Abstract

Over- and under-reaction of prices are phenomena that have been extensively documented in the literature. However, the existing theories of over- and under-reaction are criticized because they generate unsupported empirical predictions. In this paper I ask the question of whether it is possible to create a theoretical framework with a single behavioral bias which explains both phenomena, does not rely on existing frameworks, does not generate unsupported predictions but generates new, unique and testable predictions. I show that it is indeed the case. My model incorporates a behavioral investor who selectively chooses whether to incorporate new information. Information avoidance generates under-reaction while information incorporation generates over-reaction. I show that over- (under-) reaction is stronger (weaker) when the new information is extreme (not-extreme) and when the belief of the behavioral agent are more (less) uncertain. I provide support for these predictions using winner-loser portfolios.

Keywords: Over-reaction, under-reaction, learning, beliefs.

JEL classification: G11, G12, G14, G41

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1 Introduction

One of the most fascinating phenomena in the financial literature is the co-existence of both over- and under-reaction of prices to information. These phenomena have been proposed as explanations for short-term momentum and long-term reversal. This coexistence is paradoxical in nature because it is hard to build a theoretical framework in which agents both over-react and under-react to new information. In order to generate both, the current theoretical literature employs the following building blocks: multiple behavioral biases, information asymmetry and exogenous distortion of beliefs. The reason for the assumption of several behavioral biases is the theoretical difficulty to reconcile both phenomena. Each bias is assumed to generate one of the phenomena, while their combination generates both. Information asymmetry is utilized for a similar reason. By controlling the information sets of different agents, it is possible to “manipulate” them in such a way that both phenomena emerge. Finally, the by-product of all behavioral approaches is that the beliefs of agents are exogenously distorted. This approach implicitly assumes that agents never learn that they are wrong and they remain at a disadvantage even after observing a long history of realizations.

I propose a new model of over- and under-reaction that is based on a behavioral approach to learning. The model doesn’t build on the theoretical components of other models of over- and under-reaction, and thus shows that both phenomena can occur under more general conditions. In my model I embed the updating behavior proposed by Kominers, Mu, and Peysakhovich (2017). In their framework, agents face a (mental) updating cost which they pay each time they update their beliefs. Agents selectively choose to update their beliefs when they observe signals that they deem “valuable” and ignore “invaluable” signals. One novel feature of this approach is that agents condition the updating decision on the realization of the signal itself. While Kominers, Mu, and Peysakhovich (2017) study the updating behavior in a static setting, I extend their approach to a dynamic setting with a market equilibrium. The model predicts that agents update when the new information is extreme$^1$ relative to the prior since it indicates that their beliefs are not in line with reality. Agents choose to not update when the information is not extreme since it

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$^1$I define what extreme means precisely in Proposition 2.2
indicates that their beliefs are “good enough”. When agents choose to ignore information, prices under-react. Furthermore, by ignoring some information, agents’ beliefs become, on average, more uncertain (relative to a rational agent). Thus, once agents choose to update, they overweight new information and react more strongly than a rational agent would.

There are several theoretical frameworks that propose explanations for the existence of over- and under-reaction. First, Barberis, Shleifer, and Vishny (1998) show that when agents are subject to the representativeness bias and the conservatism bias, prices under-react most of the time but with periodic episodes of over-reaction. Second, Hong and Stein (1999) show that an economy with a special type of information asymmetry and two types of investors, news watchers and momentum traders can generate both over- and under-reaction. The slow information percolation in the news-watchers population generates under-reaction while the existence of momentum traders generates over-reaction. Third, Daniel, Hirshleifer, and Subrahmanyam (1998) studies an economy with agents that are subject to two different behavioral biases; the over-confidence bias and the self-attribution bias. In their model agents overweight private information, which creates a moderate over-reaction of prices. If the information is confirmed by the public signal, agents become even more overconfident and prices over-react even more. These mechanisms generate under-reaction in the short term and over-reaction in the long term. Finally, Rabin and Vayanos (2010) employ a behavioral bias of the belief in the law of small numbers coupled with a non-trivial form of parameter uncertainty with a learning agent.

The existing literature is able to explain the co-existence of over- and under-reaction. Moreover, the assumed behavioral biases are supported by experimental evidence. Thus the existing models are an important step toward a unifying framework of over- and under-reaction. However, there are some predictions that are implied by these theories which are not supported by empirical data. First, Hong and Stein (1999) and Daniel, Hirshleifer, and Subrahmanyam (1998) both predict that prices first under-react only to over-react later. This phenomenon is called the “life-cycle hypothesis” of over- and under-reaction. Early empirical literature documented that momentum occurs over shorter time-frames than reversal, thus it was hypothesized that a single shock generates both phenomena, only over different time frames. However, later empirical liter-
ature shows that momentum and reversal occur in different sets of stocks (see, e.g., George and Hwang (2004) and Da, Gurun, and Warachka (2014)). This evidence is at odds with the “life-cycle hypothesis”. The lack of evidence for the hypothesis is used to cast doubt over the frameworks that generated them (see, e.g., the discussion in George and Hwang (2004) and Conrad and Yavuz (2017)). Second, Barberis, Shleifer, and Vishny (1998) and Rabin and Vayanos (2010) predict the occurrence of over- and under-reaction after a history of identical signals. To the best of my knowledge, there is no empirical evidence in favor of this prediction.

Relative to the existing literature, the model that I propose shows that a single deviation from a rational behavior of some agents can generate both over-reaction and under-reaction. Moreover, my model does not rely on information asymmetry. Different agents in the model have different beliefs due to selective learning, but they have access to the same information. In addition, my behavioral agent can sometimes have beliefs that are inferior to the rational agent. However, when beliefs become “too wrong” is the time when she chooses to update. Thus, instead of following the existing literature and imposing exogenous distortion of beliefs, my behavioral agent endogenously closes the gap between beliefs and “reality” once it becomes too big. Although it’s hard to argue that agents are not subject ot multiple biases which persist over time and that there is a substantial information asymmetry in financial markets, my model shows that they are not necessary conditions for over- and under-reaction to occur.

In terms of empirical predictions, my model does not imply that over- or under-reaction are generated by a sequence of identical signals or the life-cycle hypothesis. The model that I propose shows that over- and under-reaction can be generated by different signals and can be temporally decoupled. Furthermore, my model can be differentiated from competing theories since it makes new and unique predictions about the occurrence of over-reaction and under-reaction. More specifically, it predicts that over- (under-) reaction is more likely to occur when the information shock is extreme (non-extreme) and when the behavioral agent faces higher (lower) uncertainty.

One of the robust findings in the literature is the co-existence of momentum and reversal. Momentum is a phenomenon that occurs over relatively short time periods (up to 12 months) in which the portfolio of past winners outperforms the portfolio of past losers. On the other hand,
reversal is a phenomenon that occurs over longer periods (between 1 and 5 years) in which the portfolio of past losers outperforms the portfolio of past winners\(^2\). One of the leading explanations\(^3\) is that momentum is generated by price under-reaction and reversal is generated by price over-reaction. The idea is quite intuitive. If there is an information shock that is not incorporated into prices immediately (under-reaction), but is incorporated in the long-run, the price follows the trend of the initial shock for some time. On the other hand, if the price over-reacts to the initial shock, eventually it adjusts back to the fundamental value which creates reversal. In my empirical approach I adopt the notion that momentum is a manifestation of under-reaction and that reversal is a manifestation of over-reaction. Thus, when a portfolio with a long position in past winners and a short position in past losers generates a positive return (momentum), I call this an instance of under-reaction. When a portfolio with long position in past winners and short position in past losers generates a negative return (reversal), I call this an instance of over-reaction.

Aside from the behavioral approach, there are some rational theories which try to explain momentum and reversal. For example, Berk, Green, and Naik (1999) analyses a model which explains momentum and Fama and French (1996) does the same for reversal. The only framework which incorporates both is Andrei and Cujean (2017). In their framework, similarly to Hong and Stein (1999), they employ gradual information percolation through the investor community. Information percolation decreases uncertainty over time and is the mechanism behind momentum. Reversal is obtained due to the existence of noise traders. Since rational agents demand a risk premium for holding excess inventory, a (positive) supply shock that is generated by a noise trader both decreases current prices and increases the future risk premium. Although an elegant way of generating both momentum and reversal in one rational framework, I provide evidence that is not in line with the model of Andrei and Cujean (2017). More specifically, in the model of momentum becomes weaker over time because uncertainty is resolved and reversal becomes more prominent. However, I show empirically that momentum is stronger in assets which are associated

\(^{2}\)There also exists a short-term reversal which operates over horizons shorter than 1 month. However, this type of reversal is attributed to other explanations like market micro-structure or asset salience (see Cosemans and Frehen (2020))

\(^{3}\)see, e.g., Barberis, Shleifer, and Vishny (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998) for a detailed discussion
with lower uncertainty.

The first prediction of my model is that non-extreme shocks generate under-reaction and extreme shocks generate over-reaction. Therefore, my model predicts that momentum should be more pronounced in stocks which experienced a series of non-extreme shocks and reversal should be more pronounced in stocks which experienced an extreme discrete shock. I identify the extremity of information shocks with the approach of Da, Gurun, and Warachka (2014). I double-sort portfolios using a shock extremeness measure and past 6 months returns. In line with the theory, over a holding period of 1-6 months after portfolio formation (when momentum is active) I find that the winner-loser portfolio of stocks that experienced non-extreme information shocks (which are predicted to under-react) generates a positive statistically significant return and it outperforms the winner-loser portfolio of stocks that experienced an extreme discrete shock (which are predicted to over-react). However, over a holding period of 13-18 months (when reversal is active), I find that the winner-loser portfolio of stocks that experienced non-extreme information shocks (which are predicted to under-react) does not produce statistically significant result while the winner-loser portfolio of stocks that experienced an extreme discrete shock (which are predicted to over-react) generates a negative statistically significant result. Furthermore, I use the methodology of Conrad and Yavuz (2017) to separate momentum stocks from reversal stocks. This method reveals that in momentum stocks, the winner-loser portfolio of the bottom extremeness quintile generates a significant positive return and outperforms the top quintile. On the other hand, in reversal stocks, the winner-loser portfolio in the top extremeness quintile generates a significant negative return and underperforms the bottom quintile, in line with my theory.

The second prediction is that high prior belief uncertainty leads to a bigger over-reaction and a smaller under-reaction. Therefore, my model predicts that momentum should be more pronounced in stocks that are traded by behavioral agents with a high beliefs uncertainty while reversal should be more pronounced is stocks that are traded by behavioral agents with a low beliefs uncertainty. I use the analyst forecast dispersion measure from I/B/E/S as a proxy for belief uncertainty. I find evidence for the first part of this prediction. I show that momentum is stronger in stocks with a low analyst forecast dispersion, in line with my theory. The sample that I use does not provide
support for the second part of the prediction. In Section 3 I discuss potential reasons for the lack of evidence for reversal.

This paper contributes to the literature on several fronts. On the theoretical side, it provides a new and comprehensive theory that explains both over- and under-reaction of prices to information. It shows that over- and under-reaction can still occur even without assuming the usual theoretical building blocks (multiple biases, information asymmetry and distortion of beliefs). On a broader spectrum, the model relates to the literature of behavioral belief formation in financial markets. Two prominent recent examples are Nagel and Xu (2019) and Bordalo, Gennaioli, and Shleifer (2018). However, the assumed behavioral bias in my model is different. Nagel and Xu (2019) study the effect of fading memory in which agents update beliefs in a distorted way that prevents them from learning the true stochastic process even after observing a long history of realizations. Under their assumption the agent has a more diffuse prior than a purely rational agent which can generate over-reaction to information, but not under-reaction. On the other hand, Bordalo, Gennaioli, and Shleifer (2018) use the notion of “diagnostic expectations” in which agents overweight future outcomes if they agree with current data. This again can generate over-reaction but not under-reaction.\textsuperscript{4}

On the empirical side, the paper contributes to the literature on momentum (Jegadeesh and Titman (1993), Jegadeesh and Titman (2001)) and reversal (De Bondt and Thaler (1985)). More specifically, the empirical approach builds on the recent finding of Da, Gurun, and Warachka (2014) that it’s not only the level of the past return that determines the subsequent returns, but also the path to this level. While their paper analyzes momentum, I extend their methodology to study reversal as well. In addition, my paper contributes to the growing strand of literature that shows that momentum and reversal are separate phenomena. Da, Gurun, and Warachka (2014) show that stocks that exhibit momentum do not revert. George and Hwang (2004) show that portfolios constructed using a sort of the nearness of the price to historical maximum exhibit momentum but not reversal. Conrad and Yavuz (2017) show that stocks that were classified as

\textsuperscript{4}In a recent paper, Bordalo, Gennaioli, Ma, and Shleifer (2020) combine diagnostic expectations with noisy information and show that individual beliefs over-react while the consensus of beliefs under-reacts. Although an important result on its own, since it is not clear how such behavior affects prices, I don’t consider this model as reconciliation of over- and under-reaction of prices.
“winners” (“losers”) and realized higher (lower) returns in the subsequent 6 months continue to exhibit momentum, while stocks that reverted in the subsequent 6 months continue to exhibit reversal. My findings are in line with this literature, and I find that momentum and reversal occur in different subsets of stocks. This finding contradicts the proposed “life-cycle” hypothesis of over- and under-reaction.

The paper is constructed as follows: In Section 2 I present my model of over- and under-reaction. In Section 3 I present empirical evidence in favor of the theory developed in this paper. Section 4 concludes.

2 A Model of Over- and Under-Reaction

In this section I present a model with a financial market. First, I describe the market and the assets that are available to investors. Next, I discuss the agents that populate the economy and derive the equilibrium price, conditional on the beliefs that each agent has. Finally, I discuss the way agents update their beliefs.

2.1 Market

The market consists of a risky asset and a risk-free one. The risky asset generates a cash-flow \( X_t \) in period \( t \). For simplicity, I create a setting without hedging demands and risk premia. I assume that the risky asset is a stock which is liquidated at the end of each period, thus there is no resale value, which precludes any hedge demands. In addition, I assume that the risky asset is in zero net supply which implies that the asset does not generate a risk premium, but is only a vehicle to trade beliefs. Let \( \mu_t \) denote the (dynamic) mean of the cash-flow. Conditional on \( \mu_t \), the cash-flow is normally distributed with a fixed (and known) variance \( \sigma^2_\varepsilon \)

\[
X_{t+1} = \mu_{t+1} + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2_\varepsilon)
\]  

(1)
The mean parameter is time varying, and evolves as a random walk

\[ \mu_{t+1} = \mu_t + u_{t+1}, \quad u_{t+1} \sim \mathcal{N}(0, \sigma_u^2) \]  

where \( \sigma_u^2 \) is the variance of \( \mu_{t+1} \) conditional on \( \mu_t \). I assume that the two random shocks are not correlated. I study the economy at a steady-state\(^5\), thus I need a varying parameter so that there is always some parameter uncertainty remaining in the market. If \( \mu_{t+1} \) were to be constant, the agents eventually learn the true parameter and the price converges to a fixed value. In addition, I assume that there exists a risk-free asset with a (gross and time-invariant) return \( R_f \). The risk-free asset is in infinite supply, and for simplicity the gross rate is assumed to be 1\(^6\).

### 2.2 Agents, Portfolio Choice and Equilibrium

I assume that there are two (representative and price-taking) agents in the market. Both agents don’t know \( \mu_t \), however they can learn about \( \mu_t \) from observing \( X_t \). The first agent is denoted as the rational (\( r \)) agent and she always updates her beliefs by applying Bayes rule. The second agent is denoted as the behavioral (\( b \)) one. This agent selectively updates her beliefs. The exact formulation of the updating behavior is in Section 2.3 where I discuss the modeling approach for the behavioral investor.

Both agents are risk averse and myopic. Let \( \alpha_t^{(i)} \) be the portfolio (in units of stock) of agent \( i \in (r, b) \) of the risky stock. The utility of both agents is of a mean-variance type and is given by

\[ U_t^{(i)} = \omega_0^{(i)} + \left( \mathbb{E}_t^{(i)} (X_{t+1}) - P_t \right) \alpha_t^{(i)} - \frac{\gamma}{2} \alpha_t^{(i)} \mathbb{V}_t^{(i)} (X_{t+1}) \]  

where \( \omega_0^{(i)} \) is the wealth of agent \( i \), and \( \mathbb{E}_t^{(i)} (X_{t+1}) \) and \( \mathbb{V}_t^{(i)} (X_{t+1}) \) are the expected value and variance of the next-period cash-flow that is perceived by each agent and \( P_t \) is the price of the risky asset. Let the beliefs of each agent on \( \mu_t \) be normal with parameters \( m_t^{(i)} \) and \( s_t^{(i)} \). In other

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\(^5\)see Lemma 2.1 for the definition of a steady-state

\(^6\)All the results hold for a general \( R_f \) with more cumbersome mathematical expressions.
words,

\[
\text{for agent } i: \quad \mu_t \sim \mathcal{N}\left(m_t^{(i)}, s_t^{(i)2}\right). \tag{4}
\]

I use the following short-hand notation \( v_t^{(i)} \equiv \mathbb{V}_t^{(i)}(X_{t+1}) \). Using the dynamics in (1) and (2), it’s easy to show that

\[
\mathbb{V}_t^{(i)}(X_{t+1}) = v_t^{(i)} = s_t^{(i)2} + \sigma_u^2 + \sigma_\varepsilon^2 \tag{5}
\]

\[
\mathbb{E}_t^{(i)}(X_{t+1}) = m_t^{(i)} \tag{6}
\]

To ease the discussion, I refer to the time varying component \( s_t^{(i)} \) as uncertainty while the structural parameters \( \sigma_\varepsilon^2 \) and \( \sigma_u^2 \) are referred to as risk. Next, maximizing (3) with respect to the portfolio \( \alpha_t^{(i)} \) gives us the solution to the portfolio in closed form

\[
\alpha_t^{(i)} \left(m_t, v_t^{(i)}, P_t\right) = \frac{m_t^{(i)} - P_t}{\gamma v_t^{(i)}} \tag{7}
\]

which is a known result that the demand for the risky asset is the dollar premium divided by the product of risk and risk aversion. The risky asset is in zero net supply, thus the equilibrium condition is

\[
\alpha_t^{(r)} + \alpha_t^{(b)} = 0. \tag{8}
\]

Which is solved by the equilibrium price

\[
P_t^* = \frac{v_t^{(b)} m_t^{(r)}}{v_t^{(b)} + v_t^{(r)} m_t^{(b)}} + \frac{v_t^{(r)} m_t^{(b)}}{v_t^{(b)} + v_t^{(r)} m_t^{(b)}} \tag{9}
\]

It’s easy to see that the price in this case is a weighted average of the mean belief of the two agents where the agent that perceives a lower variance gets a higher weight.

### 2.3 Learning

In this section I discuss the updating behavior of the agents in the model. First, I discuss the approach of Kominers, Mu, and Peysakhovich (2017). Second, I discuss how this model differs from other models of beliefs in the literature. And why this model can generate both an over- and
under-reaction while other beliefs models fail to do so.

When it comes to beliefs formation, the prevailing paradigm in the literature is the one of “Rational Expectations”. This paradigm postulates that agents utilize Bayes’ law when they update their beliefs. However, recent evidence suggests that agents tend to deviate from the said law. A good recent review on experimental studies that analyze the deviation from Bayesian updating can be found in Benjamin (2019). Some of the robust deviations that the literature pinpoints are the “Conservatism” bias which states that agents update too little upon receiving new information and “Anchoring” which states that agents deviate too little from their priors upon updating. Another robust observation is that when agents form beliefs, they overweight the extremeness of the signal and underweight it’s importance (Griffin and Tversky (1992)). This means that agents react more severely to extreme signals and disregard the fact that it might be an outlier of the data generating process.

Motivated by these observations, I utilize the approach of Kominers, Mu, and Peysakhovich (2017) to behavioral updating. In their framework agents choose whether to incorporate information or not. Each time agents choose to update, they pay a mental cost. The novel feature of their approach is that the updating decision is made conditional on the signal itself. Intuitively, agents update their beliefs when not doing so entails a cost in terms of making sub-optimal choices. In other words, the agent calculates the possible posterior and compares the loss of utility from ignoring information to a cost of updating. If the loss of utility is higher than the cost of updating, the agent chooses to update. If the loss is lower, the agent does not update.

\[
\hat{m}_t^{(i)} = \frac{s_t^{(i)2}}{s_t^{(i)2} + \sigma_\varepsilon^2} s_t^{(i)2} + \frac{\sigma_\varepsilon^2}{s_t^{(i)2} + \sigma_\varepsilon^2} m_t^{(i)}
\]

\[
\hat{s}_t^{(i)2} = \frac{\sigma_\varepsilon^2}{s_t^{(i)2} + \sigma_\varepsilon^2} s_t^{(i)2}.
\]

In the approach of Kominers, Mu, and Peysakhovich (2017) agents selectively choose whether to incorporate information or not. Each time agents choose to update, they pay a mental cost. The novel feature of their approach is that the updating decision is made conditional on the signal itself. Intuitively, agents update their beliefs when not doing so entails a cost in terms of making sub-optimal choices. In other words, the agent calculates the possible posterior and compares the loss of utility from ignoring information to a cost of updating. If the loss of utility is higher than the cost of updating, the agent chooses to update. If the loss is lower, the agent does not update.
Denote by \((\tilde{m}_t^{(i)}, \tilde{s}_t^{(i)})\) the belief system of the agent after she chooses whether to incorporate information or not. By adapting the formulation of Kominers, Mu, and Peysakhovich (2017) to the preference structure I assume in (3), the updating rule is

\[
\begin{cases}
\{\tilde{m}_t^{(b)}, \tilde{s}_t^{(b)}\} & \text{if } \left(\tilde{m}_t^{(b)} - P_{t-1}\right) \left(\alpha_t^{(b)} - \hat{\alpha}_t^{(b)}\right) - \frac{\gamma}{2} \left(\hat{\alpha}_t^{(b)} - \alpha_t^{(b)}\right)^2 \leq -c \\
\{m_t^{(b)}, s_t^{(b)}\} & \text{if } \left(\tilde{m}_t^{(b)} - P_{t-1}\right) \left(\alpha_t^{(b)} - \hat{\alpha}_t^{(b)}\right) - \frac{\gamma}{2} \left(\hat{\alpha}_t^{(b)} - \alpha_t^{(b)}\right)^2 > -c
\end{cases}
\]

(12)

where

\[
\alpha_t^{(b)} = \frac{m_t^{(b)} - P_{t-1}}{\gamma \left(s_t^{(i)} + \sigma_u^2 + \sigma_e^2\right)}
\]

(13)

\[
\hat{\alpha}_t^{(b)} = \frac{\hat{m}_t^{(b)} - P_{t-1}}{\gamma \left(\hat{s}_t^{(i)} + \sigma_u^2 + \sigma_e^2\right)}
\]

(14)

and where \(c \in \mathbb{R}^+\) is the updating cost. The idea behind this simple decision rule is quite intuitive. The agent does not update her beliefs if the loss of utility from making a sub-optimal action is lower than the cost \(c\). In addition, if we assume that \(c = 0\), then due to the optimality of \(\alpha (q)\), the LHS is always non-negative. Thus the agent always updates her beliefs and becomes fully rational and

\[
\{\tilde{m}_t^{(r)}, \tilde{s}_t^{(r)}\} = \{m_t^{(r)}, s_t^{(r)}\}
\]

(15)

holds in every period. Since the model that I build has a time dimension, an assumption is needed about the ability to “retrieve” ignored information. I assume that once an observation is ignored, the agent cannot retrieve it anymore. I implicitly assume that in each period the agent has access only to the information in the current period, and all the information which was ignored in the past is lost. I make this assumption to not lose tractability, since otherwise that agent must evaluate every possible combination of past signals and its affect on the posterior. A similar behavior may arise under a slightly weaker assumption that the cost of updating information is increasing in the number of observations that are incorporated (mental overload). Under some cost functions, the agent never chooses to incorporate more than one observation.

The idea that evaluation of information and updating of beliefs can be separated in two distinct
steps may seem unintuitive at first sight. If the investor is exposed to the information in the
comparison stage, then it shouldn’t be hard to incorporate it in her belief system. However,
I argue that the functional form in (12) is consistent with the psychological view proposed by
Kahneman (2011).

There is a long-standing notion that the human brain works on two levels. The first level
is the fast and inaccurate mode, which is responsible for decisions that are more subconscious.
Some examples are the fight-flight mechanism, sensitivity to pain and reaction to sudden noises.
This system can also perform simple mathematical calculations like $2 + 2$. On the second level
there is the slow and rational mode. This mode comes into play when the decision becomes
more complicated, and relies on memory and intellectual abilities. This system is used when the
calculation is more involved (e.g., multiplication). In the framework that I build, the two systems
work in unison in the following way; Assume that the agent sees a signal and is given the choice of
updating beliefs or not. Updating entails a cost in terms of mental effort, thus the agent first makes
a quick judgment of the “value” of information. This judgment is done by comparing the signal
to a set of signals that the agent deems “valuable” (I assume that this comparison is costless). If
the signal “passes” the test, the agent pays the cost $c$ and accesses the slow and rational mode of
thinking. This allows the agent to perform two tasks. First, the rational brain calculates a new
posterior. Second, the rational brain creates a new list of “valuable signals” for the fast brain to
use in later encounters. The set of valuable signals in my model has the form of “Realizations that
are outside a certain interval are valuable and realizations inside the interval are not”. On the
other hand, if the signal does not “pass” the test, the agent ignores it, keeping the belief system
without change while avoiding paying the cost $c$.

This approach has some similarity with the model of “Optimal Expectations” of Brunnermeier
and Parker (2005) and on a more general level to models where agents derive utility from beliefs.\(^7\)
However, in their framework, the agent weights the errors of making mistakes vs. choosing the
belief system that gives the agent a “good feeling”. In the framework that I use, the expected
error is compared to a cost the agent pays upon updating the information set. Thus it will not

\(^7\)see Golman and Loewenstein (2018)
generate over-optimistic views akin to Brunnermeier and Parker (2005).

Two similar strands of literature that also discuss potential information avoidance are the behavioral inattention strand which is summarized in Gabaix (2019) and the rational inattention strand that is surveyed in Sims (2010). Although the source of information avoidance in these two strands are different, the “mechanics” of the underlying models are quite similar. The model that I propose resembles in spirit these two literature branches, albeit my approach differs along two dimensions. The first is the way in which deviation from rationality occurs. In both behavioral and rational inattention, the agent constructs posteriors that are closer\(^8\) to priors (relative to a fully rational agent), thus both these frameworks can generate only under-reaction. In the setting that I propose, the posterior is either equal to the prior, or very far from the prior, and this allows the model to generate both phenomena. The second dimension is the timing of the decision to pay attention. In the setting of behavioral and rational inattention, the decision on how much to update is made before the signal is revealed, thus the agent chooses the level of distortion of the posterior before observing the signal. In fact, the agent cannot choose which signal she wants to incorporate and which not. In both frameworks, after making the decision on how much to update, the agent updates regardless of whether the market does not move at all, or a new market crash occurs. This seems at odds with our daily behavior in which we selectively choose which news to read or which information we learn. Put differently, an agent with a pure “rational/behavioral inattention” who is committed to listen to the signal cannot neglect it, while we often do so in reality.

The only framework that uses the idea that an agent can evaluate the signal before incorporating it is the “motivated beliefs” framework (see e.g., Bénabou (2013)) in which the agent incorporates signals that support her prior inclination. In the “motivated beliefs” framework the agent is subject to mental biases (like denial and rationalization) and sometimes may choose to update the information set in a way that is opposite to what is implied by Bayes rule just to support her prior views. In my setting there is no such “irrationality”. In the absence of a cost of updating the information set, the behavior collapses to a pure “Bayesian” updating agent.

\(^8\)In terms of Kullback–Leibler divergence, see Sims (2010) for details.
A related notion to the one that I propose is the theory of salience of Bordalo, Gennaioli, and Shleifer (2013). As is shown in the rest of the paper, agents react to extreme information which can also be called “salient” and disregard more “lukewarm” information. However, the idea proposed here is different from the theory of salience because the latter refers to agents preferences rather than beliefs. In reality, it’s hard separate the observable outcomes of the two theories, like in any other beliefs-vs-preferences story. For instance, after seeing an extreme return realization, the agent might choose to invest in the asset because it became salient relative to the market, or because the realization induced an updating of beliefs. However, salience by itself cannot generate over- and under-reaction. For example, Cosemans and Frehen (2020) show that a positively salient asset has a negative expected return while a negatively salient asset has a positive expected return. It means that salience theory can predicts something akin to over-reaction but not under-reaction. Moreover, in the setting that I build in this paper, salience will not play any role since it needs comparison between several assets, and I model an economy with only one.

2.4 Solution of the Model

In this section I discuss additional assumptions about the timing of updating relative to market clearance and about the reference price that feeds into (12).

Due to the unique structure of the updating behavior, there are two issues that need to be resolved before the model can be solved. The first is the timing of the decision to update information. If I assume that the decision to update information is made simultaneously with market equilibrium, then, under some conditions, an equilibrium does not exist. The reason is quite intuitive. Under some realizations of $X_t$, if the agent decides to update, the equilibrium price changes a lot, which makes updating not worthwhile. On the other hand, if the agent chooses to ignore the new information, the price does not move much, which makes updating profitable. Because of this, I assume that agents make the decisions sequentially. In every period the behavioral agent first sees a realization of $X_t$ and decides whether to incorporate information or not. Then, after the decision is made, agents trade based on their beliefs and the market equilibrates. This approach introduces another technical difficulty. Since the decision to update depends on the price,
an assumption is needed as to which price feeds into the decision problem. I assume that it is
the price from the last period. This implies that the agent compares the realization \( X_t \) with the
price that was paid for the asset in the previous period \( P_{t-1} \). In other words, the agent makes the
decision to update based on the \textit{realized profit} in each period. This characteristic has an intuitive
appeal since often investors react when they see significant profits and losses.

Finally, I note that due to (2), the prior beliefs of the agent in period \( t + 1 \) are related to the
posterior beliefs in period \( t \) by the following formula:

\[
\begin{align*}
m^{(i)}_{t+1} &= \tilde{m}^{(i)}_t \\
s^{(i)}_{t+1}^2 &= \tilde{s}^{(i)}_t^2 + \sigma_u^2
\end{align*}
\]  

\(16\)

\[2.5\text{ Testable Predictions of the Model}\]

In this section I provide empirical predictions which are implied by the model. I present the
predictions in terms of propositions, while all the proofs are given in the Appendix.

First, I show that there exists a steady-state level for the uncertainty of beliefs for the rational
agent.

\textbf{Lemma 2.1} \textit{There exists a steady-state level of prior and posterior beliefs uncertainty of the
rational agent which can be calculated as}

\[
\tilde{s}^{(r)}_s^2 = \frac{(\sigma_u^2)^2 + 4\epsilon^2\sigma_u^2 - \sigma_u^2}{2} \\
S^{(r)}_s^2 = \tilde{s}^{(r)}_s^2 + \sigma_u^2
\]

\(18\)

\(19\)

The implication of Lemma 2.1 is that it does not matter with which level of uncertainty the
rational agent starts, the (posterior) uncertainty eventually converges to \( \tilde{s}^{(r)}_s^2 \).

In the next two propositions I show that as long \( c \in (0, \infty) \), there are realizations \( X_t \) which
are ignored by the behavioral agent and there are realizations \( X_t \) which are incorporated in the
beliefs of the behavioral agent.
Proposition 2.2 For a given \( m_t^{(b)}, s_t^{(b)2}, P_{t-1}, c \in (0, \infty) \), there exist \( \infty < X_t < \overline{X}_t < \infty \) (which depend on \( m_t^{(b)}, s_t^{(b)2}, P_{t-1}, c \)) such that for all \( X_t \notin (\overline{X}_t, X_t) \) the agent updates beliefs, and for all \( X_t \in (\overline{X}_t, X_t) \) the agent does not update beliefs.

From Proposition 2.2 we can conclude that in each period there is a non-zero probability for the behavioral agent to incorporate information and a non-zero probability to ignore information.

The next proposition shows that if both agents start from the same beliefs uncertainty, and there is at least one period in which the behavioral agent forgoes information, then in all subsequent periods the uncertainty of beliefs of the behavioral agent is higher than the uncertainty of beliefs of the rational agent.

Proposition 2.3 Assume that \( \tilde{s}_t^{(b)2} = \tilde{s}_t^{(r)2} \). Let \( \{t+1, \ldots, t+T\} \) be \( T \) time periods. If there is some \( k \in \{t+1, \ldots, t+T\} \) such that the behavioral agent does not update in period \( k \), then \( \tilde{s}_T^{(b)2} > \tilde{s}_T^{(r)2} \).

One of the consequences of Proposition 2.2 is that there is a positive probability of the behavioral agent to forgo information in each period. Thus, if both agents start with the same uncertainty of beliefs and the number of periods goes to infinity, the probability that the uncertainty of beliefs of the behavioral agent becomes higher than the uncertainty of beliefs of the rational agent converges to 1. The conclusion of Proposition 2.3 is that, on average, the behavioral agent faces a higher uncertainty level relative to the rational agent.

Definition 2.4 I define the case of \( c = 0 \) for both agents as a rational economy, and the case of \( c \in (0, \infty) \) for one type of agent as a behavioral economy. The equilibrium prices in both economies are denoted respectively as \( P_t^{(R)} \) and \( P_t^{(B)} \).

Definition 2.5 I define the return in economy \( j \in \{B, R\} \) in period \( t \) as the difference in prices between periods \( t \) and \( t - 1 \).

\[
R_t^{(j)} = P_t^{(j)} - P_{t-1}^{(j)} \quad \text{for} \quad j \in \{R, B\} \tag{20}
\]

Definition 2.6 I define the reaction variable \( \Delta_t \) as the difference between the absolute return between the rational economy and the behavioral economy.

\[
\Delta_t \equiv \left| R_t^{(B)} \right| - \left| R_t^{(R)} \right| \tag{21}
\]
The case when $\Delta_t > 0$ is defined as an over-reaction and $\Delta_t < 0$ is defined as an under-reaction.

The next proposition is the main proposition on the occurrence of under-reaction.

**Proposition 2.7 (Under-reaction)** Let the posterior mean belief in period $t-1$ be the same for the two agents $\tilde{m}_{t-1}^{(b)} = \tilde{m}_{t-1}^{(r)} \equiv \tilde{m}$. And let the rational agent beliefs uncertainty be at the steady state level $s_t^{(r)2} = \tilde{s}_t^{(r)2}$. Then, if $X_t \in (\underline{X}_t, \overline{X}_t)$ (which implies that the behavioral agent does not update), we observe an under-reaction, i.e., $\Delta_t < 0$.

The idea behind the mechanism of under-reaction is a simple one. Since only the rational agent updates using the realization of $X_t$, the mean belief of the behavioral agent remains the same as before. Since the price is a weighted average of the mean beliefs of the two agents, the new price reflects the new information only partially.

The next proposition is the main proposition on the occurrence of over-reaction

**Proposition 2.8 (Over-reaction)** Let the posterior mean belief in period $t-1$ be the same for the two agents $\tilde{m}_{t-1}^{(b)} = \tilde{m}_{t-1}^{(r)} \equiv \tilde{m}$. Let the rational agent beliefs uncertainty be at the steady state level and the beliefs uncertainty of the behavioral agent be greater than the beliefs uncertainty of the rational agent $s_t^{(b)2} > s_t^{(r)2}$. Then, if $X_t \notin (\underline{X}_t, \overline{X}_t)$ (which implies that the behavioral agent updates), we observe an over-reaction.

Thus an over-reaction occurs if the behavioral agent chooses to update, and she has an uncertainty of beliefs greater than the rational agent. In a long-lived economy this is indeed the case as was seen in Proposition 2.3.

The conclusion of Proposition 2.7 and Proposition 2.8 is that under-reaction occurs when the behavioral agent chooses not to update and over-reaction occurs when the behavioral agent chooses to update. This is summarized in the following Corollary

**Corollary 2.9 (Over/under reaction and shock extremeness)** Over-reaction occurs when the absolute value of the shock is above a certain threshold (extreme) and under-reaction occurs when the absolute value of the shock is below a certain threshold (not extreme).
Figures 1 and 2 show the evolution of prices in an economy following a permanent and transitory shocks. The shock is realized in period $t = 2$ and is calculated to be of the magnitude which equals the upper threshold for updating in the behavioral economy $\overline{X}_t$. To induce updating, the shock is set by adding a small number to the threshold threshold (so it will be just above it), and to induce not updating the number is subtracted (so it will be just below it). I assume that all the other shocks after period $t = 2$ are zero.

From the graph of the transitory shock (Figure 1) it’s easy to see that the price of the rational economy in period $t = 2$ is between the over-reacting behavioral economy, and the under-reacting behavioral economy. Since the shock is transitory, it generates a temporary over-valuation of the asset in all economies. However, in the behavioral economy, if the agent chooses to update, the over-valuation becomes more extreme. It is important to note that the effect of over-reaction is at the most conservative case since it is just above the updating threshold. All other realizations above the threshold produce a more severe over-reaction. In all economies the price converges back to the initial value. However the convergence is much quicker in rational and the under-reacted economy. The over-reacting economy induces both a stronger reversal, and a longer period over which the reversal occurs. From the graph of the permanent shock (Figure 2) it is possible to see the effect of under-reaction. The price of the rational economy in period $t = 2$ is again bounded by the prices of the over-reacting and under-reacting economies. Since the shock is permanent, in all economies there is a temporary under-valuation. However the under-valuation is stronger in the under-reacting behavioral economy. As in the previous case, the prices of both economies eventually converge to the same value, thus the behavioral economy exhibits a stronger positive auto-correlation.

[Figure 1 about here.]

[Figure 2 about here.]

Next, the following two propositions analyze how the prior uncertainty of the behavioral agent affect the probability and severity of over- and under-reaction.
Proposition 2.10 Assume that both agents end period $t-1$ with the same posterior mean belief which is set on the true state $\tilde{m}_{t-1}^{(r)} = \tilde{m}_{t-1}^{(b)} = \mu_{t-1} = m$. When the prior uncertainty belief of the behavioral agent $s_t^{(b)2}$ increases, the probability of updating increases.

The intuition behind Proposition 2.10 is straightforward. When the uncertainty increases, the agent is less “certain” about her estimate of $\mu_t$ and thus is more prone to accepting new information.

Proposition 2.11 Let all the conditions of Proposition 2.7 (under-reaction) hold. Then, for a higher level of prior uncertainty $s_t^{(b)2}$, under-reaction becomes weaker. Alternatively, let all the conditions of Proposition 2.8 (over-reaction) hold. Then, for a higher level of prior uncertainty $s_t^{(b)2}$, over-reaction becomes stronger.

For under-reaction, the intuition is as follows. If the uncertainty of beliefs is high, the behavioral agent receives less weight in the pricing of the asset (see (9)). Thus, the price is set mostly by the rational agent, and the effect of ignoring information (and under-reaction) is diminished. For over-reaction, if the uncertainty of beliefs is high, then over-reaction becomes stronger. This is due to two mechanisms. First, since the behavioral agent chooses to update, the posterior uncertainty is lower than the prior uncertainty, which makes her weight bigger in the pricing of the asset. Second, a higher prior beliefs uncertainty implies that the updated beliefs are further from the prior since the prior receives a low weight in the updated beliefs. The conclusion of the two propositions is that when the uncertainty of beliefs of the behavioral agent becomes higher, the probability of an over- (under-) reaction becomes higher (lower) and the severity of an over- (under-) reaction becomes higher (lower). This is is summarized in the following Corollary:

Corollary 2.12 (Over/under reaction and prior uncertainty) For higher levels of prior beliefs $s_t^{(b)2}$, under-reaction becomes less prominent and over-reaction becomes more prominent.

Based on the previous propositions, under-reaction occurs when agents receive signals that don’t move the prior much, and over-reaction occurs when there is a signal which moves the prior a lot. This can be summarized in the following two empirical predictions of the model:
• Over- (under-) reaction is more likely to occur when the signal is extreme (non-extreme).

• Over- (under-) reaction is more likely to occur with diffuse priors (tight priors).

My empirical approach is to identify over- and under-reaction by momentum and reversal which are phenomena related to returns autocorrelations. I show that even if there is no autocorrelation in returns in the rational economy (which occurs under very specific conditions), the unconditional returns autocorrelation in the behavioral economy has an ambiguous sign. However, the autocorrelation conditional on the behavioral agent updating is negative while the autocorrelation conditional on the behavioral agent not updating is positive.

**Proposition 2.13 (Returns Autocorrelation)** Let the posterior mean belief in period \( t - 1 \) be the same for the two agents which are also set on the true state variable \( \tilde{m}_{t-1}^{(b)} = \tilde{m}_{t-1}^{(r)} \equiv \tilde{m} = \mu_{t-1} = m \). Let the beliefs uncertainty of the behavioral agent be greater than the beliefs uncertainty of the rational agent \( s_t^{(b)} > s_t^{(r)} \). Let there be a realization of \( \varepsilon_t \) and \( u_t \) in period \( t \), and let \( \varepsilon_s, u_s = 0 \ \forall \ s > t \). Let \( P_{\infty}^{(j)} \equiv P_{t \to \infty}^{(j)} \). If, in addition, \( s_t^{(r)} = \sigma_u^2 \) then\(^9\),

1. In the rational economy;

\[
\text{corr}(P_t^{(R)} - P_{t-1}^{(R)}, P_\infty^{(R)} - P_t^{(R)}) = 0. \tag{22}
\]

2. In the behavioral economy;

\[
\text{corr}(P_t^{(B)} - P_{t-1}^{(B)}, P_\infty^{(B)} - P_t^{(B)}|\text{updating}) < 0 \tag{23}
\]

\[
\text{corr}(P_t^{(B)} - P_{t-1}^{(B)}, P_\infty^{(B)} - P_t^{(B)}|\text{not updating}) > 0 \tag{24}
\]

\[
\text{corr}(P_t^{(B)} - P_{t-1}^{(B)}, P_\infty^{(B)} - P_t^{(B)}) \gtrless 0. \tag{25}
\]

Figure 3 shows the autocorrelation in returns in the rational and behavioral economies. First, due to the parameter choice, the correlation of the rational economy is 0. This is not a neces-

\(^9\)I note that this condition is not supported in the steady-state since \( \sigma_u^2 < s_\ast^{(r)} \). I make this assumption because it is the only case that generates 0 autocorrelation in the rational economy. This case is used to show that even if the rational economy exhibits no autocorrelation, the behavioral economy does. The general case is provided in the Appendix in Proposition A.1.
sary condition for my results to hold, but a very intuitive benchmark. It is clear that both the positive autocorrelation following not updating and negative autocorrelation following updating is increasing in \( c \) which is the main novelty of my model. As was proved formally, the unconditional autocorrelation changes signs from negative to positive when \( c \) increases. This happens because for higher values of \( c \), the probability of not updating (and thus under-reaction) increases. Because of this, positive autocorrelation occurs more frequently than negative autocorrelation.

Figure 4 supports Proposition 2.11 and the second prediction of the model. For higher levels of the belief uncertainty \( s_{t-1}^2 \), the agent becomes more susceptible to new information and thus updates and over-reacts more which increases (in absolute terms) the negative autocorrelation following the decision to update. On the other hand, for higher uncertainty levels, the weight of the behavioral agent in the market becomes smaller, which makes all future information avoidance matter less, and the positive autocorrelation following the decision to not update becomes smaller. The effect on the unconditional correlation is similar in spirit to the effect of the updating cost \( c \). Since the uncertainty affects the probability of updating, it also affects the sign of the unconditional correlation.

These predictions are unique to the model of over- and under-reaction that I propose in this paper. The first prediction already has some support in the empirical literature. First, Baker, McElroy, and Sheng (2018) show that agents update more after an extreme event of a natural disaster. However, the direct evidence from asset prices is provided by Da, Gurun, and Warachka (2014). In their empirical setting, they show that the momentum return is higher in a portfolio of stocks which experienced a series of small returns relative to a portfolio of stocks which experienced a few episodes of large returns (holding the total return equal). However, since the authors study only momentum, it provides partial support to the first prediction. In Section 3 I utilize their methodology and extend it to show that stocks that experienced a few episodes of large returns are not only less prone to momentum, but are more prone to reversal. As for the second prediction, to
the best of my knowledge, my model is the first that creates a clear cut prediction about over- and under-reaction with relation to the uncertainty level. I show supportive evidence in line with this prediction using the I/B/E/S dataset. I proxy the uncertainty level with the time-series average forecast dispersion, and show that momentum is weaker for higher uncertainty levels. I don’t find support for the effect of uncertainty on reversal. The only existing empirical paper that studies the relationship between uncertainty measures and momentum is Zhang (2006). They find a positive relationship between uncertainty and momentum which at first sight seems to contradict my prediction. In Section 3.2 I discuss the difference between our empirical approaches, and I argue that their empirical result is not, in fact, contradictory to my theory.

3 Empirical Analysis

In this section I provide empirical evidence in line with the proposed theory. I focus on the two empirical predictions derived from the theory. The first prediction that I test is that over- (under-) reaction is more prominent when new information is extreme (non-extreme). The second prediction is that over- (under-) reaction is more prominent when the uncertainty of the prior of the behavioral agent is high.

One of the manifestations of under-reaction is the existence of the momentum anomaly. It was first discovered by Jegadeesh and Titman (1993) in the cross-section of equity, but was also confirmed in the time-series of aggregate returns and many asset classes (see Moskowitz, Ooi, and Pedersen (2012)). The anomaly postulates that returns calculated over the last 6-12 months predict the subsequent returns over the following 6-12 months. Thus, buying stocks in the top decile of past returns sort and selling stocks in the bottom decile generates a significant positive return. This finding is robust to other risk-based explanations to the point that the momentum factor is often included as a control variable when other asset-pricing models are tested. One explanation to this phenomenon is that momentum is a proxy of an additional (unobserved) risk factor. However, recent evidence shows that investors perceive momentum as a strategy with a lower risk (see Merkle and Sextroh (2020)). An alternative explanation is that momentum is a result of mispricing and the existence of a friction that prevents information from being
instantaneously incorporated into prices. Thus, if information percolates slowly into prices, a positive shock (that increases prices) is followed by subsequent price increases.

This relation reverts when the sorting and predicted return is on a longer scale. This finding is also robust and is present in many markets and time periods (see Poterba and Summers (1988) and Cutler, Poterba, and Summers (1991)). This finding implies an over-reaction to information. It seems that over very long periods agents react too strongly to information and eventually their over-reaction reverts which generates a negative correlation\(^\text{10}\). However, a question remains as to whether over-reaction is always preceded by under-reaction. Some models that predict over- and under-reaction also predict a life-cycle of the two. Namely, that those assets that under-reacted will always later over-react. Thus over-reaction cannot occur without the first under-reaction. Some recent papers tested this prediction and came to the conclusion that over-reaction and under-reaction occurs in different assets. Thus the life-cycle hypothesis does not hold. In the framework that I propose, although both over- and under-reaction are a result of the same underlying mechanism, they are not produced by the same initial shock which implies that they can be temporally separated.

To test the first prediction I build upon the methodology proposed in Da, Gurun, and Warachka (2014) and show that stocks which experienced an extreme discrete shock in prices are more prone to exhibit reversal while stocks which experienced a series of non-extreme continuous shocks exhibit momentum. Furthermore, I use the approach of Conrad and Yavuz (2017) to separate the winner-loser portfolio into momentum stocks and reversal stocks and study how shock extremeness affects both groups of stocks. I argue that the results are in line with the model that I propose, however they are not compatible with Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999).

The second prediction is that over- (under-) reaction is more prominent when the prior belief of the behavioral agent is more diffuse (tight). To test this prediction I use analyst forecast dispersion from the I/B/E/S summary dataset as a proxy for prior uncertainty. In line with theory, I show that momentum is more pronounced in stocks with a lower prior belief uncertainty.

\(^{10}\)In my empirical analysis I cast doubt over the fact that reversal occurs only over long periods
I don’t find evidence that prior beliefs uncertainty measure affect reversal returns. I discuss the possible reasons for that.

3.1 Over- and Under-Reaction and Shock Extremeness

The first prediction of the model is that small shocks are ignored by the behavioral investor. Thus if we observe a stock that experienced a sequence of non-extreme information shocks that are ignored by the behavioral agent, this stock will eventually exhibit momentum once prices adjust to the “rational” level. On the other hand, an extreme discrete shock makes the behavioral agent update, but since she has a more uncertain prior relative to a rational agent, she over-weights the information, and thus over-reacts, which induces a subsequent reversal in prices. The evidence for the first part is already provided in Da, Gurun, and Warachka (2014) and is corroborated in the current paper\(^{11}\). The authors limit the scope of their study to momentum but not to reversal. My empirical analysis is set to fill the gap. In order to be consistent with the literature, I adopt the methodology of measuring the extremeness of information from Da, Gurun, and Warachka (2014) directly. The measure quantifies whether the return over some period was realized by small increments or few big shocks. Let \( R_{t-k:t-1} \) be the compounded monthly return over months \( t-k \) to \( t-1 \). I drop one month between the formation period and portfolio holding to avoid market micro-structure issues like the bid-ask bounce. Define \( p_{t-k:t-1}^{(+)} \) as the percentage of days over the formation period in which the return was positive, and \( p_{t-k:t-1}^{(-)} \) as the percentage of days in which the return was negative. The extremeness variable is defined as

\[
D_{t-k:t-1} = \text{sgn} \left( R_{t-k:t-1} \right) \left( p_{t-k:t-1}^{(-)} - p_{t-k:t-1}^{(+)} \right)
\]

(26)

where \( \text{sgn}(\cdot) \) is the sign function. The idea behind such formulation is as follows. If the return \( R_{t-k:t-1} \) is positive (negative), and the period was dominated by days with a positive (negative) return, it means that the period had non-extreme continuous shocks and the measure \( D_{t-k:t-1} \) will be closer to \(-1\). However, if the return \( R_{t-k:t-1} \) is positive (negative), but there are more negative

\(^{11}\)In addition, my model provides the underlying mechanism for the hypothesis tested in Da, Gurun, and Warachka (2014).
(positive) returns, it means that there was a big shock in the direction of $R_{t-k:t-1}$. In this case $D_{t-k:t-1}$ will be closer to 1.

I test my theory using the monthly and daily CRSP dataset for the period of 1963-2019. As a robustness test I also use a subsample of the later period of 1990-2019. I only keep stocks that are traded on one of the three leading stock exchanges (NYSE, NASDAQ and AMEX). I only keep ordinary stocks (codes 10 and 11). At the end of each month I drop stocks that don’t have the full sample for the formation and holding periods and stocks that are cheaper than $5 (penny stocks).

I correct monthly returns for de-listings. Each month I construct portfolios by sorting them first into formation period return deciles and then into $D_{t-k:t-1}$ quintiles (50 portfolios in total). To be consistent with Da, Gurun, and Warachka (2014), the formation period is taken as 12 months (thus $k = 13$ after skipping one month). In order to analyze the life cycle of momentum and reversal, I construct portfolios that are held for the period $t + l + 1 : t + l + 6$ for $l \in \{0, 6, 12\}$\(^{12}\).

Each choice of $l$ defines a holding period of 6 months which begins $l$ months from the formation of the portfolio. Since I generate a new portfolio every month, for each value of $l$, there are 6 overlapping portfolios in each point of time. The monthly return for each value $l$ in time $t$ is calculated as the average return of all the overlapping portfolios in time $t$. This method is in line with the one used in Jegadeesh and Titman (2001). In Panel A of Table 1 I show the average raw monthly return of the long-short portfolio of buying winners and selling losers for different levels of the extremeness variable and holding periods. In Panel B I show the risk-adjusted return (alpha) of the portfolio using the Fama-French 3 factor model. All standard errors are adjusted using the Newey-West approach with a maximum lag of 12 months. The analysis is made using the full sample over the period 1963-2019 and a subsample over the period 1990-2019.

The model predicts that momentum should be more pronounced in stocks that are in the low decile of the extremeness variable, and reversal should be more pronounced in the top decile of the extremeness variable. Prior literature documents that momentum occurs over shorter periods than reversal. Thus the model implies the following hypotheses:

- **Over a short period after portfolio formation (when momentum is active):**

\(^{12}\)In an unreported table, I extend to analysis also to $l \in \{18, 24, 30\}$. Most of the results over these time frames are insignificant and are dropped for brevity.
A winner-loser portfolio of the bottom extremeness quintile generates a positive return (momentum is active).

A winner-loser portfolio of the top extremeness quintile generates a lower return relative to a winner-loser portfolio of the bottom quintile (momentum is stronger when information is non-extreme).

- Over later periods (when reversal is active):

  - A winner-loser portfolio of the top extremeness quintile generates a negative return (reversal is active).
  
  - A winner-loser portfolio of the top extremeness quintile generates a lower return relative to a winner-loser portfolio of the bottom quintile (reversal is stronger when information is extreme).

From this table, for the holding period of 1-6 months, it’s apparent that winner-loser portfolio returns are statistically significant for most of information extremeness quintiles for both raw and risk-adjusted returns. However, the return is monotonically decreasing from 1.43% per month to 0.26% per month (alpha of a Fama-French model is decreasing from 1.86% p.m. to 0.38% p.m.) as $D$ becomes higher (extreme shock). This is in line with the hypothesis that momentum should be stronger in stocks that exhibit a flow of non-extreme shocks. A portfolio that sells the momentum portfolio of the bottom quintile and buys the momentum portfolio of the top quintile generates a statistically significant return of $-1.17\%$ per month (alpha of $-1.48\%$). This result is in line with Da, Gurun, and Warachka (2014). For the holding period of 7-12 months, the general pattern is similar to the holding period of 1-6 months. The alpha of the portfolio that buys the momentum portfolio of the top quintile and sells the momentum portfolio of the bottom quintile is $-0.85\%$ p.m. and is statistically significant. For both holding periods 1-6 and 7-12 the negative return of the long top quintile and short bottom quintile is generated by the short leg. Which means that that the positive return (momentum) is stronger in stocks that experienced a series of small shocks. For the holding period of 13-18 months, the difference in alphas between the winner-loser portfolio of the top quintile and the bottom quintile is still negative and statistically significant.
(−0.67% p.m.) however, now they are generated by the long leg of the difference portfolio. This means that the negative return (reversal) of the portfolio of stocks that experienced few big shocks is higher (in absolute value) than the return of the bottom quintile. Thus, overall, the evidence is in line with the proposed hypothesis. The general result is also the same for a later period 1990-2019 which supports the idea that both momentum and reversal still occur in recent years.

There is some evidence that stocks that exhibit momentum and stock that exhibit reversal are different stocks. Thus, stocks that are identified as “momentum” stocks should exhibit the same pattern as in the holding periods 1-6 and 7-12, while stocks that are identified as “reversal” stocks should exhibit a pattern closer to the holding period 13-18. To test this prediction, I separate momentum stocks from reversal stocks using the methodology of Conrad and Yavuz (2017). The authors show that by separating winners and losers into realized winners and realized losers, it’s possible to identify stocks that exhibit momentum and stocks that revert. More precisely, after forming 10 past return decile portfolios, the cross-section is observed for an additional 6 months. Then stocks are sorted again into 2 portfolios, those stocks that were above the cross-sectional median return, and those stocks that were below the cross-sectional median return.\textsuperscript{13} Finally, as before, each portfolio is divided in 5 quintiles based on the extremeness variable \((D)\). This procedure creates 100 portfolios. By looking only at the stocks that belong to either the top or the bottom decile of past returns I identify 4 portfolios which are called winner-winner(ww), winner-loser(wl), loser-winner(lw) and loser-loser(ll). For example, a winner-winner portfolio are stocks that are in the top return decile for the initial formation period and in the top half in the first 6 months after the formation period. I call the portfolio that buys the \textit{ww} portfolio and sells the \textit{ll} portfolio as “realized momentum” portfolio and the portfolio that buys the \textit{wl} portfolio and sells the \textit{lw} stocks as the “realized reversal” portfolio. In this case, the implied hypotheses are:

- \textit{In the realized momentum portfolios:}
  
  A \textit{winner-loser} portfolio of the bottom extremeness quintile generates a positive return \textit{(momentum is active)}.\textsuperscript{13}

\textsuperscript{13}Using 10 deciles in the post-formation period will result in very poorly diversified portfolios, and even portfolios that don’t contain stocks at all
A winner-loser portfolio of the top extremeness quintile generates a lower return relative to a winner-loser portfolio of the bottom quintile (momentum is stronger when information is non-extreme).

- In the realized reversal portfolios:
  - A winner-loser portfolio of the top extremeness quintile generates a negative return (reversal is active).
  - A winner-loser portfolio of the top extremeness quintile generates a lower return relative to a winner-loser portfolio of the bottom quintile (reversal is stronger when information is extreme).

Table 2 shows the returns and alphas of realized momentum and realized reversal stocks for different holding periods. The results in Table 2 provide an additional support for the hypothesis that extreme shocks generate over-reaction, and a series of non-extreme shocks generate under-reaction. For both holding periods of 7-12 and 13-18: In the realized momentum stocks group, the bottom extremeness decile generates a positive and significant alpha (1.21% p.m. and 0.58% p.m. respectively) and the alphas for the difference portfolio between the top extremeness decile and the bottom extremeness decile is negative and statistically significant (−0.71% p.m. and −0.73% p.m. respectively). In the “realized reversal” portfolios, the top extremeness decile generates a significant negative alpha (−0.60% p.m. and −0.66% p.m. respectively) and the difference alpha between the top and bottom decile are negative and significant (−0.80% p.m. and −0.57% p.m. respectively). This observation is in line with the prediction. In “momentum” stocks, lower information extremeness generates a higher (positive) momentum return. But in “reversal” stocks, higher informational extremeness generates a higher (negative) return.

There is an additional implication of the results that was neglected by the literature thus far. It was believed that momentum and reversal work on different time horizons. Momentum was believed to be a short-term phenomenon while reversal was believed to be generated over longer

Note that I use the period 1-6 return after initial formation to identify stocks as realized momentum and realized reversal. Over this time period, momentum stocks exhibit momentum and reversal stocks exhibit reversal by construction. Thus, reporting the return on period 1-6 is meaningless.
terms. This can be visible in Table 1 that negative returns to the winner-loser portfolio start to emerge in a robust manner only in the holding period 13-18\textsuperscript{15}. However, in Table 2, momentum and reversal are concurrent in different groups of stocks. This is an additional evidence that momentum and reversal occur not only in different stocks, but they also occur over similar time periods. This observation is another evidence against the life-cycle hypothesis.

[Table 1 about here.]

[Table 2 about here.]

3.2 Over- and Under-Reaction and Prior Uncertainty

To test my second prediction, I would need to acquire a measure of belief uncertainty of the behavioral agent. One of the oldest and most widely used proxies for information uncertainty is the dispersion in the earnings per share forecast of financial analysts which is obtainable from the I/B/E/S dataset (see, e.g., Ajinkya and Gift, Michael (1985)). I obtain the processed summary statistics of analyst forecasts and I calculate the measure of the normalized standard deviation of forecasts as a proxy for prior uncertainty of the behavioral agent. The normalization calculated as a ratio of the standard deviation of EPS predictions and mean EPS prediction. In cases where the mean predicted EPS is 0, the normalized dispersion is set to a positive infinity. The implicit assumption is that at least a subgroup of financial analysts is subject to the behavioral bias and sometimes chooses to forgo some non-extreme information, which results in a higher dispersion of forecasts in the analyst population\textsuperscript{16}. The hypotheses that are implied by the model are

- **Over a short period after portfolio formation (when momentum is active):**
  
  - A winner-loser portfolio of the bottom forecast dispersion quintile generates a positive return (momentum is active).

\textsuperscript{15}The existing literature reports even longer periods. See, e.g., De Bondt and Thaler (1985)

\textsuperscript{16}This assumption is supported by recent evidence in Bourveau, Garel, Joos, and Petit-Romec (2020) which shows that analyst attention affects precision and frequency of their forecasts
A winner-loser portfolio of the top forecast dispersion quintile generates a lower return relative to a winner-loser portfolio of the bottom quintile (momentum is stronger when prior is diffuse).

- Over later periods (when reversal is active):
  - A winner-loser portfolio of the top forecast dispersion quintile generates a negative return (reversal is active).
  - A winner-loser portfolio of the top forecast dispersion quintile generates a lower return relative to a winner-loser portfolio of the bottom quintile (reversal is stronger when prior is tight).

For each month-firm, if there are several observations of analyst dispersion, I choose the last one (chronologically). Since I construct winner-loser portfolios there is an issue of variable timing. Since I cannot identify the precise moment of the signal that generates over- and under-reaction, I use an aggregate method in which I calculate the average dispersion of analyst forecasts over the formation period (12 months with one month lagged). As for the CRSP sample, I employ the same data filters as before. As before, in month $t$ I create conditional portfolio sorts. First I sort on the cumulative return over period $t - 13$ to $t - 1$ and create 10 portfolios. Within each of the 10 portfolios I create a sort into 5 portfolios based on the average analyst forecast standard deviation which results in 50 portfolios in each period.

The results are in Table 3 and are in line with the first part of the hypothesis. The winner-loser portfolio in the bottom dispersion quintile generates a statistically and economically significant positive return (momentum). However there is also a statistically and economically significant difference between the top and the bottom quintiles. The bottom quintile outperforms the top quintile by %0.72 p.m. (%0.78 p.m.) in terms of raw returns (risk-adjusted returns). This implies that indeed when the dispersion is low, the momentum return (under-reaction) is stronger.

The second part of the hypothesis is not supported by the sample I use. As is visible from Table 3, almost no return is statistically significant past the first 6 months. The reason for this can be twofold. First, it might be the case that over-reaction is a much rarer event than under-reaction.
To show that it is indeed the case in my theoretical framework, I simulate the model I build in Section 2 for 100,000 periods for different levels of the updating cost $c$ and plot the density of the reaction variable $\Delta_t$ which is defined in (21). As can be seen in Figure 5, the distribution of the reaction variable is not symmetric, especially for high levels of $c$. While under-reaction is prevalent, over-reaction becomes a more rare (and extreme) event, which makes it hard to pinpoint empirically. Second, there is a need in a more precise alignment of the measure of prior uncertainty and the signal that induces over- and under-reaction. Since I use a methodology of winner-loser portfolios, the identification of a winner stock requires the calculation of a return over an interval of time (12 months in my case). However it does not pinpoint exactly when an over- and under-reaction occurs. Since I want to remain close to accepted notions of under- and over-reaction (momentum and reversal), I leave the pursuit after alternative measures which can confirm the relationship between prior uncertainty and reversal for future work.

One important note is whether the evidence that I provide in my paper is in line with the existing evidence of the relationship between uncertainty and momentum\textsuperscript{17}. One paper which studied this relationship extensively is Zhang (2006). The paper has only an intuitive motivation and doesn’t rely on a structural model. According to that paper, higher uncertainty should lead to stronger behavioral biases, and this should translate into higher momentum returns. To support the claim, Zhang (2006) uses 6 proxies for information uncertainty. The proxies are firm size (market value), firm age, analyst coverage, dispersion in analyst forecasts, stock return volatility and cashflow volatility. The measures of age, size and analyst coverage are measures that my model is silent about. However, it was proposed that size captures other risk-based explanations than purely behavioral biases (Fama and French (1992)). And since both age and analyst coverage are correlated with size, it’s unclear what these measures capture. Next, both return volatility and cashflow volatility has an ambiguous effect within the context of my model since the variation in these volatilities can come both from the permanent part $\sigma_u^2$ and the transitory part $\sigma_\varepsilon^2$. However, a higher transitory variance increases uncertainty while a higher permanent variance decreases it. Thus, the effect from within my model is ambiguous.

\textsuperscript{17}I am not familiar with any empirical work which studied the relationship between uncertainty and reversal
The last measure, analyst dispersion, is used both in my empirical approach and in Zhang (2006). However, there is a key difference in the timing of the measure. I measure the uncertainty as the time-average dispersion over the return formation period while Zhang (2006) measures it at one point in time - at the portfolio formation period. The empirical prediction of my model is that momentum is stronger for a higher prior uncertainty. Interestingly, within the scope of my model, a higher posterior uncertainty can be a sign that the agent dismissed all the information over the formation period, and prices under-reacted (which results in momentum) which is in line with the empirical finding of Zhang (2006). By calculating an average over the formation period, like I do in my approach, I take into account not only the posterior uncertainty at the end of the formation period, but the level of uncertainty over the whole formation period.

[Table 3 about here.]

[Figure 5 about here.]

4 Conclusion

This paper proposes a new framework that explains price over- and under-reaction in financial markets. The mechanism relies on a behavioral approach to beliefs updating. The model that I propose builds on the updating behavior studied by Kominers, Mu, and Peysakhovich (2017) in which agents condition the updating choice on the signal itself and thus update only to extreme signals while ignoring mild signals that has little affect on the prior. I show that, in a simple market economy, this behavior can already generate over- and under-reaction of prices to information. Although there were some theoretical attempts to reconcile the two, I show that they build on various building blocks (several behavioral biases, information asymmetry, and mental distortion of the stochastic process) and make empirical predictions that are not supported empirically (reaction to sequences and the life-cycle hypothesis). My model, however, can accommodate the lack of these empirical prediction while still generating both over- and under-reaction. On top of that, I derive new and unique predictions. I show that over- and under-reaction is related to the extremity of the shock and to the level of uncertainty the behavioral agent faces.
I test these predictions of my model empirically. The first prediction which relates over- and under-reaction to shock extremeness is tested with the methodology of Da, Gurun, and Warachka (2014). I complement their empirical approach by analyzing not only momentum but also reversal. I show that momentum returns are higher when the information comes in non-extreme continuous pieces and reversal returns are higher when the information comes in extreme discrete shocks. Furthermore, I employ the methodology of Conrad and Yavuz (2017) to separate the winner-loser portfolio to sub-portfolios of realized momentum and realized reversal. I show that the same pattern (high information extremeness generates reversal and low information extremeness generates momentum) is present in the sub-portfolios as well. For the second prediction, I use the analyst disagreement measure from the I/B/E/S dataset as a proxy for prior uncertainty. I show that, in line with my theory, momentum is stronger in stocks with a low dispersion in analyst forecasts (low prior uncertainty).
A Appendix

Proof of Lemma 2.1 For the rational agent it’s true that \( \tilde{s}_t^{(r)} = \tilde{s}_t^{(r)} \). By combining (11) and (17) we get the following mapping \( \tilde{s}_t^{(r)} \to \tilde{s}_{t+1}^{(r)} \):

\[
\tilde{s}_{t+1}^{(r)} = \tilde{s}_t^{(r)} + \frac{\sigma^2}{\tilde{s}_t^{(r)} + \sigma^2_\epsilon + \sigma^2_u} \left( \tilde{s}_t^{(r)} + \sigma^2_u \right)
\]

(27)

Imposing \( \tilde{s}_*^{(r)} = \tilde{s}_{t+1}^{(r)} = \tilde{s}_t^{(r)} \) reduces the equation to

\[
\left( \tilde{s}_*^{(r)} \right)^2 + \sigma^2_\epsilon \tilde{s}_*^{(r)} - \sigma^2_u = 0
\]

(28)

which is solved by (18) (taking the negative root implies a negative variance and thus ignored).

In addition,

\[
\frac{\partial \tilde{s}_{t+1}^{(r)}}{\partial \tilde{s}_t^{(r)}} = \left( \frac{\sigma^2}{\tilde{s}_t^{(r)} + \sigma^2_\epsilon + \sigma^2_u} \right)^2 > 0
\]

(29)

and

\[
\frac{\partial^2 \tilde{s}_{t+1}^{(r)}}{\partial \left( \tilde{s}_t^{(r)} \right)^2} = -2 \left( \frac{\left( \sigma^2_\epsilon \right)^2}{\left( \tilde{s}_t^{(r)} + \sigma^2_\epsilon + \sigma^2_u \right)^3} \right) < 0.
\]

(30)

Thus \( \tilde{s}_*^{(r)} \) is the unique fixed point.

Proof of Proposition 2.2 The LHS of the inequality in (12) is quadratic in \( \hat{m}_t^{(b)} \). The solution of

\[
\left( \hat{m}_t^{(b)} - P_{t-1} \right) \left( \frac{m_t^{(b)} - P_{t-1}}{\gamma \hat{v}_t^{(b)}} \right) - \frac{\gamma \hat{v}_t^{(b)}}{2} \left( \frac{m_t^{(b)} - P_{t-1}}{\gamma \hat{v}_t^{(b)}} \right)^2 + c = 0
\]

(31)

is

\[
\hat{m}_{t,1,2} = \left( m_t^{(b)} - P_{t-1} \right) \frac{\hat{v}_t^{(b)}}{\hat{v}_t^{(b)}} \pm \sqrt{2 \gamma c \hat{v}_t^{(b)}} + P_{t-1}
\]

(32)

Thus for all \( \hat{m}_t^{(b)} < \hat{m}_t^{(b)} < \hat{m}_t^{(b)} \) the agent will not update. And for all the other points, she will.
Finally, using the inverse mapping implied by (10) to solve for \( X_t \) to get

\[
\begin{align*}
\bar{X}_t &= \hat{m}_t^{(b)}_1 + \frac{\sigma^2}{s_t^{(b)2}} \left( \hat{m}_t^{(b)}_1 - m_t^{(b)} \right) \\
\bar{X}_t &= \hat{m}_t^{(b)}_2 + \frac{\sigma^2}{s_t^{(b)2}} \left( \hat{m}_t^{(b)}_2 - m_t^{(b)} \right)
\end{align*}
\]

(34)

(35)

The only unbounded variables in (33), (34) and (35) are \( s_t^{(b)2} \) and \( v_t^{(b)} \). Since according to (11), \( \hat{s}_t^{(b)2} \) is bounded both from below (by 0) and above (by \( \sigma^2 \)), letting \( s_t^{(b)2} \to \infty \) reduces the above equations to

\[
\begin{align*}
\hat{m}_t^{(b)}_{1,2} &= \pm \sqrt{2\gamma \sigma^2 \varepsilon} + P_{t-1} \\
\bar{X}_t &= \hat{m}_t^{(b)}_1 \\
\bar{X}_t &= \hat{m}_t^{(b)}_2
\end{align*}
\]

(36)

(37)

(38)

Thus both \( \bar{X}_t \) and \( \bar{X}_t \) exist.

**Proof of Proposition 2.3** First, since both agents start from the same belief and update until period \( k - 1 \), \( s_k^{(b)2} = s_k^{(r)2} = s_k^2 \). Using the updating formulae

\[
\hat{s}_k^{(r)2} = \frac{\sigma^2}{s_k^2 + \sigma^2} \hat{s}_k^2 < s_k^2 = \hat{s}_k^{(b)2}.
\]

(39)

Which implies

\[
\hat{s}_{k+1}^{(r)2} = \hat{s}_k^{(r)2} + \sigma_u^2 < \hat{s}_k^{(b)2} + \sigma_u^2 = s_{k+1}^{(b)2}.
\]

(40)

In addition, using (11) it’s easy to show

\[
\frac{\partial \hat{s}_t^{(b)2}}{\partial s_t^{(b)2}} = \left( \frac{\sigma^2}{\sigma^2 + \hat{s}_t^{(b)2}} \right)^2 > 0
\]

(41)

Thus, in the next period, the behavioral agent (regardless of the updating behavior) has a higher belief uncertainty than the rational agent. By induction, it also translates to all subsequent periods. The second part of the proof is easily calculated recursively by using \( \hat{s}_k^{(b)2} = s_k^{(b)2} \) and
Proof of Proposition 2.7 According to (9), in the rational economy the price simplifies to

$$ P_t^{(R)} = \tilde{m}_t^{(r)}. $$

(42)

Thus the return is

$$ R_t^{(R)} = \tilde{m}_t^{(r)} - \tilde{m}_{t-1}. $$

(43)

Using (10) and (16) it’s possible to show that

$$ R_t^{(R)} = \frac{s^{(r)}_t}{s^{(r)}_t + \sigma^2} (X_t - \tilde{m}) $$

(44)

In the behavioral economy, the behavioral agent chooses to forgo the observation, and thus $\tilde{m}_t^{(b)} = \tilde{m}_{t-1}^{(b)}$. The price in a behavioral economy is given in (9). The return is thus

$$ R_t^{(B)} = \frac{\tilde{v}_t^{(b)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \tilde{m}_t^{(r)} + \frac{\tilde{v}_t^{(r)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \tilde{m}_{t-1}^{(b)} - \tilde{m}_{t-1}^{(r)} $$

(45)

By plugging in the assumption $\tilde{m}_{t-1}^{(b)} = \tilde{m}_{t-1}^{(r)} \equiv \tilde{m}$ together with (10) and (16), we can calculate

$$ R_t^{(B)} = \frac{\tilde{v}_t^{(b)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \frac{s^{(r)}_t}{s^{(r)}_t + \sigma^2} (X_t - \tilde{m}) $$

(46)

By a simple calculation, the reaction variable is

$$ \Delta_t \equiv \left| R_t^{(B)} \right| - \left| R_t^{(R)} \right| = -\frac{\tilde{v}_t^{(r)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \frac{s^{(r)}_t}{s^{(r)}_t + \sigma^2} |X_t - \tilde{m}| < 0 $$

(47)

Proof of Proposition 2.8 According to (44), the return in the rational economy is

$$ R_t^{(R)} = \frac{s^{(r)}_t}{s^{(r)}_t + \sigma^2} (X_t - \tilde{m}). $$

(48)
In the behavioral economy the return is

\[ R_t^{(B)} = \frac{\tilde{v}_t^{(b)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \bar{m}_t^{(r)} + \frac{\tilde{v}_t^{(r)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \tilde{m}_t^{(b)} - \bar{m}_{t-1} \quad (49) \]

By plugging in the assumption \( \tilde{m}_t^{(b)} = \bar{m}_t^{(r)} \equiv \bar{m} \) together with (10) and (16), we can calculate

\[ R_t^{(B)} = \left[ \frac{\tilde{v}_t^{(b)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \frac{s_t^{(r)2}}{s_t^{(r)2} + \sigma^2} + \frac{\tilde{v}_t^{(r)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \frac{s_t^{(b)2}}{s_t^{(b)2} + \sigma^2} \right] (X_t - \bar{m}) \quad (50) \]

And the reaction variable is

\[ \Delta_t \equiv \left| R_t^{(B)} \right| - \left| R_t^{(R)} \right| \quad (51) \]

\[ = \frac{\tilde{v}_t^{(r)}}{\tilde{v}_t^{(r)} + \tilde{v}_t^{(b)}} \left( \frac{s_t^{(b)2}}{s_t^{(b)2} + \sigma^2} - \frac{s_t^{(r)2}}{s_t^{(r)2} + \sigma^2} \right) |X_t - \bar{m}| > 0 \quad (52) \]

since we assumed \( s_t^{(b)2} > s_t^{(r)2} \) (which is a viable assumptions due to Proposition 2.3),

**Proof of Proposition 2.10** By setting \( P_{t-1} = m \) and using (33), (34) and (35)

\[ X_t = m + H_t \quad (53) \]

\[ X_t = m - H_t \quad (54) \]

where

\[ H_t = \frac{s_t^{(b)2} + \sigma^2}{s_t^{(b)2}} \sqrt{2\gamma c \tilde{v}_t^{(b)}}. \quad (55) \]

Taking the following derivative shows

\[ \frac{\partial H_t}{\partial s_t^{(b)2}} = -\frac{\gamma c \sigma^2}{(s_t^{(b)2})^2 (\sigma^2 + s_t^{(b)2}) \sqrt{2\gamma c \tilde{v}_t^{(b)}}} \left( \sigma^2 s_t^{(b)2} + 2(\sigma^2 + \sigma_u^2) \left( s_t^{(b)2} + \sigma^2 \right) \right) < 0 \quad (56) \]
Since $u_t$ and $\varepsilon_t$ are both normal, the probability of updating is

\[
Prob(\text{update}) = Prob\left(X_t > \overline{X}_t\right) + Prob\left(X_t < \underline{X}_t\right)
\]

\[
= 1 - \Phi\left(\overline{X}_t; \mu_t, \sigma^2_\varepsilon + \sigma^2_u\right) + \Phi\left(\underline{X}_t; \mu_t, \sigma^2_\varepsilon + \sigma^2_u\right)
\]

where $\Phi\left(\bullet; \mu, \sigma^2\right)$ is the normal cdf. Taking a derivative gives us

\[
\frac{\partial Prob(\text{update})}{\partial s_t^{(b)2}} = -\phi\left(\overline{X}_t; \mu_t, \sigma^2_\varepsilon + \sigma^2_u\right) \frac{\partial \overline{X}_t}{\partial \overline{s}_t^{(b)2}} + \phi\left(\underline{X}_t; \mu_t, \sigma^2_\varepsilon + \sigma^2_u\right) \frac{\partial \underline{X}_t}{\partial \overline{s}_t^{(b)2}} > 0
\]

where $\phi\left(\bullet; \mu, \sigma^2\right)$ is the normal pdf.

**Proof of Proposition 2.11** For the first part, take derivative of (47)

\[
\frac{\partial \Delta_t}{s_t^{(b)2}} = \left(\frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + s_t^{(b)2}}\right)^2 \frac{-\overline{v}_t^{(r)}}{\overline{v}_t^{(r)} + \overline{v}_t^{(b)}} \frac{s_t^{(r)}_2}{s_t^{(r)}_2 + \sigma^2_\varepsilon} |X_t - \tilde{m}| > 0
\]

Thus an under-reaction becomes smaller (in absolute value). For the second part, take the derivative of (47)

\[
\frac{\partial \Delta_t}{s_t^{(b)2}} = \left(\frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + s_t^{(b)2}}\right)^2 \frac{-s_t^{(r)}_2}{-\overline{v}_t^{(r)} + \overline{v}_t^{(b)}} \frac{s_t^{(r)}_2}{s_t^{(r)}_2 + \sigma^2_\varepsilon} \left(1 - \frac{\sigma^2_\varepsilon}{\overline{v}_t^{(r)} + \overline{v}_t^{(b)}} \left(\frac{s_t^{(b)2}_2}{s_t^{(b)2}_2 + \sigma^2_\varepsilon} - \frac{s_t^{(r)}_2}{s_t^{(r)}_2 + \sigma^2_\varepsilon}\right)\right)
\]

\[
\times |X_t - \tilde{m}|
\]

since

\[
\frac{\sigma^2_\varepsilon}{\overline{v}_t^{(r)} + \overline{v}_t^{(b)}} \in (0, 1)
\]

and

\[
0 < \frac{s_t^{(b)2}_2}{s_t^{(b)2}_2 + \sigma^2_\varepsilon} < \frac{s_t^{(r)2}_2}{s_t^{(r)2}_2 + \sigma^2_\varepsilon} < 1
\]

implies that

\[
\frac{s_t^{(b)2}_2}{s_t^{(b)2}_2 + \sigma^2_\varepsilon} - \frac{s_t^{(r)2}_2}{s_t^{(r)2}_2 + \sigma^2_\varepsilon} \in (0, 1).
\]

The conclusion is that (61) is positive.
Proof of Proposition 2.13  Since both agents start from the same mean belief, it’s easy to show by direct plugging that in both economies

\[ P_{i-1}^{(j)} = m \]  \hspace{1cm} (66)

Since we assumed that the only shocks \( u_t \) and \( \varepsilon_t \) occur in time \( t \) and zero afterwards, only the permanent component remains, and in both economies the price converges to

\[ P_{\infty}^{(j)} = m + u_t \]  \hspace{1cm} (67)

The only difference between the rational and the behavioral economy occurs in period \( t \). I first discuss the rational economy. In the rational economy, all agents update using the new information, thus:

\[ P_t^{(R)} = \kappa^{(r)} X_t + (1 - \kappa^{(r)}) m = m + \kappa^{(r)} (\varepsilon_t + u_t) \]  \hspace{1cm} (68)

Where

\[ \kappa^{(r)} = \frac{s_t^{(r)2}}{s_t^{(r)2} + \sigma_\varepsilon^2} \]  \hspace{1cm} (69)

Thus:

\[ P_t^{(R)} - P_{t-1}^{(R)} = \kappa^{(r)} (\varepsilon_t + u_t) \] \hspace{1cm} (70)
\[ P_\infty^{(R)} - P_t^{(R)} = u_t + \kappa^{(r)} (\varepsilon_t + u_t) \] \hspace{1cm} (71)

Since \( E(\varepsilon_t) = E(u_t) = 0 \),

\[ \text{cov} \left( P_t^{(R)} - P_{t-1}^{(R)}, P_\infty^{(R)} - P_t^{(R)} \right) \]
\[ = E[\kappa^{(r)} (\varepsilon_t + u_t) \left( u_t + \kappa^{(r)} (\varepsilon_t + u_t) \right)] \]
\[ = \kappa^{(r)} \left( (1 - \kappa^{(r)}) \sigma_u^2 - \kappa^{(r)} \sigma_\varepsilon^2 \right) \]  \hspace{1cm} (72)
\hspace{1cm} (73)
\hspace{1cm} (74)

which equals 0 if and only if \( \kappa^{(r)} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} \) which holds by assumption. This finishes the proof for
the rational economy.

For the behavioral economy the proofs depends on whether the agent chooses to update or not. For this, define the sum of the two random shocks as

\[ z_t = \varepsilon_t + u_t \]  

(75)

Since both \( \varepsilon_t \) and \( u_t \) are normally distributed, their distribution conditional on \( z_t \) is

\[
\begin{pmatrix}
\varepsilon_t \\
u_t
\end{pmatrix} \sim_{|z_t} N\left[
\begin{pmatrix}
\frac{\sigma^2_e}{\sigma^2_e + \sigma^2_u} z_t \\
\frac{\sigma^2_u}{\sigma^2_e + \sigma^2_u} z_t
\end{pmatrix},
\begin{pmatrix}
\sigma^2_e \left(1 - \frac{\sigma^2_u}{\sigma^2_e + \sigma^2_u}\right) & -\frac{\sigma^2_u}{\sigma^2_e + \sigma^2_u} \\
-\frac{\sigma^2_u}{\sigma^2_e + \sigma^2_u} & \sigma^2_u \left(1 - \frac{\sigma^2_e}{\sigma^2_e + \sigma^2_u}\right)
\end{pmatrix}\right]
\]

(76)

By setting \( P_{t-1}^{(B)} = m \) and using (33), (34) and (35)

\[ X_t = m + H_t \]  

(77)

\[ X_t = m - H_t \]  

(78)

where

\[ H_t = \frac{s_t^{(b)} + \sigma^2_u}{s_t^{(b)2}} \sqrt{2\gamma c_{t}^{(b)}}. \]  

(79)

The agent chooses to not update when \( X_t < X_t < X_t \) which is equivalent to \( -H_t < z_t < H_t \). Thus, \( z_t \) follows a symmetric truncated normal distribution. In this case,

\[ P_t = \theta \left( \kappa^{(r)} X_t + (1 - \kappa^{(r)}) m \right) + (1 - \theta) m = m + \theta \kappa^{(r)} z_t \]  

(80)

where

\[ \theta = \frac{v_t^{(b)}}{v_t^{(b)} + \hat{\nu}_t^{(r)}} \in (0,1). \]  

(81)
Thus:

\[ P_t^{(B)} - P_{t-1}^{(B)} = \theta \kappa^{(r)} z_t \]  
(82)

\[ P_t^{(B)} - P_t^{(B)} = u_t + \theta \kappa^{(r)} z_t \]  
(83)

Next, calculate:

\[
E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \middle| z_t \right) 
\]
(84)

\[
= E \left( \theta \kappa^{(r)} z_t \left( u_t + \theta \kappa^{(r)} z_t \right) \middle| z_t \right) 
\]
(85)

\[
= \theta \kappa^{(r)} \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} - \theta \kappa^{(r)} \right) z_t^2 
\]
(86)

Since \( z_t \) is symmetrically distributed,

\[
cov \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_t^{(R)} - P_t^{(R)} \right) \middle| \text{not update} \right) 
\]
(87)

\[
= E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \middle| \text{not update} \right) 
\]
(88)

\[
= E_z \left( E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \middle| z_t \right) \middle| \text{not update} \right) 
\]
(89)

\[
= \theta \kappa^{(r)} \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} - \theta \kappa^{(r)} \right) E_z \left( z_t^2 \middle| \text{not update} \right) 
\]
(90)

Since by assumption \( \kappa^{(r)} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} \) and \( \theta \in (0, 1) \), we can conclude that the covariance is positive.

Next, the agent chooses to update when \( X_t < X_t \) or \( \overline{X}_t > X_t \) which is equivalent to \( z_t < -H_t \) or \( H_t < z_t \). Thus, \( z_t \) follows a symmetric distribution which is a composite of the two tails of the normal distribution. In this case,

\[
P_t = \hat{\theta} \left( \kappa^{(r)} X_t + (1 - \kappa^{(r)}) m \right) + \left( 1 - \hat{\theta} \right) \left( \kappa^{(b)} X_t + (1 - \kappa^{(b)}) m \right) = m + \lambda z_t 
\]
(91)
where

\[ \hat{\theta} = \frac{\hat{v}_t^{(b)}}{\hat{v}_t^{(b)} + \hat{v}_*^{(r)}} \in (0, 1). \]  

(92)

\[ \kappa^{(b)} = \frac{s_t^{(b)2}}{s_t^{(b)2} + \sigma_\varepsilon^2} \]  

(93)

\[ \lambda = \kappa^{(r)} \hat{\theta} + \kappa^{(b)} \left( 1 - \hat{\theta} \right) \]  

(94)

Due to the assumption \( s_t^{(b)2} > s_t^{(r)2} \) we can conclude that \( \lambda > \kappa^{(r)} \). Next, calculate the returns,

\[ P_t^{(B)} - P_{t-1}^{(B)} = \lambda z_t \]  

(95)

\[ P_\infty^{(B)} - P_t^{(B)} = u_t + \lambda z_t \]  

(96)

Next, calculate:

\[
E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \left| z_t \right. \right) \\
= E \left( \lambda z_t \left( u_t + \lambda z_t \right) \left| z_t \right. \right) \\
= \lambda \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} - \lambda \right) z_t^2
\]

(97)

(98)

(99)

Since \( z_t \) is symmetrically distributed,

\[
cov \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \left| \text{update} \right. \right) \\
= E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \left| \text{update} \right. \right) \\
= E_z \left( E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \left| z_t \right. \right) \left| \text{update} \right. \right) \\
= \lambda \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} - \lambda \right) E_z \left( z_t^2 \left| \text{update} \right. \right)
\]

(100)

(101)

(102)

(103)

Since we know that \( \lambda > \kappa^{(r)} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} \), we can conclude that the covariance is negative. Finally,
the unconditional covariance is

\[ \text{cov} \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \right) \]

\[ = E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \right) \]  

\[ = \text{Prob}(\text{update}) \ E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \right| \text{update} \]  

\[ + \text{Prob}(\text{no update}) \ E \left( \left( P_t^{(R)} - P_{t-1}^{(R)} \right) \left( P_\infty^{(R)} - P_t^{(R)} \right) \right| \text{no update} \]  

(104) \hspace{1cm} (105) \hspace{1cm} (106) \hspace{1cm} (107)

By varying \( c \), the probability of updating varies between 0 and 1. Since the two terms have opposing signs, the sign of the unconditional covariance depends on the value of \( c \).

**Proposition A.1 (General Case of Proposition 2.13)** Let all the conditions of Proposition 2.13 except that \( s_t^{(r)^2} = s_t^{(r)^2} > \sigma_u^2 \). Then:

1. In the rational economy;

\[ \text{corr}(P_t^{(R)} - P_{t-1}^{(R)}, P_\infty^{(R)} - P_t^{(R)}) > 0. \]  

(108)

2. In the behavioral economy;

\[ \text{corr} \left( P_t^{(B)} - P_{t-1}^{(B)}, P_\infty^{(B)} - P_t^{(B)} \right| X_t \in (X_t, X_t)) > \text{corr} \left( P_t^{(R)} - P_{t-1}^{(R)}, P_\infty^{(R)} - P_t^{(R)} \right| X_t \in (X_t, X_t)) \]  

\[ \text{corr} \left( P_t^{(B)} - P_{t-1}^{(B)}, P_\infty^{(B)} - P_t^{(B)} \right| X_t \notin (X_t, X_t)) < \text{corr} \left( P_t^{(R)} - P_{t-1}^{(R)}, P_\infty^{(R)} - P_t^{(R)} \right| X_t \notin (X_t, X_t)) \]  

(109) \hspace{1cm} (110)

**Proof of Proposition A.1** Using the same logic as in the proof of Proposition 2.13, define the variable \( z_t = \varepsilon_t + u_t \). Then, the conditional moments of returns if \( z_t \) is restricted to be in a
symmetric interval can be written as:

\[ E \left( P_{t}^{(j)} - P_{t-1}^{(j)} \right)^2 = Q^2 E \left( z_t^2 \right) \]  
\[ E \left( P_{\infty}^{(j)} - P_{t}^{(j)} \right)^2 = \frac{\sigma_u^2 \sigma_{\varepsilon}^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} + \left( Q - \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} \right)^2 E \left( z_t^2 \right) \]  
\[ E \left( P_{t}^{(j)} - P_{t-1}^{(j)} \right) \left( P_{\infty}^{(j)} - P_{t}^{(j)} \right) = Q \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} - Q \right) E \left( z_t^2 \right) \]

Where

\[ Q = \begin{cases} 
\kappa^{(r)} & \text{in a rational economy} \\
\theta \kappa^{(r)} & \text{in a behavioral economy if } X_t \in (X_t, X_t) \\
\lambda & \text{in a behavioral economy if } X_t \notin (X_t, X_t) 
\end{cases} \]

where \( \kappa^{(r)} \), \( \theta \) and \( \lambda \) are defined in the proof of Proposition 2.13. The conditional correlation is thus:

\[ \text{corr} \left( P_{t}^{(j)} - P_{t-1}^{(j)}, P_{\infty}^{(j)} - P_{t}^{(j)} \right) = \frac{Q \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} - Q \right) E \left( z_t^2 \right)}{\sqrt{Q^2 E \left( z_t^2 \right) \left( \frac{\sigma_u^2 \sigma_{\varepsilon}^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} + \left( Q - \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} \right)^2 E \left( z_t^2 \right) \right)}} \]

\[ = \text{sgn} \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} - Q \right) \left[ \frac{1}{\left( \frac{\sigma_u^2 \sigma_{\varepsilon}^2}{\sigma_u^2 + \sigma_{\varepsilon}^2} - Q \right)^2 E \left( z_t^2 \right) + 1} \right] \]

where \( \text{sgn}() \) is the sign function. A straightforward differentiation shows that it is a monotonically decreasing function in \( Q \). From the proof of Proposition 2.13 we know that \( \theta \kappa^{(r)} < \kappa^{(r)} < \lambda \), which finishes the proof.

**References**


Figure 1: This graph shows the evolution of prices in the rational economy and the behavioral economy following a transitory shock. The shock is set to equal the positive updating threshold for the behavioral economy. The shock occurs in period $t = 2$ and the shocks in subsequent periods are set to 0. The yellow line is the price as a function of time in the rational economy. The red line is the price in a behavioral economy following not updating. The blue line is the price in a behavioral economy following updating. The parameters used are $m_1 = 1$, $\tilde{s}_1^{(r)} = \sigma_x^2 = 0.0618$, $\tilde{s}_1^{(b)} = \tilde{s}_1^{(r)} + \sigma_u^2 = 0.1$, $\sigma_u^2 = 0.1$, $\gamma = 2$, $c = 0.1$. 

\[
\tilde{s}_1^{(b)} = \tilde{s}_1^{(r)} + \sigma_u^2 = 0.1
\]
Figure 2: This graph shows the evolution of prices in the rational economy and the behavioral economy following a permanent shock. The shock is set to equal the positive updating threshold for the behavioral economy. The shock occurs in period $t = 2$. The yellow line is the price as a function of time in the rational economy. The red line is the price in a behavioral economy following not updating. The blue line is the price in a behavioral economy following updating. The parameters used are $m_1 = 1$, $\tilde{s}_1^{(r)2} = \sigma^2 = 0.0618$, $\tilde{s}_1^{(b)2} = \tilde{s}_1^{(r)2} + \sigma^2$, $\sigma^2 = 0.1$, $\sigma^2_u = 0.1$, $\gamma = 2$, $c = 0.1$
Figure 3: This graph shows the autocorrelation of returns for the behavioral and the rational economies as a function of the updating cost $c$. The blue line is the correlation in a rational economy. The red line is the unconditional returns autocorrelation. The yellow line is the return autocorrelation conditional on the behavioral agent updating. The purple line is the return autocorrelation conditional on the behavioral agent not updating. The parameters used are $m_{t-1} = 1$, $\tilde{s}_{t-1}^{(r)} = \sigma_u^2$, $\tilde{s}_{t-1}^{(b)} = \tilde{s}_{t-1}^{(r)} + \sigma_u^2$, $\sigma^2 = 0.1$, $\sigma_u^2 = 0.1$, $\gamma = 2$. 

\[
\tilde{s}_{t-1}^{(r)} = \tilde{s}_{t-1}^{(b)} = \tilde{s}_{t-1}^{(r)} + \sigma_u^2, \quad \sigma^2 = 0.1, \quad \sigma_u^2 = 0.1, \quad \gamma = 2
\]
Figure 4: This graph shows the autocorrelation of returns for the behavioral and the rational economies as a function of the uncertainty of beliefs of the behavioral agent $s_{t-1}^{(b)}$. The blue line is the correlation in a rational economy. The red line is the unconditional returns autocorrelation. The yellow line is the return autocorrelation conditional on the behavioral agent updating. The purple line is the return autocorrelation conditional on the behavioral agent not updating. The parameters used are $m_{t-1} = 1$, $s_{t-1}^{(r)} = \sigma_u^2$, $\sigma_e^2 = 0.1$, $\sigma_u^2 = 0.1$, $\gamma = 2$, $c = 0.1$.
Figure 5: This graph shows the empirical density function of the relative reaction (which is calculated in (21)). The economy is simulated for 100,000 periods. The density is calculated for different levels of $c$. The parameters used for the simulations are $m_0 = 1$, $\sigma_0^2 = \sigma_*^2 = 0.0618$, $\sigma_*^2 = 0.1$, $\sigma_u^2 = 0.1$, $\gamma = 2$. 
Table 1: Information extremeness and winner-loser returns

This table shows the average monthly return (Panel A) and Fama-French 3 factor monthly alpha (Panel B) of a long winner stocks short loser stocks. Winner (loser) stocks are in the top (bottom) return decile. The return is calculated for different quintiles of information extremeness variable which is defined in (26). Low (high) $D$ values mean a return that is formed over time in many non-extreme continuous shocks (few extreme discrete shocks). In the parentheses are Newey-West corrected t-stats with a maximum lag of 12.

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Table 2: Information extremeness and realized momentum/realized reversal returns

This table shows the average monthly return (Panel A) and Fama-French 3 factor monthly alpha (Panel B) of a long winner stocks short loser stocks for realized momentum and realized reversal stocks. Realized winner (loser) stocks are in the top (bottom) return decile and above (below) the median post-formation 6 months return. Reverted winner (loser) stocks are in the top (bottom) return decile and below (above) the median post-formation 6 months return. The return is calculated for different quintiles of information extremeness variable which is defined in (26). Low (high) $D$ values mean a return that is formed over time in many non-extreme continuous shocks (few extreme discrete shocks). In the parentheses are Newey-West corrected t-stats with a maximum lag of 12. This table is based on the full sample 1963-2019.

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Panel B: Risk-adjusted returns

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Table 3: Forecast dispersion and winner-loser returns

This table shows the average monthly return (Panel A) and Fama-French 3 factor monthly alpha (Panel B) of a long winner stocks short loser stocks. Winner (loser) stocks are in the top (bottom) return decile. The return is calculated for different quintiles of analyst forecast dispersion which is defined as the annual average standard deviation of analyst forecast where the average is taken over the winner-loser formation window. In the parentheses are Newey-West corrected t-stats with a maximum lag of 12.

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Panel B: Risk-adjusted returns

| Forecast Dispersion quintile |                      |                         |                         |              |               |             |
| (1)            | 1.62***                | 0.21                    | -0.04                   | 1.41***      | 0.03         | -0.35       |
|                | (5.68)                 | (1.11)                  | (-0.20)                 | (3.74)       | 0.14         | (-1.26)     |
|                | 0.89***                | 0.23                    | -0.18                   | 0.59         | 0.13         | -0.35       |
| (2)            | (3.25)                 | (1.21)                  | (-0.99)                 | (1.59)       | (0.48)       | (-1.53)     |
|                | 0.92***                | 0.16                    | -0.32*                  | 0.59**       | -0.09        | -0.59***    |
| (3)            | (4.04)                 | (0.83)                  | (-1.62)                 | (2.03)       | (-0.37)      | (-2.65)     |
|                | 0.71***                | -0.01                   | 0.05                    | 0.40         | -0.27        | -0.02       |
| (4)            | (3.00)                 | (-0.06)                 | (0.25)                  | (1.31)       | (-1.08)      | (-0.08)     |
|                | 0.85***                | 0.15                    | -0.00                   | 0.57*        | -0.01        | -0.04       |
| (5)            | (3.68)                 | (0.74)                  | (-0.01)                 | (1.90)       | (-0.03)      | (-0.15)     |
| (5)-(1)        | -0.77****              | -0.06                   | 0.04                    | -0.83**      | -0.04        | 0.31        |
|                | (-2.73)                | (-0.25)                 | (0.13)                  | (-2.15)      | (-0.13)      | (0.85)      |