1 - e^{n-1})B$$

where the left hand side is the payoff when contributing and the right hand side is the expected payoff when not contributing expecting that there is at least one altruistic agent who will contribute. This condition depends on size of the group. We must distinguish two cases.

Case 1. *high altruism $e \leq C/B$*

Thus $B - C \leq (1 - e)B$ and since $e < 1$ it follows that $B - C \leq (1 - e^{n-1})B$ for all $n \geq 2$ and then it is a dominant strategy for any egoistic agent of not contributing. In other words a sufficiently high probability that any single agent be altruistic induces all egoistic agents to free ride as a dominant strategy. Hence the probability that the public good be provided in a group of size $n \geq 2$ is

$$Q(n, e) = 1 - e^n$$

The chance of public good provision increases with altruism and group size (different from Palfrey and Rosenthal) Summarizing, we have the following proposition.

Proposition 1. *If $e \leq C/B$, there is a unique equilibrium (in a dominant strategy) in which all the egoistic players free ride. The resulting probability of providing the public good is increasing with the group size and with the probability of altruism.*

This finding is consistent with our intuition that altruism should facilitate the provision of public goods. However we now show that it needs not be the case if altruism is low.

Case 2. *Low altruism: $e > C/B$*

Then $B - C > (1 - e)B$ and since $e < 1$ and $C > 0$ there exists n^* (with $2 < n^* < \infty$) such that

$$\begin{array}{rcll} B - C & > & (1 - e^{n-1})B & \text{for all } n < n^* \\ B - C & \leq & (1 - e^{n-1})B & \text{for all } n \geq n^* \end{array}$$

where $n^* = 1 + \frac{\ln(C/B)}{\ln(e)} > 2$. (We ignore integer problems)[7]

- If $n \geq n^*$then clearly we have the same result as in proposition 1. The size of the group is large enough to make it sufficiently likely that there will be at least one altruistic player in the group to contribute so that it is a dominant strategy for the egoistic players to free ride.
- If $n < n^*$ we have pure strategy equilibria, each with one egoistic player contributing and other egoistic players not contributing. Indeed there cannot be an equilibrium in which two egoistic players contribute because one contributor is enough and more is redundant. Also there cannot be an equilibrium in which none of the egoistic players contributes, because it would be profitable for one of them to contribute since $B - C > (1 - e^{n-1})B$ for all $n < n^*$. In addition to these pure strategy equilibria in which essentially identical egoistic players follow different strategies, there is also a mixed strategy equilibrium where all these players follow the same strategy. From now on we set aside the (asymmetric) pure strategy equilibria to focus on the (symmetric) mixed strategy equilibrium. To play a mixed strategy each egoistic player must be indifferent between the two pure strategies (contributing and not contributing). Let p be the probability that any one will not contribute. Contributing gets him $B - C$ for sure. Not contributing gets him B if someone else contributes and 0 otherwise. The probability that someone else will contribute among the $n - 1$ other players is the probability that there is at least one altruist among the other players, $1 - e^{n-1}$, plus the probability that there is no altruist but that at least one egoist will contribute,

[7]If we are to give the integer equilibrium number then it is $1 < n^* < \infty$ such that $(1 - e^{(n^*-1)-1})B < B - C \leq (1 - e^{n^*-1})B$.

$e^{n-1}(1-p^{n-1})$. Hence any egoistic player is indifferent between contributing or not when

$$\begin{aligned} B-C &= [1-e^{n-1}+e^{n-1}(1-p^{n-1})]B \\ &= (1-(ep)^{n-1})B \end{aligned}$$

This gives the mixed strategy equilibrium

$$ep = \left(\frac{C}{B}\right)^{\frac{1}{n-1}} \quad \text{for all } n < n^*$$

It follows that lower e is exactly offset by higher p to maintain the product ep constant; that is altruism completely crowds out egoistic contributions. It is also easily seen that p is increasing with the size of the group n. We can summarize this finding in the following proposition.

Proposition 2. *For all $n < n^*$, the larger the group the less likely is anyone to contribute. Furthermore, the larger the probability of altruism, the less likely anyone of the egoists will contribute.*

Using the equilibrium mixed strategy we can easily calculate the chance of providing the public good as a function of altruism and the size of the group. Since it needs only one to contribute to secure provision of the public good, the chance of providing the public good is obtained by finding the chance that the public good is not provided (which implies that everyone is egoistic and none of them contribute) and then substracting this from 1. So the probability of providing the public good in a group of size $n < n^*$ with probability $0 < 1-e < 1$ of altruism is,

$$\begin{aligned} Q(e,n) &= 1-(ep)^n \\ &= 1-\left(\frac{C}{B}\right)^{\frac{n}{n-1}} \quad \text{for all } n < n^* \end{aligned}$$

Surprisingly, altruism does not affect the chance of providing the public good provided that the size of the group is not too high; for otherwise the probability to have at least one altruistic person in the group is so high that it is impossible to sustain the mixed strategy equilibrium and it is a dominant strategy for every egoist to free ride. The corresponding chance of providing the public good is then

$$Q(e,n) = 1 - e^n \text{ for all } \ n \geq n^*$$

To summarize our finding.

Proposition 3. *If $e > C/B$ there exists a critical group size $n^* = 1 + \frac{\ln(C/B)}{\ln(e)} > 2$ such that*

(i) for any $n < n^*$ the chance of providing the public good is independent of altruism and decreasing with the size of the group

(ii) for any $n \geq n^*$ the chance of providing the public good is increasing both with altruism and with the group size.

So on the *effect of altruism*, the conjecture that altruism facilitates the provision of public goods is not correct when the group is small. The reason is that for a small group, the chance of having some altruistic players is sufficiently small to keep egoistic players contributing (at least with some probability). However, the chance of anyone egoistic player contributing decreases sufficiently to offset the increase in the chance of having at least one altruist. Similarly on the *effect of group size*, although it is more likely to have at least one altruist among a larger group, the chance of anyone egoist contributing decreases sufficiently to overcome this effect. As the group size increases the free rider problem is more important. Only if the size of the group is sufficiently high to eliminate any incentive for the egoists to contribute, then the intuition is restored; that is altruism and larger group facilitate the provision of public goods.

3. Several contributors needed

We extend the analysis by allowing for the possibility that it needs more than one person to contribute to secure public good provision. When the provision of the public good requires several volunteers (instead of one) then the decision of contributing no longer guarantees the provision of the public good. Contributing is powerless if there are too few contributors and redundant when there are too many contributors. What matters is the probability of making a pivotal contribution (when there is just one contributor missing). Let $w \geq 1$ be the number of contributors required to provide the public good and let m_n denote the actual number of contributors in a group of size n, then for any $1 \leq w \leq n$ the probability that there are exactly $w - 1$ contributors in a group of size $n - 1$ is

$$prob\{m_{n-1} = w - 1\} = \binom{n-1}{w-1} (ep)^{n-w} (1 - ep)^{w-1}$$

where $ep \in (0, 1)$ is the probability that anyone drawn randomly from the population will not contribute given the frequency e of egoists and their equilibrium probability p of not contributing.

Assuming no-refund if public good is not provided, the indifference condition between contributing and not contributing is

$$prob\{m_{n-1} \geq w - 1\}B - C = prob\{m_{n-1} \geq w\}B$$

or

$$prob\{m_{n-1} = w - 1\}B = C$$

where the probability of making a pivotal contribution is the probability of obtaining exactly $w - 1$ contributors among the $n - 1$ other players. Using the expression of the probability of being pivotal, the mixed strategy equilibrium is given by

$$\binom{n-1}{w-1} (ep)^{n-w} (1 - ep)^{w-1} = \frac{C}{B}$$

Therefore a higher frequency of altruism (lower e) will increase the equilibrium probability p of not contributing of any egoist so as to maintain ep fixed. It follows that the probability of any randomly chosen individual not contributing is independent of the frequency of altruism.

Using this mixed strategy equilibrum we can calculate the probability that the public good is not provided in a group of size n when w,$(1 \leq w \leq n)$ contributors is necessary. This is given by

$$\begin{aligned} P_w^{not}(e, n) &= \sum_{s=0}^{w-1} prob\{m_n = s\} \\ &= \sum_{s=0}^{w-1} \binom{n}{s} (ep)^{n-s} (1 - ep)^s \end{aligned}$$

Thus under the mixed strategy equilibrium, altruism does not affect the chance of providing the public good. This is because actual public good provision depends only on the probability that any randomly drawn individual will contribute, $1 - ep$, which as previously shown is independent of the frequency of altruism.

Proposition 4. *If $w \geq 1$ contributors are necessary to provide the public good in a group of size n (with $w \leq n$), under a mixed strategy equilibrium the chance of providing the public good is independent of altruism.*

We now turn to the effect of group size when several contributors are needed. We begin with the extreme case of unanimous contribution to illustrate the difference in the sharpest way and we maintain our assumption of no-refund.[8] Suppose that participation of all is required, then setting $w = n$ the mixed strategy equilibrium is

$$\frac{C}{B} = (1 - ep)^{n-1}$$

and the probability of no provision is

$$\begin{aligned} P^{not}_{w=n} &= 1 - (1 - ep)^{n} \\ &= 1 - \left(\frac{C}{B}\right)^{\frac{n}{n-1}} \end{aligned}$$

Comparing this unanimous contribution problem to the single contribution problem in section 2 we have the following proposition.

Proposition 5. *In a symmetric mixed strategy equilibrium and no refund rule, the probability of no provision when unanimous participation is required is exactly equal to the probability of provision when the participation of only one agent is required. Furthermore, with unanimous participation, the probability of provision increases as the size of the group increases.*

[8]With no refund and unanimous contribution, the incentive to free ride on others' contributions is replaced by the fear of wasting contribution if the public good is not provided. This fear may refrain agents from contributing (see Palfrey and Rosenthal, 1984, p.185).

This proposition sharply illustrates that participation of all is more likely in a larger group while from proposition 3 participation of only one is less likely. Thus the Olson conjecture (that the free rider problem is more important in larger group) is correct when the initiative of just one person is needed to secure provision but not when unanimous participation is necessary.

The next proposition is about the effect of group size for the intermediate case where the participation of several but not everyone is required ($1 < w < n$).

Proposition 6. *In a symmetric mixed strategy equilibrium with no refund rule, for any $1 < w < n$, (i) the probability of anyone contributing is decreasing with the size of the group, but (ii) the effect of group size on the probability of provision is indeterminate.*

Proof. See Appendix ∎

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4. Large Groups

In this section we assume that the group is large and we derive an approximation result on the effect of the group size upon the individual probability of contributing and the chance of providing the public good.

Let $r = 1 - ep$ be the mixed strategy equilibrium probability of contributing, From Proposition 6 we know that this probability is decreasing in n. So when the value of n is large the value of r is small and the binomial distribution with parameters n and r can be approximated by a Poisson distribution with mean $\lambda = nr$ (representing the expected number of contributors in a group of size n).[9] Now consider a larger group $n_1 > n$, and let r_1 be the corresponding equilibrium probability of contributing. Then by the indifference condition the mixed strategy equilibrium solves[10]

[9] How good is this approximation? Following Yamane (1969), the rule of thumb is that for most cases, when $nr \leq 5$, we can use the Poisson distribution as an approximation of the binomial distribution. As an example, $r \leq 0.20$ requires $n > 10$.

[10] Note the binomial property that the probability of having any exact number of contributors tends to zero as the size of the group increases to infinity. So the cost to benefit ratio (C/B) must be sufficiently small to sustain mixed strategy equilibrium in large groups.

$$prob\{m_{n-1} = w-1\} = prob\{m_{n_1-1} = w-1\} = \frac{C}{B}$$

Approximation by the Poisson distribution gives for any integer $1 < w < n$ that

$$\frac{e^{-(n-1)r}((n-1)r)^{w-1}}{(w-1)!} = \frac{e^{-(n_1-1)r_1}((n_1-1)r_1)^{w-1}}{(w-1)!}$$

Letting $\lambda_{n-1} = (n-1)r$ and $\lambda_{n_1-1} = (n_1-1)r_1$ it follows from this equation that $\lambda_{n-1} = \lambda_{n_1-1}$ and thus $r_1 = \frac{n-1}{n_1-1}r < r$ for $n < n_1$. That is for large n i ncreasing the group size decreases the equilibrium probability of contributing. Furthermore, $\lim_{n\to\infty} r = \lim_{n\to\infty} \frac{\lambda_{n-1}}{n-1} = 0$ since $\lambda_{n-1} = \lambda_{n_1-1}$for any $n \neq n_1$. If we let $\lambda = nr$ and $\lambda_1 = n_1 r_1$, then it follows from $(n-1)r = (n_1-1)r_1$ that $\lambda_1 = \lambda - (r - r_1) < \lambda$. That is the expected number of contributors decreases with the group size.[11]Therefore, the number of contributors can be approximated by a Poisson distribution for which the mean is lower $\lambda_1 < \lambda$ in the larger group $n_1 > n$. Let $f(x \mid \lambda)$ and $f(x \mid \lambda_1)$ denote the Poisson distribution (of the number of contributors x) for which the mean is respectively λ and λ_1. Then applying a theorem on Poisson distributions[12] (see Schmetterer, 1974, Theorem 33.2, p.92) we get that for any integer w and for any $\lambda_1 < \lambda$,

$$P_w^{not}(\lambda) = \sum_{x=0}^{w-1} f(x \mid \lambda) < \sum_{x=0}^{w-1} f(x \mid \lambda_1) = P_w^{not}(\lambda_1)$$

Therefore the probability of no provision is increasing with group size.

The next proposition summarizes our finding about large groups.

[11]Furthermore, $\lim_{n\to\infty} \lambda = \lim_{n\to\infty}(n-1)r + r = \lambda_{n-1}$. That is the expected number of contributors converges to a finite value.

[12]The Theorem states that given the Poisson distribution $f(x \mid \lambda)$ with mean $0 \leq \lambda < \infty$, for each $0 \leq k < \infty$ the function $S_k(\lambda) = \sum_{x=0}^{k} f(x \mid \lambda)$ is strictly monotone decreasing in λ.

Proposition 7. *For large n, in a symmetric mixed strategy equilibrium with no refund rule we have that for each integer $1 < w < n$:*

(i) the probability of anyone contributing decreases with n;

(ii) the probability of anyone contributing tends to zero as n increases to infinity;

(iii) the expected number of contributors decreases with n;

(iv) the probability of provision is decreasing with n.

5. Conclusion

We have shown that aggregate contribution to public goods is independent of altruism. The reason is that altruistic contributions crowd out selfish contributions dollar-for-dollar. This neutrality result has been obtained for the voluntary provision of discrete public goods identifying altruism as a unconditional commitment of contributing. The invariance to altruism stands in sharp contrast with continuous public goods where altruism tends to increase provision. We have also examined the effect of group size on the equilibrium contribution rates and provision level. The Olson conjecture that larger groups encourage free riding and lead to lower supply has been confirmed. The interpretation is that individual contributions will fall more rapidly than the increase in the number of potential contributors. Lastly, we have shown that the average number of contributors is decreasing with the size of the group.

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6. Appendix: Proof of Proposition 6

6.1. **impact of size on mixed strategy equilibrium.** Let $q = ep$ denote the equilibrium probability that any randomly drawn agent will not contribute in a group of size n and let $q_1 = ep_1$ be the corresponding equilibrium probability in a group of size $n+1$. Thus, q and q_1 solve

$$\binom{n-1}{w-1} q^{n-w}(1-q)^{w-1} = \binom{n}{w-1} q_1^{n-w+1}(1-q_1)^{w-1} = \frac{C}{B}$$

This implies

$$q^{n-w}(1-q)^{w-1} = \left[\frac{nq_1}{n-(w-1)}\right] q_1^{n-w}(1-q_1)^{w-1}$$

Define the function $f : (0,1) \to (0,1)$ where $f(x) = x^a(1-x)^b$, with $a, b > 0$ and $f'(x) \geq (<) 0$ for $a(1-x) \geq (<) bx$. Define also $\phi = \frac{nq_1}{n-(w-1)}$ where $\phi \geq 1$ for $(1-q_1)n \leq w-1$. Using this notation we can rewrite the above equation as follows

$$f(q) = \phi f(q_1)$$

where $f(q) = f(q_1)$ for $q = q_1$, and $f(q_1)$ is increasing for $w - 1 < (1-q_1)(n-1)$, and decreasing otherwise. Thus,

- either $\phi \geq 1$ (that is, $(1-q_1)n \leq w-1$) which implies

$$f(q) \geq f(q_1) \Longrightarrow q \leq q_1 \text{ since } (1-q_1)(n-1) < (1-q_1)n \leq w-1$$

- or $\phi < 1$ (that is, $(1-q_1)n > w-1$) which implies
$$f(q) < f(q_1) \Rightarrow \begin{array}{ll} q > q_1 \text{ for } & (1-q_1)(n-1) < w-1 < (1-q_1)n \\ q < q_1 \text{ for } & w-1 < (1-q_1)(n-1) \end{array}$$

So we have the result that increasing the group size from n to $n+1$ increases the probability of anyone not contributing ($q < q_1 \Leftrightarrow p(n) < p(n+1)$) except for the very special case where this size increase makes the expected number of contributors switch from less to more than $w-1$. This proves part (i) of proposition 6.

6.2. **Impact of size on provision.** We need to show that for any integer $1 < w < n$, increasing group size has an ambiguous effect on the probability of provision. That is,

$$P_w^{not}(e,n) \gtrless P_w^{not}(e,n+1)$$

Using our notation this requires

$$\sum_{s=0}^{w-1} \binom{n}{s} (q)^{n-s} (1-q)^s \gtrless \sum_{s=0}^{w-1} \binom{n+1}{s} (q_1)^{n-s+1} (1-q_1)^s$$

From the first part of the Appendix $q < q_1$, then applying an inequality theorem on binomial distributions (see Schmetterer, 1974, Theorem 32.3, p.90).we get

$$\sum_{s=0}^{w-1} \binom{n}{s} (q)^{n-s} (1-q)^s < \sum_{s=0}^{w-1} \binom{n}{s} (q_1)^{n-s} (1-q_1)^s$$

But for the same (positive) probability of anyone contributing $1-q_1 \in (0,1)$, it is also less likely to have less than w contributors in a larger group and thus,

$$\sum_{s=0}^{w-1} \binom{n+1}{s} (q_1)^{n-s+1} (1-q_1)^s < \sum_{s=0}^{w-1} \binom{n}{s} (q_1)^{n-s} (1-q_1)^s$$

So we have two opposite effects of group size on the probability of provision and (due to the non-differentiability of the binomial coefficient) it is not possible to say which effect dominates in general. This completes the proof.

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