Some theorems and conjectures about extremal finite set structures

A.J.W. Hilton

I shall discuss a number of recent analogues of two theorems of mine from 1976. One of these, generalizing the Erdos-Ko-Rado theorem, is that if $\mathcal{A}_1, \dots, \mathcal{A}_t$ are t cross-intersecting families of k-subsets of $\{1, 2, \dots, n\}$, where $k \leq \frac{n}{2}$, then

$$|\mathcal{A}_1| + \ldots + |\mathcal{A}_t| \leq \max \left\{ \binom{n}{k}, \; t\binom{n-1}{k-1} \right\}.$$

Here "cross-intersecting" means that if $A_i \in \mathcal{A}_i$ and $A_j \in \mathcal{A}_j$ and $i \neq j$, then $A_i \cap A_j \neq \phi$.

The other is that if $\mathcal{A}$ and $\mathcal{B}$ are two mutually strictly incomparable complement-free families of subsets of $\{1, 2, \dots, n\}$, then

$$|\mathcal{A}| + |\mathcal{B}| \leq 2^{n-1}.$$

Here "$\mathcal{A}$ is complement-free" means that $\mathcal{A}$ does not contain any set and its complement. "$\mathcal{A}$ and $\mathcal{B}$ are mutually strictly incomparable" means that if $A \in \mathcal{A}$ and $B \in \mathcal{B}$ then $A \not\supsetneq B$ and $B \not\supsetneq A$.

The recent analogues are joint work by myself, Jie Zheng and John Goldwasser.

a.j.w.hilton@reading.ac.uk

Reading and QMUL