Market structure, managerial ethics, and a theory of corporate misconduct*

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Abstract

Misconduct is widespread in Finance alongside other industries. Mis-selling, mis-representation and negligence harm consumers, or society, while raising profits. I develop a model in which managers are ethical. Managers must find a balance between their ethics, the actions which maximise their profits, any risk of regulatory sanction, and risk of reputational cost. Market structure raises two issues. First concentrated markets allow managers to capture large rents from small levels of misconduct, hurting many people only a little. But in dispersed competitive markets profits may be meagre without misconduct. Modelling these competing tensions theoretically I characterise when competition encourages misconduct.

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1 Introduction

Corporate scandals are widespread. Recent cases which have been prosecuted and resulted in fines being levied include the fixing of the foreign exchange markets and the mis-selling of mortgage related products: Since 2013 the European Union has fined 6 banks for corrupt behaviour in the interest rate derivative market, 7 banks for corruption in the trade of euro derivatives, 3 banks for collusion on the Swiss franc, and has accused 8 banks for corruption with respect to the Eurozone government bond market.\textsuperscript{1} Financial misconduct has led to a Royal Commission in Australia; to billions of fines being issued by UK regulators; to heated cross-examinations of Finance CEOs in the US Senate.\textsuperscript{2} Consumers are not the only group to be affected, and Finance is not the only industry to be implicated. In Europe the ‘horse-meat’ scandal saw a number of producers of burgers in multiple countries illicitly substitute horse-meat for beef. In the oil industry BP’s alleged safety negligence has resulted in harm to the environment and tragically even to worker deaths.\textsuperscript{3} The use by builders of sub-standard concrete has been blamed for building collapse across the developing world and resulted in many deaths.

In each case the senior managers who authorised the behaviour which ran the risk of such scandals would have been trading off a number of considerations: competitive pressure on margins, risk of regulatory sanctions, risk of reputational costs, and as they are human, the ethical distaste from conducting malpractice for profit. In this paper I model these competing effects theoretically. In so doing I study which market structures are most conducive to corporate misconduct and corporate malpractice.

Understanding whether competitive markets increase or reduce the risk of misconduct would be valuable for all regulators, and perhaps especially financial regulators, to know. It would facilitate the allocation of scarce monitoring budgets to maximal effect. However a tension is created which makes the answer \textit{a priori} unclear. One might suspect that competitive markets are most conducive to malpractice as firms earn small profits without

\textsuperscript{1}Cases cited in Easley and O’Hara (2019).
\textsuperscript{2}For references see footnotes 4, 5, and 13.
\textsuperscript{3}Respectively the Deepwater Horizon Oil spill, and the Texas City Refinery Disaster which left 15 workers dead.
malpractice, and any individual manager might reason that the harm they do only affects a small number of consumers as market shares are low. However one might instead suspect that large firms in concentrated markets have the greatest incentive to malpractice: their market shares are so large that even very small amounts of malpractice per consumer, which are hard to spot, can generate very substantial extra profits.

Casual empiricism also fails to provide a guide – misconduct has been prominently reported in both competitive and concentrated industries.

For example, consider the sale of a form of mortgage default insurance known in the UK as PPI. The market for such insurance was served by many sellers: in 2008, 6,619 regulated firms were active in the UK. This was therefore a competitive market. However the sales techniques used across the industry at that time were misconduct: they included encouraging unemployed buyers to purchase insurance for loss of job; encouraging elderly buyers over the maximum insured age to buy the insurance; and training in ‘disturbance techniques’ which will be described below. These practices were endemic – a consumer body estimated in 2008 that one in three such policies were ‘worthless’. Hence the subsequent scandal and fines, with total costs to the industry exceeding £50 billion.4

But misconduct can and does occur in concentrated markets also. The London interbank offered rate (LIBOR) is set by between 7 and 18 member banks, each of which is large and international – a market structure at the opposite extreme from the PPI example. Between 2005 and 2007 these banks were habitually gaming the LIBOR setting process so as to manipulate the rate. This allowed the banks to profit from financial products whose prices were linked to this rate; it also allowed banks to mis-represent the true value of their balance sheet, and so appear better capitalised than they really were. These LIBOR banks have since collectively paid billions of dollars of fines, and Barclays dismissed its CEO immediately after admitting its involvement.5

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4 For evidence see links at Table 1, Commission et al. (2009) §2.54, https://www.theguardian.com/business/2011/may/05/how-ppi-scandal-unfolded, https://www.which.co.uk/news/2008/05/one-in-three-with-ppi-may-find-it-worthless-144107/, https://www.ft.com/content/78da80e0-d2c9-11e9-8367-807ebd53ab77

The contention that human decision makers have moral qualms is, I believe, not controversial. There is some evidence that those who work in Finance might be less ethical than their peers (Cohn et al. (2014)). But nonetheless senior managers in Finance profess an attachment to integrity in their professional lives. For example, Bob Diamond, the former CEO of Barclays says of the LIBOR scandal: “Oh, it was horrible, ... I’ve never had to manage something more challenging for my family, my friends, my integrity.”\(^6\) While Tim Sloan, CEO of Wells Fargo after the scandal of fraudulent checking accounts, declared his commitment to “act with honesty, integrity, and accountability.”\(^7\) Thus the personal, reputational as well as regulatory costs of unethical behaviour can be substantial; both Bob Diamond and John Stumpf, the latter formerly CEO of Wells Fargo, lost their positions in the wake of the LIBOR and checking account scandal respectively.

I embed ethical concerns in a model of firm competition in a repeated market. In each period the firms set and compete in prices, and can simultaneously also decide to engage in some malpractice. The malpractice raises profits by lowering production costs, but harms one of the other stakeholders in the production process: consumers, workers, and/or the environment. Managers internalise some ethical displeasure in their utility function in a manner informed by Philosophical discourse and supported by psychological research. A market regulator can potentially detect malpractice with sufficient evidence to fine, and the probability that this happens grows in the extent of malpractice a firm undertakes. If malpractice is discovered then the firm manager loses their position and so faces a substantial reputational cost. Consumers are unaware that the product is, or even might be, less good than described – this captures that consumers assume their financial counterparts are not explicitly lying, that safety laws have been appropriately followed, that firms have not engaged in corruption. Thus, in the context of this model, consumers do not have rational expectations. The model also applies if consumers do have rational expectations but are uncaring of potential abuses towards distant workers, or far-off environmental damage.


\(^7\)Contained in https://stories.wf.com/ceo-speak-congressional-leaders?cid=wfsshare_em_112771
In the sections that follow we solve this model to derive and study both the symmetric market equilibrium, and the asymmetric equilibrium when firm managers differ in their levels of ethical observance as well as their costs.

The equilibrium level of malpractice balances the ethical cost, the risk of discovery leading to sanctions and reputational costs, and the profit effect. As competition increases I show that the costs associated with the risk of discovery shrink more rapidly than the profit and ethics effects. This result depends on the shape of demand, but when it holds, and I show it does so in most ‘standard’ scenarios, two results follow. The first is that we can offer an answer as to whether competitive market structures which are concentrated (e.g. LIBOR) or dispersed (e.g. PPI) are most conducive to malpractice: it is the latter. As more firms enter a market the break on malpractice offered by the risk of regulatory sanctions and the risk of reputational harm shrinks more rapidly than the incentive to malpractice created by profit concerns – though this too is shrinking. The net effect is that, at least beyond a competitive threshold, market equilibrium misconduct levels rise. The second result that follows is that in competitive markets the level of misconduct is predominantly a balance between the leaders’ ethics, i.e. their integrity, and the profit incentive. The regulatory and reputational effects are less germane. Hence leaders’ integrity is particularly important in competitive markets.

One reason competition is desirable is that it forces prices down and so increases consumers’ surplus. However we now have a countervailing effect: competition increases malpractice, at least in standard settings. It is therefore natural to wonder if increasing competition can lower consumer surplus overall, or is any misconduct harm to consumers outweighed by reductions in firms’ margins? I demonstrate that it is possible for competition to reduce consumer surplus, but only when firm numbers cross a critical threshold.

Not all leaders are equally ethical. Indeed one argument advanced in favour of opening markets to foreign investment is that if a more ethically minded foreign multinational buys an otherwise corrupt firm then it can raise the ethical levels, not just of the acquired firm, but of the whole market. Our framework allows us to study this contention and characterise whether it holds, or whether on the contrary, the entrant’s ethical observance
will just regress to match that of its rivals.

The organisation of this paper is as follows. First we discuss the related literature (Section 2). Then we offer a simple motivating example (Section 3), before presenting the model (Section 4). Symmetric competition is analysed first (Section 5), then asymmetric duopoly (Section 6). Extensions including simulations follow (Section 7) and then the concluding remarks (Section 8). Technical material is contained in the Appendices.

2 Related literature

Many have noted the desirability of introducing moral reasoning into economic modeling (Arrow (1973), Hausman and McPherson (1993)), however I believe this is the first attempt to try to interact ethical decision making with Nash product market competition. A literature exists which studies the principal-agent problem when agents have ethics but abstracts from strategic interactions between firms (Bénabou and Tirole (2006), Morrison and Thanassoulis (2017), Carlin and Gervais (2009)). While, in the spirit of ethics, Besley and Ghatak (2005) and Gorton and Zentefis (2019) introduce some equilibrium interaction through agents’ labour choices which arise when culturally minded workers are matched with similar firms. Easley and O’Hara (2019) show in a network model, and Parsons et al. (2018) empirically, that unethical behaviour tends to cascade so as to involve all market participants who are connected; for example being in the same geographic market. The approach here of studying market behaviour is consistent with these latter results, yet none of these works studies a model of product market competition, as I do here.

A rich branch of research in psychology studies settings in which agents face a conflict between their values and material interests. It has been, I believe compellingly, established that a tension does exist (Loewenstein et al. (2011), Sah and Loewenstein (2010)); and further the influence of morals on actions appears to depend upon the extent of competition (Falk and Szech (2013)). Focusing on lying games, experiments have established that agents have an aversion to lying, and an aversion to being thought of as liars
(Abeler et al. (2019), Dufwenberg and Dufwenberg (2018)). The contribution here is to explore the effect of competition on malpractice, including mis-selling, which is lying to the consumer.

In this work consumers do not have rational expectations – they do not anticipate the mis-selling and malpractice to which they are exposed. This fits many of the scandals described above (mortgage insurance fraud, horse-meat scandal, mis-selling of mortgages), as well as others described below. Thus I am not offering a signalling model between firms and consumers over product quality (e.g. Daughety and Reinganum (2008), Rhodes and Wilson (2018)), nor a model of reputation construction (e.g. Ely and Välimäki (2003)). Gabaix and Laibson (2006) study competition for consumers without rational expectations. My contribution here is to study the propensity for unethical behaviour including the risks of regulatory and reputational cost.

Choosing to conduct misconduct so as to lower the costs of production might be interpreted as a type of process innovation. This study can therefore be compared to the literature on competition and investment in R&D (Vives (2008), Schmutzler (2013)). In studies such as these there is no regulatory enforcement parallel, and the game is not repeated leaving no role for reputation effects. Sanction and reputation effects are important when studying ethics and misconduct, and these differences mean that the forces at play in this study are different. Further these differences lead to material differences in the predictions. In standard settings this study predicts that misconduct increases in the extent of competition; in Vives (2008) investment declines with competition, the opposite result.  

Moving away from formal modelling, in widely cited work Shleifer (2004) reflects on what follows from an assumption that ethical behaviour is a normal good. A normal good is one which an agent demands less of as their income declines. It is then immediate that if income declines with competition, then so will the demand for ethical behaviour: people,

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8Further, Vives (2008) is typical in modelling a cost to developing an innovation which is independent of volumes supplied, but grows in the extent of innovation. It is difficult to reinterpret this in an ethics framework as one would then be making the assumption that the disutility from misconduct was not altered by the number of people the misconduct was perpetrated on. This would contradict the tenets of utilitarianism (see on) at the minimum, and would likely be unappealing as an assumption to many.
it is assumed, would care less for ethics if they were poorer, and would care more for ethics if they were richer. It is easy to think of counter-arguments to such an assumption: wealthy individuals such as Bernie Madoff and Harvey Weinstein did not become more ethical as they became wealthier. In this work I do not make an assumption on whether or not ethics are a normal good. My approach is closer to the Becker (1968) tradition in which the costs and benefits of behaviour are all modeled. Even if one accepted that owner-managers who are poorer valued ethics less, it also follows that they will be subject to fines if they are unethical and they could lose all their future business. So a model is important to disentangle the effects.

Shleifer (2004) offers five examples which he suggests have unethical behaviour coinciding with competition: child labour, corruption of government officials, high executive pay, earnings manipulation, and the commercialisation of the market for education in the US. We will explore the evidence between misconduct and competition more fully in Section 5.4 below.

This study explores how ethical considerations interact with competitive pressure to yield misconduct. An important setting in which competition has been linked to unwanted outcomes concerns the relationship between competition and financial stability (Keeley (1990) empirically and Allen and Gale (2004) theoretically). Empirically whether competition increases or reduces financial crises is unclear as competitive effects interact with regulatory scrutiny in an opaque manner (Beck et al. (2006)), and the theoretical analyses do not address the propensity for unethical behaviour.

The work here complements research which models competition between firms in the labour market, not the product market. This literature argues that as talented labour becomes more scarce, misconduct can result. This may arise because firms choose to willingly risk regulatory sanction (Song and Thakor (2019)) or myopic behaviour (Bénabou and Tirole (2016), Thanassoulis (2013)) so as to be able to offer a more lucrative compensation package to a potential employee; or because a surplus of possible employers allows workers to run away and hide from the outcome of their poor prior behaviour (Acharya et al. (2016)). Labour market competition raises firms’ costs as compensation
rises, while product market competition lowers firms’ prices; in either case profits fall with competition, and competitive product markets tend to be ones where salaries are highest (Guadalupe (2007)). However the mechanisms at work are different in the two literatures and so the results differ beyond this initial similarity. The analysis of product market competition offered here allows for owner-managers’ ethics. Ethical distaste is related to the amount of harm done, and as volumes change with product market competition the ethical force in decision making is endogenised. Next, though product market competition lowers profits, it need not lower margins, and so competition in the product market need not result in more malpractice.

3 Motivating example

Consider two symmetric firms competing along a Hotelling line of length one. Consumers are uniformly distributed along this line and have transport cost \( t \). As \( t \) rises the firms are more differentiated, and so the firms exert a weaker competitive constraint on each other. If the firms set prices \( p_1 \) and \( p_2 \) then equating the disutility of transport costs and prices yields the location of the indifferent consumer, and this is identical to the demand of firm 1 due to the uniform distribution assumption:

\[
q_1(p_1, p_2) = \frac{1}{2} + \frac{p_2 - p_1}{2t}
\] (1)

Extend the standard Hotelling objective function of each firm to allow for malpractice and for ethics. In the model section that follows we will develop the formulation of these effects. To motivate the work let us simplify as much as possible and merely assert firm 1’s objective function to be:

\[
\Pi_1(p_1, y_1; p_2) = q_1 (p_1 - (c - y_1)) \left( 1 - \Phi \tilde{\delta} y_1 \right) - \omega q_1 y_1
\] (2)

where \( y_1 \) is malpractice undertaken by firm 1 which has the effect of lowering costs to \( c - y_1 \). Such malpractice increases the probability of being detected at a rate \( \tilde{\delta} \); and if
detected results in a fine increasing in profits at a rate $\Phi$. The parameter $\omega$ captures the ethics cost to the owner-manager of firm 1, which for now is increasing in the level of malpractice and the number of consumers affected.

Let us suppose that the firms are at a symmetric equilibrium $(p^e, y^e)$. We will seek to establish how the competition parameter $t$, and the symmetric equilibrium level of malpractice are related.

At equilibrium neither firm would have an incentive to alter their prices or malpractice slightly. The first order conditions will therefore hold. First we evaluate the first order condition with respect to prices. Note from (1) that at a symmetric equilibrium

$$q^e = \frac{1}{2} \quad \text{and} \quad \frac{\partial q_1}{\partial p_1} = -\frac{1}{2t}.$$  

Therefore

$$\left. \frac{\partial \Pi_1}{\partial p_1} \right|_{p_1 = p^e} = 0 \Rightarrow (p^e - c + y^e) \left(1 - \Phi \delta y^e \right) - \omega y^e = t \left(1 - \Phi \delta y^e \right).$$

(4)

The first order condition with respect to malpractice yields:

$$\left. \frac{\partial \Pi_1}{\partial y_1} \right|_{p_1 = p^e} = 0 \Rightarrow 1 - \Phi \delta y^e - \Phi \delta (p^e - c + y^e) - \omega = 0.$$  

(5)

Combining (4) and (5) so as to remove prices we have the following condition which must hold at symmetric equilibrium:

$$1 - \Phi \delta y^e - \omega = \Phi \delta \cdot \left[ t + \frac{\omega y^e}{1 - \Phi \delta y^e} \right].$$

(6)

Equation (6) allows us to readily develop the desired comparative static between competition $t$ and malpractice $y^e$. Observe that the left hand side (LHS) of (6) is declining in $y^e$, while the right hand side (RHS) of (6) is increasing in $y^e$. The equilibrium level of
malpractice is given by the intersection of the LHS and RHS functions; this is plotted in Figure 1.

![Figure 1: Motivating example](image)

If the transport cost parameter $t$ increases, then the RHS function moves upwards as depicted in Figure 1. This proves that:

**Proposition 1** *In the symmetric Hotelling case, equilibrium malpractice is declining in the level of the transport cost:*

\[
\frac{dy^e}{dt} < 0.
\]

As the transport cost parameter rises there is less competition. The firms exert a weaker competitive constraint on each other as they are more differentiated. Therefore Proposition 1 establishes that, in the simple Hotelling example, as competition declines the equilibrium level of malpractice also declines.

Proposition 1 is suggestive, and I hope interesting, but it leaves numerous questions open. What is the intuition for the result? Next competition is more naturally identified with the number of competing firms, rather than a transport cost, while the transport cost is perhaps better thought of as capturing a feature of the firms’ demand functions. So does misconduct depend on competition or on the shape of demand? This motivating example has taken the simplest possible model of ethics, how would the results vary as such modelling became richer. Further, what can we conclude when competition is between firms which are not symmetric?

All of these questions are addressed in this study.
4 The Model

I consider a repeated game in which competition takes place at integer times: $t = 1, 2, \ldots$.

Each individual stage of competition is subdivided into two sub-periods. Price decisions, malpractice decisions, and consumer demand occur in the first sub-period. Then there follows a second sub-period in which the regulatory authority may detect malpractice with sufficient evidence to fine.

I suppose that all market participants discount payoffs one period ahead by $e^{-r}$, and that there is no discounting between sub-periods. I assume that if evidence is found that an owner-manager is guilty of malpractice at time $t$ then she pays the resultant penalty, and further, she is replaced for all future periods. Thus she has to relinquish the firm’s assets to another owner-manager who will run the firm from $t + 1$ onwards. This captures that misconduct carries the risk of a reputational cost. An example of such an outcome is the dismissal of the CEOs of Barclays and Wells Fargo described above. Such outcomes can be enforced if a regulatory authority determines the former owner-manager is not a “fit and proper person” to run the enterprise. These assumptions imply that the utility gained by the initial owner-manager would cease after time $t$, though there would continue to be $n$ firms competing.

I now formally introduce all these parts of the model and I describe how ethical considerations are woven into the analysis.

4.1 The competition stage

4.1.1 First sub-period of each competition stage

There are $n \geq 2$ competing firms indexed by $i$. Each firm has marginal cost $c$ of production. The firms discover a new (unethical) practice by which they can raise profits, but at potential cost to one or more stakeholders. I will describe such malpractice shortly. The

\[9\text{Gill and Thanassoulis (2016) embed a two-stage competition model in a repeated interaction in an analogous manner.}\]

\[10\text{Such fit and proper tests are common as detailed by Boskovic et al. (2010). In Europe the test is a subjective one undertaken by the competent authorities, for example through MiFID requirements (see https://www.handbook.fca.org.uk/handbook/FIT.pdf). The US has a rules based approach in which evidence of specific malpractice is required to bar individuals.}\]
firms are each run by an owner-manager. Each owner-manager simultaneously decides on an amount of unethical practice \( y_i \in [0, \bar{y}] \) with \( \bar{y} < c \), and on a retail price \( p_i \).

**Malpractice** The unethical practice lowers the marginal cost of firm \( i \) to \( c - y_i \). The unethical practice harms one of the stakeholders in the production process: consumers, workers or the environment. Examples of such unethical practices are mis-selling, risking workers by circumventing costly safety processes, or risking the environment by saving costs.

The history of misconduct in financial markets provides multiple candidates for such misconduct. For example a financier could practice *cherry picking* by which the owner-manager illicitly assigns the best performing trades to her own account, and leaves the less good trades for the clients (e.g. FCA 2014. Aviva. or SEC 2015. Mark P. Welhouse and Welhouse & Associates Inc.); or the owner-manager could use *wash trades*, the practice of pairing trades so as to cancel out any change in beneficial ownership with one leg conducted in secret, so as to deceive the client and create the illusion of market demand and liquidity (a prominent example is US 1935. United States v. Brown et al.). Other prominent examples of misconduct affecting consumers, workers and the environment are given in Table 1.

Consumers are assumed to be unaware of the possibility of malpractice and not to form rational expectations as to the losses they might suffer. This captures many of the examples in Table 1: for example in the mis-selling of mortgage default insurance (PPI) consumers accepted sellers’ assertions as to the coverage of the insurance and failed to anticipate they were being deceived; in the horse-meat scandal consumers had no inkling that their beef-burger might contain horse. In these examples, once consumers are made aware of the widespread malpractice, it stops.

This model would also fit situations in which consumers, though aware of the malpractice, decline to alter their purchasing decisions. This is known as the *intentions – behaviour gap* in marketing science (Auger and Devinney (2007), Carrington et al. (2010)).

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11 This modelling convention allows me to circumvent principal-agent issues within the firm.
12 The source for these cases is “Behavioural Cluster Analysis”, 2018, FICC Market Standards Board.
The owner-managers estimate that had the injured parties been aware of the malpractice at level $y$, their utility would have been reduced by the function $\hat{\alpha}(y)$. More malpractice is more harmful, $\hat{\alpha}'(y) > 0$, and no malpractice leaves the consumer unaffected, $\hat{\alpha}(0) = 0$. If $\hat{\alpha}(y) = y$ then the injury is dollar for dollar commensurate with the seller’s gain; this is the case with theft for example.

The utility loss may be consciously experienced by the stakeholder in some cases, such as a de prioritisation of worker safety. However, in many prominent cases, such as those involving consumer harm, the loss need not be consciously experienced by the stakeholder. Thus in the Brazilian fraud scandal (Giannetti et al. (2017)) a borrower might mistakenly assume her inability to get credit was for good reasons as opposed to the banks’ fabrication of repayment history. The firm owner-manager will however be aware that her actions have harmed a stakeholder and the magnitude of this effect is captured by $\hat{\alpha}(y)$.

**Price Competition** I model competition in prices with differentiated goods using the discrete choice random utility framework of Perloff and Salop (1985). I assume that there exists a unit measure of consumers who desire one unit of the good. The $n$ firms each sell a version of this good. The firms are horizontally differentiated so that $X^j$ is the random match utility of a consumer for firm $j$. The $X^j$ of any consumer are iid so that the firms are ex ante symmetric. The variables $X^j$ are distributed according to a cumulative distribution function $F(x)$ with support $[\underline{x}, \bar{x}] \subseteq \mathbb{R}$, and bounded and differentiable density function $f(x)$. In common with much of the analysis using this framework, I assume that the market is fully covered so that all consumers purchase one product.

The analysis will first focus on symmetric equilibria. Let $p^e$ denote the symmetric equilibrium price. Suppose firm $i$ deviates to $p_i$. Consumers, as noted above are unaware of the malpractice and so do not infer the use of the malpractice technology from this price change. Note that $\Pr(\max_{k \neq i} X^k \leq x) = F(x)^{n-1}$ so $F(x)^{n-1}$ is the cumulative distribution of the highest match utility amongst $n−1$ firms. The demand for firm $i$’s
product is therefore,

\[ q_i(p_i) = \Pr \left( X^i - p_i > \max_{k \neq i} X^k - p^e \right) = \int_{\bar{x}}^{\bar{x}} [1 - F(x + p_i - p^e)] dF(x)^{n-1} \quad (7) \]

Define the function

\[ P(n) := n \int_{\bar{x}}^{\bar{x}} f(x) dF(x)^{n-1}. \quad (8) \]

In the Perloff and Salop (1985) model, without ethics, the equilibrium price is inversely related to \( P(n) \). We can now write the derivative of demand with respect to firm \( i \)'s price at the equilibrium price level \( p^e \), using (7), as

\[ q_i'(p^e) = -\int_{\bar{x}}^{\bar{x}} f(x) dF(x)^{n-1} = -\frac{P(n)}{n}. \quad (9) \]

This model develops the motivating example of Section 3. In that example the Hotelling model provided a microfoundation for firm demand functions with the properties at symmetric equilibrium given by (3). These properties were sufficient to allow the motivating example to be solved. We can now demonstrate that the richer Perloff and Salop (1985) model here replicates this feature of the motivating example. Consider symmetric duopoly so that \( q_i(p^e) = 1/2 \). Suppose consumers’ match utilities are uniformly distributed on [0, 2t]. In this case \( f(x) = 1/2t \) and \( F(x) = x/2t \). It follows that \( P(2) = 1/t \) so (9) yields \( q_i'(p^e) = -1/2t \). The model therefore embeds the relevant conditions of Hotelling demand.

### 4.1.2 Second sub-period of each competition stage

If malpractice is discovered with sufficient evidence to convict, then the authorities usually levy a fine. The fines for a number of examples of malpractice are listed in Table 1.\(^\text{13}\)

\(^{13}\)For press references please see:

https://www.usatoday.com/story/money/2016/07/14/bp-deepwater-horizon-costs/87087056/
<table>
<thead>
<tr>
<th>Unethical practice aimed at:</th>
<th>Misconduct</th>
<th>Regulatory Fine</th>
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<tbody>
<tr>
<td><strong>Consumers</strong></td>
<td></td>
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<tr>
<td>Mis-selling products, e.g. horse-meat in burgers scandal</td>
<td>Aware of no major fines</td>
<td></td>
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<tr>
<td>Abacus scandal (Goldman Sachs)</td>
<td>SEC fine of $550 million (disgorgement $15m and civil penalty $535m)</td>
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<tr>
<td>Mis-selling techniques, e.g. PPI Insurance for mortgages scandal</td>
<td>£19 billion for largest 4 UK banks.</td>
<td></td>
</tr>
<tr>
<td>Market manipulation, e.g. Forex fix; LIBOR fix; Consumer credit-score manipulation scandal in Brazil</td>
<td>$5.8 billion fines in US for both scandals for 6 large banks.</td>
<td></td>
</tr>
<tr>
<td><strong>Workers</strong></td>
<td></td>
<td>$80 million fine.</td>
</tr>
<tr>
<td>Saving on safety costs, e.g. BP’s Texas City Refinery disaster</td>
<td>$62 billion of fines.</td>
<td></td>
</tr>
<tr>
<td><strong>The environment</strong></td>
<td></td>
<td>$470 million out of court fine.</td>
</tr>
<tr>
<td>Cost cutting leading to environmental disaster, e.g. BP Deepwater Horizon Oil Spill The Bhopal Disaster</td>
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I assume in this model that the regulator has the right to inspect firms and the technology to detect malpractice. The probability of a regulator identifying malpractice with sufficient evidence to levy a fine (or convict) is assumed to be an increasing function of how much malpractice is being conducted:

$$ \Pr (\text{detection with enough evidence to fine}) = \delta (y) $$ \hspace{1cm} (10)

where $\delta (0) = 0$, $\delta(\bar{y}) \leq 1$, and $\delta' > 0$.\(^{14}\) I do not assume that the probability of detection is a function of the volume of production as once a firm has decided to implement a level of unethical behaviour, then one or more processes will need to be established to put the decision into effect; for example, the creation of a training regime for sales people in ‘disturbance techniques’ to ensure PPI is sold aggressively;\(^{15}\) instructions given to foremen as to how frequently production should be stopped to allow safety checking, and

\(^{14}\)This extends the motivating example in which $\delta(y) := \bar{\delta} \cdot y$

\(^{15}\)See *PPI exposé: how the banks drove staff to mis-sell the insurance* in The Guardian, 8 November 2012.
so on. Regulatory authorities will seek evidence of the existence of such processes to substantiate a fine.

The detection technology function $\delta(\cdot)$ is exogenous to the model. An outcome of this analysis will be whether a regulator can improve ethical standards by linking enforcement effort to the industrial structure, captured here by the level of competition. In case of detection, fines are assumed to be a multiple $\Phi \geq 0$ of profits made. The setting $\Phi = 0$ captures practices which, though harmful to some and unethical, are not illegal. Examples might be releasing gases into the atmosphere which contribute to global warming. Such processes, though legal, with some probability attract the negative attention of consumers and cause reputational damage. I make the following assumption:

**Assumption 1:**

$$\delta(y)\Phi \leq 1.$$  \hspace{1cm} (11)

This condition is immediate if $\Phi \leq 1$. Otherwise we require the authority’s ability to detect and acquire evidence of wrong-doing to not be too great.

### 4.2 Ethics

In the first part of this study, I assume that each owner-manager has a common utility function. This is relaxed in the duopoly analysis in Section 6. The utility function will allow for numerous different interpretations of ethics.

Denote the utility within a single competition stage, and ignoring all anticipated payoffs from future time periods, by:

$$U^i := q_i(p_i)[(p_i - c + y_i)(1 - \Phi\delta(y_i)) - \omega\alpha(y_i)]$$  \hspace{1cm} (12)

All except the final term are standard: the owner-manager will enjoy her profits arising from volumes times profit margin $(p_i - c + y_i)$ unless the regulator discovers evidence of any malpractice. The probability of being detected is $\delta(y_i)$. In this case a fine proportional to profits is extracted.

The final term, $q_i \cdot \omega \cdot \alpha(y)$, captures the ethical component of the owner-manager’s
preferences, and introduces a new function $\alpha(y)$ and new parameter $\omega$. This formulation is rich enough to cover a number of different possible philosophical traditions and approaches to ethics.

*Act Utilitarianism*

Act utilitarianism captures situations in which the agent believes it is ethically justified to consider both the potential harm done to a stakeholder as well as the potential cost savings to the firm. As an example consider a financial advisor who markets an actively traded fund to her client, but once the mandate is secured, seeks to avoid trading costs by purely following an index. (This is not a solely academic example.) The financial manager might reason that this secret switch in approaches is ethically justified as the saving in trading costs outweighs the likely reduction in the investment return. Such ethical reasoning which sets firm benefits against the costs imposed on others is consistent with that professed by many business managers (Premeaux (2004)).

Such reasoning can be modeled here by assuming:

$$\alpha(y) = \tilde{\alpha}(y) - y$$

(13)

The change in aggregate surplus caused by the act of firm $i$ raising her malpractice from zero to $y_i$, leaving all other decision variables, including the price, unchanged is $q_i(p_i)(y_i - \tilde{\alpha}(y_i))$ as misconduct $y_i$ harms stakeholders according to $q_i(p_i)\tilde{\alpha}(y_i)$ but also lowers costs by an amount $y_i$ per unit produced. (13) imports the change in aggregate surplus into the utility function (12), and weights it according to the parameter $\omega$. The parameter $\omega$ is the agent’s willpower and measures their propensity to act in accordance with their moral reasoning (Roberts (1984)).

*Deontological*

Observe that act utilitarian moral reasoning in the financial advisor example above

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16See *EU probe into asset managers uncovers potential mis-selling* in the Financial Times, February 2, 2016.

17Note that the fine paid does not form part of the aggregate surplus calculation as it is a transfer to the government and so nets out of the surplus calculation.
gives no special weight to the fact that the financial advisor lied: she secured the investment mandate with a promise to be an active fund, but she will subsequently not hold to that promise. I study a form of deontological ethics which captures this.

A deontological ethic should capture that an owner-manager would feel it is wrong to use a stakeholder as a means purely for their own financial gain. In experiments it is common for agents to profess a desire for fairness (e.g. the ultimatum game, see Rabin (1993) and Camerer and Thaler (1995)). This is reminiscent of the second formulation of Kant’s Categorical Imperative, namely: “Act in such a way that you treat humanity, ..., never merely as a means to an end, but always at the same time as an end.” (Kant (1785)) If an owner-manager misleads customers to increase her profit then she is denying each customer the chance of making an informed choice, and so is treating the customer as a means and not as an end; she is not being fair. Thus the financial advisor described above would, if she professed a deontological ethic, believe that lying to the customer as to the nature of the investment strategy was unethical, even if the trading costs saved were substantial.

Such reasoning can be modelled here by assuming:

\[ \alpha(y) = \tilde{\alpha}(y) \]  

(14)

The total harm done by the misconduct of firm \( i \) when she raises her malpractice from zero to \( y_i \) is \( \tilde{\alpha}(y_i) \) per unit produced. Using (14) in (12) imports this into the utility function weighted according to the propensity to be ethical, \( \omega \). This term is not moderated by any considerations of firm profit.\(^{18}\)

4.3 The repeated game

As noted above the competition stage game repeats for the owner-manager of firm \( i \) each time period with probability \( 1 - \delta(y_i) \) which captures the possibility that consumers

\(^{18}\)In addition the disutility from malpractice grows in the harm done, and in this respect is distinct philosophically from Kant’s reasoning. A more faithful representation of Kant’s logic is analysed in Section 5.3.
become informed of the malpractice and punish the firm, or it captures the possibility that the regulator detects the malpractice and forces the owner-manager out.

The utility function of the owner-manager of firm $i$ can be denoted by $V^i$. In a stationary equilibrium this utility satisfies

$$V^i (p^e, y^e) = U^i (p^e, y^e) + e^{-r} (1 - \delta (y_e)) V^i (p^e, y^e)$$

as the owner-manager of firm $i$ loses control of her firm with probability $\delta (y_e)$. It follows that at equilibrium values:

$$V^i (p^e, y^e) = \frac{U^i (p^e, y^e)}{1 - e^{-r} (1 - \delta (y_e))}.$$  \hspace{1cm} (15)

4.4 Solution Concept

We search for pure strategy Nash equilibria. We will study the characteristics of such equilibria which satisfy the further condition of asymptotic stability (Dixit (1986), Anishchenko et al. (2014) Chapter 2). An equilibrium is asymptotically stable if, were each firm to alter its actions at a rate proportional to the local first order gain, then small deviations from equilibrium would be damped and lead the system back to the equilibrium values. If such stability were not present then there exist small deviations which would lead firms’ actions to diverge from the equilibrium permanently.

The game is depicted graphically for reference in Figure 2.

5 Link between competition and misconduct

We pause to recall the tension in the model. We wish to identify whether competition between a few large firms, or between many small firms, is more conducive to malpractice. Perhaps a few large firms in competition are more likely to engage in misconduct. In support of such a view one might argue that for large firms even a small amount of misconduct creates a large increase in profits, and a small amount of misconduct is hard for a regulator to catch. The opposite view would contend that perhaps many
small firms in competition are more likely to engage in misconduct. Proponents of this view might argue that such firms make only small profits without misconduct creating a strong incentive to misconduct, and further as each firm has a small market share, any misconduct would only harm a small number of people and so be more palatable ethically.

This model offers an answer to this tension which we capture in this first main result:

**Theorem 2** In any symmetric stable equilibrium of the competition and misconduct game:

1. If $1 - F(x)$ is log-concave then the level of malpractice is positive and increasing in the number of competing firms for $n > n^*$; and there is no malpractice if few enough firms compete ($n \leq n^*$).

2. If $1 - F(x)$ is log-convex then the results are reversed: no malpractice for $n > n^*$; and a positive level of malpractice which declines in the number of competing firms when $n \leq n^*$.

In each case the critical threshold $n^*$ is unique and is given by the solution to

$$P(n^*) = \frac{\delta'(0) \left( \Phi + \frac{\epsilon - r}{1 - e^{-r}} \right)}{1 - \omega \alpha'(0)}$$

(16)
The critical threshold may lie below 2, or at infinity, in which case the relevant region applies for all \( n \geq 2 \).

We prove this theorem in Section 5.1. In Section 5.2 we discuss the intuition for the result, we explore the interaction of ethics and misconduct in Section 5.3 and discuss empirical evidence in Section 5.4.

5.1 Proof of Theorem 2

An interior symmetric equilibrium \((p^e, y^e)\) must satisfy firm \( i \)'s first order conditions at the equilibrium values. To derive these, suppose that firm \( i \) considered deviating from the equilibrium values \((p^e, y^e)\) to some other set of prices and malpractice, \((p_i, y_i)\), for one period. Firm \( i \)'s utility would then be:

\[
V^i(p_i, y_i) = U^i(p_i, y_i) + e^{-r(1 - \delta(y_i))} V^i \bigg|_e
\]  

(17)

The expression \( _e \) denotes that the function should be evaluated at equilibrium values. In this case we capture that the deviation alters the payoff this period and also the probability that the game repeats for this owner-manager. The first order conditions are:

\[
V^i_{y_i} \bigg|_e = 0 \quad \text{and} \quad V^i_{p_i} \bigg|_e = 0.
\]  

(18)

Variables in subscripts denote partial differentiation. Any equilibrium is characterised by (18).

Taking differentials of conditions (18) with respect to \( dy^e, dp^e \) and the parameter of interest, \( dn \), we can write:

\[
\begin{pmatrix}
\left( V^i_{y_i} \bigg|_e \right)_{y^e} & \left( V^i_{y_i} \bigg|_e \right)_{p^e} \\
\left( V^i_{p_i} \bigg|_e \right)_{y^e} & \left( V^i_{p_i} \bigg|_e \right)_{p^e}
\end{pmatrix}
\begin{pmatrix}
dy^e \\
dp^e
\end{pmatrix} =
\begin{pmatrix}
\left( V^i_{y_i} \bigg|_e \right)_{n} \\
\left( V^i_{p_i} \bigg|_e \right)_{n}
\end{pmatrix} dn.
\]  

(19)

Comparative static analysis hinges on the behaviour of the Hessian matrix, \( \mathcal{H} \), and in
particular the sign of its determinant.

As $V^i(p_i, y_i)$ in (17) is not a function of others’ malpractice ($\{y_{j\neq i}\}$) we have that

$$\left( V^i_{y_i}|_e \right)_{y_e} = V^i_{y_i,y_i}|_e < 0, \quad (20)$$

the inequality following from the second order condition for firm $i$. The second order conditions for each individual firm do not offer further insights into $\mathcal{H}$ as the matrix is an amalgamation of the first order effects from all the $n$ competing firms, and these do not aggregate in a convenient fashion.

Lemma 3 below offers an extension to asymptotic stability theory by showing that the Hessian matrix $\mathcal{H}$ inherits useful structure from the stability of the full $n$ player game:

**Lemma 3** If an interior symmetric equilibrium of the $n$ firm competition game is stable, then all the eigenvalues of the $2 \times 2$ matrix $\mathcal{H}$ have negative real parts, and in particular

$$\det \mathcal{H} > 0.$$  

**Proof.** See Appendix A. 

The assumption of stability is that if firms individually adjust their actions in proportion to their first order gain, any deviations from the equilibrium values will be damped (Dixit (1986)). As each of the $n$ firms have 2 actions, a price and a level of malpractice, there are $2n$ variables. Using Taylor expansions, the speed of change of each variable depends on changes in each of the $2n$ game variables. This yields a $2n \times 2n$ transition matrix, denoted $\mathcal{A}$, which shares many terms in common with the Hessian of interest, $\mathcal{H}$.

Stability implies that the transition matrix $\mathcal{A}$ has negative eigenvalues (Anishchenko et al. (2014)). The proof of Lemma 3 proceeds by assuming, for a contradiction, the existence of an eigenvector of $\mathcal{H}$ with positive eigenvalue. If such an eigenvector exists, then the main work of the proof is to construct an eigenvector of the larger transition matrix $\mathcal{A}$ which shares the same eigenvalue. Once this is done stability yields the sought-after contradiction; and the lemma is proved.

Having established that the determinant of $\mathcal{H}$ is positive we have a route to proving the required comparative static, if we can also sign the other terms featured in equation
Using (17) and (18) we have:

\[ V_{yi} \big|_e = -e^{-r}\delta'(y_i) \ V^i \big|_e + U_{yi} \big|_e = 0 \tag{21} \]

where, from (12)

\[ U_{yi} \big|_e = \frac{1}{n} \left[ 1 - \Phi \delta(y_e) - \Phi \delta'(y_e)(p^e - c + y_e) - \omega \alpha'(y_e) \right]. \tag{22} \]

The first order condition with respect to prices, using (17), yields

\[ V_{pi} \big|_e = U_{pi} \big|_e = 0, \tag{23} \]

as a price deviation in one period doesn’t alter prices in any future periods. Using (12) and (9) we have

\[ U_{pi} \big|_e = \frac{-P(n)}{n} \left[ (p^e - c + y_e)(1 - \Phi \delta(y_e)) - \omega \alpha'(y_e) \right] + \frac{1}{n} \left[ 1 - \Phi \delta(y_e) \right]. \tag{24} \]

We begin by establishing that

\[ \left( V_{yi} \big|_e \right)_n = 0. \tag{25} \]

To establish this observe that

\[ \frac{\partial}{\partial n} \left( U_{yi} \big|_e \right) = -\frac{1}{n} \cdot U_{yi} \big|_e, \quad \text{and} \]

\[ \frac{\partial}{\partial n} \left( V^i \big|_e \right) = \frac{1}{1 - e^{-r}(1 - \delta(y_e))} \frac{\partial}{\partial n} \left( U^i \big|_e \right) = -\frac{1}{n} \frac{U^i \big|_e}{1 - e^{-r}(1 - \delta(y_e))} = -\frac{1}{n} V^i \big|_e. \tag{27} \]

Combining these results in (21) gives (25).
Next we establish from (23) and (24) that

\[
\left( V^i_{p_i e} \right)_n = \left( U^i_{p_i e} \right)_n = -\frac{1}{n} \left( U^i_{p_i e} \right)_e^0 \frac{P'(n)}{n} \left[ (p^e - c + y^e)(1 - \Phi \delta(y^e)) - \omega \alpha(y^e) \right] \\
= -P'(n) \left[ \frac{1}{P(n)} (1 - \Phi \delta(y^e)) \right] \\
= \text{sign} - P'(n).
\] (28)

Line (28) uses (23) and (24) to substitute out for prices. Line (29) follows from assumption (11).

Now inverting (19) using (25) and (29) yields

\[
\begin{pmatrix}
\frac{dp^e}{dn} \\
\frac{dy^e}{dn}
\end{pmatrix} = \text{sign} \frac{1}{\det \mathcal{H}} \begin{pmatrix}
\left( V^i_{p_i e} \right)_{p^e} - \left( V^i_{y_i e} \right)_{p^e} & 0 \\
-\left( V^i_{p_i e} \right)_{y^e} & \left( V^i_{y_i e} \right)_{y^e}
\end{pmatrix} \begin{pmatrix}
0 \\
P'(n)
\end{pmatrix}
\] (30)

\[\Rightarrow \frac{dy^e}{dn} = \text{sign} - \left( V^i_{y_i e} \right)_{p^e} \cdot P'(n)
\] (31)

where we have used Lemma 3 to sign the determinant of the Hessian.

The next step is to use (21) and observe that

\[
\left( V^i_{y_i e} \right)_{p^e} = -e^{-r} \delta'(y^e) \left( V^i_{p_i e} \right)_{p^e} + \left( U^i_{y_i e} \right)_{p^e}
\] (32)

Each of these two terms can be signed:

\[
\left( U^i_{y_i e} \right)_{p^e} = \frac{1}{n} \left[ -\Phi \delta'(y^e) \right] < 0 \text{ from (22)},
\]

\[
\left( V^i_{p_i e} \right)_{p^e} = \frac{\left( U^i_{p_i e} \right)_{p^e}}{1 - e^{-r}(1 - \delta(y^e))} = \text{sign} \frac{\partial}{\partial p^e} \left\{ \frac{1}{n} [(p^e - c + y^e)(1 - \Phi \delta(y^e)) - \omega \alpha(y^e)] \right\} > 0.
\]

Combining in (32) we establish that

\[
\left( V^i_{y_i e} \right)_{p^e} < 0.
\] (33)
Which then substitutes into (31) and proves that at an interior equilibrium

\[
\frac{dy^e}{dn} = \text{sign} P'(n).
\] (34)

We now appeal to an important Lemma proved elsewhere in the literature:

**Lemma 4** [Anderson et al. (1995) (Proposition 1), and Zhou (2017) (Lemma 1)] If \( 1 - F(x) \) is log-concave then \( P(n) \) increases in \( n \). If \( 1 - F(x) \) is log-convex then \( P(n) \) decreases in \( n \).

So we establish the required comparative static linking malpractice with number of competitors at an interior equilibrium.

To complete the proof of the characterisation of the equilibrium we now study the boundaries of malpractice, \( y = 0 \) or \( y = \bar{y} \). At any equilibrium, prices will be interior and so the first order condition (24) is identically zero. This yields

\[
[1 - \Phi \delta(y^e)] = P(n) [(p^e - c + y^e)(1 - \Phi \delta(y^e)) - \omega \alpha(y^e)].
\] (35)

As the number of competing firms varies, it is possible that the equilibrium level of malpractice moves in to and out of the boundary values. Let \( n^* \) be one of these firm numbers such that the equilibrium is interior but with no malpractice: \( y^e = 0 \). From (35) we see that

\[
p^e - c = \frac{1}{P(n^*)}.
\] (36)

Substituting this into (12) and (15), the equilibrium utility in this case will satisfy:

\[
V^i(p^e, y^e = 0) = \frac{1}{(1 - e^{-r})n^*P(n^*)}
\]

So we can write the first order condition in malpractice, (21), as

\[
0 = -e^{-r} \delta'(0) \left( \frac{1}{(1 - e^{-r})n^*P(n^*)} \right) + \frac{1}{n^*} \left[ 1 - \Phi \delta'(0) \frac{1}{P(n^*)} - \omega \alpha'(0) \right].
\] (37)

Simplifying (37) gives (16). If \( 1 - F(x) \), is either log-concave or log-convex, then \( P(n) \) is
monotonic in $n$ (Lemma 4) and so multiple solutions $n^*$ to (16) cannot exist. Theorem 2 now follows.

5.2 Intuition for ethics, misconduct & competition result

In this section I will first offer an intuition for Theorem 2. It will be apparent that it also explains the motivating example, Proposition 1. I will then explore the role log-concavity is playing in this model.

5.2.1 Intuition discussion

The economics underlying Theorem 2 can perhaps most readily be seen by explicitly studying the marginal incentive an owner-manager has to deviate away from a competitive equilibrium without malpractice. In other words, from writing out the first order condition evaluated when $g^e = 0$. Using (21) and (15) we have

$$V_{g^i}|_{e,g^e=0} = -e^{-r}\delta'(0)\frac{U_i|_{e,g^e=0}}{1 - e^{-r}} + U_{g^i}|_{e,g^e=0}.$$  

Writing this out explicitly using (12) and (22) we establish the first order conditions as:

$$V_{g^i}|_{e,g^e=0} = \frac{1}{n} \left[ 1 - \omega(0) - (p^e - c)\delta'(0)\left(\Phi + \frac{e^{-r}}{1 - e^{-r}}\right) \right] \tag{38}$$

Expression (38) parallels (5) in the motivating example. The scaling factor $1/n$ is the volume served by each firm in equilibrium. The incentive to increase misconduct, ignoring the sanctions regime and any lost future profits, comes from the balance of the ethical consideration (the term $\omega(0)$ in (38)) against the direct increase in profits which can be achieved through lowering costs (the term 1 in (38)). Both of these terms grow proportionally with volumes: an extra unit of malpractice raises profits by 1 per unit of volume and lowers utility by $\omega(0)$ per unit of volume scaled by the agents’ willpower $\omega$. As these effects are proportional to volumes, their relative ranking is not altered as the
number of competing firms rises or falls.

In equilibrium the net incentive to increase malpractice from direct profit and ethical considerations is set against the expected cost of the regulatory sanction, and potential reputational costs from future periods. These final two effects are captured by the term (†).

If the owner-manager increases her level of misconduct marginally above zero then the probability of being sanctioned, by the authorities or by future consumers, increases at a rate \( \delta'(0) \). Both of these effects are proportional to the profits earned: regulatory sanctions by assumption (see on for revenue based sanctions); and the reputational cost is proportional to equilibrium profits as in repeated periods of competition these profits are the ones which will be earned in the future. As profit equals volumes times margin, the relative importance of the two terms in (†) as competition changes depends upon the relationship of the margin and volumes to the number of competing firms.

In standard settings increasing the number of competitors lowers margins in equilibrium. In this case profits, which are volumes times margins, fall more rapidly than volumes alone do with increases in the number of competitors. So the deterrence of the reputation and sanctions part of the utility function will decline more rapidly than the other parts of the utility function. Two results then follow. The first is that in competitive markets equilibrium misconduct levels rise as two of the breaks against misconduct (sanctions and reputation) shrink in their potency more than the incentive to misconduct (profit). This begins to illuminate Theorem 2. The second implication is that in competitive markets the equilibrium level of misconduct becomes predominantly a balance between ethics and profit incentives alone. We return to this shortly.

Thus the impact of the competitive structure on whether malpractice commences in equilibrium turns on whether competition increases or decreases margins. The equilibrium margins in turn depend upon the shape of the distribution of consumers’ tastes, and this is captured by the log-concavity requirement. Hence we establish a connection between the pattern of consumers’ valuations, the margins charged in a no-malpractice equilibrium, and hence the incentive to begin malpractice. This logic underlies Theorem
Numerically we can explore how the balance of forces created by profits, ethical considerations, reputation effects and sanctions, evolves at equilibrium levels of malpractice. Such a numerical analysis allows us to move away from the boundary case of zero misconduct. Figure 3 conducts this exercise. We extract the four components of the change in utility generated by a marginal increase in malpractice using (21) and (22):

\[ V_{yi}^{\prime} \bigg|_e = -e^{-r\delta'(y_i)} V_{yi}^{\prime} \bigg|_e + \frac{1}{n} \left[ \underbrace{1 - \Phi\delta'(y^e)}_{\text{Reputation}} - \underbrace{\omega\alpha'(y^e)}_{\text{Profits}} - \underbrace{\Phi\delta'(y^e)(p^e - c + y^e)}_{\text{Ethics}} \right] \]

The left hand panel of Figure 3 depicts these four forces in absolute terms in a given example. It is clear that ethical considerations are always a substantial force pushing away from misconduct, but when the market is concentrated this force might not be the strongest – in the example the reputation effect is more important. However as competition increases the impact of reputation considerations drops to second and then last place while ethical considerations become the most important break on misconduct. This is depicted more forcefully in the right panel of Figure 3 which expresses the ethics, sanctions and reputation effects as a proportion of the profit effect. In this panel it becomes clear that ethics is the most important break on owner-managers’ misconduct when the market is competitive. So as noted, the equilibrium level of misconduct in competitive markets is mostly determined by the leaders’ ethics versus profit considerations.

5.2.2 The role of log-concavity

Gabaix et al. (2016) demonstrate that with enough firms in the Perloff and Salop (1985) competition model, the equilibrium price is proportional to the expected gap a consumer has between the highest and the second highest draw from their taste distribution. Intuitively each competing firm will only make a sale to a consumer if the consumer likes their product the most, that is has the highest taste draw; and the amount which will be charged is equal to the expected gap between this valuation and the valuation the

\[^{19}\text{Compare the limit in Gabaix et al. (2016) Proposition 2 to that in their Theorem 1.}\]
consumer has for the next best product – that is the second highest taste draw. If the tails of the taste distribution $F$ are thin then this gap is decreasing in the number of draws, i.e. the number of firms, $n$. The thickness of the tails can be captured by the sign of $\lim_{x \to \bar{x}} \frac{d}{dx} \left(1 - \frac{F(x)}{f(x)}\right)$, and this entity is known as the tail index. But log-concavity of $1 - F(x)$, the expression used in Theorem 2, is determined by the sign of $\frac{d}{dx} \left( -\frac{f(x)}{1-F(x)} \right)$ and this shares the sign of the tail index. Hence log concavity of the reliability function $1 - F(x)$ is measuring the thickness of the tails of the taste distribution, and therefore the expected gap between the highest and next highest valuation, and therefore captures the relationship between equilibrium margins and entry.

Bagnoli and Bergstrom (2005) show that log-concavity of $1 - F(x)$ is implied by log-concavity of the density, and this holds for a long list of well-known distributions, including the normal, uniform, and logistic. With log-concavity the above discussion explains why the equilibrium margins fall as the number of competing firms increase and so in this case misconduct increases with competition. This link between entry and declining margins tallies with our intuition gained from competition models such as Cournot. However it does not hold universally.

Figure 3: Balance of forces in owner-managers’ incentive to misconduct
Notes: The balance of ethical considerations, sanctions deterrent, reputational concerns, and profits, within the first order condition for more malpractice. All evaluated at equilibrium levels of misconduct. The horizontal axis depicts the number of firms, the vertical axis corresponds to the value within the first derivative of utility with respect to misconduct. Numerical parameters are given by the leading example of Section 7.2 with parameters in footnote 29. In this example at five firms or below the equilibrium is of no misconduct; the profit incentive is smaller than the deterrent created by the sum of ethics, sanctions and reputation. At six firms and above malpractice is positive arising from an interior equilibrium; at equilibrium the sum of the ethics, sanctions and reputation effects are exactly counterbalanced by the profit incentive.
Bagnoli and Bergstrom (2005) show that a number of distributions instead generate reliability functions, $1 - F(x)$, which are log-convex. If valuations were drawn from these distributions then equilibrium margins rise as firms enter, and so misconduct falls with competition. Prices rising in response to entry is perhaps not common, but it is also not rare. Ward et al. (2002) document cases in which entry caused incumbent brands to raise their prices in the retail food industry; Perloff et al. (2006) present similar findings in the pharmaceutical industry (anti-ulcer drugs). Economically such settings arise when entry causes firms to alter their pricing so as to appeal more squarely to those consumers with the highest match value for their product.\footnote{The observation that entry can cause prices to rise has also been made in theoretical models (Chen and Riordan (2008)) and in simulation models (Thomadsen (2007)).}

## 5.3 Link between ethics and misconduct

### 5.3.1 Nonlinear harm functions

It is easy to imagine arguments which would suggest that the harm suffered by stakeholders is non-linear in the amount of misconduct. For example, if the misconduct is the use of unsafe ingredients then typically a consumer might be unaffected by small amounts of unsafe food within a product; but as the amount of such food rises the probability of suffering food poisoning might increase exponentially.

Theorem 2 is remarkably clear in its prediction: whatever the shape of the harm function $\tilde{\alpha}(y)$, with log-concave tails to consumers’ valuations, competition weakly increases the equilibrium level of misconduct.

A more naive approach of extending the Hotelling case used in the motivating example would give misleading results. For example, suppose that we extend the Hotelling example to the single stage utility function (12), and let $\alpha'(y)$ be convex.\footnote{So that $\alpha'' > 0$.} It follows that the LHS function plotted in Figure 1 would be concave. This would give more than one candidate for equilibrium, and the candidates would differ in their comparative static between the extent of malpractice and the competitive pressure.

However Theorem 2 is clear that any candidate equilibrium which does not obey the
comparative static between competition and malpractice is not stable. Thus the model offered here allows us to extract the most empirically relevant predictions. And these apply even if the harm function is not linear.

5.3.2 More ethical owner-managers

As ethics become more pronounced one might expect the equilibrium level of misconduct to fall. However this is reasoning is incomplete, and potentially misleading, for at least two reasons.

Firstly one needs to be careful of the distinction between act utilitarian ethics and a deontological tradition. In the latter case misconduct lowers utility as one might expect. But in the case of act utilitarian ethics, it is possible that at some levels of malpractice an extra unit of malpractice is aggregate surplus increasing. This would be the case if the extra unit of malpractice lowered costs by more than the harm suffered by the stakeholder: \( \tilde{\alpha}'(y) < 1 \). In this case we would not expect more ethically minded owner-managers to conduct less of the malpractice at all.

Let us set this case aside and assume that whatever the owner-managers’ ethical tradition, more malpractice lowers utility so that \( \alpha'(y) > 0 \). There now follows a negative feedback loop. Suppose that ethics improve and this drives equilibrium malpractice down. It follows that long run profits decline as production costs are increased. But then the value of reputation is also reduced. However this latter effect acts to raise misconduct. Which force dominates?

**Proposition 5** Assume that \( \alpha'(y) > 0 \) so that more misconduct lowers utility ceteris paribus. The equilibrium level of misconduct is declining in ethical willpower, \( \omega \), if the discount rate is high enough:

\[
\frac{e^{-r}}{1 - e^{-r}(1 - \delta(y^e))} < \frac{\alpha'(y^e)}{\alpha(y^e)} \cdot \frac{1}{\delta'(y^e)}. 
\]

(39)
Condition (39) is easier to interpret if we rewrite it as:

\[
\frac{e^{-r} \delta(y^e)}{1 - e^{-r}(1 - \delta(y^e))} < \frac{\alpha'(y^e)y^e}{\alpha(y^e)} \cdot \frac{\delta(y^e)}{y^e \delta'(y^e)} = \frac{\text{elasticity of ethical cost } \alpha(y^e)}{\text{elasticity of detection } \delta(y^e)}.
\]

So we see that Proposition 5 establishes that if the sensitivity of the agents’ utility to misconduct is sufficiently large as compared to the sensitivity of detection to misconduct, then increasing owner-managers’ ethics leads to lower levels of equilibrium misconduct. The former begins the feedback loop of ethical observance on malpractice, while the latter prevents the effect being reversed from too great an impact on future expected profits.

### 5.3.3 Kantian agents and the dangers of oligopoly

Both the utilitarian and deontological approach I have modelled have the feature that the disutility term grows continuously in the number of consumers (or stakeholders) who are harmed. In particular a minor bit of malpractice is assumed to alter utility from the ideal of no malpractice by only a small amount. This may seem natural on casual inspection, but it is a contentious assumption. Agents who were truly Kantian would consider that any malpractice, however minor, is wrong full stop. As Kant would claim, lying is unethical, and whether the lie is relatively large or small has no bearing on this judgement.

I approach Kantian ethics by creating a substantial utility difference between no malpractice and any positive level of malpractice. To this end consider adding in a Kantian disutility term for malpractice, \( K \), to the single stage utility given in (12):

\[
U^i := q_i(p_i) [(p_i - c + y_i)(1 - \Phi \delta(y_i)) - \omega \alpha(y_i)] - K I_{y_i > 0}.
\]

The function \( I_{y_i > 0} \) is the indicator function taking the value 1 if the owner-manager of firm \( i \) engages in any malpractice at all. If the Kantian term, \( K \), in the utility function is large then the owner-manager will not engage in any malpractice. However, with a finite \( K \) value it is possible that the utility gain from engaging in malpractice is great enough to outweigh the Kantian dislike for behaving unethically.
If an interior positive malpractice equilibrium exists in this case, then its properties with respect to firm entry will coincide with those of Theorem 2 as the second derivatives used to prove Theorem 2 are unchanged.\textsuperscript{22} In particular if consumers’ valuations have log-concave upper tails then at a positive malpractice equilibrium the level of malpractice will grow with the number of competing firms.

A Kantian dislike of malpractice does however alter the equilibrium outcome in one significant extent:

**Proposition 6** If $1 - F$ is log-concave and owner-managers have Kantian elements to their utility then a symmetric positive malpractice equilibrium can only exist under oligopolistic market structures with the number of competing firms satisfying $n < \bar{n} < \infty$.

**Proof.** Suppose otherwise that a positive malpractice equilibrium with $n$ competing firms exists such that $n$ is arbitrarily large. Equilibrium prices will be interior and so given by (35). Substituting this into (15) we have:

$$V^i|_e = \frac{1}{1 - e^{-r}(1 - \delta(y^r))} \cdot U^i|_e$$

$$= \frac{1}{1 - e^{-r}(1 - \delta(y^r))} \cdot \left[ \frac{1}{n} \left( \frac{1 - \Phi\delta(y^r)}{P(n)} \right) - \mathcal{K}_{y^r > 0} \right] \text{ using (35) and (40).}$$

If $1 - F$ is log-concave then $P(n)$ is increasing in $n$ (Lemma 4), and so we have:

$$\lim_{n \to \infty} V^i|_e \leq -\mathcal{K}_{y^r > 0} < 0$$

Which is a contradiction as deviating to no malpractice would allow a positive utility to be derived. \hfill \blacksquare

Hence in the prominent case of $1 - F$ log-concave, we establish that malpractice is most likely for oligopolies: if there are many firms competing the profits available from malpractice are too small to outweigh the Kantian distaste for engaging in malpractice. This result is not obvious however; it need not hold when consumers’ valuations have log-convex upper tails as though volumes decline with competition, margins do not.

\textsuperscript{22}The level of malpractice in equilibrium is changed as the long run value of being in business to the owner-manager is reduced by the term $\mathcal{K}$.  

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5.4 Empirical predictions and evidence

5.4.1 Empirical predictions

To link Theorem 2 directly to an empirical prediction would require including a test on the shape of consumers’ valuation functions. That said however, margins declining as competition increases is the more standard case – and this arises with log-concave and not log-convex demand. If one accepts this then a test of Theorem 2 would further need a setting to be found which held other factors constant: the sanctions regimes; owner-managers’ average level of ethical observance; consumer tastes; and the size of the market.

These final requirements suggest that a test would likely need an exogenous cause of a change in the number of competing firms within a market. Such a change can come about from regulatory intervention and licensing rules:

**Prediction 1:** If the number of authorised competitors within a market shrinks, then equilibrium levels of misconduct should fall.

Regulators often alter the number of permitted firms. In finance entry barriers can be raised by mandating particular qualifications for entry (e.g. Australia increasing the required professional qualifications for Financial Advisors\(^{23}\)) or by adjusting the difficulty of applying for a license (e.g. for banking licenses\(^{24}\)). Outside of Finance, taxis and pharmacies are typically licensed professions.

A second empirical prediction of this work arises via Proposition 5:

**Prediction 2:** An increase in the ethical willpower of firms’ leaders in a competitive market reduces the equilibrium amount of misconduct when the discount rate is high enough.

This test asserts that ethics matter for market outcomes and is a test of the validity of including ethics in such a competition model. The empirical literature has been creative in measuring ethical willpower and has considered behaviour such as: the propensity to pay parking fines incurred abroad (Fisman and Miguel (2007)); surveys to ascertain the

\(^{23}\)See Australian Securities & Investment Commission *Professional standards for financial advisors.*

\(^{24}\)See *UK regulators launch unit to help challenger banks enter market*, Financial Times, July 10, 2015.
propensity to bribe or misreport (Fritzsche and Becker (1984)); or use of self-reporting in lab experiments to detect cheating on average (Cohn et al. (2014)).

Next one could base a test on the following corollary:

**Corollary 7 (to Theorem 2)**  
*Prediction 3:* In an interior equilibrium prices and malpractice vary inversely to each other in the number of competitors.

**Proof.** At an interior equilibrium

\[
\frac{dp^e}{dn} = \text{sign} \left( \frac{P'(n)}{(V^i_{yi|e})^y_e} \right) - \text{sign} \left( \frac{dy^e}{dn} \right).
\]

Comparing prices to malpractice obviates the need to identify the shape of valuation functions. In the more typical case of log-concave demand the first and third hypothesis together predict that increasing the number of licensed competitors in a market should lower prices but also increase the equilibrium level of malpractice.

### 5.4.2 Empirical evidence

If one accepts log-concavity as the most likely shape of consumers’ valuations, then Theorem 2 would predict a link between malpractice and many small firms competing in a market. We observe that anecdotally such a link is credible. A rich source of historical examples conforming to this is available in Rashid (1988) including: the milk industry of Bangladesh; the rice industry in India; the cotton industry in England.

These examples may however be driven by the lack of producers’ unions which became prominent in England for example. More recent examples are harder to dismiss however. The case of the misselling of mortgage default insurance (PPI) in the UK was described in the introduction and fits the empirical prediction, as does the Financial Advisor scandal which has recently been the subject of a Royal Commission in Australia.\(^{25}\) We have also noted the five cases that Shleifer (2004) outlined as consistent with this empirical

\(^{25}\text{See Banking royal commission told 90% of financial advisers ignored clients’ best interests, Guardian, 16 April 2018.}\)
hypothesis: child labour, corruption of government officials, high executive pay, earnings manipulation, and the commercialisation of the market for education in the US.

There exists an empirical literature on misconduct and competition, albeit ignoring the impact of ethics. Many of these find an increasing relationship between misconduct and competition, which would align with Theorem 2. This is the case for: conducting review fraud for themselves or denigrating rivals (Luca and Zervas (2016)); waiving required testing standards for automobiles (Bennett et al. (2013)); and avoiding corporate tax in China (Cai and Liu (2009)).

However some studies have found an inverted-U relationship between competition and misconduct which would be more consistent with the Kantian extension (Section 5.3.3). This is the case for: industrial pollution (Polemis and Stengos (2019)), and the quality of reported earnings (Guo et al. (2019)).

6 Asymmetries: the market impact of a more ethical firm

Thus far we have considered a setting in which all owner-managers shared the same ethical stance and all firms were equally efficient. We relax both of these restrictions in this section. For tractability we focus on a duopoly setting, and to simplify the exposition we focus on a single period of competition. This section therefore studies the market outcomes when an ethical owner-manager competes with a less ethical one.

6.1 Model extensions to asymmetric duopoly

Consider a duopoly with firms $i \in \{1, 2\}$; firm $i$ has marginal cost $c_i$ and its owner-manager has ethical willpower $\omega_i$ allied to a personal ethics function $\alpha_i(\cdot)$. Suppose the firms compete once. The objective function of firm $i$ follows from the benchmark case.

\footnote{So working in the limit of the discount rate $r \to \infty$.}
with the personalisation of costs and ethics:

\[ U^i := q_i(p_i) \left[ (p_i - c_i + y_i)(1 - \Phi \delta(y_i)) - \omega_i \alpha_i(y_i) \right] \]  

We will characterise equilibrium behaviour and so in this section we make the following assumption:

**Assumption A:** In equilibrium both firms are active \((q_i > 0)\) and equilibrium is at an interior level of malpractice: \(y_i \in (0, \bar{y})\).

Assumption A rules out settings where one of the firms is excluded from the market, and delivers that at equilibrium values the first order conditions apply:

\[ U^i_{y_i} (y^e_i, p^e_i; y^e_j, p^e_j) = 0 = U^i_{p_i} (y^e_i, p^e_i; y^e_j, p^e_j) \]  

Each firm is optimising against the rival, and so the second order conditions are satisfied. These require the Hessian for each firm to be negative definite:

\[ U^i_{y_iy_i} < 0, U^i_{p_ip_i} < 0 \]  

\[ U^i_{y_ip_i} \cdot U^i_{p_iy_i} - (U^i_{p_ip_i})^2 > 0 \]  

Beyond the second order conditions, the structure of our problem yields some further relations. As in the symmetric case \(U^i_{y_j} = 0\) for \(j \neq i\). This captures that malpractice of rival firms does not have a direct effect on an owner-manager’s objective function. The effect is indirect via any concurrent changes in prices. Secondly observe that at an equilibrium with positive production by both firms, the condition \(U^i_{y_i} = 0\) requires

\[ \frac{\partial}{\partial y_i} \left( (p_i - c_i + y_i)(1 - \Phi \delta(y_i)) - \omega_i \alpha_i(y_i) \right) \bigg|_{(y^e_i, p^e_i; y^e_j, p^e_j)} = 0 \]  

It follows that \(U^i_{y_ip_j} = 0\) for \(i \neq j\) at equilibrium prices.

We now invoke the assumption that the equilibrium is stable. This requires that the Hessian matrix for the whole system evaluated at the equilibrium values is negative
definite. Labeling this matrix $\tilde{A}$ and ordering the variables by firm, we have:

$$
\tilde{A} := 
\begin{pmatrix}
U_{y_1 y_1}^1 & U_{y_1 p_1}^1 & 0 & 0 \\
U_{y_1 p_1}^1 & U_{p_1 p_1}^1 & 0 & U_{p_1 p_2}^1 \\
0 & 0 & U_{y_2 y_2}^2 & U_{y_2 p_2}^2 \\
0 & U_{p_2 p_1}^2 & U_{y_2 p_2}^2 & U_{p_2 p_2}^2
\end{pmatrix}
$$

(46)

where we have used the insights above to replace some Hessian entries with zeros. Taking
differentials of the first order conditions (42) and focusing on the ethics of firm 2 we have:

$$
\tilde{A} \begin{pmatrix} dy_1^c \\ dp_1^c \\ dy_2^c \\ dp_2^c \end{pmatrix} = - \begin{pmatrix} U_{y_1 \omega_2}^1 \\ U_{p_1 \omega_2}^1 \\ U_{y_2 \omega_2}^2 \\ U_{p_2 \omega_2}^2 \end{pmatrix} d\omega_2 = \begin{pmatrix} 0 \\ 0 \\ q_2 \alpha_2'(y_2) \\ \frac{\partial q_2}{\partial p_2} \alpha_2(y_2) \end{pmatrix} d\omega_2.
$$

(47)

To evaluate the comparative statics we therefore need to invert the Hessian matrix $\tilde{A}$.
Stability yields that $\det \tilde{A} > 0$. We will, in the following, use the result that the inverse
of a matrix can be expressed in terms of its adjoint. The details are in Appendix B.

### 6.2 Market impact of a more ethical firm

We can now give the main result of this section.

**Theorem 8** Suppose that both owner-managers dislike malpractice so that it lowers utility ceteris paribus: $\alpha_i' > 0$ for $i \in \{1, 2\}$. The comparative statics in duopoly competition
with respect to the ethics of firm 2, $\omega_2$, satisfy:

1. $\frac{dp_2^c}{d\omega_2} > 0$;

2. 

$$
\frac{dy_1^c}{d\omega_2} = \text{sign} \left( - \frac{dp_1^c}{d\omega_2} \right) \begin{cases} < 0 & \text{if } 1 - F(x) \text{ is log-concave} \\ = \text{sign} - \frac{\partial^2 \ln q_1}{\partial p_1 \partial p_2} & \text{otherwise} \end{cases};
$$

3. If in addition $\alpha_2'' \geq 0$ then $\frac{dy_2^c}{d\omega_2} < 0$.  

39
**Proof.** See Appendix B. ■

Theorem 8 delivers the result that if the owner-manager of firm 2 becomes more ethical so that $\omega_2$ increases, then firm 2 will raise its retail prices. This is because the increased morality causes the owner-manager to dislike her existing level of malpractice and the harm she is doing to the firm’s stakeholders. This increased subjective distaste for malpractice is as if the owner-manager has incurred a higher cost of production. This drives the manager to bring down volumes by raising her price, and so reduce the harm done.

The rival firm 1’s behaviour can now be signed. Given firm 2 responds to her increased morality by raising retail prices, she becomes a less effective competitor. As a result firm 1 seeks to alter her price to increase her profits. The direction in which prices change depends upon whether firm 1’s log demand displays increasing differences in prices, or otherwise.\(^{27}\)

In the prominent setting of consumer tastes coming from the class of log-concave density functions (e.g. normal, double exponential, uniform), then the proof of Theorem 8 demonstrates that prices become strategic complements: an increase in firm 2’s price results in firm 1 raising its price also.

Firm 1’s level of malpractice moves in the opposite direction to its prices. The explanation parallels the logic in the symmetric equilibrium case discussed above. If the equilibrium dynamics cause firm 1’s prices to rise, margins grow. As margins grow the effect of the sanctions becomes more prominent in the decision calculus. Had we been in a repeated game setting, this effect would be reinforced by the increased value of future profits, and so the increased value of reputation. Hence the owner-manager of firm 1 lowers her level of malpractice.

Thus in the prominent setting of consumers’ tastes coming from the set of log-concave density functions, malpractice at firm 1 declines as firm 2 becomes more ethical.

These first two results of Theorem 8 lend themselves directly to empirically testable

\(^{27}\)The expression $\partial^2 \ln q_1 / \partial p_1 \partial p_2$ captures whether the log of firm 1’s realised demand has increasing differences in prices. If this second derivative is positive then should firm 2 raise her price, $\partial \ln q_1 / \partial p_1$ will increase. As this derivative is negative, this implies that the log of realised demand becomes less sensitive to firm 1’s own prices if the rival’s price goes up.
predictions as the observable prices, and potentially observable misconduct levels are linked to the ethical willpower of firm 2’s owner-manager, and measuring this is potentially possible as discussed in Section 5.4.

Firm 2’s level of malpractice is more delicate to sign (and so to empirically test) as ex hypothesi firm 2’s ethical willpower has changed. Absent this effect it would be natural for misconduct to fall as the firm raised its prices and margins for the same reasons as described above. However the change in willpower has increased the relevance of ethical effects in the owner-manager’s decision calculus. Thus suppose, against the assumption in part 3 of Theorem 8, that the harm function was concave. It follows that an increase in malpractice would lower the sensitivity of harm to the malpractice level: \( \alpha' \) would decline. As the harm component of utility is more dominant this would create a force to raise misconduct, making the final outcome ambiguous. This effect does not arise if the harm function for owner-manager 2 is weakly convex – such as in the case of food-poisoning or theft (e.g. mis-selling). In this setting firm 2 lowers her equilibrium level of malpractice.

6.3 Discussion and empirical evidence

If a less corrupt, or more ethical, owner should acquire a competitor in a perhaps corrupt local market, will the ethical newcomer raise the ethical conduct of rivals, or will her own ethical conduct become polluted? This is an open question, and I believe an interesting one. The analysis above offers some insight into this question. In the prominent setting of consumers’ tastes being drawn from the class of log-concave densities, the acquisition by a more ethical owner of firm 2 will lower malpractice at both the rival firm 1, and for a wide range of ethical harm functions, the newly acquired firm 2. This is consistent with evidence, such as Kwok and Tadesse (2006), which argues that entry into a market of a (more ethical) multinational lowers corruption and malpractice amongst home firms.

However, does corruption spread and damage the behaviour of the ‘ethical’ firm? Kartner and Warner (2015) argue through a case study of Siemens that corruption does indeed spread to the incoming multinational. Hence thinking of Siemens as an example of an ethical MNC, its behaviour in corrupt markets fell to below the levels it would tolerate
elsewhere. This is consistent with Theorem 8 as in studies such as Kartner and Warner (2015) one is comparing a MNC’s behaviour in one market (abroad) to its behaviour in another (home). These markets would be expected to have different levels of equilibrium malpractice as the competitive conditions between home and abroad would likely differ; and this paper has demonstrated market structure affects equilibrium malpractice.

7 Robustness of the analysis

7.1 Revenue based fines

The model so far has assumed that regulatory fines are proportional to profits. Further, one important effect has been that as these fines are proportional to profits, their influence on owner-managers’ behaviour as competition changes develops differently to the force created by ethics which hinges instead on volumes. This begs the question of whether the results would be robust under a revenue fine framework.

Revenue based fines are prominent. In the Goldman Sachs Abacus scandal recounted in Table 1 the fine of $550 million went an order of magnitude beyond the profit element of a comparatively paltry $15 million. In antitrust cases the European authorities are clearer with a base fine level set at 30% of the revenue accrued from sales benefiting from wrongful behaviour.\(^{28}\)

Our results are robust to revenue based fines:

**Theorem 9 (Theorem 2 with revenue fines)** Suppose the model is adjusted so that the regulatory fine, denoted \(\Phi^{\text{rev}}\) is proportional to revenues and not profits. Then Theorem 2 holds with the critical threshold number of firms changing to:

\[
P(n^*) = \frac{\delta'(0) \left( \Phi^{\text{rev}} + \frac{e^{-r}}{1 - e^{-r}} \right)}{1 - \omega \alpha'(0) - \delta'(0) \Phi^{\text{rev}} c}.
\] (48)

The proof of Theorem 9 is given in Appendix C. The result outlined in Theorem 2 is

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\(^{28}\)See Factsheet “Fines for breaking EU Competition law” available at https://ec.europa.eu/competition/cartels/overview/
seen to hold, even though fines are no longer proportional to profits. The rationale for this is as follows. Revenues can be split in an accounting sense into profit margin plus costs of production. The costs of production depend upon the competitive equilibrium only through the volumes demanded. These costs therefore scale in proportion to volumes, and they share this characteristic with the ethical distaste and with the profit incentive from a marginal bit of malpractice. The latter two effects were discussed in detail in Section 5.2. The cost elements of revenue therefore do not alter the balance which determines the level of misconduct. We are therefore left with the profit elements of revenue; but the effect of profit sanctions was explored in the core model. Therefore the economics of revenue based sanctions, in at least the sense of Theorem 2, parallel those of profit based sanctions.

7.2 Simulations

The analysis so far leaves a critical question unanswered: can increased competition damage consumer surplus?

In the leading case of consumers’ valuations being drawn from distribution functions with log-concave upper tails, we established (Theorem 2) that misconduct was positive in symmetric equilibrium when the number of competitors was above a critical threshold. The discussion above also established that the equilibrium margins decline as competition increases (Corollary 7). Therefore below this critical competitive threshold more firms competing in a market lowers prices, improves the match between consumers and firms, and has no effect on malpractice – so it must raise consumer surplus.

Once competition increases beyond the critical threshold however, more competition implies lower prices and a better consumer match, but it also implies greater amounts of misconduct. There are therefore competing effects and so, at least in principle, it is possible that the increase in misconduct outweighs the reduction in prices and leaves consumers worse off.

This section explores numerically whether competition can lower consumer surplus.

In conducting this analysis it is important that we focus on stable equilibria. Thus one
cannot naively program the two first order conditions in misconduct (21) and in prices (23) directly into a computer based numerical solver routine. We therefore proceed as follows. In any equilibrium prices will be interior, and so the first order condition will be satisfied. Thus prices will be given by (35). These prices can then be substituted into the first order condition in misconduct, (21). This delivers:

$$n \cdot V_{y_i}^i|_{y_e} = \frac{-e^{-r}\delta'(y^e)}{1 - e^{-r}(1 - \delta(y^e))} \cdot \frac{1 - \Phi\delta(y^e)}{P(n)} - \Phi\delta'(y^e) + 1 - \Phi\delta(y^e)$$

(49)

For this study we use the Salop circle formulation of our model (Salop (1979)). The Salop circle is a simple extension of Hotelling and has been widely used in theoretical work in Finance and Economics (e.g. Grossman and Shapiro (1984), Hakenes and Schnabel (2010)). To generate this assume that tastes are drawn from the uniform distribution on $[0, t]$ so that $f(x) = 1/t$ with distribution function $F(x) = x/t$. It follows that

$$P(n) = \frac{n}{t},$$

and so $q_i'(p^e) = -\frac{1}{t},$  

(50)

so that the derivative of demand is identical to the Salop circle formulation with $n$ symmetric firms and transport cost $t$.

For a given number of firms $n$ we can use (50) to determine the graph of $V_{y_i}^i|_{y_e}$ against misconduct $y^e$ using (49). If $V_{y_i}^i|_{y_e}(y^e = 0) \leq 0$ then no-malpractice can be sustained as an equilibrium outcome. If $V_{y_i}^i|_{y_e}(y^e = 0) > 0$ then equilibrium requires a positive amount of malpractice. Candidate equilibra occur at the zero points of the function (49). Stability requires the graph of $V_{y_i}^i|_{y_e}$ to cut the $y^e$ axis from above. This ensures that if firm $i$ deviated slightly to alter her malpractice then myopically following her first order condition would drive her back to the equilibrium; yielding stability. This condition is satisfied at the smallest positive root of (49). We identify this root in the numerical analysis.

To complete the numerical analysis we select some standard functional forms and parameter values. We set the owner-managers’ ethics disutility function to be linear:
\( \alpha(y) := 2y. \) We relegate other choices to a footnote as these are not critical to the exposition.\(^{29}\) We can therefore establish equilibrium misconduct \( y^e \) as a function of the number of competing firms \( n. \) Substituting back into (35) we can establish the equilibrium margins. Equilibrium margins and malpractice as a function of competitor numbers are plotted in Figure 4.

![Figure 4: Equilibrium misconduct, \( y^e \), and margins \( (p^e - c) \) as a function of competitor numbers.](image)

We see in Figure 4 that no misconduct can be sustained as an equilibrium outcome if the number of competitors is weakly below a critical threshold; five competitors in this numerical example. We also observe that equilibrium margins decline in the number of competing firms. These observations were predicted by Theorem 2 and Corollary 7. However one other observation is striking. If the number of competing firms grows high enough, equilibrium margins, \( p^e - c \), become negative. It is only through misconduct to lower the marginal cost that a firm can remain in business at all.

We can now establish the welfare and consumer properties of the solution. To do so we must calculate the welfare gain associated with the match between consumers and firms. With \( n \) competing firms in a Salop circle of unit perimeter, consumers travel a

\(^{29}\text{We set } \Phi \delta(y) := 1 - e^{-y} \text{ and use parameter values } \Phi = t = 1, \omega = 1/4, \text{ and } r = 1/2.\)
distance of between 0 and $\frac{1}{2n}$, and an average distance of $\frac{1}{4n}$. We label the perfect match value $v$. Then allowing for malpractice we have:\(^{30}\)

\[
\text{Welfare} = v - \frac{t}{4n} - c + y^c - \hat{\alpha}(y^c) \\
\text{Consumer Surplus} = v - \frac{t}{4n} - p^c - \hat{\alpha}(y^c)
\]

If the owner-managers are deontological – they care only for stakeholder harm – then the harm function is equal to the ethics function as (14) indicates. However if the owner-managers are act utilitarian, then they set their profits against the harm done. Thus for a given level of managerial ethical distaste, the actual harm done to stakeholders is greater by the costs saved. That is, re-arranging (13) we have $\hat{\alpha}(y) = \alpha(y) + y$. Substituting these into (51) we can determine the relationship of welfare and consumer surplus to the number of competing firms. This is plotted in Figure 5.\(^{31}\)

![Figure 5: Welfare and consumer surplus as a function of the number of competitors](image)

Figure 5 offers an answer to the question we set at the start of this section: can increased competition damage consumer surplus? We see that it can. In the case we study, once the number of competing firms crosses the critical threshold at which equilibrium malpractice begins, the increase in misconduct is great enough that it outweighs both the reduction in prices, and the improved product-consumer match, generated by increased

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\(^{30}\)In the microfoundation of Section 4.1 the representative consumer had a match $\in [v - t, v]$. The highest of the $n$ draws from the uniform distribution on $[0, t]$ is expected to be $\frac{n - t}{n + 1}$. Therefore the expected match value of the representative consumer when served by $n$ firms is $v - \frac{t}{n + 1}$. This doesn’t alter any of the conclusions which are described in the Salop circle setting.

\(^{31}\)We set $v - c = t = 1$ in the numerical simulation.
competition. So overall the effect of increased competition is to lower, not raise, consumer surplus. In the case captured by Figure 5 consumer surplus is quasi-concave; it is maximised at an interior amount of competition. In the case we have studied consumer surplus is maximised at the critical threshold number of firms, $n^*$.

We see that increased competition also lowers overall welfare. This reflects the fact that, by assumption, misconduct is aggregate surplus reductive, and so as misconduct increases, the losses associated with misconduct mount.

We next study the implications of competition and ethics on profits at an industry and firm level. Profits are the difference between welfare and consumer surplus; using (51) we can construct Figure 6. We see that at an industry level the commencement of misconduct arrests the decline in profits and instead allows the industry profits to grow with competition. As competition increases the increase in malpractice lowers the costs of production so as to outweigh the reduction in margins from reduced prices, and the reduction in the overall pie available from the welfare destructive nature of misconduct. At the firm level, depicted in the right hand graph in Figure 6, we see that the commencement of malpractice substantially reduces the decline of per firm profits from competition.

![Figure 6: Profits at the industry (left) and firm level (right)](image)

The result that equilibrium malpractice can cause consumer surplus to decline in competition is, I think, interesting. We therefore conclude this section by exploring this effect under a variety of ethics and harm functions: linear, convex and concave.
7.2.1 Linear harm function

We first expand our study by considering linear harm functions. The numerical results are given in Figure 7.

Figure 7: Linear harm and ethics functions
Notes: Owner managers are deontological so that the stakeholder harm function and disutility function coincide: $\alpha(y) = \tilde{\alpha}(y)$. The parameter values are as in footnote 29. The horizontal axis measures the number of competing firms. Note the vertical axis is identical in the right hand graph of welfare and consumer surplus. However the vertical scale is resized in the left hand graph of margins and misconduct to adapt to the level of each.

The effect that greater competition above the misconduct threshold lowers consumer surplus manifests for the harm functions $\alpha(y) = 2y$ and $3y$; for the steeper harm function malpractice is introduced after 10 firms compete as opposed to after five. The reduction in consumer surplus is more gentle for the steeper harm function however. This arises as the level of malpractice is less pronounced in equilibrium. The increased harm function acts
as a deterrent to the owner-managers from malpractice. At even steeper harm functions, such as $\alpha(y) = 4y$, there is no malpractice for 20 or fewer competitors, and so more competition increases consumer surplus.

At the shallowest harm function considered, $\alpha(y) = y$, the temptation to malpractice is very pronounced as the harm done to the stakeholders is less than in the other cases of Figure 7. However this creates an interesting secondary effect. The first effect is that with more than three firms competing, with the given parameter values, malpractice rises rapidly and so consumer surplus declines. However malpractice rapidly approaches its limiting level. This results in the secondary effect, more competition does not increase malpractice enough to keep reducing consumer surplus, and so consumer surplus begins to rise with further competition. Further note that welfare is not reduced by malpractice; the consumer harm is outweighed by the change in margins.

### 7.2.2 Exponential harm functions

A deeper analysis is revealed when ethics and harm are not linear.

**Convex harm function**

The numerical analysis for this case is given in Figure 8. Let us first consider the
exponential harm function: $\alpha(y) = e^y - 1$. Such a harm function is small at low levels of malpractice, but then rises gradually. An example of a situation which might generate such a harm function is use of unfit ingredients in food; small volumes of bad materials might not affect the consumer, but beyond a critical volume food poisoning might result. Similar effects to those described with a linear harm function occur. Firstly, malpractice begins at even modest levels of competition; beginning with more than three firms in the upper row of Figure 8. The increase in malpractice is rapid, and the amount of consumer surplus falls. However, as the number of competitors and so malpractice rises, the harm function grows exponentially. The result is that the level of malpractice plateaus in equilibrium. As a result further competition has a stronger effect on margins than on malpractice and so consumer surplus begins to rise again, albeit imperceptibly in the upper row.

With the more pronounced exponential harm function $\alpha(y) = e^{2y} - 1$, there is an interesting development. As the rate of change of harm is even more pronounced at low levels of malpractice, the equilibrium level of malpractice is reduced, and this softens the reduction in margins which now remain positive, even with twenty competitors. As a result the price contribution to consumer surplus is reduced, and so competition lowers consumer surplus due to the misconduct effect.

*Concave harm function*

A concave harm function can arise if initial amounts of malpractice are painful to one or more of the stakeholders, but then subsequent malpractice has more subdued effects. One example of this setting might be the practice of permitting an abusive environment to exist for staff in which, for example, a very productive but otherwise abhorrent worker is tolerated by the owner-manager as part of a team. Here even low levels of exposure to the abusive colleague can cause significant personal harm as workers’ dignity is affected, whilst at greater levels of exposure to the colleague the workers are somewhat desensitised as the harm is already done. The numerical analysis of this case is contained in Figure 9.

To explore the consumer surplus effects of such a setting we consider the harm function $\alpha(y) = \ln(1 + y)$. The results are on the top row of Figure 9. Once malpractice arises
\[ \alpha(y) = \ln(1 + y) : \]

\[ \alpha(y) = \ln(1 + 2y) : \]

Figure 9: Concave harm and ethics functions

Notes: Owner managers are deontological so that the stakeholder harm function and disutility function coincide: \( \alpha(y) = \tilde{\alpha}(y) \). The parameter values are as in footnote 29. The horizontal axis measures the number of competing firms. Note the vertical axis is not identical in each column, as the graphs are resized to best depict the data in each case.

in equilibrium, here at more than three competing firms, the level of malpractice rapidly moves towards its limiting level. This is an effect which was shared with the strongly convex harm function \( e^y - 1 \) explored in the top row of Figure 8. Once again, the effect of more firms competing then acts mostly through the effect on margins, and so competition acts to raise consumer surplus albeit modestly.

These results are reinforced if the rate of increase of pain experienced by the owner-manager (and by the stakeholders) caused by beginning malpractice is increased. This is depicted in the second row of Figure 9 which studies the harm function: \( \alpha(y) = \ln(1 + 2y) \). This acts to delay the implementation of malpractice to greater levels of competition, and keeps the level of malpractice lower. However, once malpractice begins it has a very significant effect on consumer surplus. This is partly because the benefits to margins have already been partially exhausted at the greater number of competing firms.

8 Concluding remarks

The relationship between malpractice and competition hinges on the shape of consumers’ taste distribution and on owner-managers’ ethics in a manner which can be characterised.
The most standard distributions (normal, uniform, extreme value) and the most standard ethical views (utilitarian, deontological) yield a positive link between competition and malpractice. These results are robust whether the competition is one-off or repeated, whether the harm function perpetrated on the stakeholders (consumers, workers, environment) takes a concave or convex form, and they apply whether sanctions for misconduct are profit or revenue based. The results are qualified when taste distributions have log-convex upper tails, and when ethics include a substantial (Kantian) disutility from any wrong-doing, however trivial. The former case reverses the link between competition and malpractice; the latter change causes competition and malpractice to become inverse-U shaped. If competition is asymmetric duopoly, the prominent case holds that improved ethics in one firm lowers equilibrium malpractice in both, but it acts to raise prices across the market also. The misconduct induced by competition is not minor in magnitude; it can overwhelm improvements in consumer surplus from reduced prices.

Finally we conclude with two policy remarks. Firstly, an outstanding yet important policy question is how should financial regulators with scarce resources deploy their attention so as to maximise welfare and minimise incidents of corruption and malpractice. Would it be wiser to focus regulatory attention on market structures of intense competition between many smaller service providers, or to focus on the interactions of the largest financial institutions with correspondingly large market shares? With the caveat that bad people will be bad in any market structure, this model argues that regulators should prioritise markets which exhibit intense competition between many smaller players. Examples of this being the mortgage default insurance market (PPI) discussed above, and the market for IFAs in the US.\(^{32}\) This does not mean that there should be no enforcement against large firms. Rather the regulator can dial down the resources on large firms as she benefits from the fact that the ethical and reputational deterrence are larger for the larger firms, and large enough to compensate for the profit incentives.

Secondly the model offers insights into how policy makers can deter unethical but non-criminal activity, and so activity which does not incur explicit regulatory sanction.

\(^{32}\)In the case of IFAs, malpractice is an endemic feature of the sector in the US (Egan et al. (2016)).
For example, what market structures are most likely to encourage adoption of low-carbon technologies? Squandering energy in production, or failure to make full use of carbon-capture and carbon-offsetting technologies is not illegal. However if high-carbon activities are discovered then special interest groups may succeed in inflicting a reputational cost on the firm. The question is whether concentrated industries would be best suited to encouraging green production; or might the high profits at risk from more expensive green technology cause concentrated industries to be too reluctant to be truly green. Theorem 2 of this study gives a clear steer whilst allowing for managers’ ethical distaste to contribute to climate change, and even though there are no official sanctions (Φ = 0). One need only map the unethical action as the removal of carbon-abatement activity. It then follows that concentrated markets are the ones most suited to the maintenance of a higher-cost, lower-carbon production equilibrium: once again such market structures maximise the ethical and reputational pressure to such behaviour, outweighing the profit concerns.

A Proofs of symmetric analysis

Proof of Lemma 3. It will be helpful to denote, in this proof only:

\[
a := V^i_{p,p_i} \bigg|_e, \quad b := V^i_{p,p_j} \bigg|_e \quad \text{for } j \neq i, \quad \tilde{c} := V^i_{y,p_i} \bigg|_e \\
d := V^i_{p,y_i} \bigg|_e \quad \text{for } j \neq i, \quad \epsilon := V^i_{y,y_i} \bigg|_e
\]

It then follows we can write

\[
\mathcal{H} = \begin{pmatrix} e & \tilde{c} + (n - 1) \epsilon \\ \epsilon & a + (n - 1) b \end{pmatrix}
\]

Now consider the requirements of stability (Dixit (1986)). Suppose the firms find themselves at a non-equilibrium point \( \{\tilde{p}_j, \tilde{y}_j\} \), which is close to the equilibrium values \( p^e, y^e \). Suppose each firm updates its prices and malpractice proportionally to the first order gain achieved by changing the variable. Using a Taylor Expansion for firm \( i \), for points close to the equilibrium we have:

\[
\hat{p}_i := V^i_{p_i} \bigg|_{\tilde{p}, \tilde{y}} = (\tilde{p}_i - p^e) V^i_{p,p_i} \bigg|_e + \sum_{j \neq i} (\tilde{p}_j - p^e) V^i_{p,p_j} \bigg|_e + (\tilde{y}_i - y^e) V^i_{y,y_i} \bigg|_e
\]
A similar expression can be established for $\dot{y}_i$:

$$\dot{y}_i := V^i_{y_i} | (\hat{p}_i, \hat{y}) = (\hat{p}_i - p^c) V^i_{y_i, p_i} | e + \sum_{j \neq i} (\hat{p}_j - p^c) V^i_{y_i, p_j} | e + (\dot{y}_i - y^c) V^i_{y_i, y_i} | e$$

The system path near to an equilibrium point is therefore captured by the following $2n \times 2n$ matrix:

$$\begin{pmatrix}
\dot{y}_1 \\
\dot{p}_1 \\
\dot{y}_2 \\
\dot{p}_2 \\
\vdots \\
\dot{y}_n \\
\dot{p}_n
\end{pmatrix} = \begin{pmatrix}
e & c + (n-1) d \\
c & a + (n-1) b \\
0 & d & e & c & 0 & d & \cdots & e & c \\
0 & b & c & a & 0 & b & \cdots & e & c \\
0 & b & 0 & b & 0 & b & \cdots & e & c
\end{pmatrix} \begin{pmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_1 \\
\omega_2
\end{pmatrix} = \lambda \begin{pmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_1 \\
\omega_2
\end{pmatrix}.$$

We have denoted the transition matrix $\mathcal{A}$. Stability of the equilibrium at $p^c, y^c$ requires that all the eigenvalues of $\mathcal{A}$ have negative real parts (Dixit (1986), Anishchenko et al. (2014) Chapter 2).

Suppose now that $\lambda$ is an eigenvalue of $\mathcal{H}$ with eigenvector $(\omega_1, \omega_2)^T$. Thus

$$\begin{pmatrix}
e & c + (n-1) d \\
c & a + (n-1) b \\
0 & d & e & c & 0 & d & \cdots & e & c \\
0 & b & c & a & 0 & b & \cdots & e & c \\
0 & b & 0 & b & 0 & b & \cdots & e & c
\end{pmatrix} \begin{pmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_1 \\
\omega_2
\end{pmatrix} = \lambda \begin{pmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_1 \\
\omega_2
\end{pmatrix} = \begin{pmatrix}
e \omega_1 + \omega_2 (c + (n-1) d) \\
\omega_2 \\
\vdots \\
\omega_1 \\
\omega_2
\end{pmatrix} = \lambda \begin{pmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_1 \\
\omega_2
\end{pmatrix}.$$

We show that $\lambda$ must be an eigenvalue of the matrix $\mathcal{A}$.

Consider the $2n$ vector

$$W := (\omega_1, \omega_2, \omega_1, \omega_2, \cdots, \omega_1, \omega_2)^T.$$

We have

$$\mathcal{A}W = \lambda W,$$

as required.

Stability of the system thus implies that $\lambda$ has negative real parts and so the result is proved. ■
Proof of Proposition 5. Inverting the analogue of (19) as in (31) we have:

\[
\left( \frac{dy^e}{dp^e} \right) = \frac{1}{\det H} \left( \begin{vmatrix} -\left( V_{yi|e}^i \right)_{pe} & \left( V_{yi|e}^i \right)_{pe} \\ -\left( V_{yi|e}^i \right)_{yse} & \left( V_{yi|e}^i \right)_{yse} \end{vmatrix} \right) \left( \begin{vmatrix} -\left( V_{yi|e}^i \right)_{pe} \\ \left( V_{yi|e}^i \right)_{yse} \end{vmatrix} \right) d\omega \quad (53)
\]

To establish \( dy^e / d\omega \) we need only sign four of these terms. Two are immediate: from (23) and (24) \( \left( V_{yi|e}^i \right)_{pe} = \frac{P(n)}{\alpha(y^e)} > 0 \); from (33) \( \left( V_{yi|e}^i \right)_{pe} < 0 \). Next we expand (21) using (22), (15) and (12) to establish:

\[
\left( V_{yi|e}^i \right)_{pe} = -e^{-r} \delta'(y^e) \cdot \frac{\alpha(y^e)}{n} - \frac{\alpha'(y^e)}{n} \quad (54)
\]

The next step is to establish that at a stable equilibrium we have \( \left( V_{yi|e}^i \right)_{pe} < 0 \). To see this we must extend Lemma 3. Following Dixit (1986) stability of equilibrium must hold even if the adjustment speeds of the \( n \) players are not identical. Thus the equilibrium must be stable even if \( \dot{p}_i = s \cdot V_{pi|e}^i \) for some constant \( s > 0 \), and similarly \( \dot{y}_i = \tilde{s} \cdot V_{yi|e}^i \) for a potentially different constant \( \tilde{s} \). Repeating the proof of Lemma 3 we establish the corollary that the matrix

\[
\tilde{H} := \left( \begin{array}{cc} \tilde{s} \cdot \left( V_{yi|e}^i \right)_{yse} & \tilde{s} \cdot \left( V_{yi|e}^i \right)_{pe} \\ s \cdot \left( V_{pi|e}^i \right)_{yse} & s \cdot \left( V_{pi|e}^i \right)_{pe} \end{array} \right)
\]

must have eigenvalues with negative real parts for all \( s, \tilde{s} > 0 \). This implies that the trace is negative for any values of \( s, \tilde{s} > 0 \), and so \( \left( V_{yi|e}^i \right)_{pe} < 0 \).

Combining we have established that \( dy^e / d\omega < 0 \) if \( \left( V_{yi|e}^i \right)_{pe} < 0 \). Using (54) we establish (39).  

B Proofs of duopoly analysis (Section 6)

Before proving Theorem 8 we first establish a lemma on the inverse of the Hessian, \( \tilde{A} \), given in (46).

It is known (Lancaster and Tismenetsky (1985)) that \( \tilde{A}^{-1} = \frac{1}{\det \tilde{A}} \cdot \text{adj}(\tilde{A}) \) where the adjoint is the transposed matrix of cofactors of \( \tilde{A} \), and in turn the cofactor is plus or minus the minor \( M_{pq} \) which is the determinant of the submatrix of \( \tilde{A} \) obtained by striking out the \( p^{th} \) row and \( q^{th} \) column.\[^{33}\] That is:

\[
\tilde{A}^{-1} = \frac{1}{\det \tilde{A}} \begin{pmatrix} M_{31} & -M_{41} \\ -M_{32} & M_{42} \\ M_{33} & -M_{43} \\ -M_{34} & M_{44} \end{pmatrix}
\quad (55)
\]

Where \( \pm M_{ij} \) is the cofactor of the Hessian \( \tilde{A} \) evaluated at matrix entry \( \{i, j\} \). Given the zero entries in the right hand side of equation (47) we do not calculate all the terms of

\[^{33}\]The factor is +1 if the sum \( p + q \) is even.
the inverse of $\hat{A}$.

**Lemma 10** The signs of the entries in the matrix $\hat{A}^{-1}$ satisfy:

$$M_{43} < 0, M_{34} < 0, M_{44} < 0$$

$$M_{31} = \text{sign } M_{41} = \text{sign } M_{32} = \text{sign } M_{42} = \text{sign } \frac{\partial^2 \ln q_1}{\partial p_1 \partial p_2}$$

The sign of $M_{33}$ is ambiguous.

**Proof.** The cofactors of $\hat{A}$ can be calculated as:

$$M_{41} = \begin{vmatrix} U_{11p1} & U_{1p1} & 0 \\ U_{1p1} & 0 & U_{1p2} \\ 0 & U_{2y2} & U_{2y2} \end{vmatrix} = -U_{11p1} \cdot U_{1p1} \cdot U_{1p2} \cdot U_{2y2}$$

$$M_{42} = -U_{11p1} \cdot U_{1p2} \cdot U_{2y2}$$

$$M_{43} = U_{2y2} \cdot (U_{11p1} \cdot U_{1p1} - (U_{11p1})^2)$$

$$M_{44} = U_{2y2} \cdot (U_{11p1} \cdot U_{1p1} - (U_{11p1})^2)$$

The proof now uses the second order conditions (43) and (44) and two further results. First

$$\Rightarrow U_{2y2} = -q_2 \cdot \Phi \delta'(y_2) < 0. \quad (60)$$

And analogously for $U_{11p1}$.

The second observation arises by using the first order condition $U_{1p1} = 0$ to substitute into the second derivative term $U_{1p1p2}$. We have

$$U_{1p1p2} = (1 - \Phi \delta(y_1)) \cdot \left( \frac{\partial^2 q_1}{\partial p_1 \partial p_2} \cdot \frac{q_1}{\partial q_1} + \frac{\partial q_1}{\partial p_2} \right)$$

Now recalling Assumption A and observing that

$$\frac{\partial^2 \ln q_1}{\partial p_1 \partial p_2} = \frac{1}{q_1^2} \left( q_1 \frac{\partial^2 q_1}{\partial p_1 \partial p_2} - \frac{\partial q_1}{\partial p_1} \frac{\partial q_1}{\partial p_2} \right)$$

we have that

$$U_{1p1p2} = \text{sign } \frac{\partial^2 \ln q_1}{\partial p_1 \partial p_2}$$

The result now follows algebraically by inspection of the cofactors (56)–(59), and similarly for the third column of $\hat{A}^{-1}$ in (55). For ease of replication note that

$$M_{31} = -U_{11p1} U_{1p1} U_{2y2}$$

$$M_{33} = (U_{11p1} U_{1p1} - (U_{11p1})^2) U_{2y2} - U_{11p1} U_{1p1} U_{2y2}$$

$$M_{32} = -U_{11p1} U_{1p1} U_{2y2}$$

$$M_{34} = U_{2y2} (U_{11p1} U_{1p1} - (U_{11p1})^2)$$

**Proof of Theorem 8.** From (47) and (55) we have:

$$\begin{pmatrix} dy_1^c \\ dp_1^c \\ dy_2^c \\ dp_2^c \end{pmatrix} = \frac{1}{\det \hat{A}} \begin{pmatrix} \cdot & M_{31} & -M_{41} \\ \cdot & -M_{32} & M_{42} \\ \cdot & M_{33} & -M_{43} \\ \cdot & -M_{34} & M_{44} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ q_2 \delta'(y_2) \\ \frac{\partial y_2}{\partial p_2} \delta_2(y_2) \end{pmatrix} d\omega_2.$$
One can therefore read off that

\[
\frac{dy_i}{d\omega} = \frac{1}{\text{det } A} \left[ M_{31} \cdot \frac{q_2 \alpha_2(y_2)}{+ve} - M_{41} \cdot \frac{\partial q_2}{\partial p_2} \cdot \alpha_2(y_2) \right] = \text{sign using Lemma 10} - \frac{\partial^2 \ln q_1}{\partial p_1 \partial p_2}
\]

We can now appeal to Quint (2014), Theorem 1, to establish that if the consumers’ tastes generate a log-concave reliability function, \((1 - F(x))\), then the log of each firm’s realised demand has increasing differences in prices in the discrete choice model studied here. Proceeding in this way, using Lemma 10 yields all the results except the sign of \(dy_2/d\omega_2\). For this observe that

\[
U_{y_2} = q_i \cdot [1 - \Phi\delta(y_i) - \Phi\delta'(y_i)(p_i - c_i + y_i) - \omega_i\alpha_i'(y_i)]
\]

Now suppose we begin at equilibrium in which the first order condition is satisfied \((U_{y_2}^2 = 0)\). For a contradiction suppose that \(dy_2/d\omega_2 \geq 0\). If \(\omega_2\) were to rise then we know from the already proved comparative statics that at the new equilibrium \(p_2\) is higher, and by assumption \(y_2\) is also weakly higher. Note that these imply that \(\delta'(y_2)\) has grown, as has \(\delta(y_2)\). But then collectively if \(\alpha_2'(y_2)\) has also grown we must then have \(U_{y_2}^2 < 0\) which is a contradiction to equilibrium. This proves the third and final result.

C Further proofs

Proof of Theorem 9. The proof proceeds analogously to Theorem 2 and so here I highlight only the differences. The revenue based fines alter the single stage payoff (12) to

\[
U^i := q_i(p_i) [(p_i - c + y_i) - \delta(y_i)\Phi^{rev} p_i - \omega\alpha_i(y_i)]
\]

This alters the first derivatives at equilibrium (22) and (24) respectively to:

\[
U_{y_i}^i = \frac{1}{n} \left[ 1 - \delta'(y_i)\Phi^{rev} p_i - \omega\alpha'(y_i) \right]
\]

\[
U_{p_i}^i = - \frac{P(n)}{n} \left[ (p^e - c + y^e) - \delta(y^e)\Phi^{rev} p^e - \omega\alpha(y^e) \right] + \frac{1}{n} [1 - \delta(y^e)\Phi^{rev}].
\]

The proof then proceeds directly with the culmination of a new (37) which is altered to

\[
0 = \frac{-e^{-r}\delta'(0)}{(1 - e^{-r})n^* P(n^*)} + \frac{1}{n^*} \left[ 1 - \delta'(0)\Phi^{rev} \left( \frac{1}{P(n^*)} + c \right) - \omega\alpha'(0) \right].
\]

Simplification of (62) yields the remaining result.

References


