

Department of Economics

A Benchmark Estimate for the Capital Stock.
An Optimal Consistency Method

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Abstract. There are alternative methods to estimate a capital stock for a benchmark year. These methods, however, do not allow for an independent check, which could establish whether the estimated benchmark level is too high or too low. I propose here an optimal consistency method (OCM), which may allow estimating a capital stock level for a benchmark year and/or checking the consistency of alternative estimates of a benchmark capital stock.

Key words: Benchmark Capital, Perpetual Inventory Method (PIM), Optimal Consistency Method (OCM).

JEL Classification: O4, B4

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1. Introduction

The estimation of fixed capital stock series is useful for work that uses production functions, such as applications of growth models or growth accounting studies, including technological change and international comparisons (Hofman, 2000a; Maddison, 1993; Denison, 1993). There are two main alternative methods to establish the level of the capital stock for a benchmark year. These are either based on direct observation, via surveys, balance sheets, insurance and the like, for a benchmark year or on a depreciation-discounted accumulation of historical investments up to a benchmark year. The former method can be highly costly, demanding a major use of resources, but it may also be unreliable for chronological or international comparisons, as both the quality of the sources across countries and the compiling instruments are likely to be inconsistent or at least uneven.

In turn, the latter method demands less effort and is currently the most used. Apart from others, most OECD countries actually use it, which allows for international comparisons, as the methodological procedure is standard and therefore transparent. This is normally known as the “perpetual inventory method” (PIM) (Hofman, 2000a; 2000b; Blades, 1993; Goldsmith, 1951). But also here, especially when historical investments are not fully available and/or their sources and definition are not fully consistent over time, a good chunk of past investment may come from rough estimates, rule of thumb, other countries experience and the like. This would make the resulting benchmark-capital stock only accurate within a wide range.

Despite these unavoidable errors, neither method allows for an independent check, which could establish whether the estimated benchmark capital level is too high or too low. I propose here an optimal consistency method (OCM), based on a PIM-derived equation, optimised via linear programming, which may allow estimating a capital stock level for a benchmark year and/or checking the consistency of alternative estimates of a benchmark capital stock. Notice that if the former aim were appropriately satisfied, it would then constitute also a third method for estimating a benchmark capital stock.

2. Reference Cases and PIM

I use the work of Hofman (2000a) as a reference to check against my proposed method. I hasten to add that Hofman is fully aware of the range of inaccuracy of his results, as for certain periods, especially before 1925, the data “may well be substantially revised when further research is done” (p.183). Hofman applies a PIM to six Latin American countries so as to set up a 1950-benchmark for the Gross and Net Fixed Capital Stocks, disaggregated into Business Fixed Capital, Non-Residential Fixed Capital and Residential Capital. We will apply our method only to the aggregate Net Fixed Capital Stock (NFCS), but both the disaggregated net capitals above can be estimated in the same way and the Gross Fixed Capital Stock can also be estimated with some necessary modifications, as is shown below.

To clarify, the PIM for the NFCS of a benchmark year starts by defining this year’s capital as equal to last year’s capital minus its depreciation (Dep) plus this year’s gross fixed capital formation (GFCF):

$$(1) K_1 = K_0 - \text{Dep} + I_1$$

Where $K = \text{NFCS}$ and $I = \text{GFCF}$ so as to carry less notation. Hofman then assumes a straight-line depreciation pattern⁽¹⁾. Therefore, equation (1) can be restated as:

$$(2) K_1 = K_0 (1 - \lambda) + I_1$$

Where λ is the rate of depreciation. And then capital is accumulated retrospectively over its service life up to the 1950-benchmark year. For example, following OECD practice (see Hofman, 2000a), the service life of Residential Capital is assumed to be 50 years on average. Thus, the accumulation should start in 1900 and end up in 1950, the capital in 1900 being the GFCF on residential capital of that year alone. In turn, Non-Residential Capital is assumed to live an average of 40 years and Business Fixed Capital 15 years. Hence, the later two should start accumulating in 1910 and 1935 up to 1950, respectively. Therefore, the three types of capital can be accumulated according to the following series:

$$(3) K_{1950} = \sum I_i (1 - \lambda)^{(t-i)}$$

Where the sum subindex i ranges from either 1900 or 1910 or 1935 to 1950, according to capital type. The equation above represents a weighted average of historical investments, the weights being the reciprocal of the depreciation, powered by the number of elapsed years. To arrive at the NFCS for the benchmark year the three series should be added up. It can be seen that the minimum requirement to

achieve the benchmark year is the availability of the three types of GFCF from their starting year to the benchmark year. In most countries, especially developing ones, this is not always available or, if it is, consistent with modern definitions of GFCF, especially before the Second World War. Therefore, this is the main drawback, as gaps have to be filled with a variety of estimating methods (see Hofman 2000a, especially Appendix D), which may not result in accurate figures and can create significant deviations from the true value, whatever this is. It is for this reason, and also for the lower data requirement, that the method proposed below may be a valuable alternative⁽²⁾.

3. Proposed Method

Let us start with the definition of the first difference for income or output (i.e. $\Delta Y = Y_1 - Y_0$), which can be re-arranged as

$$(4) Y_1 = Y_0 + \Delta Y$$

Where “ Δ ” means variation, Y is income and the subindices “1” and “0” represent the terminal and the initial period, respectively. Assuming a rate of capital depreciation λ , then

$$(5) Y_1 = Y_0(1 - \lambda) + \Delta Y$$

Where Y_1 is Gross Domestic Product and ΔY represents incremental Gross Domestic Product, i.e. it contains additions to net output to compensate for past depreciation.

Let us now assume that the production function of the economy is well proxied by capital alone *a la* Harrod-Domar (Jones, 1975) or alternatively that there are relatively stable relationships between average output and average capital and between medium-term variations in output and medium-term variations in capital. The long-term and medium-term stability of capital-output ratios is well supported by empirical studies that use actual data. Empirical evidence also shows that the yearly variations of capital output ratios are normally smooth and moderate. That is, when a country exhibits an average capital-output ratio of, say, 2.5, it is highly likely that it will keep around 2.5 over the medium and even long terms (see Hofman, 2000a, especially Appendix E).

Our method only requires that the capital-output ratios be smooth and stable over the 16-year period, 1951-1966, as will be clarified later. Hofman's capital-output ratios show here only a small average variability for the six countries in our sample. It shows 6 per cent variability around the mean, ranging from 9 percent in the case of Brazil to only 4 per cent in the cases of Chile and Colombia, the standard deviation being 2 per cent around the mean. This implies that for any annual average growth rate of GDP and a given investment coefficient, the growth estimates that result from the average capital-output ratio would have only a small variability over this period. For example, if the resulting average annual growth rate were 5 per cent, this would be contained within the interval 4.7 to 5.3, which is quite narrow.

Indeed, the correlation coefficient between actual GDP and NFCS varies from 0.97 (Colombia) to nearly 1.0 (Mexico) for the same period. Even for a period as long as 1950-1994, this coefficient shows to be always larger than 0.95. These high

correlations respond partly to the fact that the increment to the stock of capital, GFCF, is also a component of GDP. In our sample, they also appear to respond to either the possibility that capital is an all-dominant factor or that capital appears to proxy well the other productive factors that contribute to the generation of GDP. That is, these other factors appear to be dragged by capital, moving in similar proportions and directions. This would make appear that capital exhibits constant rather than diminishing returns, i.e. if capital increased by a given proportion, then output would increase by a similar proportion, at least over our sample period.

Hence, the capital-output ratios do appear to show a reasonable stability. Therefore, given that we are aiming at establishing approximate NFCS levels for a benchmark year, say, around 10 of the “true value”, then the capital-output ratios can be considered as fairly constant for the sample period. This may also allow us to use the actual variability around the mean as an acceptable interval for the average productivity and the benchmark capital that come from our optimisation exercise. The long-term relationship is normally represented by the average capital-output ratio, while the medium-term relationship is represented by the incremental capital-output ratio. That is,

$$(6) k_0 = K/Y \text{ and}$$

$$(7) k_1 = \Delta K/\Delta Y$$

Where k_0 is the average capital-output ratio and k_1 is the incremental capital-output ratio. These two represent the inverse of the average productivity of capital in the

economy in the long and medium terms, respectively.

Assuming that investment takes a lag of one year to become productive, and substituting (6) and (7) into (5) accordingly, then

$$(8) Y_1 = (1/k_0)(1 - \lambda)K_{-1} + (1/k_1)\Delta K_0$$

Let $(1/k_0) = \alpha_0$, $(1/k_1) = \alpha_1$, $\Delta K_0 = I_0$ and $(1 - \lambda) = \beta$ then

$$(9) Y_1 = \alpha_0 \beta K_{-1} + \alpha_1 I_0$$

Notice that I_0 and β are normally available. We attempt to estimate α_1 and $\alpha_0 K_{-1}$ and therefore Y_1 at optimal levels. Therefore, if $\alpha_0 K_{-1}$ could be estimated, then the benchmark capital K_{-1} could also be estimated under different assumptions for α_0 . A first assumption could be that $\alpha_0 = \alpha_1$. That is, the average and the incremental productivities are the same. This would correspond to a Harrod-Domar production function (Jones, 1975) or the AK endogenous growth model (Aghion & Howitt, 1998), which according to Solow (1994) is mostly the Harrod-Domar model with “bells and whistles”. A second, more general, assumption would be that $\alpha_0 \leq \alpha_1$, that is, the average productivity is smaller than or equal to the incremental productivity of capital. This would allow for the normal expectation that capital formation of later vintages is likely to have a higher productive quality than that of earlier vintages (see Denison, 1993; Hulten, 1992). Hence, the productivity of capital is likely to increase over time, which has been observed empirically (e.g. Hulten, 1992).

Therefore, if the yearly average growth in productive quality over given periods could be estimated (e.g. Hofman 2000a), then an estimated correction coefficient could be applied, as

$$(10) \alpha_0 = c\alpha_1$$

Where $c \leq 1$ is the correction coefficient⁽³⁾.

4. Estimation Procedure

With a view to estimate α_1 and $\alpha_0 K_{-1}$ at optimal levels, i.e. avoiding fluctuation-affected estimates, we use a simple linear programming model based on the generalisation of equation (9) $Y^*_t = \alpha_0 K_{t-2} \beta + \alpha_1 I_{t-1}$. Therefore, letting $K_{-2} = K_b$, the iterative solution for t periods is

$$(11) Y^*_t = \alpha_0 K_b \beta^t + \alpha_1 \sum I_{i-1} \beta^{(t-i)}$$

Where the period t ranges from 1 to n , K_b is the benchmark capital stock, “*” denotes “optimal” and the sum subindex i ranges from 1 to t . The base year product $[\alpha_0 K_b]$ and the productivity coefficient α_1 are therefore the two parameters to estimate. The linear programme then takes the following shape (see also Albala-Bertrand, 1999):

$$(12) \text{ Minimise: } Z = \sum (Y^*_t - Y_t) = (\alpha_0 K_b \sum \beta^t + \alpha_1 \sum \sum I_{i-1} \beta^{(t-i)}) - (\sum Y_t)$$

$$\text{Subject to: } Y^*_t \geq Y_t$$

$$(\alpha_0 K_b) \text{ and } \alpha_1 \geq 0$$

Where the series Y^*_t is calculated via equation (11), the subindex t for the first sum Σ and also Y^*_t and Y_t range from 1 to n , and the second subindex i for the second sum Σ ranges from 1 to t . Notice that, making the necessary changes, this methodology can also be used for the Gross Fixed Capital Stock (GFCS) level⁽⁴⁾. But in such a case the method would more appropriate for checking the consistency of other methods than as a third estimating approach⁽⁵⁾.

5. Application and Results

We can now apply the above procedure, using the same data as that used by Hofman (2000a, 2000b) so as to assess whether his benchmark estimates from the perpetual inventory method (PIM) are in agreement with those from our optimal consistency method (OCM). Following Hofman, we use the same six Latin American countries and the 1950-benchmark year for the net fixed capital stock (NFCS). The OCM estimation of our parameters uses the data for GFCF and GDP between 1951 and 1966. That is, following equation (9) or (11), the initial year for GDP is 1952 and that for CFCF is 1951, while the benchmark year for capital is 1950. To prevent a single odd year influencing the optimal (extreme) point, we use a three-year moving average for both series over the sample period. This fifteen-year period is then long enough to cover a cycle, so that we do not expect that either an odd sample or a particular odd year can over-influence the estimations. Table 1 below presents the estimates from our Optimal Consistency Method (OCM) against those from Hofman's Perpetual Inventory Methods (PIM) for the six Latin American countries in study.

Table 1: OCM Estimates against PIM Estimates for Six Countries

	$\lambda=0.024$	$\lambda=0.029$	$\lambda=0.025$	$\lambda=0.027$	$\lambda=0.026$	$\lambda=0.03$
	Argentina (1980a.u.\$m)	Brazil (1980a.zn)	Chile (1980\$m)	Colombia (1980\$m)	Mexico (1980\$m)	Venezuela (1980b.v.\$m)
$\alpha_0 K_b$	13514	1769	400500	379530	719870	50448
α_1	0.333	0.575	0.300	0.460	0.538	0.395
OCM K_b ($\alpha_0 = \alpha_1$)	40583	3077	1336000	825065	1338048	127716
Interval(+/-) %	4.8	9.7	3.4	3.8	7.8	6.2
PIM K_b (Hofman)	26089	1560	862636	662418	837728	94446
Ratio OCM/PIM	1.56	1.97	1.55	1.25	1.60	1.35
Interval	1.48-1.63	1.78-2.16	1.49-1.60	1.20-1.29	1.47-1.72	1.27-1.44

It can be seen that, assuming $c = 1$ (i.e. $\alpha_0 = \alpha_1$), the implied net fixed capital stock NFCS from our OCM (OCM K_b in the table) is larger than that proposed by Hofman's PIM (PIM K_b in the table). For example, the mid-point of our implied NFCS for Argentina is 40583, while the Hofman's PIM NFCS for that country is only 26089. That is, allowing for the small intervals⁽⁶⁾, the former is around 60 per cent larger than the latter, just as is the case of Mexico. For Brazil the implied NFCS appears to be around twice as large as the PIM NFCS, and for Chile and Venezuela around 50 and 60 percent as large, respectively. For Colombia the two differ by around 25 percent. Given that c , as indicated above, is likely to be smaller than unity, then this set of disparities is likely to be larger. Therefore, the perpetual inventory method, as applied by Hofman (2000), appears to underestimate systematically *the optimal consistency* level of the Net Fixed Capital Stock for the benchmark year.

I have therefore shown, first, that an optimally consistent benchmark capital should be of a given magnitude order (i.e. around 40 billion for Argentina, 3 billion for Brazil, and so on) and, second, that Hofman's PIM estimates might generally be too low for consistency. This may be due to either an underestimation of the pre-benchmark year GFCF series that feeds the PIM or too high a depreciation rate⁽⁷⁾, or both.

6. Conclusion

Most methods currently used to establish a capital stock for a benchmark year heavily depend on both the availability of data and the estimating method to fill gaps. The former is often incomplete and definitionally inconsistent over time, while the latter is normally rough and depends on convenient assumptions. My proposed method for the Net Fixed Capital Stock (NFCS) overpasses such difficulties, as all that it requires is good GDP and GFCF series for the 15 years after the chosen benchmark year. Checking against an independent study for six Latin American countries, I showed that the NFCS estimates coming from the “Perpetual Inventory Method” (PIM) appeared to underestimate systematically those coming from the “Optimal Consistency Method” (OCM) as proposed here. Therefore, this method, based on parametric estimates of a simple and stable equation via linear programming, appears to be strong enough to estimate an optimally consistent capital stock level for a benchmark year and/or to check the consistency of alternative estimates of the benchmark capital stock.

Notes

(1) There are various possible assumptions about depreciation to choose. For example, there could be a variable depreciation over time, diminishing at the beginning and accelerating after a trough, or just variable according to other economic factors. In turn, the straight-line depreciation assumes that efficiency declines linearly over the lifetime of capital, therefore, the same depreciation rate applies over the capital-service life. This means that the rate of depreciation is equal to the inverse of the capital life, i.e. if the capital life was 40 years, then the depreciation rate would be 1/40 a year.

(2) The method can also be used for a benchmark year for the Gross Capital Stock (GFCS) level, as is indicated in note (4) below. The GFCS for any given year is normally defined as $GFCS_i = GFCS_{i-1} - KR_i + GFCF_i$. That is, the GFCS of any given year is equal to the capital stock of the previous year minus the retirement of capital (KR) that ceased its useful life in the same year plus the gross fixed capital formation (GFCF) of that year. Notice that this formulation, contrary to that of the Net Fixed Capital Stock, assumes that capital does not depreciate over its service life, but it switches off its service once its working life finishes, and therefore should be discounted (retired) from the capital stock (e.g. 50 years for residential capital). For example, a light bulb has about the same efficiency until it bursts. But most types of capital actually require a good deal of maintenance and repairs, over their service life, so as to delay productivity losses, e.g. lorries or roads. The problem with considering this capital as the one that should be used in growth studies is that the expenditure in maintenance and repairs, which clearly is there to counteract depreciation, is not

counted as (replacement/restoration) investment. Therefore, there is a massive amount of investment outlays that are simply ignored. This may make sense for taxation or other accounting reasons, but it may not make much economic sense. In addition, most investment actually depreciates even allowing for maintenance and repairs, i.e. they lose productive quality over their working lives, like dams, housing, and most machinery and equipment, which is what the NFCS considers.

(3) For example, if for a focus period the capital quality contributes to the total growth of GDP in 0.15 percentage points annually, and the average life of the benchmark capital is, say, 16 years, then the correction coefficient would be 0.976, i.e. $1/(1.0015)^{16}$.

(4) The Gross Capital Stock (GFCS) level for a benchmark year is usually defined as: $GFCS_b = \sum GFCF_i$, where “b” is the benchmark year, and the sum subindex i ranges from the year of the initial investment up to the benchmark year, matching its service life length. For example if $b = 1950$ and the capital life is 50 years, then the sum index would accumulate all investment from 1901 to 1950. Thereafter, this capital will also undergo retirements, corresponding to previous investments as they cease their working life. Therefore, the GFCS for any year after the benchmark year would be: $GFCS_{b+t} = GFCS_{b+t-1} - KR_{b+t-d} + GFCF_{b+t}$. Where KR is the value of capital retired due to service life exhaustion. Accordingly, the subindex “t” corresponds to additional years after the benchmark year and the subindex “d” to the service life at the moment of its inception. For example, if we want to know the residential GFCS for 1955, assuming that the residential capital life is $d = 50$ years and that the benchmark year is $b = 1950$, then the equation will look like: $GFCS_{1955} = GFCS_{1954} - RK_{1905} + GFCF_{1955}$.

Now, following a similar iterative procedure to that we used for the NFCS, the equivalent equation would take the form: $GFCS_{b+t} = \alpha_0 GFCS_b - \alpha_2 \sum RK_i + \alpha_1 \sum GFCF_i$. Where the range of the first sum index I , for RK , is from $[b-d]$ to $[b-d+t]$ and that of the second sum index I , for $GFCF$, is from $[b]$ to $[b+t-1]$. So the parameters $[\alpha_0 GFCS_b]$, α_1 and α_2 can be estimated via linear programming as before, and an optimally consistent $GFCS_b$ can be generated to check against the $GFCS_b$ coming from alternative methods.

(5) Notice that in this case we do require knowing at least the initial fifteen years of capital accumulation before the benchmark year for $GFCS_b$. Therefore, it requires more information than that for NFCS, making it less useful as an independent third way for a benchmark capital estimation.

(6) The percentage dispersion from the mean, which makes the intervals in the table, corresponds to the variation coefficient that was calculated with the actual data supplied by Hofman (2000a), as indicated in page 6 above.

(7) It is more probable that the initial GFCF series was underestimated than the depreciation rate was too high. If the depreciation rate were decreased for the PIM it would also decrease for our method, so the optimally consistent benchmark capital stock would also increase. It is however not clear which capital estimate would increase faster, making it less safe to alter the depreciation to correct the PIM.

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