Monetary Policy and Bond Prices with Drifting Equilibrium Rates*

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Abstract

We study drift and cyclical components in U.S. Treasury bonds. We find that bond yields are drifting because they reflect the drift in monetary policy rates. Empirically, modeling the monetary policy drift using demographics and productivity trends, plus long-term inflation expectations, leads to cyclical deviations of bond prices from their drift that predict bond returns in- and out-of-sample. These bond cycles can originate from term premia or temporary deviations from rational expectations in a behavioral framework. Through the lens of our model, we detect a significant role of the latter in determining the cyclical properties of yields with short maturities.

JEL codes: E43, E52, G12.

Keywords: Monetary Policy Rule, Secular Trends, Term Structure, Diagnostic Expectations, Bond Return Predictability.

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1 Introduction

Bond prices are drifting: they are non-stationary.\footnote{Bond prices have been drifting in the last forty years because their secular drivers have been drifting. As we shall see later, we find that the age structure of the population, potential output growth, and long-term inflation expectations jointly capture the stochastic trend in yields. Throughout the paper, we use the words trend and driver interchangeably.} In turn, when the stochastic trend has been removed from yields, cyclical (i.e., stationary) components naturally emerge. The fact that bond prices are drifting has important implications for modeling monetary policy, the term structure of interest rates, and holding period excess bond returns.\footnote{The relevance of investigating the drift in the term structure of yields is not restricted to Treasury bonds. For example, Farhi and Gourio (2018) propose a macro-finance neoclassical growth model to account for drifting real rates and stable return to private capital. van Binsbergen (2020) finds that accounting for secular trends in interest rates is fundamental for assessing long duration dividend risk. Campbell and Sigalov (2020) derive a model of reaching for yield and show that agents take more risk when the real interest rate declines while the risk premium remains constant. Also, see Campbell (2019) for a discussion (available here) on the importance of drifting prices for long-term investing.} Moreover, understanding the drift in bond prices is of essential importance in the current scenario in which several fiscal authorities are considering issuing very long dated bonds or pension funds would like to buy long-term Treasuries for duration matching purposes;\footnote{See Need discount debts: try 50-year bonds (WSJ, 2021).} indeed it is at the long-end of the curve where the implications for a drifting term structure become more relevant.\footnote{When the model is stationary, long-term forecasts inevitably converge to the unconditional average. In line with this fact, Giglio and Kelly (2018) document that long-maturity Treasury bond prices are significantly more volatile with respect to short-maturity prices relative to what standard affine stationary models predict.} However, these implications have been so far unexplored since both standard factor models for the term structure and (empirical models built on) monetary policy rules are designed for stationary variables. Only recently, the non-stationarity of bond yields has been acknowledged (Kozicki and Tinsley, 2001) and modeled within an arbitrage-free dynamic term structure model (DTSM) with a shifting endpoint (Bauer and Rudebusch, 2020). Despite these seminal contributions, the exact nature of the drivers of the stochastic trend in yields, the relation between the cyclical components of yields with the term premium
and expectation errors about the short-term rate, and the extent of interest rates and bond returns predictability in a model of drifting yields, all warrant further research.

This paper shows that reconstructing the term structure starting from a simple monetary policy rule with an equilibrium rate driven by productivity and demographics trends, together with long-term inflation expectations, goes a long way in capturing the stochastic trend in yields. Our framework establishes a set of novel facts about Treasury bonds, while offering the possibility to revisit classic questions related to bond predictability and monetary inertia.

First, our monetary policy rule (with a target rate modeled by fluctuations in potential output, demographics, and long-term inflation expectations) tracks well the evolution of the short-term rate both in- and out-of-sample. Importantly, by being explicit about the non-stationary drivers of rates, our model is purposely transparent and simple (i.e., not involving any filtering). We find that policy inertia can be overestimated if the drivers of the drifting equilibrium policy rates are not included in the monetary reaction function, contributing to the debate on monetary policy inertia as a result of omitted factors in the Fed’s reaction function or interest rate smoothing (see, e.g., Rudebusch, 2002, 2006; Coibion and Gorodnichenko, 2012).

Second, we derive the implications of our monetary policy rule specification for the entire term structure of Treasury bond yields. Our approach decomposes bond yields at any maturity into a drifting component—the average expected sequence of monetary policy rates over the life of the bond—and a residual cyclical component—the deviation of yields from their drift. We show that our framework with drifting bond prices implies a battery of mis-specification tests such as parametric restrictions on yields and their drift that are analogous to the restriction between prices and dividends in the Campbell and Shiller (1988) present-
value model. Specifically, when the (non-stationary) drivers of the monetary policy rates have been correctly specified, deviations of bond prices from their estimated drift should be stationary with a co-integrating vector of $(1, -1)$, generating the cyclical components of yields. Our empirical analysis confirms these predictions.

Having analyzed the statistical properties of our model, and confirmed it is well-behaved, we turn to address the, admittedly challenging, question of the economic interpretation of the cyclical components. Within our stylized framework, we supply an upper bound to the role of deviations from rational expectations (in the form of diagnostic expectations) for the fluctuations in the cyclical component of yields. In particular, when we test for the role of Diagnostic Expectations (overreaction of agents to deviations of the monetary policy rate from its trend), we find that up to 40% of the fluctuations in yield cycles can indeed be attributed to this mechanism for bonds with a 2-year maturity. However, the explanatory power of diagnostic expectations for bond cycles declines with the maturity of the bond, leaving a potential important role for term premia. Indeed, when we compare the cyclical component in the 10-year bond implied by our framework to the state-of-the-art term premia generated by the no-arbitrage DTSM in Bauer and Rudebusch (2020), we find a correlation as high as 80% in the last thirty years. Interestingly, this high correlation is achieved despite the fact we do not impose no-arbitrage restrictions. This finding, in turn, suggests that no-arbitrage restrictions are empirically of second order importance in trend-cycle models of yields. At a minimum, however, we strongly reject any evidence of non-stationary term premia.

Finally, we show that a framework with drifting bond prices naturally implies bond risk premia predictability. Specifically, we formally show that stationary deviations of bond prices from their drift should predict excess bond returns. Empirically, our model generates large
$R^2$ of about 30\% (10\%) when it is used to predict the one-year (one-quarter) ahead excess returns on bond with maturities ranging from 2 to 10 years. We also construct a single yield-based cycle factor and find that our return-forecasting factor subsumes common bond risk premia predictors, such as the Cochrane and Piazzesi (2005) and Cieslak and Povala (2015) factors. Importantly, our results survive out-of-sample and hold internationally.

**Related Literature.** Our evidence that bond prices are drifting is in line with several papers documenting a slow-moving component common to the entire term structure (see, for example, Balduzzi et al., 1998 and Fama, 2006).

Stationarity of returns and non-stationarity of prices is common to many asset classes. In the equity space, standard factor models focus on returns and leave prices undetermined. In a related paper focusing on stock prices, Favero et al. (2020) show that modeling the drift in stock prices leads to an equilibrium correction term in a model relating returns to factors; however, this term is invariably omitted in standard factor model of stock prices. Interestingly, in the fixed income space, standard factor models concentrate on bond prices rather than on holding period returns but ignore their drifts. The evidence in this paper shows that a stationary (factors) framework cannot be adopted for yields-to-maturity. In this regard, our analysis supports the literature that models Treasury yields using shifting endpoints (Kozicki and Tinsley, 2001), vector autoregressive models (VAR) with common trends (Negro et al., 2017), and slow-moving averages of inflation (Cieslak and Povala, 2015) and consumption (Jørgensen, 2018).

Standard ATSMs for bond yields assume stationarity, thus ruling out (by design) the drift in bond prices. Hence, our evidence is in line with Bauer and Rudebusch (2020) who propose a term structure model for interest rates with four state variables, one of which being an (unobserved) stochastic trend common across Treasury yields. Importantly, none
of the above cited papers explores the implications of drifting equilibrium rates for monetary policy, Treasury yields, and bond returns predictability within a cohesive framework.\(^5\)

Finally, our paper fits into the literature that studies the role played by (shifts in) the monetary conduct in determining the dynamics of bond yields.\(^6\) Berardi et al. (2020) show that the stance of monetary policy—as proxied by the difference between the natural rate of interest and the current level of short term rate—contains valuable information for bond predictability. Ang et al. (2011) show that the evolution of the Fed’s response to inflation affect long-term yields. Similarly to Ang et al. (2011), we propose to model monetary policy and the term structure of interest rates jointly. However, our modeling of the policy rule with a drifting equilibrium rate is different from their model with time-varying policy coefficients. In turn, our approach has implications for interest rates comovement and bond returns predictability induced by deviations of bond prices from their drift. These testable implications are unique to our framework and not shared by Ang et al. (2011).

2 Modeling Monetary Policy

Monetary policy rules specify the dynamics of the short-term rate, \(y_t^{(1)}\). The following specification is general and encompasses most of the rules that have been proposed in the

\(^5\)Also, in our framework stationarity of bond returns naturally co-exists with non-stationary bond prices. Bond returns are predicted by the stationary deviations of bond prices from their drift. Interestingly Bauer and Rudebusch (2020) note that, even when no-arbitrage is imposed, the loading of returns on the unobserved common stochastic trend is an order of magnitude smaller than the loading of prices. They also report that predictive regressions of yields on de-trended yields and trend proxies lead to coefficients on the trend that are not significantly different from zero.

\(^6\)An important literature (see, for example, Bernanke and Kuttner, 2005; Ozdagli, 2018; Chava and Hsu, 2020) investigate the impact of monetary policy shocks on equity prices and the cross-section of stock returns. Koijen et al. (2017) propose a three-factor model for stocks and bond returns. The investigation of a factor model with drifting bond and equity prices is an interesting avenue for future research.
where \( y_t^* \) is the equilibrium monetary policy rate, \( X_t \) is a vector of stationary monetary policy drivers, and \( u_t^{(1)} \) is a monetary policy residual following an AR(1) process with persistence \( \rho \), as in, e.g., Rudebusch (2006) and Pasten et al. (2020). Arguably, the most famous special case of this specification is the Taylor (1993) rule. In this case, the vector \( X_t \) is composed of the output gap and the percentage deviation of inflation from its target. Furthermore, the Taylor (1993) rule assumes a constant equilibrium policy rate (i.e., \( y_t^* = y^* \)) and provides a natural benchmark for our analysis.

With a constant equilibrium rate, the estimate of the AR(1) persistence parameter is often close to one. This is to be expected since, if monetary policy rates are drifting, any attempt to model them only by means of stationary factors such as the output and inflation gaps naturally leads to a highly persistent process for \( u_t^{(1)} \). This fact has spurred an important literature debating the sources and the implications of monetary policy inertia. One common narrative is that monetary policy inertia is fictitious and stems from omitted variables in the Fed’s reaction function (e.g., Rudebusch, 2002, 2006). Others have argued that the central bank conducts sluggish partial adjustment of short-term policy interest rates, modeled through interest smoothing in the policy rule (e.g., Coibion and Gorodnichenko, 2012).

\(^7\)The “natural” level of real interest rates is often referred to as the “natural”, “equilibrium” or “neutral” real rate of interest. Interestingly, the possibility of a non-stationary equilibrium rate is rarely entertained in the traditional literature. A notable exception is Woodford (2001) who shows that the optimal policy response to real disturbances requires including a time-varying real rate in monetary policy rules. See Giammarcoli and Valla (2004) and Kiley (2015) for a review of the various concepts and estimation methods adopted in the literature.

\(^8\)For a detailed discussion on optimal monetary policy inertia see Woodford (2001, 2003).
We contribute to this debate showing that monetary policy inertia is overestimated when the time-varying drivers of the drifting equilibrium rate are not included in the monetary policy rule. Importantly, regardless of the origins of monetary policy inertia, long-term forecasts of interest rates produced by a policy rule with constant equilibrium rate will inevitably rapidly converge to the unconditional sample mean over the estimation period.\footnote{Rudebusch (2002) highlights the contradiction between apparent high-persistence and low-predictability of policy rates.}

Interest rates are sometimes modeled in first-difference which removes the stochastic trend in policy rate at the cost of leaving the equilibrium level of the policy rate undetermined (e.g., Orphanides, 2003). The model in first-difference is a special case of our general specification when $\rho = 1$. Specifying the monetary policy rule in first-difference comes with benefits and costs.\footnote{Cochrane (2007) provides a thorough discussion on the effects of specifying a model in level vs. first-difference to compute long-term yield-curve decomposition.} The benefit of making the rule independent from the challenging estimation of the level of the equilibrium rate has to be traded-off against the cost of accepting that any monetary policy shock (i.e., any deviation from the rule) has a permanent effect on policy rates. Indeterminacy is a major concern for long-term forecasting, because as the unconditional distribution of policy rates is not defined, the long-run policy rate is also left undetermined.

We propose a “cointegrating” approach to drifting policy rates, where the stationarity of residuals of the monetary policy reaction function is taken as an indication of a valid specification for $y_t^*$. Equivalently, a valid specification for the equilibrium rate requires that $y_t^*$ is the stochastic trend that drives drifting policy rates.
Specifically, we propose to model drifting policy rates as follows:

\[
y_t^{(1)} = y_t^* + \beta_1 E_t(\pi_{t+1} - \pi_t^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)}
\]

\[
y_t^* = \gamma_1 MY_t + \gamma_2 \Delta x_{t,pot}^* + \gamma_3 \pi_t^*
\]

\[
u_t^{(1)} = \rho u_{t-1}^{(1)} + \epsilon_t^{(1)},
\]

where \(y_t^{(1)}\) is the one-period (three-month) yield, \(y_t^*\) is the equilibrium nominal rate, \(\pi_t\) is the percentage annual log change in Personal Consumption Expenditures (PCE), \(\pi_t^*\) is the Fed perceived target rate (PTR), and \(x_t\) is the output gap (log percentage difference between real GDP and potential GDP). The monetary policy residual with a drifting equilibrium rate \(u_t^{(1)}\) is stationary under cointegration, i.e., \(|\rho| < 1\); \(\epsilon_t^{(1)}\) is an i.i.d innovation. The drivers of the equilibrium real rate are the age structure of population and potential output growth.\(^{11}\) We obtain the nominal equilibrium rate by adding the central bank inflation target \(\pi_t^*\). Appendix A provides details on the data source.

Following Geanakoplos et al. (2004) and Favero et al. (2016), the age structure of the population is described by the ratio of middle-aged (40-49) to young (20-29) population in the U.S. (labelled as \(MY_t\)). Potential output growth is the percentage annual log change in potential output.

\(MY_t, \Delta x_{t,pot}^*, \text{ and } \pi_t^*\) are non-stationary (i.e., their mean changes over time) and they

\(^{11}\)Including only inflation as driver of non-stationary policy rates is equivalent to assume a counterfactual stationary equilibrium real rate (see, e.g., Lunsford and West, 2019). Empirically, although inflation is the most important driver of the policy rate, using only inflation leaves a persistent component in yields unexplained (i.e., the AR(1) persistence parameter for \(u_t^{(1)}\) in equation (1) is 0.79); this evidence is in line with Bauer and Rudebusch (2020).
represent the drivers of the drifting equilibrium rate in our cointegrated specification.\textsuperscript{12,13}

Finally, in all our tests, we always compare the results from our baseline (drifting) model to the results of a restricted model that, inspired by the large body of literature on the classical Taylor (1993) rule, does not model the drift in monetary policy:\textsuperscript{14}

\begin{equation}
\begin{aligned}
y_t^{(1)} &= y^* + \beta_1 E_t(\pi_{t+1} - \pi^*_t) + \beta_2 E_t(x_{t+1}) + u_t^{(1)}. \\
u_t^{(1)} &= \rho u_{t-1} + \varepsilon_t^{(1)}.
\end{aligned}
\end{equation}

\section{Empirical Results}

Panel A of Figure 1 displays the realized nominal short-term rate, the fitted rates from our cointegrated monetary rule (c.f. Equation (1)), and the fitted monetary policy rates from a version of our model which restricts the equilibrium rate to be constant (c.f. Equation (2)). Panel B plots the monetary policy residuals implied by our proposed monetary policy rule and its restricted version. Table 1 reports the estimation results for these two rules.\textsuperscript{15}

\textsuperscript{12}Our specification is compatible with yields being non-stationary or yields appearing non-stationary from the perspective of a model that does not include regime-shifts. What matters for the validity of our specification is that the deviations of actual rates from equilibrium rates are stationary.

\textsuperscript{13}The $p$-values from the Phillips and Perron (1988) unit root test for $MY_t$, $\Delta x_{pot}^t$ and $\pi_t^*$ are respectively 0.95, 0.13, and 0.69; thus, we cannot reject the null of each series being integrated of order one.

\textsuperscript{14}We deviate in two respects from a standard empirical Taylor rule. First, the model in (2) is a forward-looking version of the policy rule. This is consistent with the perspective that monetary policy changes take time to affect the economy (see, e.g., Clarida et al., 2000; Coibion and Gorodnichenko, 2011). Second, we specify the inflation gap as deviations of inflation from a time-varying inflation target ($\pi_t^*$) rather than from a constant inflation target (e.g., 2%). This is in line with the idea that the inflation gap should measure the difference between actual inflation and the central bank’s long-run target (e.g., Cogley et al., 2010). A standard empirical Taylor rule with constant inflation target would lead to a better in-sample fit ($R^2 = 66\%$). The in-sample success of the standard Taylor rule is explained by the fact that de-trending inflation with a constant target over the full sample results in a highly persistent inflation gap, which mechanically explains better non-stationary yields. Importantly, a standard Taylor rule is inferior to our rule with drifting rates in terms of long-term forecasts, modeling the term structure, and bond risk premia predictability.

\textsuperscript{15}Our estimate of the loading on $\pi_t^*$ is in line with parameter values reported in Bauer and Rudebusch (2020, Table 1) despite the difference in the maturity of the bond analyzed (their Table 1 analyzes the 10-year bond, whereas we focus on the 3-month Treasury bill).
Figure 1: Actual vs Fitted Short-Term Rate. Panel (a) shows actual three-month yield and fitted values for our (cointegrated) model with drifting equilibrium rates (c.f. equation (1); see green dashed line) as well as for a model that restricts the equilibrium rate to be constant (c.f. equation (2); see brown dotted line). Panel (b) shows the differences between actual three-months yield and the fitted values. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.
Table 1: Short-term rate models with and without drifting equilibrium rate

This table reports the estimates for our (cointegrated) model with drifting equilibrium rates (c.f. equation (1); see column (2)) as well as estimates for a model that restricts the equilibrium rate to be constant (c.f. equation (2); see column (1)). We estimate the two rules by instrumental variables, where the instruments are lags of inflation gap and output gap. The last row reports OLS estimates for the monetary policy residuals’ persistence. Values in parenthesis are GMM standard errors that correct for autocorrelation in the residuals. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

<table>
<thead>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
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<tr>
<td>MY</td>
<td>−2.652***</td>
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<tr>
<td></td>
<td>(0.726)</td>
</tr>
<tr>
<td>∆x²</td>
<td>0.932***</td>
</tr>
<tr>
<td></td>
<td>(0.317)</td>
</tr>
<tr>
<td>π*</td>
<td>1.656***</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
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<tr>
<td>E(πt+1 − π* ²t+1)</td>
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</tr>
<tr>
<td></td>
<td>(0.519)</td>
</tr>
<tr>
<td>E(xt+1)</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(0.481)</td>
</tr>
<tr>
<td>Constant</td>
<td>4.656***</td>
</tr>
<tr>
<td></td>
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<tr>
<td>Observations</td>
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<tr>
<td>Adjusted R²</td>
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<tr>
<td>ρ</td>
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<td>(0.022)</td>
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</table>

Figure 1–Panel A shows that our monetary rule with a drifting equilibrium rate tracks well the short-term rate movements throughout the sample. Indeed, the $R^2$ for the cointegrated specification is about 95% whereas that of a model with constant equilibrium rate is just about 4% (c.f. Table 1).\textsuperscript{\textmd{16,17}} Figure 1–Panel B shows that the residuals implied by our drifting

\textsuperscript{16}Furthermore, a regression of the three-month yield on the fitted values implied by the two monetary rules (dotted and dashed lines in Figure 1–Panel A) delivers an estimate of zero on the rule with constant equilibrium rates (2), and a statistically significant estimate not different from one on the drifting rule (1).

\textsuperscript{17}Positing the following cointegration framework where the equilibrium real rate $r* = \frac{\pi}{\pi*}$ is estimated first, i.e.,
monetary policy rule are mean reverting. On the other hand, the residuals from a rule with constant equilibrium rates display a close-to-unit root behavior. This is confirmed in Table 1: the residuals from the rule with drifting (constant) equilibrium rates have an autoregressive coefficient equal to 0.67 (0.95).

Figure 2 displays the forecasts implied by the two monetary policy rules. Although the in-sample performance from the two models are similar, the long-term out-of-sample forecasts are dramatically different. Indeed, the policy rule with constant equilibrium rates generates forecasts that converge fast to the unconditional mean. On the other hand, the drifting monetary policy rule tracks well the future evolution of the short-rate for each of the three out-of-sample periods considered in the figure. Appendix Figures B.2 and B.3 confirm that allowing for interest rate smoothing in the rule with constant equilibrium rate does not alter our conclusion: The long-term forecasts converge fast to the unconditional mean, and underperform relative to the forecasts from a model with drifting equilibrium rate. This conclusion holds independently from whether interest smoothing is characterized as a first- or a second-order autoregressive process (Coibion and Gorodnichenko, 2011).

Finally, note that the fitted short rate in Figure 1(a) falls below zero only for a very short period of time, and the forecasts in Figure 2 never hit the bound. As the economy is affected by the entire path of expected future short-term rates (e.g., Swanson and Williams, 2014), our results suggest that an accurate modeling of the trend alleviates concerns related to the effectiveness of monetary policy at the zero lower bound (see Aruoba and Schorfheide, 2016 for a discussion).

\[
y_t^{(1)} = \alpha_1 r_t^* + \alpha_2 \pi_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)}
\]
\[
r_t^* = \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot}
\]

leaves our conclusions unaltered. See Appendix Figure B.1.
Figure 2: Long-Term Forecasts of Short-Term Rate. This figure shows actual three-month yield and predicted rates implied by our (cointegrated) model with drifting equilibrium rates (c.f. equation (1); green dashed line) and by a model that restricts the equilibrium rate to be constant (c.f. equation (2); brown dotted line). The forecast of the drifting rule exploits the exogeneity of the demographic variable ($MY$) and of potential output ($\Delta x_{pot}$). In particular, the rule is estimated until 1995, 2000, and 2005 in the top, mid, and bottom panels, respectively. We then use the coefficients estimates, the projections of $MY$ and $\Delta x_{pot}$ (see also Appendix A), and the forecast of inflation and output gap from a VAR(1) as in equations (6) and (7). $\pi^*$ is modeled as a random walk. Monetary policy residuals persistence $\rho$ is 0.673 and 0.949 for the drifting and the constant equilibrium rate models respectively. Dotted vertical lines represent the end of in-sample estimation period. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.
3 Modeling a Drifting Term Structure

The entire term structure is drifting. Models that parsimoniously describe the term structure by projecting rates on a set of factors and by modeling the dynamics of the factors with a VAR will be inevitably confronted with the problem generated by the presence of unit roots in the VAR. Highly persistent VAR generate imprecise forecasts at long-horizons (e.g., Giannone et al., 2019). This feature can explain mixed results from the forecasting performance of affine term structure models (see, for example, Duffee, 2002; Sarno et al., 2016). Remarkably, this problem has not been fully acknowledged until very recently (see Bauer and Rudebusch (2020), Cieslak and Povala (2015), Favero et al. (2016)). We use the drift in monetary policy rates to model the drift in the entire term structure:

\[
y^{(n)}_t = y^{(n),*}_t + \delta_0^{(n)} + u_t^{(n)}
\]

[3]

\[
y^{(n),*}_t = \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)}]
\]

Yields at all maturities are decomposed into a trend, \(y^{(n),*}_t\), and a cyclical component, \(\delta_0^{(n)} + u_t^{(n)}\). The trend is the average of expected monetary policy rates over the duration of the bond, while the cyclical component is the stationary residuals from the \((1, -1)\) cointegrating relationship between yields and their drift. We consider as valid any model of the term structure that delivers cointegration between \(y_t^{(n)}\) and \(y_t^{(n),*}\) with a \((1, -1)\) cointegrating vector and, therefore, a stationary \(u_t^{(n)}\).
Our full term structure model is specified as follows:

\begin{align}
  y_t^{(1)} &= y_t^* + \beta_1 E_t(\pi_{t+1} - \pi_{t+1}^*) + \beta_2 E_t(x_{t+1}) + u_t^{(1)} \tag{4} \\
  y_t^* &= \gamma_1 MY_t + \gamma_2 \Delta x_t^{pot} + \gamma_3 \pi_t^* \\
  u_t^{(1)} &= \rho u_{t-1}^{(1)} + \epsilon_t^{(1)} \\
  y_t^{(n)} &= y_t^{(n),*} + \delta_0^{(n)} + u_t^{(n)} \tag{5} \\
  y_t^{(n),*} &= \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} E_t[y_t^{(1),i}] \\
  (\pi_t - \pi_t^*) &= \theta_{1,1} (\pi_{t-1} - \pi_{t-1}^*) + \theta_{1,2} x_{t-1} + \theta_{1,3} (y_{t-1}^{(1)} - y_{t-1}^*) + v_{1,t} \tag{6} \\
  x_t &= \theta_{2,1} (\pi_{t-1} - \pi_{t-1}^*) + \theta_{2,2} x_{t-1} + \theta_{2,3} (y_{t-1}^{(1)} - y_{t-1}^*) + v_{2,t} \tag{7}
\end{align}

where we assume \( \text{Cov}(v_{1,t}, u_t^{(1)}) = \text{Cov}(v_{2,t}, u_t^{(1)}) = 0. \)

Projections of the equilibrium policy rates depend on productivity and demographics, which we take as exogenous; thus, we do not specify the law of motion for \( MY_t \) and \( \Delta x_t^{pot}. \) The U.S. Census Bureau and the U.S. Congressional Budget Office provide ready-to-use projections respectively for \( MY \) and potential output. Equations (6) and (7) are used to compute the projections of inflation and output gaps. The dynamics of these two stationary variables depend on their own lags and on a third stationary variable: the deviation of the short-term rate from its trend. This cycle in monetary policy enters the dynamics of output and inflation gaps with a one-quarter lag; this is consistent with the delay with which monetary policy affects these variable in our specification of the forward looking policy rule (4). Finally, note that we do not impose NA restrictions when estimating our model. Thus, our estimation strategy runs the cost of losing efficiency if NA holds to gain consistency in the case NA is violated.
3.1 Empirical Results

3.1.1 Misspecification test for term structure models

The validity of a model with drifting monetary policy rates and bond prices can be assessed by checking the existence of cointegrating relationships with parameters $(1, -1)$ between $y_t^{(n)}$ and $y_t^{(n),*}$ (see Equation (5)). Thus, in this section we investigate the strength of the cointegrating relationship, the $(1, -1)$ parametric restriction, and the behavior of the residuals for our baseline model (see Equations (4)–(7)) as well as for its restricted version where the drift in monetary policy is assumed away (i.e., $y_t^* = y^*$).

Figure 3 reports the results for the (strength of the) cointegration relationship for five maturities ranging from 2- ($n = 8$ quarters) to 10-years ($n = 40$ quarters). Panel (a) is for the restricted model whereas Panel (b) is for our model with drifting equilibrium policy rates.

Our model with drifting equilibrium monetary policy rate provides overwhelming evidence to reject the null hypothesis of absence of cointegrating relation between $y_t^{(n)}$ and $y_t^{(n),*}$ for all the considered maturities. From an economic perspective, this implies that fluctuations in productivity, demographics and long-term inflation expectations are successful in modeling not only the drift in monetary policy rates but also the drift in the entire term structure.

Furthermore, Appendix Table B.1 confirms that, within our framework with drifting policy rates, the parametric restriction $(1, -1)$ on the cointegrating relationship between yields and their drift is supported in the data for every maturities ranging from 2- to 10-years.

In all, our choice of the drivers for the equilibrium rate $y_t^*$ provides also an accurate
(a) Model with constant equilibrium rate. (b) Model with drifting equilibrium rate.

Figure 3: Engle and Granger (1987) Cointegration Test: This figure shows results for the Engle and Granger (1987) cointegration test for $u_t^{(n)}$ defined in equation (3) for different maturities. Panel (a) reports test statistics for a model that restricts the equilibrium policy rate to be constant, i.e., $y_t^* = y^*$ (c.f. equation (2)). Panel (b) reports test statistics for our (cointegrated) model with drifting equilibrium policy rates (c.f. equations (4)–(7)). The null hypothesis is absence of cointegration. The dashed red line is the critical value at 5% level of significance as suggested by MacKinnon (2010). Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

description of the stochastic trend underlying interest rates.

3.1.2 The dynamics of cyclical yields components

Next we study the behavior of the residual $u_t^{(n)}$. Specifically, Figure 4 shows the decomposition of the 10-year yield $y_t^{(40)}$ into $y_t^{(40),*}$ and $\delta_0^{(40)} + u_t^{(40)}$, as per equation (5). As before, Panel (a) refers to the restricted model whereas Panel (b) refers to our benchmark model with drifting equilibrium policy rates. It is obvious that the two models have opposite implications: the residuals (dotted line) follow a random walk under the classical model with
constant equilibrium rates, but are stationary in our model with drifting rates.\textsuperscript{18,19}

![Graph](image)

\textbf{Figure 4: Decomposing long-term rates.} Panel (a) shows the decomposition of the ten-year yield implied by a model which assumes away drifting monetary policy rates (i.e., $y_t = y^*$. Panel (b) shows the decomposition of the ten-year yield implied by our model with drifting equilibrium rates (see equations (4)–(7)). Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

Importantly, Figure 5 shows that our estimated deviations of bond prices from their drifts comove strongly with state-of-the-art term premium estimates like the one proposed by Bauer and Rudebusch (2020).

\textsuperscript{18}Replacing, in the restricted model, the perceived target rate $\pi_t^*$ with a fixed target rate at 2\%, leaves our conclusion unchanged: the 10-year residual is close to a random walk with an AR(1) coefficient of 0.98.

\textsuperscript{19}Wright (2011) argue for term premiums to decline internationally over the sample 1990–2007. Bauer et al. (2014) and Wright (2014) discuss the extent to which small-sample bias in maximum likelihood estimates of affine term structure models alters the conclusions about term premia and its (a)cyclical properties. Our evidence is complementary: we do not focus on statistical biases but we stress the importance of modeling the economic determinants of equilibrium rates. Furthermore, our framework is flexible and allows, without imposing, to interpret the (stationary) deviations of bond prices from their drifts as term premia.
Figure 5: Cyclical component from model with drifting equilibrium rates vs. term premium estimate: This figure shows the term premium component for a 10-year Treasury bond estimated following the methodology (OSE, observed shifting endpoint) proposed by Bauer and Rudebusch (2020) together with deviations of the 10-year bond yields from their drift, $\delta_0^{(40)} + u_t^{(40)}$, implied by our (cointegrated) model with drifting equilibrium rates (c.f., equations (4)–(7)). Quarterly observations. The sample period is 1980:Q1 to 2018:Q1.

This analysis is reminiscent of Joslin et al. (2013) who find that the estimated joint distribution within a macro-finance term structure model with NA is nearly identical to the estimate from an economic-model-free factor vector-autoregression. The evidence in Figure 5 suggests that this conclusion is likely to hold true also in models that accommodate a drifting term structure.

### 3.2 A Simple Test For the Role of Diagnostic Expectation

We assess the role played by deviations from rational expectations (in the form of diagnostic expectations) in explaining the cyclical component of yields within our framework.

Under Rational Expectations (RE), the cyclical component $u_t^{(n)}$ would be identified with
the term premium of the $n$-period bond. Consistently with our framework where $u^{(n)}_t$ should be stationary, Dai and Singleton (2002) argues that it is not plausible to consider the risk premium as a non-mean reverting component. However, a stationary $u^{(n)}_t$ is also consistent with, e.g., temporary deviations from Rational Expectations generated within a Diagnostic Expectations framework (see Gennaioli and Shleifer, 2018) where long rates over-react relative to change in expectations about short rates.

Following Bordalo et al. (2018) and d’Arienzo (2020), diagnostic expectations about a stationary process $\omega_t$ can be represented as follows:

$$E^{DE}\left[\omega_{t+1} \mid I_t\right] = E[\omega_{t+1} \mid I_t] + \theta\left(E[\omega_{t+1} \mid I_t] - E[\omega_{t+1} \mid I_{t-1}]\right).$$

(8)

We apply this expectation formation mechanism to the stationary deviations of the one period rate from its stochastic trend:

$$\omega_{t+1} = y^{(1)}_{t+1} - y^{*}_{t+1}.$$  

(9)

So we have:

$$E_t^{DE}\left[y^{(1)}_{t+1} - y^{*}_{t+1}\right] = E_t\left[y^{(1)}_{t+1} - y^{*}_{t+1}\right] + \theta\left(E_t\left[y^{(1)}_{t+1} - y^{*}_{t+1}\right] - E_{t-1}\left[y^{(1)}_{t+1} - y^{*}_{t+1}\right]\right).$$

(10)

Diagnostic expectations, $E^{DE}\left[y^{(1)}_{t+1} - y^{*}_{t+1} \mid I_t\right]$, differ from rational expectations, $E\left[y^{(1)}_{t+1} - y^{*}_{t+1} \mid I_t\right]$, by a shift in the direction of the information received at time $t$ on deviations of monetary policy from its (stochastic) trend. Under the diagnostic expectations hypothesis agents over-react to the stationary deviations of monetary policy from its trend.

Since we take the trend in monetary policy rates as exogenous, diagnostic expectations
on the drift coincide with the rational ones. We then write:

$$E_t^{DE} [y_{t+1}] = E_t [y_{t+1}^{(1)}] + \theta \left( E_t [y_{t+1}^{(1)} - y_{t+1}^*] - E_{t-1} [y_{t+1}^{(1)} - y_{t+1}^*] \right). \tag{11}$$

Interestingly, in this case, Equation (3) can then be re-written as follows:

$$y_t^{(n)} = \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} E_t [y_{t+i}^{(1)}] + \delta_0^{(n)} + \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} \left( E_t^{DE} [y_{t+i}^{(1)}] - E_t [y_{t+i}^{(1)}] \right) \tag{12}$$

Thus, the (stationary) component $u_t^{(n)}$ can in principle be explained by the over-reaction induced by diagnostic expectation: i.e., $u_t^{(n)}$ can be justified also if term premia are constant or even absent.

We rewrite equation (12) as follows:

$$y_t^{(n)} = \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} E_t [y_{t+i}^{(1)}] + \delta_0^{(n)} + \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} \left( E_t^{DE} [y_{t+i}^{(1)}] - E_t [y_{t+i}^{(1)}] \right) \tag{13}$$

where in the second step we exploit equation (11).

In all, we can estimate the $\theta$ parameter by running the following regression:

$$y_t^{(n)} - y_t^{(n,*)} = \delta_0^{(n)} + \theta \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} (E_t - E_{t-1}) [y_{t+i}^{(1)} - y_{t+i}^*]$$
where \( \left( \frac{1}{n} \right) \theta \sum_{i=0}^{n-1} \left( E_t \left[ y_{t+i} - y_t^* \right] - E_{t-1} \left[ y_{t+i} - y_t^* \right] \right) \) is obtained from forward simulation of our model.

Several comments are in order. First, deviations from rational expectations depend on the parameter \( \theta \) and on the persistence of the deviations of monetary policy rates from the trend. Second, the stationarity of \( (y_t^{(1)} - y_t^*) \) implies that, for large \( n \) (i.e., at long horizons), diagnostic expectations for the monetary policy rates will converge towards rational expectations.\(^{20}\) Third, and most important, the estimated value of the \( \theta \) parameter allows to assess the relevance of Diagnostic Expectations under the null of our model.

Table 2 displays the results for such test. We find that diagnostic expectations explain between 3% and 40% of the variability in the cyclical components of yields for bonds with maturity from 2 to 10 years. In line with our discussion of equation (3.2), the importance of diagnostic expectations decreases with the maturity of the bond.

The empirical relevance of overreaction has been recently documented by Cieslak (2018) for the short end of the curve. Similarly, Piazzesi et al. (2015) provide evidence that realized survey (interest rates) forecast errors as well as forecast differences relative to VAR-based measure may be responsible for the time-variation in bond premia from statistical models. We have shown that these explanations may be important even in a model that accommodates a drifting term structure. However, the contribution of overreaction decreases at long maturities; this is consistent with deviations of monetary policy rates from the equilibrium rate being fast mean-reverting.

\(^{20}\)Maxted (2019) considers a case in which convergence of DE to RE is not realized as the underlying process is non-stationary.
Table 2: Testing Diagnostic Expectations

This table reports seemingly unrelated regressions (SUR) estimates for regression (3.2) for bonds with different maturities (n). We restrict \( \theta \) to be the same across maturities. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

<table>
<thead>
<tr>
<th>( u_t^{(8)} ) (1)</th>
<th>( u_t^{(12)} ) (2)</th>
<th>( u_t^{(20)} ) (3)</th>
<th>( u_t^{(28)} ) (4)</th>
<th>( u_t^{(40)} ) (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{n} \sum_{i=0}^{n-4} (E_t - E_{t-1})[y_{t+1}^{(1)} - y_{t+i}] )</td>
<td>3.679***</td>
<td>3.679***</td>
<td>3.679***</td>
<td>3.679***</td>
</tr>
<tr>
<td>(0.299) (0.299) (0.299) (0.299) (0.299)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.717***</td>
<td>0.877***</td>
<td>1.189***</td>
<td>1.467***</td>
</tr>
<tr>
<td>(0.058) (0.067) (0.076) (0.081) (0.085)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>160</td>
<td>160</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>R^2</td>
<td>0.408</td>
<td>0.254</td>
<td>0.112</td>
<td>0.057</td>
</tr>
</tbody>
</table>

4 Predicting Holding Period Excess Returns

Predictability of interest rates on the basis of a model with a common stochastic trend in yields also implies predictability of holding period excess returns on the basis of the stationary deviations of bond yields from their drift.

To see this, write the expected excess return obtained by holding the \( n \)-period bond for one period as:

\[
E_t(rx_t^{(n)}) = y_t^{(n)} - (n-1)E_t(y_{t+1}^{(n-1)}) - y_t^{(1)}
\]

\[
= y_t^{(n)} - (n-1)\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n)}\right) - y_t^{(1)}
\]

\[
= y_t^{(n)} - y_t^{(1)} - (n-1)\left(E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}\right) - (n-1)\left(y_t^{(n-1)} - y_t^{(n)}\right), \quad (14)
\]

where \( y_t^{(n)} - y_t^{(1)} \) is the slope of the term structure, \( y_t^{(n-1)} - y_t^{(n)} \) is known as the roll-down,
and \( E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)} \) is the expected change in prices of the \((n - 1)\)-maturity bond. Since the seminal contributions by Fama and Bliss (1987) and Campbell and Shiller (1991), the slope of the term structure has played a central role for forecasting bond returns. Indeed, it is common to assume away any predictability arising from \( E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)} \), since the level of the term structure is deemed to be close to unforecastable (see, e.g., Duffee, 2013).

Our proposed “cointegrated” specification of the monetary policy rule and the term structure suggests otherwise. Using Equation (5) and assuming that the cyclical component follows an AR(1) process with persistence \( \rho \), i.e., \( u_t^{(n)} = \rho u_{t-1}^{(n)} + \epsilon_t^{(n)} \), one can express the expected price changes as

\[
E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)} = E_t(y_{t+1}^{(n-1),*} - y_t^{(n-1),*}) + (\rho_{(n-1)} - 1) \underbrace{(y_t^{(n-1)} - y_t^{(n-1),*} - \delta_0)}_{u_t^{(n-1)}}.
\]

Therefore, in our model, persistent but stationary deviations of bond prices from their drift, \( u_t^{(n-1)} \), show up as a natural predictor of excess bond returns.\(^{21}\) This term has gone unrecognized since standard models start off with stationary factor (within our framework, this is equivalent to assume a constant equilibrium rate). In turn, this leads to a close-to-unit-root residual (c.f., Figure 4(a)), or \( \rho_{(n-1)} - 1 \approx 0 \) (and the level being a random walk \( E_t(y_{t+1}^{(n-1)}) = y_t^{(n-1)} \)).\(^{22}\)

\(^{21}\)More precisely, \( u_t^{(n-1)} \) should forecast the price change component in bond returns. However, empirically the correlation between \( r x_{t+4}^{(n)} \) and the price change term, \( - (y_{t+4}^{(n-4)} - y_{t}^{(n-4)}) \), is high at 93%, 95%, 97%, 98%, and 99% for \( n = 8, 12, 20, 28, 40 \) quarters, respectively.

\(^{22}\)Cieslak and Povala (2015) and Jørgensen (2018) predict bond returns using a de-trended (term structure) level factor. Using their proposed persistence-based Wold decomposition, Ortu et al. (2020) extract a cyclical component from the level of the yield curve and show that it contains information about future excess bond returns. To our knowledge, we are the first to show that a cyclical component of the level of the term structure emerges as a natural predictor within a cointegrated framework of bond prices.
We start the evaluation of the predictive performance of our model with a drifting equilibrium rate by running the following regression:

\[ rx_{t+4}^{(n)} = \alpha + \beta E_t(rx_{t+4}^{(n)}) + \epsilon_t, \]  

(16)

where \( rx_{t+4}^{(n)} \) is the realized one-year holding period excess return of a bond with maturity \( n \)-quarters. We denote with \( E_t(rx_{t+4}^{(n)}) \) the expected excess return implied by our specification that allows for stationary deviations of bond prices from their drifts.\(^{23}\) We compare our specification to the classical model with a constant equilibrium rate.\(^{24}\) Table 3 displays the results for the model with constant equilibrium rate in Panel A, and the results for our model with drifting bond prices in Panel B. We consider maturities ranging from 2 (\( n = 8 \) quarters) to 10 years (\( n = 40 \) quarters). The regression of realized excess returns on the expected returns implied by our (cointegrated) model with drifting equilibrium rates delivers statistically significant estimates and coefficients of determination that are greater than 30% at all maturities.\(^{25}\) On the other hand, a classical model with constant equilibrium rates leads to a coefficient not significantly different from zero and to small explanatory power.

We also highlight that the model with constant equilibrium rates performs worse than a (reduced-form) model based just on the slope. This is easily explained. The realized returns \( rx_{t+4}^{(n)} \) on the left hand side of (16) are stationary whereas the expected returns \( E_t(rx_{t+4}^{(n)}) \) from the model with constant equilibrium rate is non-stationary since it inherits the drift from the residual component \( u_t^{(n)} \) (c.f. Figure 4).

\(^{23}\)We exploit equations (4)–(7) together with the exogeneity of demographics and potential output to construct the expected change in constant-maturity yield \( (E_t(y_{t+1}^{(n-1)}) - y_t^{(n-1)}) \) in equation (14).

\(^{24}\)To make our results comparable to a large literature (e.g., Cochrane and Piazzesi, 2005; Cieslak and Povala, 2015) we focus on one-year excess returns. However, our conclusions are identical when we use one-quarter holding period returns.

\(^{25}\)The constant is not statistically significant for bond with maturities \( n = 8, 12, 20 \) quarters.
Table 3: Predictive Regressions across Different Maturities

This table reports OLS estimates for the regression \( rx_{t+4}^{(n)} = \alpha + \beta E_t(rx_{t+4}^{(n)}) + \epsilon_t \) where \( rx_{t+4}^{(n)} \) is the realized one-year holding period excess return of a bond with maturity \( n \)-period and \( E_t(rx_{t+4}^{(n)}) \) is the expected excess return implied by our specifications. Panel A reports results for the classical model with a constant equilibrium rate. Panel B reports results for our model with drifting equilibrium rates. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

### Panel A: Model with constant equilibrium rate.

<table>
<thead>
<tr>
<th>( E_t(rx_{t+4}^{(8)}) )</th>
<th>( E_t(rx_{t+4}^{(12)}) )</th>
<th>( E_t(rx_{t+4}^{(20)}) )</th>
<th>( E_t(rx_{t+4}^{(28)}) )</th>
<th>( E_t(rx_{t+4}^{(40)}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.427**</td>
<td>0.308</td>
<td>0.212</td>
<td>0.171</td>
<td>0.139</td>
</tr>
<tr>
<td>(0.201)</td>
<td>(0.188)</td>
<td>(0.167)</td>
<td>(0.151)</td>
<td>(0.131)</td>
</tr>
</tbody>
</table>

| Observations             | 156                      | 156                      | 156                      | 156                      | 156                      |
| R^2                      | 0.128                    | 0.092                    | 0.069                    | 0.060                    | 0.054                    |

### Panel B: Model with drifting equilibrium rate.

<table>
<thead>
<tr>
<th>( E_t(rx_{t+4}^{(8)}) )</th>
<th>( E_t(rx_{t+4}^{(12)}) )</th>
<th>( E_t(rx_{t+4}^{(20)}) )</th>
<th>( E_t(rx_{t+4}^{(28)}) )</th>
<th>( E_t(rx_{t+4}^{(40)}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.868***</td>
<td>0.822***</td>
<td>0.693***</td>
<td>0.706***</td>
<td>0.619***</td>
</tr>
<tr>
<td>(0.187)</td>
<td>(0.159)</td>
<td>(0.151)</td>
<td>(0.120)</td>
<td>(0.120)</td>
</tr>
</tbody>
</table>

| Observations             | 156                      | 156                      | 156                      | 156                      | 156                      |
| R^2                      | 0.366                    | 0.366                    | 0.331                    | 0.361                    | 0.320                    |
4.1 Dissecting Predictive Regressions

To further dissect the unique contribution coming from our cointegrated approach, Table 4 shows that the expected change in the \((n-1)\)-maturity bond prices drives away the predictability of the slope (column (1)), and that deviations of bond prices from their drift, \(u_t^{(n-1)}\), are the most important driver of such predictability (c.f. columns (3) and (4)). Also, the loading on the cyclical component \(u_t^{(n-1)}\) is negative as predicted by our framework: if \(0 < \rho_{(n-1)} < 1\), then next period returns are negative in times when bond prices are higher than those implied by their drift.

In the Appendix, we show that the relevance of such cyclical component for forecasting excess returns is not restricted to any specific maturity or holding period. Table B.2 reports results for the predictive regressions when we use bonds with maturities ranging from 2- to 7-years. Also, Table B.3 confirms that stationary deviations of bond prices from their drift predict quarterly holding period bond returns (i.e., non-overlapping returns). Overall, this evidence suggests that the adjustment of bond prices towards their drift is a key economic mechanism for understanding bond returns predictability.

Finally, Appendix Table C.1 shows that the US cyclical component \(u_t^{(n)}\) predict UK and Canadian bond returns, even after controlling for the local slope of the term structure.\(^{26}\) This finding resonates with the evidence in Dahlquist and Hasseltoft (2013). Despite this similarity, Dahlquist and Hasseltoft (2013) attributes the international comovement in bond returns to a global (admittedly, mostly US) bond risk premium; on the other hand, we have not imposed no-arbitrage restrictions so that our cyclical component is also compatible with

\(^{26}\)Consistent with our model, we employ the local slope of the term structure as a proxy for the deviations of non-US yields from their drifts. Controlling for the local cyclical component does not change our conclusion. However, we note that the lack of an exogenous potential output series, \(\Delta x_t^{pot}\), and of a perceived target inflation rate, \(\pi_t^*\), may be responsible for the poor performance of the local cycle in Canada and UK. Further investigation on this topic is on our agenda for future research.
investors overreacting to deviations of policy rates from its trend leading to overestimation of future short rates (and lower bond returns).\footnote{Our findings are also consistent with the idea that the Fed is the leader among central banks in setting monetary policy (Brusa, Savor and Wilson, 2019). See also One Policy to Rule Them All: Why Central Bank Divergence Is So Slow (Wall Street Journal, 2016) for a recent discussion on the topic.}

### Table 4: Dissecting Predictive Regressions

This table reports OLS estimates for the regression $r_{x_{t+4}}^{(40)} = \alpha + \beta' X_t + \epsilon_t$ where $r_{x_{t+4}}^{(40)}$ is the realized one-year holding period excess return of a bond with maturity 10-year and $X_t$ contains different return predictors. Column (1) exploits equation (14) reported here for reader’s convenience:

\[
E_t(r_{x_{t+4}}^{(40)}) = y_t^{(40)} - y_t^{(4)} - (40 - 4) \left( E_t(y_{t+4}^{(40-4)}) - y_t^{(40-4)} \right) - (40 - 4) \left( y_t^{(40-4)} - y_t^{(40)} \right).
\]

Column (2) shows that the slope is a significant predictor of excess bond returns when considered in isolation. Columns (3) and (4) exploit the decomposition of expected price changes per equation (16) reported here for reader’s convenience:

\[
E_t(y_{t+4}^{(40-4)}) - y_t^{(40-4)} = E_t \left( y_{t+4}^{(40-4),*} - y_t^{(40-4),*} \right) + \left( \rho^{(40-4)} - 1 \right) u_t^{(40-4)}.
\]

In columns (3) and (4) we neglect the roll-down term which empirically is found to be insignificant. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^{(40)} - y_t^{(4)}$</td>
<td>3.217</td>
<td>2.320*</td>
<td>0.740</td>
<td>0.929</td>
</tr>
<tr>
<td></td>
<td>(2.085)</td>
<td>(1.397)</td>
<td>(2.280)</td>
<td>(1.212)</td>
</tr>
<tr>
<td>$- (40 - 4) \left( E_t(y_{t+4}^{(36)}) - y_t^{(36)} \right)$</td>
<td>0.543***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$- (40 - 4)(y_t^{(36)} - y_t^{(40)})$</td>
<td>-4.340</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.817)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$- (40 - 4) \left( E_t(y_{t+4}^{(36),<em>}) - y_t^{(36),</em>} \right)$</td>
<td></td>
<td>-0.241</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.260)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$- (40 - 4) u_t^{(36)}$</td>
<td></td>
<td>-0.640***</td>
<td>-0.626***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.164)</td>
<td>(0.119)</td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.342</td>
<td>0.060</td>
<td>0.316</td>
<td>0.320</td>
</tr>
</tbody>
</table>
4.2 The Information Content of Yield Cycles

Several bond returns predictors have been proposed in the literature since the seminal papers by Fama and Bliss (1987) and Campbell and Shiller (1991). It is then natural to ask to what extent the yield cycles $u_t^{(n)}$ capture new information not already conveyed by other variables.

Specifically, we compare the predictive power of our yield cycles to two well known return-predicting factors that are both constructed from the yield curve: (1) the Cochrane and Piazzesi (2005, CP) factor which is based on a linear combination of forward rates; (2) the Cieslak and Povala (2015, CPo) factor which relies on information contained in yields that have been detrended using a long-term moving average of inflation.

Rather than using a specific cycle for each maturity $n$ we construct a common yield cycle using a procedure akin to Cochrane and Piazzesi (2005). Specifically, we run regressions of the average (across maturity) excess return on all cycles,

$$
\frac{1}{9} \sum_{n=2}^{10} r_{x_{t+4}}^{(n)} = \gamma_0 + \gamma_1 u_t^{(1)} + \ldots + \gamma_{40} u_t^{(40)} + \varepsilon_{t+1}.
$$

Our yield-based cycle factor is given by $\tilde{u}_t = \tilde{\gamma}' u_t$.

Table 5 shows the results. In Panel A we investigate the predictive content of our cycle

---

28Several papers have found that the state of the economy also conveys information about future bond returns. E.g., Cooper and Priestley (2008) propose the output gap, whereas Ludvigson and Ng (2009) propose to extract information from a large set of macrofinancial variables. Related, Bansal and Shaliastovich (2013) document that real growth and inflation uncertainties predict, respectively, lower and higher bond risk premia, and propose a long-run risk type model for rationalizing this finding. Since our yield cycles are obtained by removing the stochastic trend (due to the equilibrium rate) in interest rates, we restrict our attention only to yield-based predicting factors.

29To construct the CP and CPo factors we follow the procedure described in the original papers. I.e., to construct the CP factors we use only one- through five-year zero coupon bond prices and estimate the loadings by running a regression of the equally-weighted average (across maturity) excess return on the forward rates. To construct the CPo factor instead we employ duration standardized returns. To be consistent with the overall empirical analysis, unlike in the original papers, both factors are constructed using quarterly observations.
relative to the CP factor, whereas in Panel B we compare it to the CPo factor. The odd columns confirm that both CP and CPo forecasts excess returns of all bonds. Importantly, Panel A shows that our yield cycle drives away the CP factor, and delivers $R^2$ that are about three times those obtained by the CP regressions. Panel B tells a similar story. Despite the large $R^2$ obtained by the CPo factor, our yield cycle continues to be a significant predictor of bond returns at all maturities ranging from 2- to 10-years. In fact, comparing the $R^2$ from the multiple regression in Panel A to those in Panel B, we see that replacing CP with CPo does not alter the predictive content of our yield-based cycle.
Table 5: Predictive Regressions: Horse race against other bond predictors

This table reports OLS estimates for the regression \( r_{x_t(n)}^{(n)} = \alpha + \beta_1 F_t + \beta_2 \tilde{u}_t + \epsilon_t \) where \( r_{x_t(n)}^{(n)} \) is the realized one-year holding period excess return of a bond with maturity \( n \)-period, \( F_t \) is the Cochrane and Piazzesi (2005) factor (CP\(_t\)) in Panel A and the Cieslak and Povala (2015) factor (CPo\(_t\)) in Panel B, and \( \tilde{u}_t \) is the single-return forecasting factor implied by our model with drifting equilibrium rates. The Cochrane-Piazzesi factor is constructed as in Cochrane and Piazzesi (2005) using quarterly zero-coupon Treasury yields from Gürkaynak et al. (2007) with maturities from 1 to 5 years. The Cieslak-Povala factor is constructed as in Cieslak and Povala (2015) using quarterly zero-coupon Treasury yields from Gürkaynak et al. (2007) with maturities from 1 to 10 years. \( \tilde{u}_t \) is the fitted value from regressing the average one-year holding-period excess returns on a \( n \)-periods Treasury bond for \( n = 4, 8, \ldots, 40 \) on our cyclical components \( u_t(n) \) \( n = 1, \ldots, 40 \) (see Eq. (16)). Values in parenthesis are standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

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<th>( r_{x_t(8)}^{(8)} )</th>
<th>( r_{x_t(12)}^{(12)} )</th>
<th>( r_{x_t(20)}^{(20)} )</th>
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<td>(5)</td>
</tr>
<tr>
<td>CP(_t)</td>
<td>0.434***</td>
<td>0.104</td>
<td>0.821***</td>
<td>0.168</td>
<td>1.554***</td>
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<td></td>
<td>(0.139)</td>
<td>(0.148)</td>
<td>(0.305)</td>
<td>(0.331)</td>
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<tr>
<td>( \tilde{u}_t )</td>
<td>0.219***</td>
<td>0.434***</td>
<td>0.807***</td>
<td>0.118***</td>
<td>1.188***</td>
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<tr>
<td></td>
<td>(0.035)</td>
<td>(0.079)</td>
<td>(0.164)</td>
<td>(0.240)</td>
<td>(0.344)</td>
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<td>156</td>
<td>156</td>
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<tr>
<td>Adjusted R(^2)</td>
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<td>0.390</td>
<td>0.132</td>
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<tr>
<td>CPo(_t)</td>
<td>1.366***</td>
<td>0.428</td>
<td>2.720***</td>
<td>0.959</td>
<td>5.230***</td>
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<td>(0.516)</td>
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<td>( \tilde{u}_t )</td>
<td>0.184***</td>
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<td>0.787**</td>
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<td>(0.240)</td>
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<td>156</td>
<td>156</td>
<td>156</td>
<td>156</td>
</tr>
<tr>
<td>Adjusted R(^2)</td>
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<td>0.342</td>
<td>0.433</td>
<td>0.472</td>
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4.3 Out-Of-Sample Predictability

As a final robustness test we consider out-of-sample predictability as measured by $R^2_{OOS}$ computed as follows:

$$R^2_{OOS} = 1 - \frac{\sum_{t=1}^{T} \left( r_{x_t+4}^{(n)} - \hat{r}_{x_t+4}^{(n)} \right)^2}{\sum_{t=1}^{T} \left( r_{x_t+4}^{(n)} - \bar{r}_{x_t+4}^{(n)} \right)^2}$$

where $\hat{r}_{x_t+4}^{(n)}$ is the fitted value from our predictive regression estimated through period $t - 1$ and $r_{x_t+4}^{(n)}$ is the historical average return estimated thorough period $t - 1$. If the $R^2_{OOS}$ is positive, then the predictive regression has lower average mean squared prediction error than the historical average return. This is always the case for all regressions reported in Table 6.

Table 6: Out-Of-Sample Tests

This table reports $R^2_{OOS}$ for the predictive regression $r_{x_t+4}^{(n)} = \alpha + \beta' \tilde{u}_t + \epsilon_t$ where $r_{x_t+4}^{(n)}$ is the realized one-year holding period excess return of a bond with maturity $n$-period and $\tilde{u}_t$ is the single-return forecasting factor implied by our model with drifting equilibrium rates. $\tilde{u}_t$ is the fitted value from regressing the average one-year holding-period excess returns on a $n$-periods Treasury bond for $n = 4, 8, \ldots, 40$ on our cyclical components $u_t^{(n)}, n = 1, \ldots, 40$ (see Eq. (16)). We use a rolling window for estimating the predictive regressions. The $R^2_{OOS}$ is computed as in Campbell and Thompson (2008); $p$-values for $R^2_{OOS}$ are computed as in Clark and West (2007). In Panel A the out-of-sample period starts in 1990; in Panel B the out-of-sample period starts in 2000. Quarterly observations.

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<td>$R^2_{OOS}$</td>
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<tr>
<td>$r_{x_t+4}^{(8)}$</td>
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<tr>
<td>(1)</td>
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<tr>
<td>$R^2_{OOS}$</td>
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5 Conclusions

This paper proposes a general framework to model a common drift in bond prices, and studies its implications for monetary policy, the term structure of interest rates, and bond returns predictability.

We start by showing that there is a drift in monetary policy rates which can be successfully modeled by fluctuations in productivity, demographics, and long-term inflation expectations. Our approach delivers monetary policy residuals that are substantially less persistent than those implied by standard policy rules. Thus, through the lens of our analysis, we find that monetary inertia is overestimated when the drift in policy rate is not modeled.

The drift in bond prices is described by the average of expected monetary policy rates over the residual life of the bond. Appropriate modeling of the drift in monetary policy should deliver stationary deviation of yields to maturity from their drift. These stationary deviations of bond prices from their drift could be explained by the presence of term premia and/or by temporary deviations from rational expectations in a behavioral framework. Our empirical evidence shows that deviations from rational expectations in the form of Diagnostic Expectations account for up to 40% of the fluctuations in yield cycles for bonds with maturities 2-year. However, the importance of DE decreases at longer maturities leaving an important role for term premia. At a minimum, when the deviations of bond prices from their drift are interpreted as term premia, our finding implies that models that misspecify the drift in monetary policy and in bond prices will fail to generate stationary term premia.

Finally, we find that persistent but stationary deviations of U.S. Treasury bond prices from their drift predict excess returns in- and out-of-sample, as well as outside the U.S. Next period returns from holding long-term bonds are negative in times when bond prices are higher than those implied by their drift. Once again this predictability can be related to
term premia or to reversion of temporary overreaction about future monetary policy. Future research should investigate the origins of bond price deviations from their drift and of the associated returns predictability documented in this paper.
References


Berardi, Andrea, Michael Markovich, Alberto Plazzi, and Andrea Tamoni (2020) “Mind the (Convergence) Gap: Bond Predictability Strikes Back!,” *Management Science (Forthcoming)*.


Online Appendix

A Data

We employ quarterly data in our empirical analysis; thus, we proxy for the 1-period bond yields using the end-of-quarter 3-month Treasury bill rates from the Federal Reserve’s H.15 release. Our sample period starts with Paul Volekers appointment as Fed chairman, because of evidence that monetary and macroeconomic dynamics changed at that time (e.g., Gertler et al., 1999).

Zero-coupon Treasury yields with 1- to 10-year maturities are from Gürkaynak et al. (2007).

The Federal Reserve’s perceived target rate (PTR) for inflation is a survey-based measure of long-run inflation expectations; PTR is used in the Fed’s FRB/US model and available at https://www.federalreserve.gov/econres/us-models-package.htm.

MY is available until 2050 and is hand-collected from various past Census reports available at https://www.census.gov/data.html. Potential output is available until 2030 and can be downloaded at https://fred.stlouisfed.org/series/GDPPOT. See also Figure A.1.

Figure A.1: Drivers of the Equilibrium Nominal Rate. This figure shows the dynamics for the drivers of the time-varying equilibrium nominal rate \( y_t^* \) in equation (1). The left panel shows the ratio of middle-aged (40-49) to young (20-29) population, \( MY \), and for potential output growth, \( \Delta x_{pot}^t \). The right panel shows the Federal Reserve’s perceived target rate (PTR) for inflation. \( MY \) is available until 2050 and is hand-collected from various past Census reports available at https://www.census.gov/data.html. Potential output is available until 2030 and can be downloaded at https://fred.stlouisfed.org/series/GDPPOT. Dotted vertical lines denote the end of our sample, i.e., 2019:Q4. Quarterly observations.
### B Additional Results

#### Table B.1: Testing Parametric Restriction on the Cointegrating Relationship between Yields and Drifting Equilibrium Rates

This table reports OLS estimates for the regression $y_t^{(n)} = \alpha + \beta y_t^{(n),*} + \varepsilon_t$, where $y_t^{(n)}$ is the observed yield at time $t$ of a bond with maturity $n$-period and $y_t^{(n),*} = (\frac{1}{n}) \sum_{i=0}^{n-1} E[y_{t+i}^{(1)} | I_t]$. Values in parenthesis are 95% confidence interval. Costant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

| $y_t^{(8),*}$ | 1.082*** |  (0.950, 1.213) |
| $y_t^{(12),*}$ | 1.059*** |  (0.914, 1.203) |
| $y_t^{(20),*}$ | 1.014*** |  (0.853, 1.174) |
| $y_t^{(28),*}$ | 0.984*** |  (0.810, 1.158) |
| $y_t^{(40),*}$ | 0.960*** |  (0.755, 1.165) |

| Observations | 160 | 160 | 160 | 160 | 160 |
| R²           | 0.944 | 0.930 | 0.914 | 0.902 | 0.888 |
Table B.2: Predictive Regressions (across different maturities): Slope versus Cyclical Component

This table reports OLS estimates for the regression \( r_{x_t+4}^{(n)} = \alpha + \beta_1(y_t^{(n)} - y_{t}^{(4)}) + \beta_2(-(n-4) \ u_{t}^{(n-4)}) + \epsilon_t, \)
where \( r_{x_t+4}^{(n)} \) is the realized one-year holding period excess return of a bond with maturity \( n \)-period, \( y_t^{(n)} - y_{t}^{(4)} \) is the slope for a \( n \)-period bond, and \( -(n-4) \ u_{t}^{(n-4)} \) is the deviation of a \( n \)-period maturity yield from its drift. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

<table>
<thead>
<tr>
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<th>( r_{x_t+4}^{(8)} )</th>
<th>( r_{x_t+4}^{(12)} )</th>
<th>( r_{x_t+4}^{(20)} )</th>
<th>( r_{x_t+4}^{(28)} )</th>
</tr>
</thead>
</table>
| \( y_t^{(8)} - y_{t}^{(4)} \) | 1.389*  
(0.832)  | 1.587*  
(0.865)  | 1.474*  
(0.773)  | 1.265  
(0.914)  |
| \(-8 \ - \ 4 \) \( u_{t}^{(4)} \) | -1.237***  
(0.301)  | -1.187***  
(0.271)  | -0.886***  
(0.189)  | -0.765***  
(0.122)  |
| \(-12 \ - \ 4 \) \( u_{t}^{(8)} \) | \( -0.857*** \) \( (0.178) \)  | \( -1.187*** \) \( (0.271) \)  | \( -0.886*** \) \( (0.189) \)  | \( -0.765*** \) \( (0.122) \)  |
| \( y_t^{(12)} - y_{t}^{(4)} \) | \( -0.886*** \) \( (0.189) \)  | \( -0.857*** \) \( (0.178) \)  | \( -1.187*** \) \( (0.271) \)  | \( -0.765*** \) \( (0.122) \)  |
| \(-12 \ - \ 4 \) \( u_{t}^{(12)} \) | \( -0.857*** \) \( (0.178) \)  | \( -1.187*** \) \( (0.271) \)  | \( -0.886*** \) \( (0.189) \)  | \( -0.765*** \) \( (0.122) \)  |
| \( y_t^{(20)} - y_{t}^{(4)} \) | \( -0.886*** \) \( (0.189) \)  | \( -0.857*** \) \( (0.178) \)  | \( -1.187*** \) \( (0.271) \)  | \( -0.765*** \) \( (0.122) \)  |
| \(-20 \ - \ 4 \) \( u_{t}^{(20)} \) | \( -0.790*** \) \( (0.148) \)  | \( -0.886*** \) \( (0.189) \)  | \( -1.187*** \) \( (0.271) \)  | \( -0.765*** \) \( (0.122) \)  |
| \( y_t^{(28)} - y_{t}^{(4)} \) | \( -0.886*** \) \( (0.189) \)  | \( -0.857*** \) \( (0.178) \)  | \( -1.187*** \) \( (0.271) \)  | \( -0.765*** \) \( (0.122) \)  |
| \(-28 \ - \ 4 \) \( u_{t}^{(28)} \) | \( -0.745*** \) \( (0.127) \)  | \( -0.886*** \) \( (0.189) \)  | \( -1.187*** \) \( (0.271) \)  | \( -0.765*** \) \( (0.122) \)  |

Observations 156 156 156 156 156 156 156 156  
Adjusted R\(^2\) 0.319 0.259 0.341 0.276 0.361 0.319 0.356 0.334
Table B.3: Predictive Regressions (quarterly holding period returns): Slope versus Cyclical component

This table reports OLS estimates for the regression $r_{x_{t+1}}^{(n)} = \alpha + \beta_1 (y_{t}^{(n)} - y_{t}^{(1)}) + \beta_2 (- (n - 1) u_{t}^{(n-1)}) + \epsilon_t$, where $r_{x_{t+1}}^{(n)}$ is the realized one-quarter holding period excess return of a bond with maturity $n$-period, $y_{t}^{(n)} - y_{t}^{(1)}$ is the slope for a $n$-period bond, and $(- (n - 1) u_{t}^{(n-1)})$ is the deviation of a $n$-period maturity yield from its drift. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors with automatic bandwidth selection procedure as described in Newey and West (1994). Constant estimates are not tabulated. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.

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<td>$y_t^{(8)} - y_t^{(1)}$</td>
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<td></td>
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<tr>
<td>$-(8 - 1) u_t^{(7)}$</td>
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<td>$y_t^{(12)} - y_t^{(1)}$</td>
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<td>$-(12 - 1) u_t^{(11)}$</td>
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<td>$R^2$</td>
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<td>0.116</td>
<td>0.108</td>
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Figure B.1: Actual vs Fitted Short-Term Rate: Equilibrium Real Rate. This figure shows actual three-months yield and fitted values for our baseline (cointegrated) model with drifting equilibrium rates (c.f. equation (1); green dashed line), and for a cointegrated rule with $r^*$ (brown dotted line). The estimated cointegrated policy rule with $r^*$ has the following coefficients (HAC standard errors in parenthesis):

$$y_t^{(1)} = 0.667^{***} r_t^* + 1.449^{***} \pi_t^* + 0.822^{***} E_t(\pi_{t+1} - \pi_{t+1}^*) + 0.318^{***} E_t(x_{t+1}), \quad R^2 = 94.3\% .$$

We denote $r^*$ as the equilibrium real rate. We get an estimate for the equilibrium real rate by regressing the real rate $r_t = y_t^{(1)} - E_t(\pi_{t+4})$ on $MY$ and potential output growth. We use as $E_t(\pi_{t+4})$ the expected one-year ahead inflation from the Survey of Professional Forecasts (SPF). The estimates for $r^*$ are:

$$r_t^* = -4.040^{***} MY_t + 1.812^{***} \Delta x_{t}^{pot}, \quad R^2 = 68\% .$$

Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.
Figure B.2: Long-Term Forecasts of Short-Term Rate: Interest Rate Smoothing (1). This figure shows actual three-months yield and predicted rates implied by equation (1) in case of the policy rule with constant equilibrium rate and interest rate smoothing (brown dotted line) or our baseline (cointegrated) model with drifting equilibrium rates (green dashed line). The estimated empirical Taylor rule with interest rate smoothing (one lag) has the following coefficients (HAC standard errors in parenthesis):

\[
y^{(1)}_t = 0.310^{***} + 0.935^{***} y^{(1)}_{t-1} - 0.034^{(0.140)} E_t(\pi_{t+1} - \pi^*_t) + 0.070^{**} E_t(x_{t+1}), \quad R^2 = 92.7\%.
\]

Dotted vertical lines represent the end of in-sample estimation period. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.
Figure B.3: Long-Term Forecasts of Short-Term Rate: Interest Rate Smoothing (2). This figure shows actual three-months yield and predicted rates implied by equation (1) in case of the policy rule with constant equilibrium rate and interest rate smoothing (brown dotted line) or our baseline (cointegrated) model with drifting equilibrium rates (green dashed line). The estimated empirical Taylor rule with interest rate smoothing (two lags) has the following coefficients (HAC standard errors in parenthesis):

\[ y_t^{(1)} = 0.253^{*} + 0.792^{***} y_{t-1}^{(1)} + 0.173^{***} y_{t-2}^{(1)} - 0.185 E_t(\pi_{t+1} - \pi^*_t + 1) + 0.052^{**} E_t(x_{t+1}), \quad R^2 = 94.9\% . \]

Dotted vertical lines represent the end of in-sample estimation period. Quarterly observations. The sample period is 1980:Q1 to 2019:Q4.
### C International Evidence

Table C.1: Predictive Regressions (across different maturities): Slope versus Cyclical Component

This table reports OLS estimates for the regression $r_{x,t+4}^{(n)} = \alpha + \beta_1(y_{t}^{(n)} - y_{t}^{(4)}) + \beta_2(-(n-4)\ u_{t}^{(n-4)}) + \epsilon_t$, where $r_{x,t+4}^{(n)}$ is the realized one-year holding period excess return of a bond with maturity $n$-period, $y_{t}^{(n)} - y_{t}^{(4)}$ is the slope for a $n$-period bond, and $(-(n-4)\ u_{t}^{(n-4)})$ is the deviation of a $n$-period maturity yield from its drift. Values in parenthesis are conservative standard errors from reverse regressions computed as in Hodrick (1992). Constant estimates are not tabulated. Quarterly observations. For UK, zero-coupon bonds data are from the Bank of England (https://www.bankofengland.co.uk/statistics/yield-curves); the sample period is 1980:Q1 to 2019:Q4. For Canada, zero-coupon bonds data are from the Bank of Canada (https://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/); the sample period is 1986:Q1 to 2019:Q4.

#### Panel A UK

<table>
<thead>
<tr>
<th></th>
<th>$r_{y,n}^{(12)}$</th>
<th>$r_{y,n}^{(20)}$</th>
<th>$r_{y,n}^{(40)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{t}^{(12)} - y_{t}^{(4)}$</td>
<td>1.152 (0.737)</td>
<td>1.537 (0.749)</td>
<td>1.970 (1.103)</td>
</tr>
<tr>
<td>$-(12 - 4)\ u_{t}^{(8)}$</td>
<td>1.152 (0.140)</td>
<td>0.336 (0.106)</td>
<td>-0.289 (0.091)</td>
</tr>
<tr>
<td>$y_{t}^{(20)} - y_{t}^{(4)}$</td>
<td>0.350 (0.140)</td>
<td>0.344 (0.109)</td>
<td>-0.393 (0.095)</td>
</tr>
<tr>
<td>$y_{t}^{(40)} - y_{t}^{(4)}$</td>
<td>0.371 (0.144)</td>
<td>0.344 (0.109)</td>
<td>-0.393 (0.095)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.126</td>
<td>0.149</td>
<td>0.159</td>
</tr>
<tr>
<td>Observations</td>
<td>156</td>
<td>156</td>
<td>156</td>
</tr>
</tbody>
</table>

#### Panel B Canada

<table>
<thead>
<tr>
<th></th>
<th>$r_{y,n}^{(12)}$</th>
<th>$r_{y,n}^{(20)}$</th>
<th>$r_{y,n}^{(40)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{t}^{(12)} - y_{t}^{(4)}$</td>
<td>1.062 (0.792)</td>
<td>1.362 (1.072)</td>
<td>1.883 (1.136)</td>
</tr>
<tr>
<td>$-(12 - 4)\ u_{t}^{(8)}$</td>
<td>0.478 (0.146)</td>
<td>0.422 (0.107)</td>
<td>0.339 (0.116)</td>
</tr>
<tr>
<td>$y_{t}^{(20)} - y_{t}^{(4)}$</td>
<td>-0.511 (0.141)</td>
<td>0.456 (0.101)</td>
<td>-0.393 (0.104)</td>
</tr>
<tr>
<td>$y_{t}^{(40)} - y_{t}^{(4)}$</td>
<td>-0.511 (0.141)</td>
<td>0.456 (0.101)</td>
<td>-0.393 (0.104)</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.132</td>
<td>0.132</td>
<td>0.132</td>
</tr>
</tbody>
</table>