Magnitude, persistence, and homology: theory and applications

Supervisor: Nina Otter

Research Group: Statistics and Data Science

Project description:

Magnitude is a cardinality-like invariant of a metric space, akin to the Euler characteristic of a topological space. Magnitude was introduced around 2010 by Tom Leinster and is currently the object of intense research, since it has been shown to encode many known invariants of metric spaces.

In topology, it is known that the Euler characteristic is related to the singular homology of a topological space, since, under some finiteness assumptions, one can express the Euler characteristic as the alternating sum of the Betti numbers of the space. One thus says that singular homology “categorifies” the Euler characteristic. Recently, Leinster and Shulman have proposed “magnitude homology”, a homology theory for metric spaces that categorifies magnitude when the metric space is finite. However, magnitude homology fails to categorify magnitude for infinite metric spaces, and the problem of finding the right categorification of magnitude is currently still open.

An open question in the field was whether magnitude homology is related to persistent homology, one of the most successful methods in Topological Data Analysis. This question was answered in the affirmative in [O], in which a further homology theory for metric spaces, called “blurred magnitude homology”, was introduced to establish this connection. Several considerations make it reasonable to think that blurred magnitude homology captures more refined information about the geometry and topology of a metric space than magnitude homology. Blurred magnitude homology has motivated the definition of a new type of invariant of metric spaces, called “persistent magnitude”, which has already proven very successful in the study of datasets exhibiting self-similar properties [GH, KMO].

The goal of this project is to study theoretical properties of magnitude homology and blurred magnitude homology, as well as their applications to real-world data sets exihibiting self-similar properties, such as the growth of fungi, and certain types of stochastic processes. Suitable candidates will have a strong background in algebraic topology or related fields.

References

- [GH] D. Govc, R. Hepworth, Persistent magnitude, Journal of Pure and Applied Algebra , 225:3, 2021
- [KMO] S. Kalisnik, M. O'Malley, N. Otter, Alpha magnitude, in preparation
- [O] N. Otter, Magnitude meets persistence. Homology theories for filtered simplicial sets, to appear in Homology, Homotopy and Applications

Further information:

[How to apply]

[Entry requirements]

[Fees and funding]