Investment in general training
with consensual layoffs

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Abstract

We study non-contractible firms’ investment in general training in a model of frictional unemployment. Since training is vested in workers, firms’ return to training is zero when a match ends. Consensual layoff provisions or large severance payments oblige firms to bargain efficiently over the joint payoff from separation. This increases employers’ incentives to train as they share workers’ outside return to general human capital. The result generalizes to all types of general investment that are vested in the non-investing party on separation.

We also show that, independently from underinvestment in training, the laissez-faire equilibrium is always inefficient for any given level of investment.

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1 Introduction

The traditional theory of human capital as pioneered by Becker (1964) predicts that in a competitive labour market workers should bear the full cost of and capture the entire return to general training. In such an environment investment in general training is fully efficient, barring borrowing constraints. Yet, there is substantial evidence that firms share both costs and proceeds of general training. For instance, Harhoff and Kane (1994) document how German firms bear a substantial part of the cost of apprenticeship training despite that apprenticeship programmes are highly standardized and provide mostly general skills. For the US Barron, Berger and Black (1997) find that productivity growth associated with training exceeds ten times wage growth, even though most of this training is deemed general by the firms providing it\(^1\). Furthermore, there is a widespread consensus epitomized by Lucas (1987, p.53) that the Walrasian framework cannot capture crucial aspects of labour markets and that search frictions are crucial to explain unemployment.

Investment in general training is lower than socially optimal when costly search implies deviations from the benchmark competitive paradigm. Search costs drive a wedge between the return to a (profitable) match and the return to seeking another partner. They thus generate a quasi-rent to continuing employment. In the absence of contracts then, bilateral bargaining determines the division of the joint surplus. This gives firms an incentive to invest in general training as long as they capture a positive fraction of the total surplus. On the other hand, the level of investment is inefficiently low as both firms and workers capture only part of the return. This is the standard hold up problem of Williamson (1985).

There are two facets to hold up. First, even if complete contracting between the current employer and the worker is possible at the time of investment, part of the return to general training will be held up by future employers if there is a positive probability of separation. Since the future employer is unknown at the time of investment the first best can never be achieved, as argued in Acemoglu (1997). Second, in the absence of contracts, investment is held up also by the current partner further depressing incentives, as shown in Grout (1984).

\(^1\)Bishop (1996) provides extensive references to the empirical evidence on the issue.
Various simple and less simple contractual solutions to this second kind of spillover have been suggested. The existing literature, though, has concentrated on investment in assets that are either specific to the relationship or general, but of the “selfish investment” type. A general selfish investment is one that increases the investing party’s benefit from trade both inside and outside the relationship (e.g. physical capital). General training does not fall in either of the above categories. It increases firm’s revenues, but it is vested in the worker in case of separation.

This paper analyses non-contractible investment in general human capital in an equilibrium search model. It takes as a stylized fact firms’ investment in general training and assumes that bargaining takes place according to a variant of Rubinstein’s (1982) strategic bargaining model. Returns are determined by relative bargaining power if they exceed outside market opportunities, but are constrained by the binding outside return otherwise. We show that institutions that allow firms to terminate the employment relationship only with workers’ consent, or that, in general, limit employers’ ability to lay workers off, improve firms’ incentives to invest in general training. The intuition is the following. Since human capital is vested in the worker, a firm’s return in case of separation is independent from its investment in the current worker. So, its marginal return to training is zero in those states of nature in which its outside market opportunity is binding whether the match is severed or not. Nonetheless, as general training increases a worker’s productivity also with other employers, the worker does capture part of the return in case of separation\(^2\). Consensual layoff arrangements prevent a firm from unilaterally terminating the employment relationship and oblige employers to bargain over the size of the payment - equivalently the share of the total payoff from separation - that induces workers to accept severance. By forcing firms and workers to share the return to training in all states of nature in which workers do not quit voluntarily, consensual layoffs improve employers’ incentives to train. For the same reason, these arrangements also boost workers’ incentives to carry out costly general investment which is vested in the firm. Examples of these investments are workers’ effort to ensure product quality and the development of products that remain the intellectual property of the firm.

\(^2\)In case of separation, the remaining part of the return is reaped by the future employer as argued by Acemoglu (1997).
Interestingly, there exist real world institutions that resemble the kind of optimal arrangements highlighted in this paper. In Germany, firms cannot legally carry out mass redundancies unless they have agreed with workers’ representatives on a social plan covering procedures and compensation packages. Some US firms such as DEC, IBM, Eli Lilly contractually commit to a zero-firing policy that effectively prevents them from laying off workers unless by mutual consent. The institution of lifetime employment in Japan has the same effects. Legislated severance payments and other job security measures may achieve some or all of the efficiency gains associated with consensual layoffs depending on their size. Large enough statutory dismissal costs effectively prevent firms from unilaterally terminating the employment relationship. Yet, whenever separation is efficient the parties will bargain efficiently on a lower voluntary severance payment which induces the worker to agree on termination. Though, job security is often blamed for distorting the allocation of workers across firms, this paper shows that not only this cannot be if wages are flexible, but that dismissal restrictions may actually induce both firms and workers to invest more in activities that benefit each other.

We also discuss the efficiency properties of the decentralized equilibrium we characterize. Independently from underinvestment in training, the laissez-faire equilibrium is always inefficient for any given level of investment. Hosios (1990) has shown that the right value of the Nash bargaining parameter can decentralize the social optimum in search models with homogeneous agents. In our environment, workers are heterogeneous along the job creation and the job destruction margins. While a firm can match with a trained or an untrained worker, all separation release a skilled employee. For this reason the sharing parameter alone cannot ensure efficiency on both margins.

This paper is related to a number of contributions in the literature. As in the literature surveyed in Acemoglu and Pischke (1999) it takes market imperfections as the reason why firms invest in general training. As in the incomplete contract literature it emphasizes contractual incompleteness within the current match as a source of underinvestment. Our result exploits the insight of Hart and Moore (1988) and further explored by MacLeod and Malcomson (1993), Che and Chung (1996) and Che and Hausch (1999). In all these papers breach remedies can restore efficiency under certain conditions. As noted above, though, these articles all restrict attention to investment which is either specific, or general but
vested in the investor. On the other hand, the kind of investment we consider is general but vested in the non-investing party. The type of breach remedy proposed in the above articles is an unconditional tax on separation. Unlike the consensual layoffs arrangements discussed here, when investment is general and vested in the non-investing party such a tax would never allow the investor to capture a share of the return in case of separation.

The idea that dismissal costs can increase firms’ incentive to invest in general training in the presence of labour market frictions has previously been explored by Jansen (1997). Her result exploits a mechanism quite different from the one used in the above mentioned articles and the present paper. She shows that, under the assumption that dismissal costs work as an unconditional tax on separation and that job destruction rates (assumed exogenous) are lower for skilled workers, firing costs may increase firms’ incentives to invest in general training for certain parameter configurations. The intuition is that, since dismissal costs cannot be bargained away, firms can reduce the probability of paying them by training their workers. The effect is reversed, though, if the separation rate is the same for both trained and untrained workers. Our result is instead unambiguous and does not require either assumption. In fact we show that in equilibrium a higher level of general training does not necessarily imply a lower separation rate, as workers’ productivity increases both inside and outside the match.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyses the equilibrium and discusses the empirical predictions. Section 4 derives conditions for steady-state efficiency and discusses the sources of inefficiency in the laissez-faire equilibrium. Section 5 concludes.

2 The model

2.1 Economic environment

Time is discrete. We adopt the notational convention $x = x(t)$ and $x' = x(t+1)$ to denote the value of a variable $x$ at the beginning of period $t$ and $t+1$ respectively. Agents are risk-neutral and discount the future at the constant rate $r$. The total labour force is constant and there is a potentially unlimited supply of productive units. At the beginning of each
period there are $u$ searching unemployed workers and $v$ firms with an open vacancy.

Production requires a fixed quantity of physical capital which has to be in place before the firm starts searching for a partner. The cost of the investment is $\kappa$ and can be fully recovered in case of separation. Alternatively, one could think of $\kappa$ as a one-off cost to the firm of entering the labour market. As shown in Fella (1999), what is crucial for the result in this paper and for any effect of firing costs in a bargaining framework is that the firm’s return to firing a worker is positive in the absence of employment protection legislation.

Because of uncertainty about the location of potential partners’ agents have to search for one. Finding a match takes at least one period. Search frictions are modelled according to a constant returns to scale, strictly concave, matching technology. So, matching probabilities depend only on market tightness $\theta = v/u$. $q(\theta)$ and $p(\theta) = \theta q(\theta)$ are respectively the proportion of firms and workers who find a match by the end of the period. Both are restricted to lie in the unit interval.

The timing of events for a matched pair is illustrated in figure 1. At the end of period $t$ a partner has been found. Before the quality of the match is discovered - at time $t.1$ - the parties can negotiate side-payments\(^3\). If the worker is untrained the firm trains her at time $t.2$. Training is fully general and takes place at a constant marginal cost normalized to one. Investment is instantaneous and third parties cannot verify neither its level nor the productivity of the match. This prevents a matched pair from writing a complete enforceable contract at time $t.1$ and implies that firms underinvest in training since investment is held up.

At the beginning of $t+1$ the pair draws a match-specific random productivity shock $z$. Shocks are independently and identically distributed across matches with support $[0, \infty)$ and continuous cumulative density function $G(z)$.

If the shock is favourable enough the pair bargains over a wage and produces in period $t + 1$ a flow of output $zf(h)$ with $f(.)$ strictly increasing, strictly concave and satisfying the Inada conditions. After one period of production the pair dies. To ensure stationarity of the environment it is assumed that every worker that is employed at the beginning of period $t + 1$ begets a son/daughter that will enter the labour market and start searching

\(^3\)In section 3.3, we discuss the consequences of relaxing this assumption.
at the beginning of the next period. Normalizing the total labour force at the beginning of each period to one implies that the flow of new entrants into the labour force at the beginning of the following period is

\[ \text{in}' = 1 - u \]  

(1)

If the shock is below a reservation level \( b \) the parties separate and start searching for a new partner. The firm has to pay a statutory severance payment\(^4\) equal to \( F \) in case it fires the worker, but no payment is due if a worker quits. Clearly, our distinction is meaningful only if third parties can distinguish between quits and layoffs.

We assume that outside parties can verify: a) whether a worker shows up for work; b) if the firm allows the worker on the premises; c) any written communication between the two parties. A separation is deemed a dismissal if the firm gives the worker written notice that it no longer wishes to continue the employment relationship. The end of the relationship is deemed a quit if the worker does not show up for work without providing a written justification (e.g. a medical certificate) or if the worker gives written notice that she no longer intends to continue in employment. Until one of these actions is taken the employment relationship is considered in existence. This seems broadly consistent with existing practices in most developed countries.

Carmichael (1983) has argued that severance payments cannot be conditioned on the identity of the party initiating separation: a firm that wanted to dismiss a worker could always induce her to quit by making her life difficult and viceversa. In practice,

\[^4\text{As shown in Fella (1999), given efficient bargaining, allowing for part of the cost born by the firm to be wasted would not affect the result.}\]
legislation often prescribes payments to employees in case of layoff, but workers are not
entitled to (and in general do not receive) any payment if they quit. So it has to be the
case that conditional severance payments are, if only imperfectly, enforceable. MacLeod
and Malcomson (1989) have shown that, if firms but not third parties can observe effort,
workers’ moral hazard problem can be solved by a wage contract with a performance-
related component. On the other hand, one would expect that, at least in the case of
collective workforce reductions, it is difficult for an employer to convince a court that a
claim of constructive dismissal filed by a works council or a group of workers is unfounded.
Furthermore, if firms could easily disguise layoffs as quits dismissal costs, and the whole
debate on their impact, would be irrelevant as firms would never pay them.

The fact that a proportion of trained matched workers becomes unemployed implies
that the unemployment pool contains both skilled and unskilled workers. Since training is
general and search costly, it also implies the presence of positive spillovers as in Acemoglu
(1997).

For simplicity, I restrict attention to symmetric, steady-state, pure-strategy equilib-
ria. To find such an equilibrium, suppose that (given the matching and bargaining pro-
cess) the level of training of the representative skilled worker equals \( h^* \). Then derive the
individually-optimal entry decision of a single unmatched firm and the investment decision
\( h \) of a single firm, matched to an unskilled worker, with the total number of vacancies
\( v \), unemployment stocks \( u \) and \( u_s \) and \( h^* \) taken as given. In equilibrium \( h = h^* \).

2.2 Flows and unemployment

The stock of unemployed workers at the beginning of each period evolves according to

\[
u' = u [1 - p(\theta) (1 - G(b))] + in'. \tag{2}
\]

\( u' \) equals the number of searching workers who were not matched in the previous period,
plus those who found employment but whose job was destroyed plus the flow \( in' \) of new
entrants into the labour force. Together with (1), equation (2) implies that steady state
unemployment is given by

\[
u = \frac{1}{1 + p(\theta) [1 - G(b)]}. \tag{3}
\]
Equation (3) is the Beveridge curve. A higher job finding rate \( p(\theta) \) and a lower rate of destruction of unproductive matches \( G(b) \) decrease steady state unemployment.

Since all the workers who lose their job are trained, the stock of skilled unemployed workers evolves according to

\[
u'_s = u_s [1 - p(\theta)] + up(\theta)G(b).
\]

(4)

The mass of skilled unemployed workers \( u'_s \) equals the number of skilled workers who did not leave unemployment in the previous period plus those workers (all trained) who were matched but lost their job in the previous period. This implies a steady state proportion of skilled workers in the unemployed pool equal to

\[
u_s = G(b).
\]

(5)

### 2.3 Search

For simplicity, we assume there are no unemployment benefits and the utility of leisure is zero. So \( U(h) \), the asset value of an unemployed worker with general human capital \( h \) at the beginning of the period, is

\[
[r + p(\theta)]U(h) = p(\theta)E_a(h),
\]

(6)

where \( E_a(h) \) is the value of accepting a match.

Our set up implies that all skilled workers have the same level of training. So, in the symmetric equilibrium \( h = 0 \) if the worker is untrained and \( h = h^* \) for a trained worker, where \( h^* \) is the optimal level of training for the representative firm.

\( V \), the value of a searching firm, depends then on the expected level and incidence of training among the unemployed population and satisfies

\[
[r + q(\theta)]V = q(\theta) [(1 - G(b)) J_a(0) + G(b)J_a(h^*)],
\]

(7)

where \( J_a(0) \) and \( J_a(h^*) \) are the values of accepting a match with an unskilled and trained worker respectively. Conditional on having contacted a worker the probability that she
is skilled is \( u_s/u = G(b) \). In equilibrium with free-entry the value \( V \) of posting a vacancy equals \( \kappa \), the investment cost.

### 2.4 Bargaining

Because of search frictions a match which is formed and/or is not destroyed yields quasi-rents. We assume that the parties will bargain over the division of these quasi-rents according to a variant of alternating offer bargaining due to Binmore (1987).

At the beginning of each bargaining round, nature selects one of the two parties to make an offer, the worker being selected with probability \( \beta \). The counterpart either accepts the offer, in which case production takes place and the game ends, or she rejects the proposal and the game moves to a new round after a delay equal to \( \Delta \). When responding to an offer each party can also unilaterally and irreversibly abandon the negotiations to trade outside (take her outside option, in the bargaining terminology), ending the game. We assume the parties cannot search for another partner during bargaining\(^5\).

The solution to the general bargaining problem is given by the following proposition.

**Proposition 1** Be \( S \) the expected value of the total surplus from reaching an agreement and \( E \) and \( J \) respectively the worker’s and firm’s share of this surplus. Then:

\[
\text{a)} \quad S = \max\{C, U + \kappa\}, \quad \tag{8}
\]

where \( C \) is the expected value of the total surplus from continuation of the match;

\[
\text{b)} \quad \text{the unique, subgame perfect equilibrium values of } E \text{ and } J \text{ satisfy}
\]

\[
E = \begin{cases} 
\beta S & \text{if } U < \beta S < S + F - \kappa \\
U & \text{if } U > \beta S \\
S + F - \kappa & \text{if } \beta S > S + F - \kappa 
\end{cases} \quad \tag{9}
\]

\(^5\)Relaxing this assumption would not alter the qualitative nature of our result. Masters (1998) allows for search during bargaining in a similar set up. He shows that, unless the employment relationship is mediated by an intermediary who pays the parties their marginal product, the underinvestment result goes through.
Proof. a) With transferable utility, sharing the higher between the joint payoff from separation and from continuation is Pareto optimal.

b) Binmore (1987) shows that in the absence of outside options the parties share the joint payoff according to the relative bargaining power $\beta$. Binmore, Shaked and Sutton (1987) prove that outside options bound bargained payoffs from below.

The first part of proposition 1 implies that the parties will bargain over the higher between the joint payoff from continuation and the total return from separation. With transferable utility, the separation decision is always efficient in the sense that it maximizes the total payoff, independently from the existence of legislated dismissal costs. This is just one more instance of the Coase theorem.

Part b) states that the parties share the joint payoff according to the relative bargaining power $\beta$ unless either party can do better by abandoning the match and searching for a new one. In this latter case, the binding outside option determines the shares. If $F > 0$, firing costs reduce the firm’s outside option and its payoff in those states in which its market return would be binding in the frictionless equilibrium.

Firing costs drive a wedge between the return to the firm’s assets outside the relationship in case the worker unilaterally abandons the match and the same return if the firm fires the worker. This wedge increases the scope for bargaining not only over the surplus from continuation, but also over the total payoff from separation. The firm cannot sever the relationship unless it pays the firing cost or bargains with the worker over a voluntary side-payment that induces him to quit. On the other hand, workers are free to quit at any time.

When a match is formed at time $t$ the ex ante expected surplus to split is

$$S_a(h) = S_e^c(h^*) - (h - h^*).$$

The ex ante surplus from meeting a worker with human capital $h$ is given by $S_e^c(h^*)$, the expected ex post surplus from being matched with a trained worker at the beginning of $t+1$, minus the cost of training the worker. The cost is obviously zero for a trained worker.
with initial human capital \( h^* \).

Using (7) and (10) we can then write the free-entry condition as

\[
\kappa \left( 1 + \frac{r}{q(\theta)} \right) = S_p^e(h^*) - G(b)E_a(h^*) - (1 - G(b))(h + E_a(0)),
\]

where the expectation of the ex post surplus equals

\[
S_p^e(h^*) = f(h^*) \int_b^\infty zdG + G(b) [U(h^*) + \kappa].
\]

The joint surplus coincides with the revenue from production if the match-specific shock is above the reservation productivity \( b \) and the total return from separation otherwise. In case the match is severed the joint payoff is given by the value \( U(h^*) \) of being a trained unemployed worker plus \( \kappa \), the value of search to the firm.

Given that all firms are identical the worker’s outside option cannot be binding at \( t.1 \), as at best she will meet an identical firm one period later.

Similarly, the firm’s outside option is not binding in case it is matched with a trained worker. In the best possible case, it will meet a similar worker with a one-period delay. Proposition 1 then implies

\[
E_a(h^*) = \beta S_p^e(h^*).
\]

Things are different in case a firm meets an unskilled worker. If the firm turns the worker down and searches for another match, with positive probability it will find a skilled worker after one period and will not have to bear the training costs. So the firm’s share of the total surplus is the higher between the return to going back to search \( \kappa \) and a proportion \( (1 - \beta) \) of the surplus. That is

\[
E_a(0) = \min \{ \beta (S_p^e(h^*) - h^*), S_p^e(h^*) - h^* - \kappa \}.
\]

Equation (15) shows that, though the firm invests in training non-cooperatively, an untrained worker shares the cost of the training that it is optimal for the firm to provide ex post. It needs to be pointed out that dismissal costs do not affect the firm’s outside option at time \( t.1 \) since they are not due if a job applicant is turned down before starting
employment.

Whatever the distribution of the ex post surplus at \( t + 1 \) side payments ensure that the ex ante distribution satisfies (14) and (15).

We can then use equations (6), (13) and (14) to solve for the reduced-form asset value of a trained unemployed worker

\[
U(h^*) = \frac{p(\theta)\beta}{r + p(\theta)[1 - \beta G(b)]} \left[ f(h^*) \int_b^\infty zdG + G(b)\kappa \right]
\]

and the ex post, expected joint payoff

\[
S_p^e(h^*) = \frac{r + p(\theta)}{r + p(\theta)[1 - \beta G(b)]} \left[ f(h^*) \int_b^\infty zdG + G(b)\kappa \right].
\]

Equations (12), (14) and (15) allow to solve for the reduced-form, free-entry condition

\[
\kappa \left(1 + \frac{r}{q(\theta)}\right) = (1 - G(b)) \max \left\{ (1 - \beta) \left( S_p^e(h^*) - h^*\right), \kappa \right\} + G(b)(1 - \beta)S_p^e(h^*). \tag{18}
\]

3 Investment and equilibrium

The firm invests in training non-cooperatively after side-payments have been exchanged and before uncertainty about the quality of the match is revealed. Optimality then requires equality between the marginal investment cost and the expected marginal return to the firm, or

\[
1 = \frac{\partial J_p^e(h^*)}{\partial h}, \tag{19}
\]

where \( J_p^e(h^*) \) is the expected post-investment payoff to the firm.

At time \( t + 1 \), once the quality of the match has been realized, the surplus from reaching an agreement is

\[
S_p(z, h^*) = \max \left\{ zf(h^*), U(h^*) + \kappa \right\}. \tag{20}
\]

From proposition 1 we know that the parties bargain over \( zf(h^*) \) as long as continuation is efficient or \( z \geq b \), where the reservation productivity \( b \) satisfies

\[
bf(h^*) = U(h^*) + \kappa. \tag{21}
\]
In general, $\beta$ determines the share of revenues that each party receives when revenues are high, but either party’s outside return may become binding for low values of $z$. The following proposition establishes the conditions under which the firm’s or the worker’s market alternative is binding with positive probability.

**Proposition 2** If $(1 - \beta)U(h^*) < \beta\kappa$ in equilibrium, then for

$$F < \kappa - (1 - \beta)bf(h^*)$$

(22)

there exists $z_r \in [b, \infty)$ satisfying

$$F = \kappa - (1 - \beta)z_rf(h^*)$$

(23)

such that $\forall z \leq z_r$, $J_p(z, h^*) = \kappa - F$.

Vice versa, if $(1 - \beta)U(h^*) > \beta\kappa$, then, $\forall F$, there exists $z_r \in [b, \infty)$ satisfying

$$U(h^*) = \beta z_rf(h^*)$$

(24)

such that $\forall z \leq z_r$, $E_p(z, h^*) = U(h^*)$.

**Proof.** See appendix A. $\blacksquare$

The condition $(1 - \beta)U(h^*) < \beta\kappa$ implies that, when the match productivity is low, the firm’s bilateral monopoly share of the highest between the surplus from production and that from separation falls short of the firm’s payoff from firing the worker and trading outside. When the match productivity is low the firm receives its outside option since the threat to fire the worker is credible and is actually carried out when separation is efficient.

Vice versa, if the inequality is reversed, it is the worker’s market return that becomes binding in bad states and independently from the size of firing costs. When separation is efficient, the worker quits the firm, since the share of the total payoff from separation he would obtain by bargaining is lower than her outside option.

In general, there is no reason to expect one condition rather than the other to prevail. In the presence of both idiosyncratic and aggregate uncertainty one would expect the first condition to prevail in recessions, when the value of being unemployed is low, and the
reverse condition to prevail in booms, when market tightness and the expected surplus from a match are high.

Given that firing costs matter only in those states of nature in which the firm’s outside option is binding in the *laissez faire* equilibrium, we will assume for simplicity in what follows that the first condition always holds.

### 3.1 Equilibrium with small severance payments

Proposition 1 and 2 together imply that if firing costs satisfy $F < \kappa - (1 - \beta)bf(h^*)$, the expected ex post payoff to the firm will be

$$J_p^*(h^*) = (1 - \beta)f(h) \int_{z_r}^{\infty} zdG + G(z_r)(\kappa - F).$$  \hspace{1cm} (25)

The firm receives a share $(1 - \beta)$ of total revenue if the match productivity is high enough and its outside option in all other states. The first-order condition for optimal investment is then

$$1 = (1 - \beta) \frac{\partial f(h^*)}{\partial h} \int_{z_r}^{\infty} zdG(z).$$  \hspace{1cm} (26)

With small or no severance payments the privately optimal level of training is independent from external conditions. Since human capital is vested in the worker the firm’s payoff when $z < z_r$ is independent from the level of training.

The level of investment is a decreasing function of $z_r$, as the higher $z_r$ the higher the probability that the firm’s outside return is binding. As equation (23) shows, severance payments reduce $z_r$. Hence they increase the range of states over which the firm shares the return from its investment and improve its incentives to train.

That breach remedies can improve the investor’s incentives through the mechanism highlighted here was first suggested by Hart and Moore (1988) and further exploited in MacLeod and Malcomson (1993), Che and Chung (1996) and Che and Hausch (1999). The only difference is that while in those articles breach penalties cannot be conditioned on the identity of the party who refuses to trade, here severance payments are not due if it is the worker that quits the firm. The reason for this difference is twofold. First, this paper focuses on the employment relationship rather than general bilateral relationships.
In practice workers do not receive any payment if they quit. Second, when firms invest in general, rather than specific, training it is not necessarily the case that imposing a lump-sum transfer on the firm if the worker quits improves the firm’s incentives to train. For example, MacLeod and Malcomson (1993) show that if fixed-wage contracts can be written, firms capture the full marginal return to training in those states in which the contract is not renegotiated. Taxing firms on quits would increase the probability that workers capture part of the return and discourage investment.\(^6\)

We can now characterize the equilibrium with zero or small severance payments. Using equations (21) and (16) we can write the reduced form job destruction condition as

\[
bf(h) = \kappa + \frac{p(\theta)\beta}{r + p(\theta) [1 - \beta G(b)]} \left[ f(h^*) \int_b^\infty zdG + G(b) \right].
\]

(27)

**Definition 3** A stationary symmetric equilibrium with zero or small dismissal costs is a vector of allocations \([\theta, u, u^*, h^*, b, z_r]\) and a value function \(S^e_p(h^*)\) such that: (i) the free entry condition (18) determines \(\theta\), (ii) \(S^e_p(z, h^*)\) is given by equation (20), (iii) the two flow equilibrium equations (3) and (5) determine \(u\) and \(u^*\), (iv) \(h^*\) solves the first order condition (26), and (v) \(z_r\) and \(b\) satisfy equations (23) and (27).

For a given level of \(h\), equilibrium can be represented graphically as the intersection of the job destruction (JD) and a job creation (JC) condition as in Mortensen and Pissarides (1994). Under the assumption that the firm’s outside option is not binding\(^7\) at \(t.1\), one can write one version of the job creation condition by using equation (21) to replace \(U(h^*)\) in (18), (13) to obtain

\[
\kappa \left(1 + \frac{r}{q(\theta)}\right) = (1 - \beta) \left[ f(h^*) \int_b^\infty zdG + G(b) bf(h^*) - (1 - G(b)) h^*\right].
\]

(28)

Figure 2 plots the two curves in the \((\theta, b)\) space. The JC locus is upward sloping.\(^8\) The JD curve - given by equation (27) - is upward sloping and convex, with a strictly positive

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\(^{6}\)On the other hand, in our model the worker’s marginal return is lower outside than within the relationship in those states in which the the worker’s frictionless outside option is binding. Taxing firms’ on quits would further increase incentives to invest.

\(^{7}\)The case in which the outside option is binding is qualitatively similar.

\(^{8}\)In Mortensen and Pissarides (1994) the JC curve is downward sloping due to the different bargaining solution adopted.
horizontal intercept at \( b = \kappa/f(h^*) \) and a vertical asymptote. Thus, provided \( JC \) lies above \( JD \) at \( b = \kappa/f(h^*) \) - that is provided vacancy posting is positive when the value of unemployment is zero - an equilibrium exists\(^9\).

The system is block recursive with equations (23) and (26) determining the level of training. An increase in severance payments, results in higher training. This induces firms to post more vacancies for given separation rate and to fire less for given market tightness. Suppose the economy is initially in equilibrium at \( E \). An increase in severance payments then moves both the \( JD \) and the \( JC \) curves up. It can be easily shown that the horizontal shift in the \( JC \) locus always exceeds the shift in the \( JD \) curve. Assuming that the equilibrium is unique, severance payments unambiguously increase market tightness and the job finding rate \( p(\theta) \), but have an ambiguous effect on the reservation productivity \( b \) and the separation rate. In case the job destruction rate increases, the net effect on equilibrium unemployment is ambiguous. Numerical simulations, though, indicate that whichever the direction of the movement in unemployment incidence, the increase in vacancy posting prevails and employment increases.

\(^9\)It is not possible to prove that the equilibrium is unique, though numerical experimentation suggests that this is the case.
3.2 Equilibrium with consensual layoffs

Small severance payments reduce the probability that the firm’s outside option is binding when continuation is efficient. Yet, they do not prevent employers from firing the worker when the match is no longer viable. So, the firm does not capture any return to its investment in case of separation.

Suppose instead that a firm can sever the employment relationship only with the worker’s consent. This effectively locks the firm in a bilateral monopoly situation. The firm does not only have to share the surplus from production. When separation is efficient the firm cannot unilaterally sever the relationship, though this would give it a larger share of the total separation payoff. Instead, it has to bargain over the size of the payment that induces the worker to agree on separation. The firm’s ex post payoff at time $t + 1$ is then

$$J_p(z, h^*) = (1 - \beta) \max \{zf(h^*), U(h^*) + \kappa\}.$$ (29)

With the expected payoff at the time of investment given by

$$J_{\text{ep}}(h^*) = (1 - \beta) \left[ f(h^*) \int_b^{\infty} zdG + G(b) (U(h^*) + \kappa) \right]$$ (30)

the firm would invest up to the point where

$$1 = (1 - \beta) \left[ f'(h^*) \int_b^{\infty} zdG + G(b) \frac{\partial U(h^*)}{\partial h} \right],$$ (31)

or, using equation (16)

$$1 = (1 - \beta) \frac{r + p(\theta)}{r + p(\theta)[1 - \beta G(b)]} f'(h^*) \int_b^{\infty} zdG.$$ (32)

Confronting equations (31) and (26) it is evident that the obligation to severe the employment relationship by mutual consent further increases investment for two reasons. First, the firm’s outside option is never binding when production is efficient: $z_r$ does not enter the investment condition any more. Second, the firm now captures a fraction $(1 - \beta)$ of the marginal return to training outside the relationship. Consensual layoff arrangements reduce the firm’s total return from separation, but by forcing employers to
share the total outside payoff they increase their marginal return to training. This second effect is the new insight of this paper. In so far as investment is general and vested in the non-investing party on separation, institutions or contractual arrangements that result in sharing of the total separation payoff improve incentives to invest.

Interestingly, institutions of the kind envisaged here do exist in practice. In Germany firms cannot carry out collective redundancies unless they have secured the works council’s approval of a social plan detailing the conditions and terms of layoffs, including the size of severance payments. The institution of lifetime employment in Japan and the voluntary commitment to a zero-firing policy in certain firms such as DEC, IBM, Eli Lilly and others achieve the same result. Dismissals are still carried out but only on terms which meet the workers’ consent. Note that, provided ex ante side payments, are unconstrained our model predicts that it is rational for firms to adopt such policies.

In other countries such as Spain and Italy, high explicit or implicit firing costs can achieve the same result. In fact, it can be shown that

**Corollary 4** If \((1 - \beta)U(h^*) < \beta \kappa \) and \(F > \kappa - (1 - \beta)bf(h^*)\) then the firm’s ex post payoff is given by

\[
J_p(z, h^*) = (1 - \beta) \max \{zf(h^*), U(h^*) + \kappa\}.
\]

**Proof.** See appendix A

Large enough severance payments achieve the same effect as a consensual layoff clause by reducing the firm payoff from firing below the bilateral monopoly outcome. The firm is then better off paying the worker a share of the total separation payoff to induce her to quit rather than unilaterally severing the relationship.

We can now characterize the equilibrium with either large severance payments or consensual layoff provisions.

**Definition 5** A stationary symmetric equilibrium with consensual layoff is a vector of allocations \([\theta, u, u^*, h^*, b]\) and value function \(S^c_p(h^*)\) such that: (i) the free entry condition \((18)\) determines \(\theta\), (ii) \(S^c_p(z, h^*)\) is given by equation \((20)\), (iii) the two flow equilibrium equations \((3)\) and \((5)\) determine \(u\) and \(u^*\), (iv) \(h^*\) solves the first order condition \((32)\), and (v) \(b\) satisfies equation \((27)\).
The equilibrium can still be represented by the job destruction and job creation loci in figure 2, but the system is no longer recursive. The optimal level of investment in equation (32) now depends on aggregate variables. Yet, one can prove that the equilibrium with consensual layoff provisions features a higher training level than the one with small or no severance payments.

We can use a continuity argument exploiting the equivalence between consensual layoffs and large enough severance payments established in corollary 4. We know from the previous subsection that in the equilibrium with small severance payments the optimal level of training in equation (26) is independent from external conditions and increasing in the size of dismissal costs. As the severance payment $F$ increases, both $z_r$ and $b$ change, but their distance decreases. For $F$ converging to its critical value $\kappa - (1 - \beta)bf(h)$ from below, $z_r$ converges to $b$. So, the integral on the right hand side of equation (26) is infinitesimally close to the first addendum in the bracket on the right hand side of (32). For $F$ equal or larger than its critical value the right hand side of (32) equals the right hand side of (26) plus a strictly positive term in $U'(h^*)$.

So, an equilibrium with consensual layoffs features, coeteris paribus, a higher training level and job finding rate while, as in the previous section, no unambiguous analytical predictions can be made on the direction of the change in the separation and the unemployment rates.

3.3 Empirical predictions and discussion

The model is too crude to allow for convincing calibration. Yet, its main insight revolves around the internalization of the externality associated with human capital being vested in the worker on separation. This aspect would survive in largely unchanged form in a more realistic model.

It is then possible to work out the percentage change in the level of training stemming from the introduction of consensual layoffs in an economy in which severance payments are large enough to ensure that firms’ outside returns are never binding when continuation is efficient, but not so large as to induce consensual layoffs. When $F$ is just below the level in (22) that results in consensual layoffs, $z_r$ in equation (26) is infinitesimally close
Table 1: Percentage increase in training level associated with consensual layoffs.

<table>
<thead>
<tr>
<th>Layoff rate</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>β 0.5</td>
<td>0.05</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td>β 0.6</td>
<td>0.06</td>
<td>0.13</td>
<td>0.22</td>
</tr>
<tr>
<td>β 0.7</td>
<td>0.07</td>
<td>0.16</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table 1 presents the percentage change in the level of training associated with consensual layoffs for different values of the sharing parameter β and the layoff rate G(b).

Assuming a Cobb-Douglas production function \( f(h) = h^\delta \), the percentage change in investment associated with the introduction of consensual layoffs is then implicitly given by

\[
\frac{h_H^*}{h_L^*} = \left(1 + \frac{p\beta G(b)}{r + p(1 - \beta G(b))}\right)^{1/\delta}.
\]  

The output elasticity \( \delta \) can be recovered from empirical studies of the impact of training on wages. Under the assumption of rent sharing, the wage and revenues elasticity with respect to training coincide. Parent (1999), using the US National Longitudinal Survey of Youth estimates a wage semielasticity with respect to training equal to 0.12 which given a mean level of training equal to one quarter gives an elasticity of 0.03. An elasticity of 0.02 can be obtained based on a similar study by Loewenstein and Spletzer (1999). We chose an intermediate value of \( \delta = 0.025 \) and set the real interest rate \( r \) to 0.04. The value of the job finding rate \( p \) is not particularly crucial. It is clear from the above equation that, as long as \( p \) is relatively large with respect to \( r \), it has little effect on the results. We set \( p = 1 \) which is consistent with an average unemployment duration not exceeding one year.
The range for $\beta$ reflects the empirically observed values for the share of labour income in total product. Since there is no sharing in the case of quits both in reality and in our model, the relevant separation rate to look for is the layoff probability rather than the total job destruction rate rate. Blanchard and Portugal’s (2000) comparative study of job and worker flows in Portugal and the US identifies the layoff rate with the rate of job destruction. They estimate the annualized (quarterly) layoff rate for Portugal to 16% and the same rate for the US to respectively 22% and 29% for the manufacturing sector and all sectors respectively.

As the table shows, the gains are small in countries with low layoff rates, but can be quite sizeable in countries in which firm-initiated turnover is higher. This is no surprise, as the extent of the externality is increasing in the rate of turnover. Also, the higher is $\beta$ the higher is the fraction of the spillover accruing to the worker on separation and the larger the incentive that consensual layoffs provide.

The size of these effects suggests that the mechanism provided cannot be the main explanation for cross-country and cross-culture variation in training levels. Yet, it is by no means negligible, at least in countries with higher layoff rates.

Some empirical support for this mechanism is provided by Bishop (1991) who finds that the likelihood and amount of formal training are higher at firms where firing a worker is more difficult.

The insight of this paper is not restricted to firm-provided training. A number of authors have conjectured that job security measures may increase workers’ contribution to firms’ value. The mechanism studied here applies equally to investment carried out by employees which is vested in the firm and general in nature. For example, the reputation for high quality and reliability of German and Japanese cars is vested in the manufacturing companies, but is largely dependent on their labour force effort. A programme developed by a software engineer employed by a firm is intellectual property of the employer. In all these cases, consensual layoff arrangements allow workers to capture part of the return to

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10 For example, the studies surveyed in Bishop (1991) document large differences in the incidence and duration of training between the US on the one hand and Germany and Japan on the other. Krafcik (1990) finds that newly hired assembly workers in the US receive an average of 48 hours of training in US-owned plants and 280 in Japanese-managed ones.

11 See, for example, Nickell (1998) and Bean (1997).
their investment on separation.

Of course, a measure which redistributes ex post payoffs from firms to workers must reduce firms’ incentives along some other line. Provided side payments from workers to firms are not required or constrained, consensual layoffs arrangement do not alter ex ante bargaining power. So they have no direct effect on any investment carried out before a match is formed\textsuperscript{12}. On the other hand, consensual layoff provisions do reduce firms’ incentives to re-invest in physical capital and other assets which may be general, but whose return is now partly captured by the worker in case of separation.

Relaxing the assumption that there are no constraints on workers’ entry fees, opens the possibility that consensual layoff arrangements may reduce firms’ ex ante bargaining power. This would result not only in lower vacancy posting, as in Garibaldi and Violante (2000), but also in lower ex ante investment by firms. It has to be noted, though, that it is not obvious that side-payments from workers to firms are required in equilibrium. This depends not only on training costs, but also on whether it is workers’ or firms’ ex post bargaining power that exceeds its ex ante counterpart. In general this depends on the probability that each party’s outside option is binding ex post\textsuperscript{13}.

Throughout our analysis, we have assumed that in the frictionless equilibrium firms rather than workers would like to unilaterally sever the relationship when productivity is low. The insight of the paper, though, applies equally to quits. Measures to prevent workers from quitting unless by mutual consent would further allow firms to capture part of the marginal return to their investment and improve incentives. We do not observe institutions of this kind, though. One would expect them not only to conflict with the natural law tenet that human capital cannot be alienated, but also to run into difficulties and possibly result in inefficient employment continuation in so far as workers are unable to buy out their jobs due to borrowing constraints. On the other hand, we do observe similar institutions when firms rather than workers are the non-investing party and natural rights or borrowing constraints are less of an issue. For example, top managers’ effort

\textsuperscript{12}Though, they may have indirect effects if general training is a complement or substitute for other forms of investment.

\textsuperscript{13}For example, under our assumption that workers’ outside returns are never binding, it is firms that should pay an entry fee to skilled workers unless consensual layoff measures ensure that ex ante and ex post bargaining power coincide.
is a typical example of general, worker-initiated investment which is vested in the firm of separation. It is quite common for companies to negotiate golden-handshakes when top managers are removed. Given the publicity that these payments often receive, it is conceivable that in these cases reputation consideration may support the mechanism highlighted in this paper even in the absence of explicit contracts.

One point that this paper does not address is why firms invest in general training in the first place. Under realistic values for \( \beta \) the level of training would be higher if workers, rather than firms, invested. MacLeod and Malcomson (1993) have shown that simple fixed wage contracts allow the firm to capture the full marginal return to its investment with a very high probability. In such a set up it would be efficient for firms to invest in training provided that the probability that the workers’ outside return is binding is low and the insight highlighted in this model would still apply. Extending the paper in this direction is a priority for future research.

4 Efficiency

It is well known that the decentralized equilibrium in a search environment without wage posting is not efficient unless the share parameter \( \beta \) happens to satisfy some variant of the Hosios (1990) efficiency condition and balance the thick market and congestion externalities. Apart from this special case, both job creation and job destruction are inefficient. This compounds the inefficiency associated with the non-contractibility of investment and discussed above. In such a second-best world consensual layoffs do not necessarily increase the flow of consumable resources. At the decentralized allocation a higher level of training boosts output net of training costs, but this may or not be offset by the increase in total search costs resulting from the increase in vacancy posting.

As it turns out, one cannot even conclude that starting from laissez-faire training increases efficiency conditional on \( \beta \) satisfying the Hosios condition. In fact, independently from hold up issues, the mere coexistence of skilled and unskilled workers introduces a form of inefficiency that is absent from search models with homogeneous workers. To better highlight this inefficiency, we will abstract from the investment decision in what follows and show that, conditional on any positive level of investment, there is no value of the
sharing parameter $\beta$ that can decentralize the social optimum. For ease of comparison we will assume that the level of investment is fully efficient in the decentralized economy of the previous section and will compare the decentralized and socially optimal job destruction and job creation decision\textsuperscript{14}.

The utilitarian social planner chooses a time path for the control variables, the beginning-of-period reservation productivity and market tightness pair $(b, \theta')$, to maximize the present value of aggregate income. The corresponding value function solves the Bellman equation

$$L(u, u_s, \theta) = \max_{b, \theta'} \frac{1}{1 + r} \times \left\{ up(\theta) A - (u - u^*) p(\theta) h - [\theta'u' - u(\theta - p(\theta))] \kappa - r\theta'u'\kappa + L' \right\}$$

(36)

s.t. \[u' = u [1 - p(\theta) (1 - G(b))] + in' \]
\[u'_s = u_s [1 - p(\theta)] + up(\theta)G(b), \]

with $A = f(h) \int_b^\infty zdG + G(b)\kappa$.

The social planner takes into account the evolution of the unemployment stock and of the number of skilled unemployed workers, but takes the demographics $in'$ as given\textsuperscript{15}. Aggregate income is defined as market output net of both investment costs and the opportunity cost $r\kappa$ of unfilled vacancies. Investment costs comprise the cost of training the number $(u - u^*) p(\theta)$ of unskilled workers who find a match - the second addendum in equation (36) - plus the cost of opening new vacancies - equal to $\kappa$ times the flow of new vacancies $\theta'u' - u(\theta - p(\theta))$. Note that $\theta$, the lagged value of the control variable $\theta'$, enters the state space.

In what follows, rather than characterizing the social optimum for arbitrary initial conditions, we solve for the steady state.

The first order necessary conditions for the socially optimal reservation productivity

\textsuperscript{14}The condition for socially optimal investment is derived in appendix B.

\textsuperscript{15}Since the demographics in our model just ensures stationarity of the environment, it seems natural to assume that it cannot be controlled by the social planner.
and tightness are respectively

\[ bf(h) = \kappa + L_u + L_{us} \]

(37)

and

\[ \kappa \left( 1 + \frac{r}{p'(\theta)} \right) = A - (1 - G(b)) (h + L_u), \]

(38)

where \( L_u \) and \( L_{us} \) are the stationary partial derivatives of the value function. The above conditions are also sufficient for an optimum under our assumptions of strict concavity and homogeneity of the matching function.

The first equation implies that separation is efficient when revenues from production fall below the value of physical capital \( \kappa \) plus the social value of a trained unemployed worker. The latter can be decomposed into the sum of the shadow price \( L_u \) of one more unskilled unemployed worker in the unemployment pool plus the value \( L_{us} \) of replacing one skilled for one unskilled worker, keeping the total size of the pool constant.

The second condition implies that the social cost of posting a vacancy, given by the investment cost \( \kappa \) plus the carryover cost - adjusted for the reduction in the duration of unemployment - must equal the expected social return. The latter takes into account the social opportunity cost \( L_u \) in case production takes place and the worker does not return to the unemployment pool\(^{16}\).

By the envelope theorem and stationarity, the steady state social values of a skilled and unskilled unemployed worker are respectively

\[ L_{us} = \frac{p(\theta)}{r + p(\theta)} h \]

(39)

and

\[ L_u = \frac{p(\theta)}{r + p(\theta) (1 - G(b))} \left[ A - \kappa \left( 1 + \frac{r}{q(\theta)} \right) \right] - \frac{p(\theta)}{r + p(\theta)} h. \]

(40)

At constant total unemployment, the only benefit from one more skilled worker in the pool is the saving of the cost \( h \) if the worker finds a job.

\(^{16}\)One may rightly note that it is a trained, not an untrained, worker that does not reenter the unemployment pool. Yet, keeping aggregate unemployment constant the steady state number of skilled unemployed is independent of market tightness as can be seen from equation (5).
The social value \( L_u \) of one more (unskilled) unemployed worker, instead, is the expected flow of output net of vacancy posting costs and of the cost of training her when she is matched with a firm for the first time.

It is useful to rewrite equation (38) by making use of the fact that \( p(\theta) = \theta q(\theta) \). If we call \( \eta(\theta) \) the elasticity of \( p(\theta) \) with respect to \( \theta \) we can write (38) as

\[
\kappa \left( 1 + \frac{r}{q(\theta)} \right) = A - (1 - G(b)) (h + L_u) - \frac{r \kappa (1 - \eta(\theta))}{q(\theta) \eta(\theta)}.
\] (41)

Equations (40) and (41) together can then be used to rewrite the shadow value of an untrained unemployed worker as

\[
L_u = \frac{\kappa \theta (1 - \eta(\theta))}{q(\theta) \eta(\theta)} - \frac{p(\theta) G(b)}{r + p(\theta)} h.
\] (42)

Let us write the private job creation condition in a form comparable to equation (41). To this purpose let us define the difference between the asset value of a skilled and unskilled matched worker as \( e = E_a(h^\ast) - E_a(0) \). Using equations (6) and (12) privately optimal vacancy posting satisfies

\[
\kappa \left( 1 + \frac{r}{q(\theta)} \right) = A + G(b) U(h) - (1 - G(b)) (h^\ast - e) - \left( 1 + \frac{r}{p(\theta)} \right) U(h^\ast).
\] (43)

Under our assumption that investment in the decentralized equilibrium is socially optimal \( (h = h^\ast) \), we are now in a position to characterize the conditions for efficiency of the decentralized equilibrium conditional on a given level of training. In what follows, all expression are evaluated at the social planner optimum. Efficient vacancy posting requires the right hand sides of (41) and (43) to be equal, or

\[
(1 - G(b)) L_u + \frac{r \kappa (1 - \eta(\theta))}{q(\theta) \eta(\theta)} = \left( 1 - G(b) + \frac{r}{p(\theta)} \right) U(h^\ast) - (1 - G(b)) e.
\] (44)

Efficient job destruction requires the private and social values of a skilled unemployed worker to be the same or, comparing equations (21) and (37),

\[
U(h^\ast) = L_u + L_{us}.
\] (45)
One can use equations (16), (39) and (40) to rewrite (45) as

\[ \Psi(\beta) = (r + p(\theta))(1 - \beta)A - (r + p(\theta) - \beta p(\theta)G(b)) \left( 1 + \frac{r}{q(\theta)} \right) \kappa = 0. \] (46)

It is easy to check that it is \( \Psi(0) > 0 \) and \( \Psi(1) < 0 \). Since \( \Psi(.) \) is continuous the mean value theorem implies that (46) is satisfied for a value \( \beta^* \) of the sharing parameter in \((0,1)\). Let us assume that \( \beta \) takes exactly this value and derive the restrictions that this imposes on the differential \( e \). This requires solving the system formed by (39), (45) and (44) for \( e \) as a function of \( L_u \). The result is

\[ L_u = \frac{\kappa \theta (1 - \eta(\theta))}{q(\theta)\eta(\theta)} - \frac{p(\theta)}{r} \left[ \frac{r + p(\theta)(1 - G(b))}{r + p(\theta)} h^* - (1 - G(b)) e \right]. \] (47)

Hence, the differential \( e \) has to ensure equality of (42) and (47). It can easily be checked that this requires \( e = h^* \).

With \( \beta \) taking care of job destruction - i.e. aligning the social and private values of a skilled worker - efficiency requires untrained workers to pay for the full cost of the training. This ensures that private and social values coincide for unskilled workers too. But, unless \( \beta = 1 \), in the decentralized equilibrium unskilled workers pay for only a fraction of the total training cost \( h \), as can be seen from equation (15). Hence, the sharing parameter \( \beta \) alone is not sufficient to ensure full efficiency in this model. This would not be the case if there were just one worker type.

With homogeneous agents all that is required to achieve efficiency on both the job creation and job destruction margins is that the sharing parameter \( \beta \) satisfies the Hosios (1990) condition equating the private and social value of an unemployed worker. This can be easily checked by considering the case in which there is no investment in training; i.e. both \( h \) and \( L_u \) are zero and all workers are identical. Then, \( e = 0 \) is necessary and sufficient to equate the value of \( L_u \) in equation (42) and (47). If the sharing parameter is such as to ensure efficient separation then also job creation is efficient\textsuperscript{17}.

The result that the Hosios condition is not sufficient to ensure efficiency in models

\textsuperscript{17}The only difference with the respect to Hosios (1990) is that the optimal value of the sharing parameter \( \beta \) does not coincide with the elasticity of the probability of filling a vacancy due to the different bargaining solution adopted.
with heterogeneous agents is not new. Bertola and Caballero (1994) show that when firms with heterogeneous productivities can choose the rate of vacancy posting at a convex cost, job creation at more productive units is inefficiently low in the absence of firm-specific subsidies. Davis (1995) extends their result to the case of heterogeneity on both sides of the market. Shimer and Smith (2000) reach similar conclusions in a very general setting in which heterogeneous agents look for a match with endogenous and possibly non-stationary search effort. They show that efficiency can only be achieved by subsidizing (taxing) the search effort of agents who are more (less) productive than average. In all these papers, the inefficiency stems from the inability of the share parameter alone to provide the correct investment or search incentives to heterogeneous agents.

Our set up not only trivially extends the above result to the case in which heterogeneity is restricted to non-investing agents - workers in our case - but, more interestingly, sheds light on the mechanism through which heterogeneity matters.

To this effect, consider the case in which \( h > 0 \) and there are both skilled and unskilled workers in the unemployment pool. If the separation rate were exogenous, there would be just one active margin - the job creation one. We show in appendix B that there exists one value for the sharing parameter \( \beta \) that again ensures full efficiency. With workers’ facing no active economic decision, apart from participation, efficiency only requires that the private return to posting a vacancy coincides with its social counterpart.

This highlights the fact that heterogeneity is not sufficient to invalidate Hosios result. In the present model there are two types of unemployed workers and two active margins - job creation and job destruction - that affect matching opportunities for other searchers. As our previous discussion has shown, either workers’ heterogeneity or the existence of more than one active margin alone would not do. It is heterogeneity across active margins\(^{18}\) that drives the inefficiency of the decentralized outcome when the social planner has only one instrument - the bargaining share parameter - at her disposal. In our model a searching firm can meet either a skilled or an unskilled worker, but all separations release a skilled unit of labour. Hence equality between the social and the private return to vacancy posting does not imply efficient reservation productivity and viceversa. Only if

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\(^{18}\)If training were fully specific, heterogeneity across active margins would disappear and an appropriate value for the share parameter would ensure efficiency.
social and private values coincide for both skilled and unskilled unemployed workers are incentives correct on both the job creation and destruction margins.

5 Conclusion

This paper has analysed non-contractible firms’ investment in general human capital in a model of frictional unemployment. General training increases workers’ productivity with other employers but is vested in the worker on separation. This depresses investment as no return accrues to the firm on separation. We have shown that consensual layoffs, by obliging firms to share the total payoff from separation, improve employers’ incentives to train. The mechanism applies to all forms of general investment that is vested in the non-investing party. It applies equally to workers’ investment to improve product quality and develop new products that remain intellectual property of their employers.

We have also shown that, independently from underinvestment in training, the *laissez-faire* equilibrium is always inefficient for any given level of investment. The coexistence of skilled and unskilled workers implies that the Hosios (1990) condition fails to ensure equality between social and private values for both skilled and unskilled workers. Since workers are heterogeneous along the job creation and the job destruction margins the sharing parameter alone cannot ensure efficiency.
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Appendix A: Proofs of propositions

Proposition 2. If \((1 - \beta)U(h^*) < \beta \kappa\), then for

\[ F < \kappa - (1 - \beta)bf(h^*) \tag{48} \]

there exists \(z_r \in [b, \infty)\)

\[ F = \kappa - (1 - \beta)z_r f(h^*) \tag{49} \]

such that \(\forall z \leq z_r, J_p(z, h^*) = \kappa - F\).

Vice versa, if \((1 - \beta)U(h^*) > \beta \kappa\), then, \(\forall F\), there exists \(z_r \in [b, \infty)\) satisfying

\[ U(h^*) = \beta z_r f(h^*) \tag{50} \]

such that \(\forall z \leq z_r, E_a(z, h^*) = U(h^*)\).

Proof. Suppose, by contradiction, there is no \(z_r \in [b, \infty)\) such that either party’s outside option is binding. Then, by proposition 1, it has to be \(J_p(z, h^*) = (1 - \beta)z f(h^*)\) \(\forall z \in [b, \infty)\). By continuity of revenues in \(z\) then

\[ U(h^*) = bf(h^*) \tag{51} \]

and

\[ \kappa = (1 - \beta) bf(h^*). \tag{52} \]

But then (51) and (52) imply \((1 - \beta)U(h^*) = \beta \kappa\) which contradicts either assumption. The inequality \((1 - \beta)U(h^*) < \beta \kappa\) implies

\[ \frac{\kappa}{1 - \beta} > U(h^*) + \kappa = bf(h^*) \tag{53} \]

or

\[ \kappa > (1 - \beta) bf(h^*). \tag{54} \]
Hence, by continuity there exists $z_r > b$ such that

$$\kappa = (1 - \beta)bf(h^*)$$

as long as $F < \kappa - (1 - \beta)bf(h^*)$.

Symmetrically, it can be shown that the reverse inequality $(1 - \beta)U(h^*) > \beta\kappa$ implies that it is the worker’s outside option $U(h^*)$ which is binding for some $z_r \in [b, \infty)$. $z_r$ is unaffected by severance payments in this case as the worker’s outside option is not. ■

**Corollary 3.** If $(1 - \beta)U(h^*) < \beta\kappa$ and $F > \kappa - (1 - \beta)bf(h^*)$ then the firm’s payoff at time $t.3$ is given by

$$J_p(z, h^*) = (1 - \beta) \max\{zf(h^*), U(h^*) + \kappa\}.$$  \hspace{1cm} (56)

**Proof.** The inequality $F > \kappa - (1 - \beta)bf(h^*)$ implies that as long as $z \geq b$ the firm’s is better of sharing the payoff from continuation rather than firing the worker. So, $z_r \notin [b, \infty)$. Remembering that $bf(h^*) = U(h^*) + \kappa$, it also implies that when $z < b$, it is optimal for the firm to negotiate a voluntary severance payment that leaves the worker a share $\beta$ of the total payoff from separation $U(h^*) + \kappa$ rather than paying the legislated severance payment $F$. The inequality $(1 - \beta)U(h^*) < \beta\kappa$ implies that the worker would not leave voluntarily without such a payment. ■
Appendix B.

Socially optimal training

Let us define by $S_U(h)$ the social value of a trained unemployed worker and by $S_E^e(h)$ the expected social surplus associated with a matched skilled worker at time $t$. We can write

$$(p + r)S_U(h) = pS_E^e(h)$$

and

$$S_E^e(h) = f(h)\int_b^\infty zdG + G(b) (S_U(h) + \kappa).$$

Solving for $S_E^e(h)$ we can write

$$S_E^e(h) = \frac{r + p(\theta)}{r + p(\theta)(1 - G(b))} f(h)\int_b^\infty zdG + G(b)\kappa.$$  

The socially optimal level of training satisfies $1 = \partial S_E^e(h)/\partial h$ or

$$1 = \frac{r + p(\theta)}{r + p(\theta)(1 - G(b))} f'(h)\int_b^\infty zdG.$$  

It is straightforward to see that investment is always inefficiently low in the decentralized equilibrium as the right hand side of (32) is always smaller than the right hand side of (60) for any value of $\beta$.

Efficiency with exogenous separation rate

The socially optimal vacancy posting condition (41) can be rewritten as

$$\kappa \left(1 + \frac{r}{q(\theta)}\right) = A - (1 - G(b))(h + L_u) - \frac{r\kappa (1 - \eta(\theta))}{q(\theta)\eta(\theta)}.$$  

Under the assumption that the firm’s outside option is not binding when a matched with an unskilled worker is formed, the worker bears a share $\beta$ of the training cost. So the different between the asset values of matched skilled and unskilled workers is $e = \beta h^*$. The privately optimal job creation condition can be rewritten by replacing $U(h^*)$ and $e$.
in equation (43) using (16 to obtain
\[
\kappa \left(1 + \frac{r}{q(\theta)}\right) = (1 - \beta) \frac{r + p (1 - G(b))}{r + p (1 - \beta G(b))} A - (1 - G(b)) (1 - \beta) h. \tag{62}
\]

It is straightforward to verify that for $\beta$ increasing in the [0,1] interval, the right hand side of equation (62) decreases monotonically from a value larger than the (positive) right hand side of (61) to a negative value. The mean value theorem implies that there exists $\beta \in (0,1)$ that equates the right hand sides of the two equations decentralizing efficient job creation.

It is tedious but straightforward to prove that the same result applies if the firm’s outside option does bind when a match with an unskilled workers is formed.