

Random Walks in Restricted Domains and Standard Young Tableaux

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Research Group: [Dynamical Systems and Statistical Physics](#)

Funding: For September 2021 entry: Funding may be available through QMUL Principal's Postgraduate Research Studentships, School of Mathematical Sciences Studentships, and EPSRC DTP, in competition with all other PhD applications.

Studentships will cover tuition fees, and a stipend at standard rates for 3-3.5 years.

We welcome applications for self-funded applicants year-round, for a January, April or September start.

Project description:

The aim of this project is to study the connections between Standard Young Tableaux (SYT) and Random Walks using enumerative and bijective methods. Combinatorial interpretations of SYT of bounded height have recently received much attention; a good survey of this active area in combinatorics is given in [1].

SYT of bounded height can both be represented using specific random walks in the positive orthant of \mathbb{Z}^d and a certain type of coloured Motzkin paths. This connection is understood well, and explicit bijections have been constructed [2].

In dimensions $d=2$ and $d=3$ there is a well-known connection with Dyck and Motzkin paths. Intriguingly, recent enumerative work [3] found that restricting the domain for the random walks is in some sense equivalent to considering lattice paths in a finite strip. This result is based on equinumeracy shown by comparing counting formulas

Very recently [4] a bijective proof has been found, opening up a much promising avenue for further research. In particular, the connection to SYT has not been clearly identified. The main open problem to be worked on is identifying an appropriate statistics for SYT. Moreover, it is unclear how to generalise these results to higher dimensions, and hence to SYT of arbitrary bounded height.

References

[1] Marni Mishna. On standard Young tableaux of bounded height. Book Chapter in Recent Trends in Algebraic Combinatorics, Helene Barcelo, Gizem Karaali, Rosa Orellana (Eds.) Springer 2019 (pp. 281-303)

[2] Sen-Peng Eu, Tung-Shan Fu, Justin T. Hou, and Te-Wei Hsu. Standard Young tableaux and colored Motzkin paths. *J. Combin. Theory Ser. A*, 120(7):1786-1803, 2013.

[3] Paul R. G. Mortimer and Thomas Prellberg. On the Number of Walks in a Triangular Domain. *Electron. J. Combinat.* 22:P1.64, 2015

[4] Julien Courtiel, Andrew E. Price, and Irene Marcovici. Bijections between walks inside a triangular domain and Motzkin paths of bounded amplitude. Preprint 2020, <https://arxiv.org/abs/2007.08868>

Further information:

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