$$X_{n+1} = a X_n \quad , a > 1$$

$X_* = 0$ unst. f.p.

TDFC:

$$x_{n+1} = a x_n + K (x_n - x_{n-1})$$

$$x_n = \mu^n \quad \mu_{1,2} = \frac{a+k}{2} \pm \sqrt{\left(\frac{a+k}{2}\right)^2 - k}$$

K: $x_{n+1} = a x_n + K_n (x_n - x_{n-1})$

k_n "random" $\quad K_n = \begin{cases} 0 & \text{w.prob } P_0 \\ k & \text{w.prob } P_1 \end{cases}$ IID

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Lyap. exp.

$$\Lambda = \left\langle \lim_{n \to \infty} \frac{1}{n} \ln \left| \frac{x_n}{x_0} \right| \right\rangle$$

$$= \left\langle \lim_{n \to \infty} \frac{1}{n} \sum_{\ell=1}^{n} \ln |z_\ell| \right\rangle \quad z_\ell = \frac{x_\ell}{x_{\ell-1}}$$

RDS: $x_{n+1} = a x_n + K (x_n - x_{n-\tau_n}) \quad \tau_n = 0, 1$

RMT:

$$\begin{pmatrix} x_{n+1} \\ x_n \end{pmatrix} = \begin{pmatrix} a+k_n & -k_n \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_n \\ x_{n-1} \end{pmatrix}$$

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$$z_{n+1} = a + k_n - \frac{k_n}{z_n}$$
$$= f_{k_n}(z_n)$$

$\rho_n(z) = \langle \delta(z - z_n) \rangle$ prob. dist. of z_n

FP eq.

$$\rho_{n+1}(z) = \int \underbrace{\langle \delta(z - f_{k_n}(z')) \rangle}_{p_0 \delta(z-a) + p_1 \delta(z - f_k(z'))} \rho_n(z') dz'$$
$$= p_0 \delta(z-a) + p_1 \int \delta(z - f_k(z')) \rho_n(z') dz'$$

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$$\rho_\infty(z) = p_0 \sum_{l=0}^{\infty} p_1^l \delta(z - f_k^{(l)}(a))$$

Lyap. exp.

$$\Lambda = \int \mathrm{Ln}|z| \rho_\infty(z) dz$$
$$= p_0 \sum_{l=0}^{\infty} p_1^l \mathrm{Ln}|f_k^{(l)}(a)|$$

Orbit of f_k: $f_k^{(l)}(a)$?

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$$f_k^{(\ell)}(a) = \frac{\xi_\ell}{\xi_{\ell-1}}$$

$$\xi_{n+1} = (a+k)\xi_n - k\xi_{n-1} \qquad \xi_0 = a$$

$$\xi_{-1} = 1$$

$$\xi_n = \mu_1 \frac{a-\mu_2}{\mu_1-\mu_2} \mu_1^n + \mu_2 \frac{a-\mu_1}{\mu_2-\mu_1} \mu_2^n$$

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$$\Lambda = P_0^2 \sum_{\ell=0}^{\infty} P_1^\ell \, \mathrm{Ln}|\xi_\ell|$$

If $\mu_1 = \mu_2^* = R_k e^{i\phi_k}$

$$\xi_n = r_0 R_k^n \sin(n\phi_k + \delta_0)$$

$$\Lambda = \dots + P_0^2 \sum_{\ell=0}^{\infty} P_1^\ell \, \mathrm{Ln}|\sin(\ell\phi_k + \delta_0)|$$

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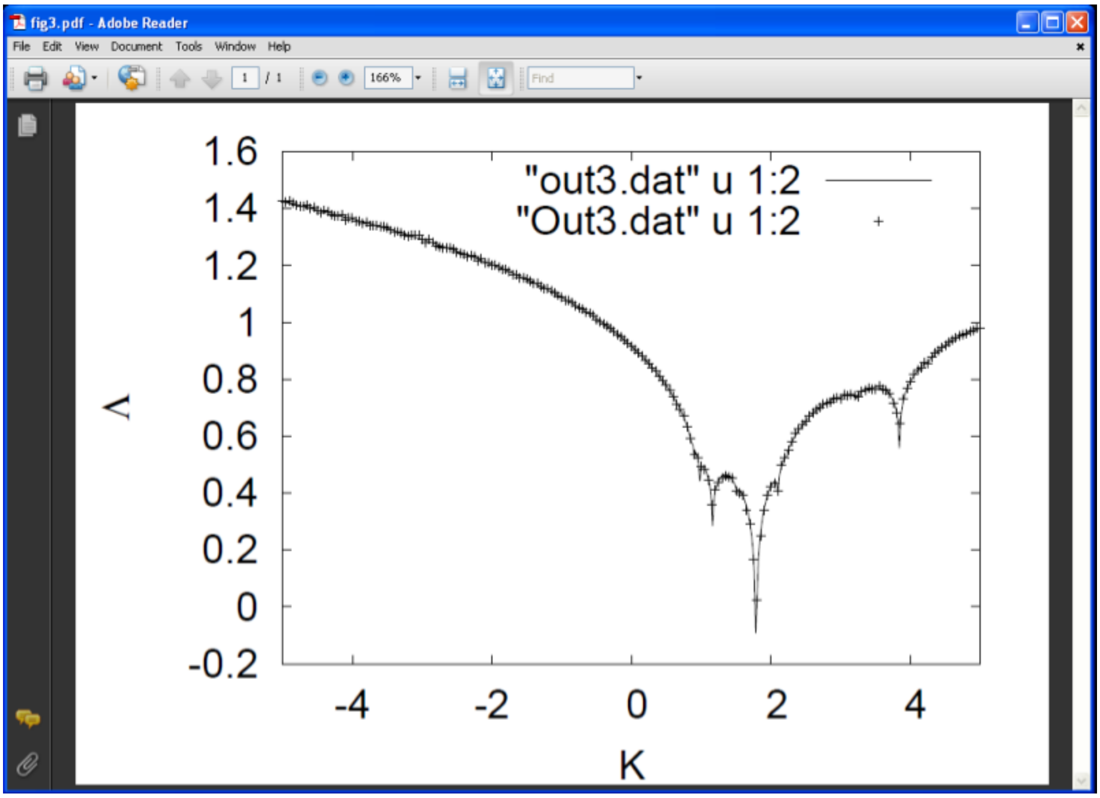

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Structure of Λ

$$g(x)=\sum_{\ell=1}^{\infty}\left(\frac{1}{2}\right)^{\ell}\ln|\sin(\ell x)|$$

$\frac{x}{2\pi}\in\mathbb{Q}$ log sing.

$$\int_0^{\pi} g(x)\,dx=\sum_{\ell=1}^{\infty}\left(\frac{1}{2}\right)^{\ell}\underbrace{\int_0^{\pi}\ln|\sin(\ell x)|\,dx}_{-\pi\ln 2}$$

$$=-\pi\ln 2$$

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