DISTRIBUTION OF VALUES OF $L$-FUNCTIONS AND MODULAR FORMS

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$L$-functions and modular appear prominently in modern number theory and also arise in other areas of mathematics. The analytic properties of these functions are closely related to arithmetic problems involving the distribution of the prime numbers, lattice points, and structure of elliptic curves. For instance, important problems such as the Generalised Riemann Hypothesis and Birch-Swinnerton Dyer Conjecture are directly related to the theory of these functions.

This project aims to further understand the analytic behaviour of $L$-functions and modular forms. In particular, the goal is to understand how the values of $L$-functions and modular forms are distributed and give potential applications to arithmetic, or, mathematical physics. Specific problems will follow the interests of the student and possible topics may include, the distribution of zeros of Hecke cusp forms, moment estimates for families of quadratic twists of modular $L$-functions, random model approximations of families of $L$-functions, and equidistribution of mass of automorphic forms.

Part of the project could potentially explore profound connections between $L$-functions and modular forms to questions in quantum chaos, which is an sub-field of mathematical physics which studies the quantum dynamics of classically chaotic systems. This is an especially exciting area of study that is still quite mysterious. Fascinatingly, much progress has been made in special arithmetic settings in which modular forms and $L$-functions play a prominent role.

Previous exposure to analytic number theory, or, algebraic number theory would be advantageous, however not necessary and may be learned during PhD research. Background in real and complex analysis along with probability theory and harmonic analysis would be desirable.