Optimal Impartial Selection
Proposal for a PhD project
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Impartial selection is the problem of selecting some of the members of a group of individuals based on nominations from other members of the group, in such a way that no individual can influence their own chance of being selected. The problem was first considered independently by Holzman and Moulin [2013] and by Alon et al. [2011] and has since then been studied quite extensively in economics, mathematics, and computer science. It has applications in peer review, committee elections, performance appraisals, and many other situations where members of a group are selected for an award or assigned a task. Of particular interest are optimal mechanisms, which subject to impartiality select individuals who receive as many nominations as possible.

If we identify individuals with the vertices of graph and nominations with directed edges in that graph, a mechanisms for impartial selection is given by a function that maps every directed graph to a probability distribution over its vertices and satisfies certain constraints across the set of all graphs with the same number of vertices. A mechanism is optimal if among all such functions it maximizes the minimum expected overall indegree of the set of vertices selected, where the minimum is taken over the set of all graphs.

Optimal impartial mechanisms are currently known only for the selection of a single individual [Fischer and Klimm, 2015] and in the limit as the number of individuals to be selected goes to infinity [Alon et al., 2011]. For all other cases upper and lower bounds have been given on the maximum quality of the individuals selected [Bjelde et al., 2017]. These results were obtained using fairly elementary techniques from discrete probability and linear optimization.

A PhD project on optimal impartial selection could for example pursue better upper and lower bounds for the selection of two or more individuals, seek improvements for the special case without abstentions, where each individual must submit at least one nomination, or study mechanisms for a more general problem where nominations can vary in intensity.

References


