Abstract For Talk
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Abstract

My project deals with critical systems coupled with random couplings to exponentially relaxing systems. A couple of example systems were studied numerically.

I investigated a recursive dynamical system of the Kaplan-Yorke type of the form:

$$f : \begin{cases} x_{n+1} = T(x_n) \\ y_{n+1} = \lambda y_n + h(x_n) \end{cases} \tag{1}$$

Where I chose the recursive logistic map for x_{n+1} and $h(x)$ such that $h(x) = \xi_n x_n$ where ξ_n is uncorrelated noise, chosen randomly on $[-1, 1]$. Further, I input different λ between $[.1, .99]$, which were focused later, so as to see the behaviour as λ approaches 1. Note, $\lambda = e^{-\tau}$. Lastly, I rescaled this system by $\tau^{1/2}$. Therefore, I can re-write f such that:

$$f : \begin{cases} x_{n+1} = 1 - \mu x_n^2 \\ y_{n+1} = \lambda y_n + \xi_n x_n \tau^{1/2} \end{cases} \tag{2}$$

with $\mu = 1.401155189\ldots$ (the Feigenbaum critical point, and 2, and $x(0)$ is chosen randomly on $[-1, 1]$. This is the system that we have chosen and analysed throughout the past year.

I initially analysed my data by simply looking at histograms of these plots and then I furthered my research numerically. I found that this system, with both μ-values, to be q-Gaussian distributed with q-values less than 1. I continued to cascade this model, but was met with little reward.

I continued this research using the recursive version of the 4th order Chebyshev Polynomial in place of x_{n+1}, i.e.,

$$x_{n+1} = 8x_n^4 - 8x_n^2 + 1 \tag{3}$$

I did this because higher-order Chebyshev Polynomials are analogous to chaotic systems. I had similar results as with the previous systems, as expected.

I will be discussing my work on these systems including possible meanings and explanations of each system and their results.