Decomposing Complete Hypergraphs

A well-known theorem of Graham and Pollak states that it is not possible to partition a complete graph on n vertices into fewer than $n-1$ complete bipartite graphs. The natural generalization for r-graphs is the following question: what is the least number $f_r(n)$ such that the complete r-graph on n vertices can be partitioned into $f_r(n)$ complete r-partite r-graphs? In 1986, Alon showed that $f_3(n) = n - 2$, and more generally for $r > 3$ that $A_r n^{\lfloor \frac{r}{2} \rfloor} \leq f_r(n) \leq B_r n^{\lfloor \frac{r}{2} \rfloor}$, for some constants $A_r < B_r$. The asymptotic bounds for f_r have not improved since. In this talk, we discuss the case of 4-graphs and some other related problems.