Unique Continuation for Geometric Wave Equations, and Applications to Relativity, Holography and Controllability.

Supervisor: Arick Shao

Research Group: Geometry & Analysis

Funding: For September 2021 entry: Funding may be available through QMUL Principal's Postgraduate Research Studentships, School of Mathematical Sciences Studentships, and EPSRC DTP, in competition with all other PhD applications.

Studentships will cover tuition fees, and a stipend at standard rates for 3-3.5 years.

We welcome applications for self-funded applicants year-round, for a January, April or September start.

Project description:

A wide variety of phenomena in science, economics, and engineering are mathematically modelled by partial differential equations, or PDEs. Wave equations form an important subclass of PDEs and are found within many fundamental equations of physics, such as the Maxwell equations (electromagnetics), Yang-Mills equations (particle physics), Einstein field equations (gravitation), and Euler equations (fluid dynamics).

A basic problem in PDEs, and for wave equations in particular, is to find a unique solution given appropriate data. This can be interpreted as being able to "predict the future" given initial conditions for a system. On the other hand, in settings where the equation may not always be solved, it remains pertinent to ask whether solutions, if they exist, remain unique; this is the problem of unique continuation.

On the abstract side, the focus of the project is on studying unique continuation questions in "degenerate" settings, for which the classical theory fails to apply. One particular goal is the development of robust geometric techniques that apply to a wide variety of curved settings.

However, the bulk of the project will deal with applying the results and techniques of this unique continuation theory toward other problems in PDEs and physics:

One ongoing direction of this research project is to apply unique continuation techniques to study holographic principles in theoretical physics. This is largely motivated by the AdS/CFT correspondence, a tremendously influential idea in theoretical physics that is also currently lacking rigorous mathematical foundations.

Another arc of this project deals with applying ideas from unique continuation theory, in particular Carleman estimates, toward problems in control theory and inverse problems. Given the prevalence of waves in physics, both control and inverse problems for waves are well-connected to important questions in science and engineering.

References

A. Shao, *On Carleman and observability estimates for wave equations on time-dependent domains, (preprint)* arXiv: http://www.arxiv.org/abs/1805.07859

G. Holzegel, *A. Shao, Unique continuation from infinity in asymptotically Anti-de Sitter spacetimes II: Non-static boundaries*, Comm. Partial Differential Equations, 42 (2017), 1871–1922

G. Holzegel, A. Shao, *Unique continuation from infinity in asymptotically Anti-de Sitter spacetimes*, Commun. Math. Phys., 347 (2016), 1–53

Further information:

How to apply

Entry requirements

Fees and funding