Geometric Analysis of Interfaces

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Research Group: Geometry, Analysis & Gravitation

Project description:

This project will investigate the relationship between the differential geometry of interfaces in the zero width limit and mean curvature flow.

This project aims to develop the theoretical framework of the singularity formation of the Mean Curvature Flow (MCF) and the Allen-Cahn flow (ACF). The MCF is a geometric flow that describes the motion of a surface. It was introduced by Mullins as a model for the formation of grain boundaries in annealing metals and appears as the flow to equilibrium of soap films, the motion of embedded branes in approximations of the renormalisation group flow in theoretical physics, boundaries of Ginzburg- Landau equations of simplified superconductivity and as a method of denoising in image processing.

The aim is to develop an analysis of the fine structure of singularities of the zero set of ACF, characterise their formation geometrically and analytically, and find ways to deal with such singularities. The development of this theory will transform applications of geometric and singular peturbations flows across geometry, analysis, the theory of partial differential equations, topology, mathematical physics and beyond.

The specific PhD project will be to develop a new monotonicity formula for the Allen-Cahn flow analogous to the frequency function monotonicity for the heat equation. This frequency function will first be used to prove estimates on the Hausdorff dimension of the singular set of ACF. Further refinement of the analysis will allow us to estimate the solutions to ACF using polynomial solutions to the heat equation. These polynomials have good control on the measure of the singular set which will pass to full solutions of ACF. Since ACF itself does not form singularities, such a result shows that ACF can be used to flow MCF through singularities. Such a result would have a tremendous number of applications in geometry.

Further information:

How to apply

Entry requirements

Fees and funding