Despite the rapid development of data analysis in the last few years, a key open challenge remains in interpreting and drawing conclusions based on the results of the analysis. This is problem is exacerbated by the “black-box” methods such as deep learning and functional methods, which while highly successful can have problematic or non-intuitive interpretations. This project will be to develop methods to find stable structures in data – that is set-theoretic structures which are stable under some perturbation model in sensible and rigorous way. The general approach will be based on applied topology, particularly on techniques based on (persistent) (co)homology as well as strengthen connections with homotopy theory.

**Stable Topological Representatives:** Algebraic invariants based on persistence, in particular persistent homology and cohomology classes, have been shown to be stable in a number of different ways. This is especially useful when these invariants are computed from finite and noisy data. Building on the stability of the algebraic invariants, the goal of this project is to understand when can these stable invariants can be stably mapped back to the underlying data through their representatives, find obstructions where this is impossible, and quantify how stable the mapping is.

**Topological Constraints in Optimization:** By understanding the mapping from algebraic invariants back to the underlying topological space, this project will examine applications of incorporating topological constraints into existing machine learning pipelines. The prototypical example is regularizing deep neural networks in order to promote certain global geometric features. This includes applications such as surface reconstruction with high codimension (which appear in time varying point clouds) and to deal with missing data (where topological constraints can be used to deal with the “holes” in the data).

**Quantifying Instability:** This project’s goal is to develop techniques to quantify instability under different models. This will look at how stability (and instability) translate across different perturbation models and how can this instability be visualized with respect to data.

These questions will combine experimentation with data along with theoretical work. The work will combine algorithms development and require at least some familiarity with programming and it would be highly desirable to have some previous exposure to topology and/or geometry. The goal is to prove theorems when stable structures exist, how to quantify how stable structures are and to understand this mapping back to the data in order to develop numerous possible applications including optimization and visualization.