Diameters of random Cayley graphs

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Given a group G and a subset S of the elements of G, the Cayley graph $\Gamma = \Gamma(G; S)$ of G with respect to S has the elements of G as its vertex set and for $g \neq h$ it has an edge between g and h if and only if $hg^{-1} \in S$ or $gh^{-1} \in S$.

The model $\mathcal{G}(G, p)$ is the probability space of all graphs $\Gamma(G; S)$ in which every element of G is assigned to the set S independently at random with probability p. This model has many similarities with the model $\mathcal{G}(n, p)$, which is the probability space of all graphs with vertex set $\{1, 2, \ldots, n\}$ in which every edge appears independently at random with probability p.

In this talk we will concentrate on the diameter of random regular graphs. It is known that for every $\varepsilon > 0$ a graph from $\mathcal{G}(n, p)$ with high probability has diameter at most 2 if $p \geqslant \sqrt{\frac{(2+\varepsilon)\log n}{n}}$ and diameter greater than 2 if $p \leqslant \sqrt{\frac{(2-\varepsilon)\log n}{n}}$. It is natural to ask whether this result generalises to random Cayley graphs. The answer is positive but it turns out that the point at which this change occurs depends on the family of groups we are working with.

In the talk I will explain how the result for random graphs is proved and then sketch some parts of the proof of the result for random Cayley graphs.