Let X_1, X_2, ... be random processes with paths in the Skorokhod space $D[0,\infty)$ and ξ_1, ξ_2,... positive random variables such that the pairs (X_1, ξ_1), (X_2, ξ_2), ... are i.i.d. We call the random process $(Y(t))$ defined by $Y(t) := \sum_{k\geq 0} X_{k+1}(t-\xi_1-\ldots-\xi_k)1_{\{\xi_1+\ldots+\xi_k\leq t\}}$, $t \geq 0$ 'random process with immigration at the epochs of a renewal process' and advocate using this term instead of a more familiar term 'renewal shot noise'.

We focus on weak convergence of random processes with immigration. There are four different regimes of convergence and each of these requires a separate treatment:

(I) the distributions of $Y(t)$ converge weakly on its own, i.e., neither normalization nor centering is needed;

(II) the distributions of $Y(t)$ converge weakly after normalization and possibly centering;

(III) the distributions of $Y(t)$ converge weakly after centering without normalization;

(IV) the random variables $Y(t)$, properly scaled, converge in probability or almost surely to random variable with a nondegenerate distribution.

In this talk, assuming that X_k and ξ_k are independent, weak convergence of $(Y(t))_{t\in\mathbb{R}}$ in the first regime will be described. In the case when the distribution of ξ_1 is nonlattice and has finite mean, we provide sufficient conditions for the weak convergence of the finite-dimensional distributions of $(Y(u+t))_{u\in\mathbb{R}}$ as $t\to\infty$ and sufficient conditions for the weak convergence of the same processes in the Skorokhod space endowed with the J_1-topology. The limits are stationary processes with immigration. In the case when the functions $\mathbb{P}\{\xi_1 > t\}$ and $Var[X_1(t)]$ are regularly varying at infinity of negative index strictly larger than -1 and asymptotically equivalent we derive sufficient conditions for the weak convergence of the finite-dimensional distributions of $(Y(ut))_{u>0}$ as $t\to\infty$. The limits are conditionally Gaussian processes, and any version of these processes takes values in the Skorokhod space $D(0,\infty)$ with probability less than one.

The talk is based on a recent paper with A. Iksanov (Kyiv) and M. Meiners (Darmstadt).