Detecting phase synchronisation in time series data sets

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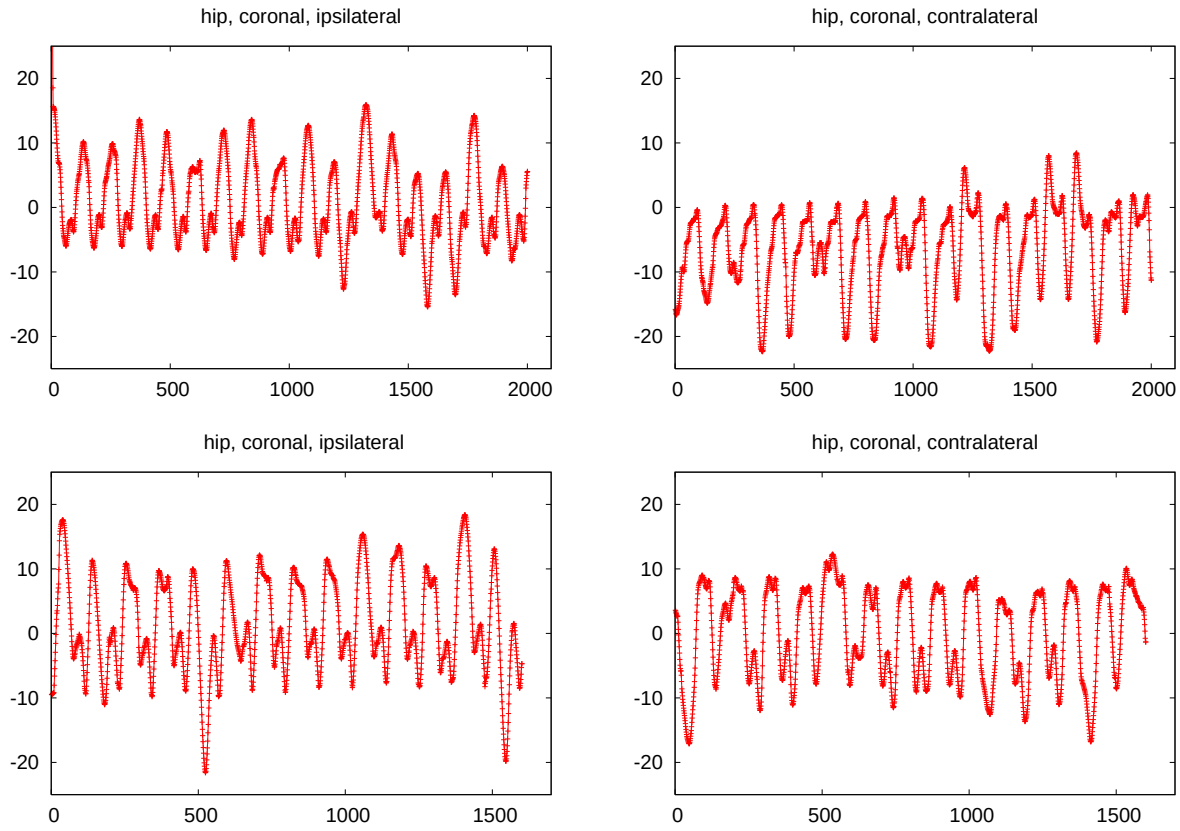
0 Content

1 Motivation

Movement data (time series)

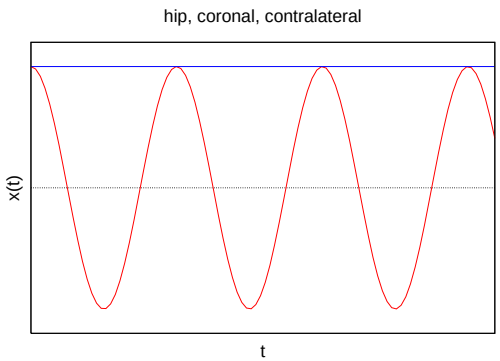


- Non stationary
- "Chaotic" oscillation
- Classification

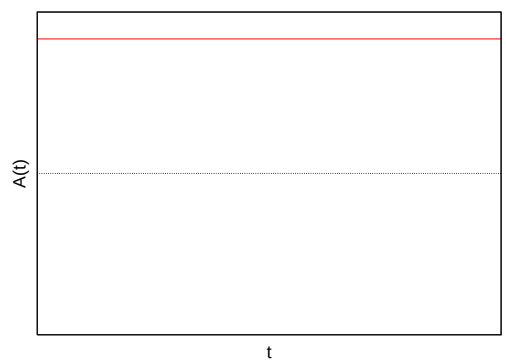
2 What is a phase ?

Idea: Rewrite the time signal $x(t)$ as $x(t) = A(t)\cos(\Theta(t))$ where $A(t)$ is called the amplitude and $\Theta(t)$ the phase. But how (e.g. lack of uniqueness etc.)?

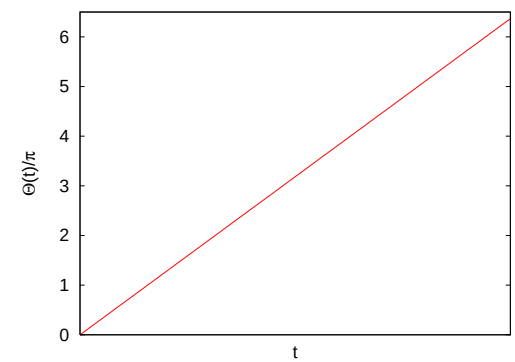
Harmonic signal: $x(t) = a\cos(\omega t)$ $(a, \omega > 0)$



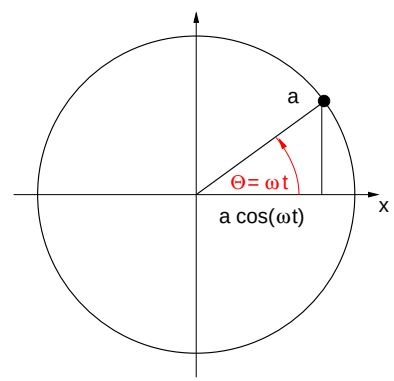
amplitude: $A(t) = a$



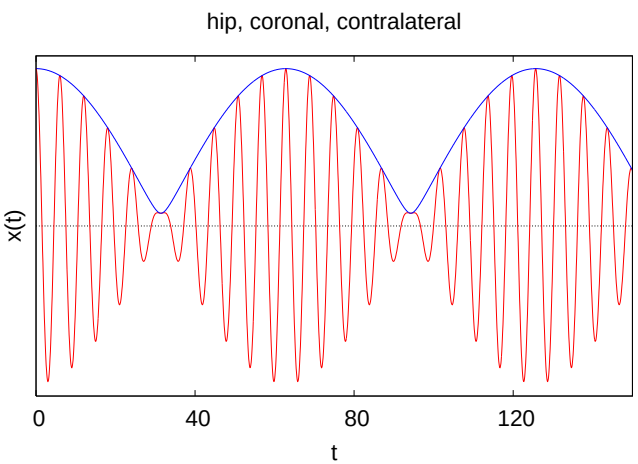
phase: $\Theta(t) = \omega t$



Observe: $x(t) = \mathrm{Re}(z(t))$ where
$z(t) = a\exp(i\omega t) = a\cos(\omega t) + ia\sin(\omega t)$
and $\Theta(t) = \arg(z(t))$.



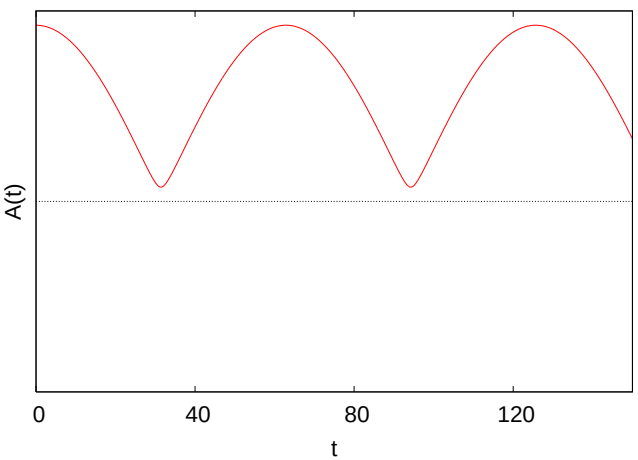
Beat: $x(t) = a\cos(\omega_1 t) + b\cos(\omega_2 t)$ $(a > b > 0)$.

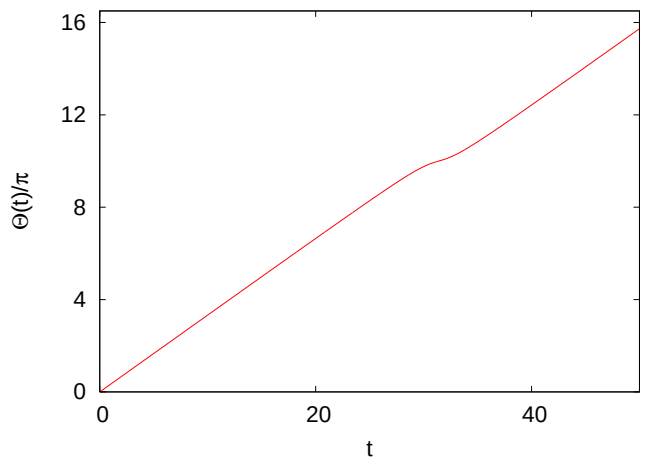


How to express this as $x(t) = A(t)\cos(\Theta(t))$ in a meaningful way?

Use $x(t) = \mathrm{Re}(z(t))$ with

$$
\begin{aligned}
z(t) &= A(t)\exp(i\Theta(t)) = a\exp(i\omega_1 t) + b\exp(i\omega_2 t) \\
A(t) &= (a^2 + 2ab\cos(\Delta\omega t) + b^2)^{1/2} \\
\Theta(t) &= \omega_1 t + \arctan(b\sin(\Delta\omega t)/(a + b\cos(\Delta\omega t)))
\end{aligned}
$$





Recall:

$$\begin{aligned} x(t) &= a\cos(\omega t) = a/2\exp(i\omega t) + a/2\exp(-i\omega t) \\ \rightarrow \quad z(t) &= a/2\exp(i\omega t) \text{ (positive frequency component !)} \end{aligned}$$

General case:

$$\begin{aligned} x(t) &\rightarrow \hat{x}(\omega) = \int_{-\infty}^{\infty} \exp(-i\omega t)x(t)dt \\ &\rightarrow \hat{z}(\omega) = \begin{cases} \hat{x}(\omega) & \text{if} \quad \omega > 0\,, \\ 0 & \text{if} \quad \omega \leq 0\,. \end{cases} \\ &\rightarrow z(t) = \int_{-\infty}^{\infty} \exp(i\omega t)\hat{z}(\omega)d\omega/(2\pi) \\ &\rightarrow \Theta(t) = \arg(z(t)) \text{ phase} \\ & \quad\; A(t) = 2|z(t)| \text{ instantaneous amplitude} \end{aligned}$$

Properties: (→ L. Cohen et al., Sig. Process. **79** (1999) 301)

$$z(t) = x(t) + i/\pi(\mathcal{H}x)(t) = x(t) + i/\pi \int_{-\infty}^{\infty} x(s)/(t-s)ds$$

- Amplitude continuity: a small change in the value of the signal $x(t)$ should induce a correspondingly small change in the instantaneous amplitude $A(t)$.
- Phase independence of scale: multiplying the real signal $x(t)$ by a real positive constant c should have no effect on the instantaneous phase and should multiply the instantaneous amplitude by the same constant.
- Harmonic correspondence: the instantaneous amplitude and phase of a pure sinusoid $a\cos(\omega t + \phi)$ should be given, respectively, by $A(t) = a$, $\Theta(t) = \omega t + \phi$.

3 Phase synchronisation

Coupled chaotic oscillators

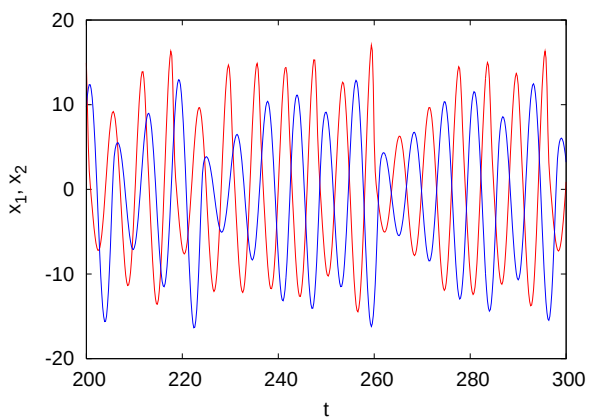
(→ M. Rosenblum et al., Phys. Rev. Lett. **76** (1996) 1804)

$$\begin{aligned}\dot{x}_1 &= -1.015y_1 - z_1 + C(x_2 - x_1)\\ \dot{y}_1 &= 1.015x_1 + 0.15y_1\\ \dot{z}_1 &= 0.2 + z_1(x_1 - 10)\end{aligned}$$

$$\begin{aligned}\dot{x}_2 &= -0.985y_2 - z_2 + C(x_1 - x_2)\\ \dot{y}_2 &= 0.985x_2 + 0.15y_2\\ \dot{z}_2 &= 0.2 + z_2(x_2 - 10)\end{aligned}$$

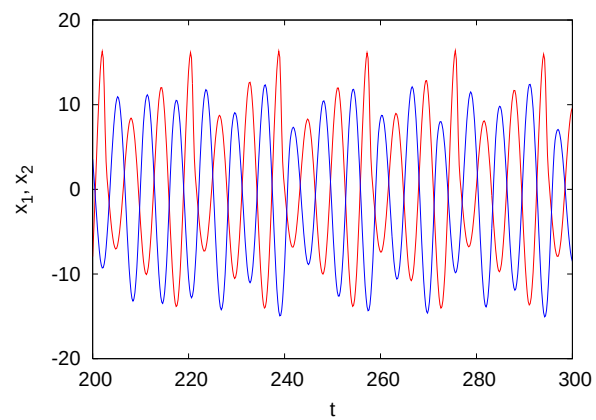
small coupling

$C = 0.015$



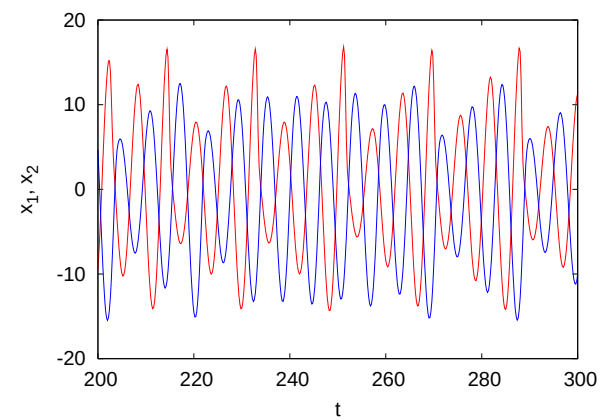
intermediate coupling

$C = 0.027$

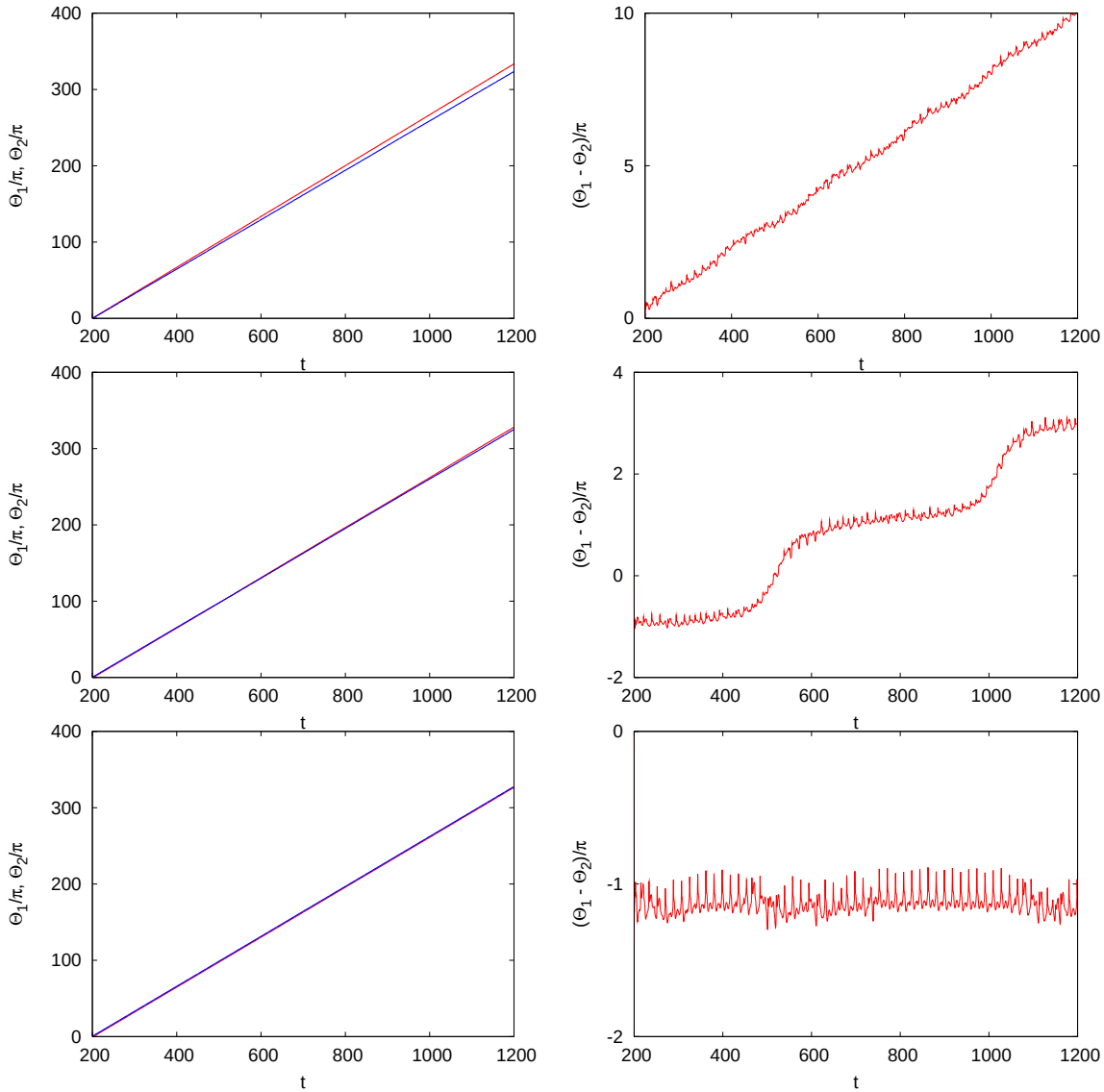


moderate coupling

$C = 0.035$



Phase representation



small coupling

$C = 0.015$

desynchronised

$$|\Theta_1(t) - \Theta_2(t)| \sim t$$

intermediate coupling

$C = 0.027$

phase slips

moderate coupling

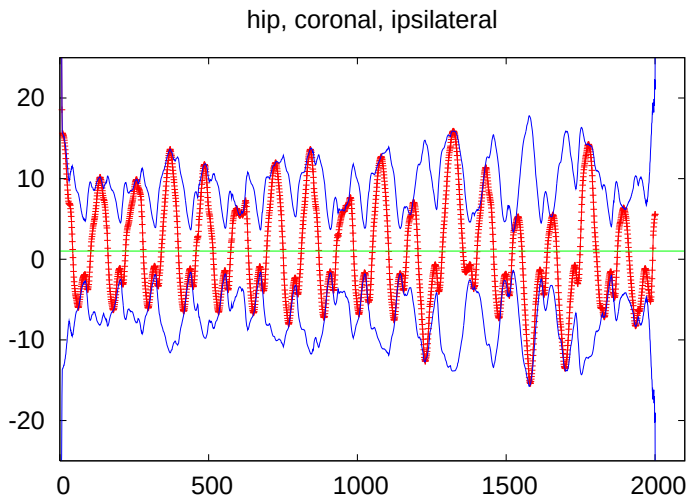
$C = 0.035$

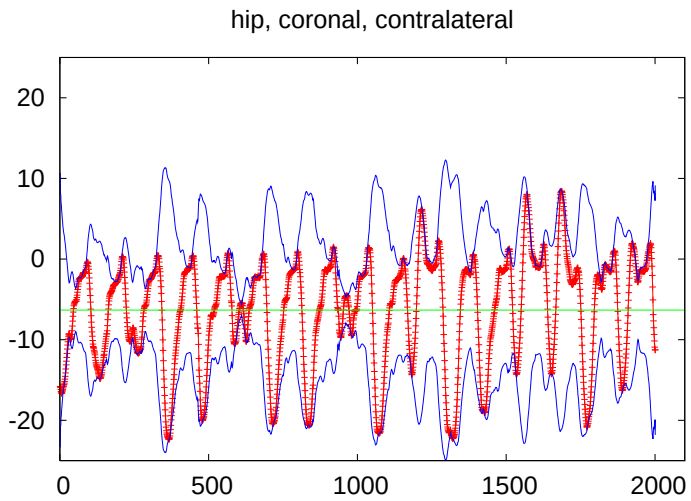
phase synchronisation

$$|\Theta_1(t) - \Theta_2(t)| < \text{const.}$$

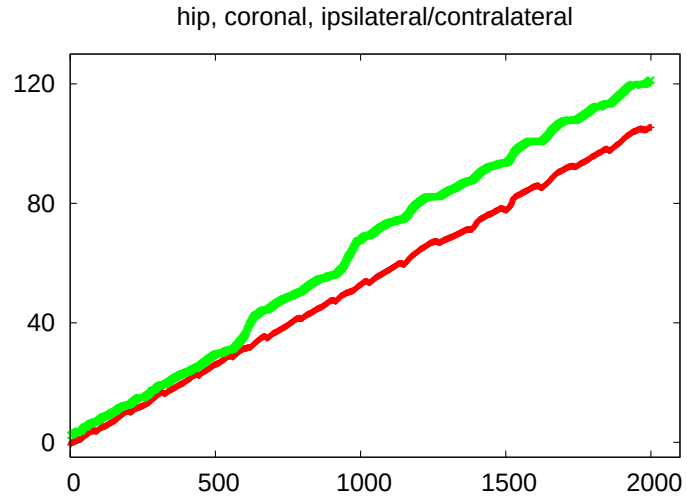
4 Analysis of movement data

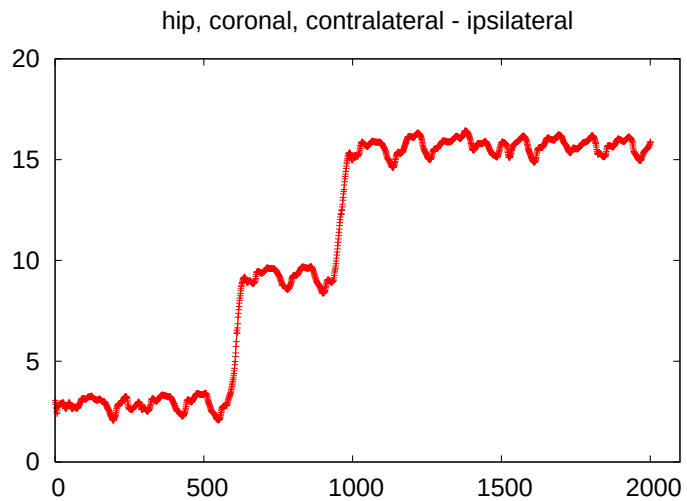
time series



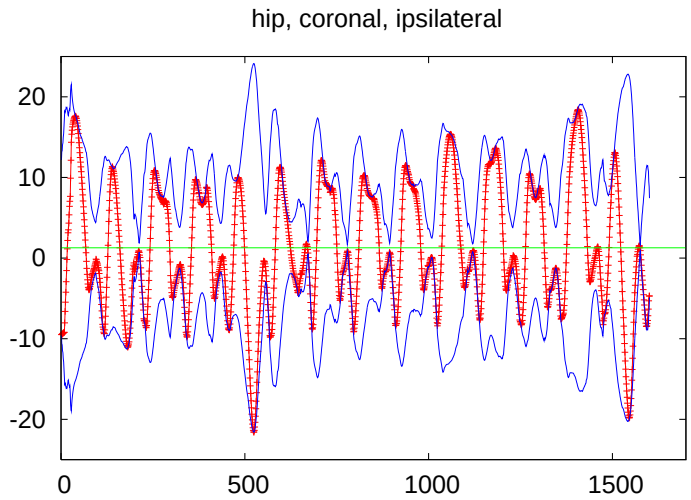


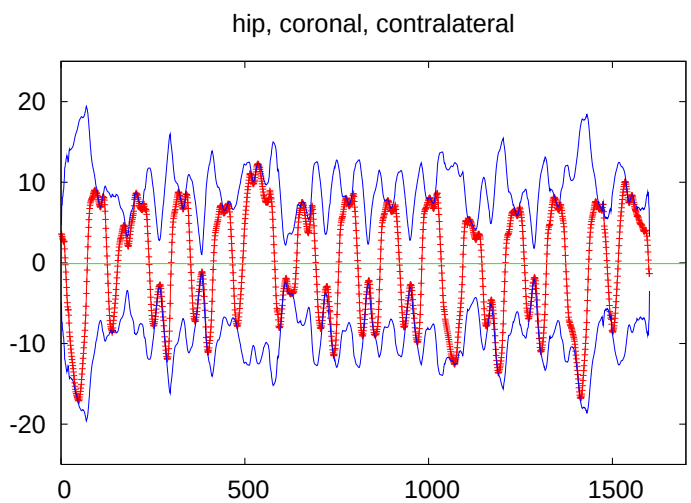
phases and phase difference



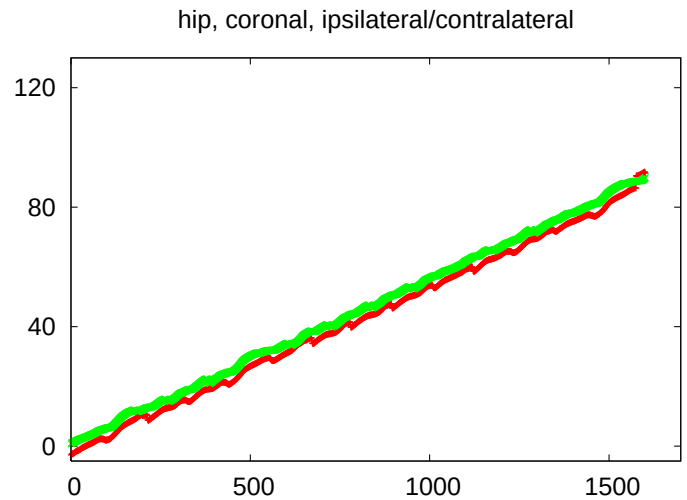


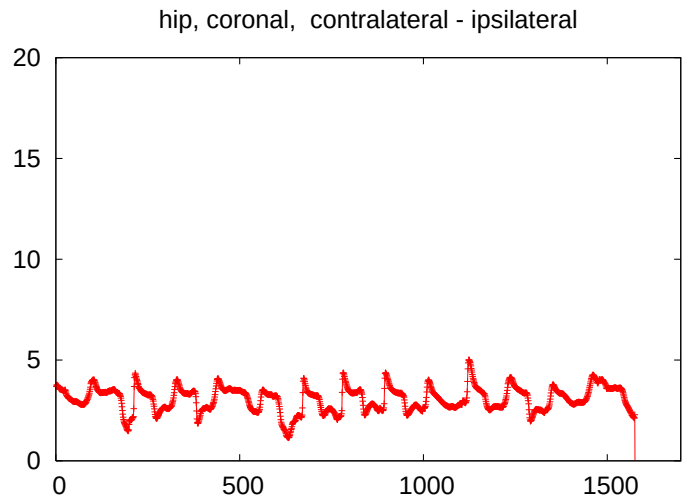
time series





phases and phase difference

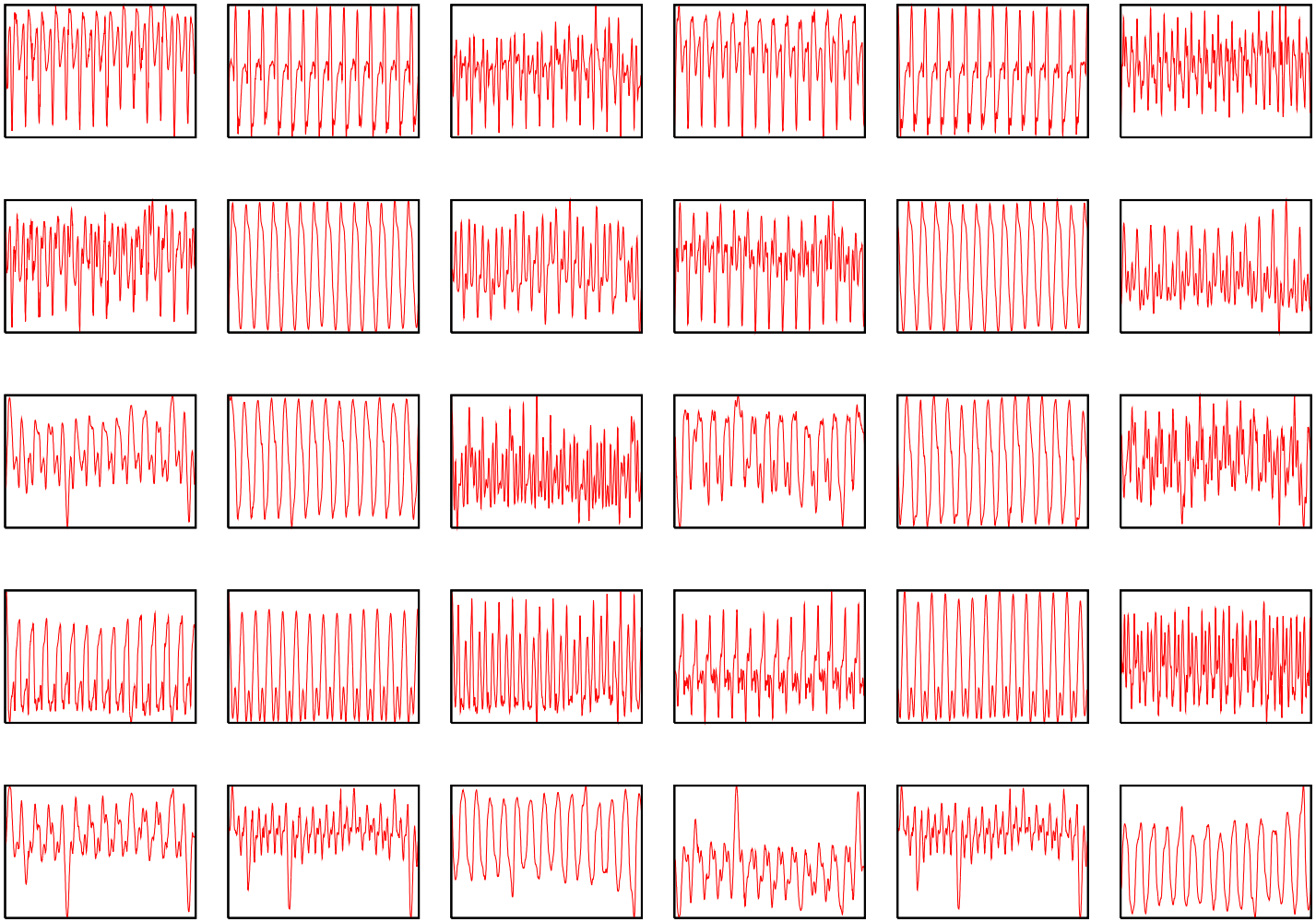




5 Epilogue

Structure of the data set

- Position: Ankle, Foot, Hip, Knee, Pelvis
- Coordinate: Coronal, Sagittal, Horizontal
- Orientation: Ipsilateral, Contralateral



Normalisation