Variance after-effects, portfolio turnover and excess volatility

Preliminary and incomplete, please do not circulate

Tony Berrada*

December 9, 2018

Abstract

Variance after-effect is a perceptual bias in the dynamic assessment of variance. Experimental evidence shows that perceived variance is decreased after prolonged exposure to high variance and increased after exposure to low variance. We introduce this effect in an otherwise standard financial model where information about variance is incomplete and updated sequentially. We show that variance after-effect can explain excessive portfolio turnover/rebalancing and excess stock volatility, two effects largely documented in the finance literature. We also introduce a modeling framework to assess the importance of these effect (and other perceptual biases) in a delegation environment, when the agent (a machine) is unbiased, the principal (a human) is biased, and resources (computational capacity) are limited. We show that while reduced, biases cannot be completely eliminated if resources are finite.

Keywords: Variance after-effect, learning, turnover, volatility

JEL Classification: G12

*University of Geneva - Geneva finance research institute (GFRI) and Swiss Finance Institute, Unime-mail, Boulevard du Pont d’Arve 40, 1211 Geneva–4, Switzerland. Email: tony.berrada@unige.ch. I thank seminar participants at the 2018 Lake Luzern Neurofinance Conference for comments.
1 Introduction

There is experimental evidence that humans’ perception of variance is biased when inferred in sequences. Perceived variance is decreased after prolonged exposure to high variance and increased after exposure to low variance. Through a series of laboratory experiments Payzan-LeNestour, Balleine, Berrada, and Pearson (2016) show that the variance after-effect appears across very different visual representations of variance, and that the effects are not sensory, but operate at a cognitive level of information processing. As such, integrating these effects when modeling economic and financial decision, where variance assessment is often pivotal, seems warranted.

In this paper we propose to introduce the variance after-effect in an otherwise standard financial setup, where information about the variance of some underlying state variable is incomplete. Agents must learn from the observation of a signal and update their prior about the relevant parametric quantities. As a benchmark learning model we use the standard bayesian approach in a simple context where the updating procedure is highly tractable. Our examples rely on a normal-inverse-gamma (NIG) setup: the variable of interest is normally distributed with known mean and unknown variance, and the agent has inverse-gamma (IG) prior about the variance. The advantage of this assumption is that the NIG is a conjugate of the normal distribution with unknown variance, and as a result the posterior distribution remains in the NIG class. We model variance after-effects through a single parameter which affect the updating procedure of the IG distribution. This parameter depends on the sequence of observed signals, or more precisely, on the sequence of their sample variances. The modeling approach further allows for varying degree of intensity, and as a special case, when the intensity is null, coincides with the standard bayesian approach.

We first consider a problem of portfolio choice and show that our setup can explain the apparent excessive portfolio turnover first documented in Barber and Odean (2000). Using realistic parametric choices for the return distribution and rebalancing frequencies,
we observe that turnover increases significantly as a function of the varying after-effect intensity. Changes in posterior variances updates are amplified by the sequential bias which also produces reversal in the variance estimates, and as a result the investor re-allocates his portfolio more significantly on average when subject to variance after-effect.

We then analyze an equilibrium model, where the traded asset price is a claim on a normally distributed payoff with known mean and unknown variance. The agent observes sequences of signals about the payoff and trades at regular frequency. Using a similar learning procedure as in the portfolio problem, we show that the bias introduced in the variance estimate translate into increased equilibrium return volatility, as documented in LeRoy and Porter (1981) and Shiller (1981). The excess volatility increases as a function of the after-effect intensity.

In an era where automated and algorithmic trading represents a large fraction of the trading volume on typical financial markets, it might be questionable to care about perceptual biases that hinder human decision making, when humans are actually not directly implementing the trading strategies. We address this issue and introduce a modeling framework to assess the importance of perceptual biases in a delegation environment, when the agent (a machine) is unbiased, the principal (a human) is biased, and resources (computational capacity) are limited. We show that while reduced, biases cannot be completely eliminated if resources are finite. We provide a simple example where the principal allocates capital between two agents based on a risk parity criteria, i.e. the principal seeks to equate the marginal risk contribution of each investment strategy. The agents operate at high frequency and are unbiased, the principal operates a low frequency and re-allocate capital based on his biased assessment of the variance of the return of the strategies generated by the agents. We quantify the misallocation and show that it can be substantial in the presence of variance after-effects.

The paper is organized as follows. Section 2 introduces two financial examples where variance after-effects distort, first, trading behavior and, second, return volatility. Section 3 describes the delegation framework and provides an example. Section 4 concludes.
2 Examples in a financial context

In this section we propose two models where information pertaining to variance is incomplete, and the sequential learning of agents displays variance after-effects. We first look at a partial equilibrium model of portfolio choice, and then describe a simple equilibrium model. The first setup focuses on incremental portfolio turnover induced by variance after effect, while the second analyzes the contribution of variance after effect to the equilibrium stock price volatility.

2.1 A portfolio choice model

We consider the problem of an investor who seeks to allocate her wealth between a risk free asset with constant return normalized at 0 and a risky asset with normally distributed return with mean $\mu$ and variance $\sigma^2$. We assume that the investor seeks to maximize the per period Sharpe Ratio, or equivalently that she has logarithmic preferences over terminal wealth. In a context of perfect information, the fraction optimally allocated to the risky asset is then given by

$$\pi_t = \frac{1}{A} \frac{\mu}{\sigma^2_t}$$

where $A$ is a measure of risk aversion.

We assume that information is incomplete and, that the investor knows the mean $\mu$ but not the variance $\sigma^2$. Before each investment decision, the investor observes a return sequence $x_1, x_2, \ldots, x_n$ where

$$x_i \mid \mu, \sigma^2 \sim N(\mu, \sigma^2) \ i.i.d.$$  

\footnote{Alternatively, we can consider the sequence as a signal with same mean and variance as the return.}
We further assume that the investor has inverse-gamma priors

$$\sigma^2 \sim \text{IG}(\alpha, \beta)$$

where $\alpha$ and $\beta$ are hyperparameters of the prior beliefs. We choose this distributional framework for tractability, the normal-IG distribution is a conjugate prior for the normal distribution with known mean and unknown variance. It follows that the posterior distribution conditional on signal observation is

$$\sigma^2 | x_1, x_2, \ldots, x_n \sim \text{IG} \left( \alpha + \frac{n}{2}, \beta + \frac{\sum (x_i - \mu)^2}{2} \right).$$

Repeating the returns sequence generates a series of posterior beliefs

$$(\alpha_1, \beta_1), (\alpha_2, \beta_2), \ldots, (\alpha_t, \beta_t), \ldots$$

A bayesian learner uses the predictive variance

$$\hat{\sigma}_t^2 = \frac{\beta_t}{\alpha_t - 1}$$

to construct her mean-variance portfolio

$$\pi_t = \frac{1}{A} \frac{\mu}{\hat{\sigma}_t^2}.$$  

We propose to introduce the variance after-effect by modifying the predictive variance through a change of the updating procedure of the hyperparameter $\beta$. Namely we depart from the bayesian approach

$$\beta_{t+1} = \beta_t + \frac{[\sum (x_i - \mu)^2]}{2}_t$$
by introducing an adjustment factor $\Theta_t$,

$$\beta^{ae}_{t+1} = \beta^{ae}_t + \Theta_t \frac{[\sum (x_i - \mu)^2]_t}{2}$$  \hspace{1cm} (1)

where

$$\Theta_t = \left[1 + \left(\frac{n - 3}{n - 1} \frac{[\sum (x_i - \mu)^2]_t}{[\sum (x_i - \mu)^2]_{t-1}} - 1\right) \lambda\right].$$  \hspace{1cm} (2)

The coefficient $\lambda \geq 0$ measures the size of the variance after-effect, and the notation $[\sum (x_i - \mu)^2]_t$ indicates a sample variance estimate during the $t^{th}$ sequence. The ratio of sample variance has an F-distribution with mean $\frac{n-3}{n-1}$, the adjustment factor is therefore on average equal to 1. However, a given sample variance estimate will appear lower (higher) if it follows a higher (lower) sample variance in the previous sequence. This is precisely what the experimental evidence on the variance after-effect has established. The coefficient $\lambda$ can be varied to account for the observed heterogeneity of the variance after-effect across subjects in experimental context.

### 2.2 Simulation results

Barber and Odean (2000) provide a detailed analysis of the trading pattern of a large group of retail investors. Ranking the investors on the basis of their average turnover, they find no significant difference in gross return, but after transaction cost, investors who trade more often earns far less. They attribute this apparent irrational behavior to overconfidence.

In this section, we analyze the quantitative implications of the model previously developed, and show that the variance after-effect might also bring some insight in understanding the trading behavior of retail investors. We simulate the trading behavior of ten groups of investors who differ only by the intensity of the variance after-effect, measured by $\lambda \in \{0, \ldots, 1.5\}$. The estimation is based on sequences of $n = 20$ signals, and the
portfolio is rebalanced $T = 30$ times. The return distribution has a mean of $\mu = 0.1$ and volatility $\sigma = 0.2$. We report the simulation results in Figure 1. There are no significant differences in term of expected return across the various groups, and we can conclude that the variance after-effect coefficient doesn’t affect the average performance. However, the turnover is an increasing function of the bias. Figure 2 shows that while it doesn’t affect expected return, variance after-effect increases the portfolio return’s volatility.

We can get a clearer understanding of the effect’s mechanism by comparing bayesian and the modified framework (AE-bayesian) learning dynamics. Figure 3 displays a sample path of the variance estimates as well as the adjustment factor when the variance after-effect intensity $\lambda = 1$, and Figure 4 displays the portfolio level and the associated turnover.

2.3 A financial equilibrium model

We consider an economy with a representative agent endowed with utility over terminal consumption only

$$U(C_T) = -\frac{1}{A}e^{-AC_T}.$$ 

There is a single risky asset, with price $P_t$ providing a terminal dividend $D_T \sim N(\mu, \sigma^2)$ and a risk free bond with maturity $T$ which is used as the numeraire.$^2$ The mean of the dividend’s distribution is known, but the agent must learn the variance $\sigma^2$. In between trading dates $t \in \{1, \ldots, T\}$ the agent observes a signal sequence $x_1, x_2, \ldots, x_n$ where

$$x_i \mid \mu, \sigma^2 \sim N(\mu, \sigma^2) \text{ i.i.d.}$$

As in the previous setup, the agent has IG prior

$$\sigma^2 \sim \text{IG}(\alpha, \beta)$$

$^2$In a model without intermediate consumption the risk-free rate is undefined, it is common to then use the arbitrary level of the risk bond as a numeraire see for exemple Berrada (2009).
and it follows that, conditional on a signal sequence, variance has an IG distribution

\[ \sigma^2 | x_1, x_2, \ldots, x_n \sim IG \left( \alpha + \frac{n}{2}, \beta + \frac{\sum(x_i - \mu)^2}{2} \right). \]

Repeating signal sequence generates a series of updates

\[(\alpha_1, \beta_1), (\alpha_2, \beta_2), \ldots, (\alpha_t, \beta_t), \ldots\]

We show in the appendix that, in equilibrium, the asset price is given by

\[ P_t = \frac{E_t \left[ e^{-AD_T} D_T \right]}{E_t \left[ e^{-AD_T} \right]} \]  \hspace{1cm} (3)

where \( D_T | \mu, \alpha_t, \beta_t \) has a student’s t-distribution.

We extend this bayesian framework by introducing the variance after-effect in the learning mechanism described above. Namely, we use equations (5) and (2) to modify the learning sequence. This yields an adjusted equilibrium price which we denote

\[ P^{ae}_t = \frac{E \left[ e^{-AD_T} D_T | F_t \right]}{E \left[ e^{-AD_T} | F_t \right]} \]  \hspace{1cm} (4)

where \( D_T | \mu, \alpha_t, \beta^{ae}_t \) has a student’s t-distribution (for a proof see for example DeGroot (1970)). The difference between the two economies stems from the dynamics of the hyperparameter \( \beta \). The after-effect intensity \( \lambda \) affects the price distribution through its impact on \( \beta^{ae}_t \), and therefore the equilibrium volatility as well.

2.4 Simulation results

LeRoy and Porter (1981) and Shiller (1981) provide the first observations that stock prices move too much to be only driven by dividend variation. The idea that stock have excessive volatility has been widely addressed in the literature.

In this section we present simulation results of the equilibrium framework previously
described. We compare the equilibrium of that economy with one populated by a standard
bayesian learner, to see if the variance after-effect can also provide a simple solution to
the excess volatility puzzle. We also vary intensity of the variance after-effect, measured
by $\lambda \in \{0, \ldots, 1.5\}$. The agent estimates the variance based on a sequence of $n = 10$
signals, and there are $T = 20$ trading periods. She has a risk aversion level of $A = 2$.
The dividend distribution has a mean $\mu = 0.2$ and a volatility $\sigma = 0.1$.

Figure 5 displays the equilibrium stock return volatility as a function of the variance
after effect intensity. We can see that it is clearly increasing, from 1% to 6%. Figure 6
provides additional insight by considering a sample path of the stock price return along
with the bayesian and AE-baysiean estimates of variance.

3 A framework for delegation to an artificial agent

In this section we introduce a simple model to assess the impact of an estimation bias
in the context of a delegated task. We borrow the terminology of the principal-agent
framework, but note that here the problem is not related to moral hazard, as the agent is
an algorithm/machine which doesn’t seek to deviate from the principal’s objective, rather
the problem is the potential principal’s bias in allocation of ressources across artificial
agents.

Without loss of generality we consider an allocation of ressources between two artificial
agents (A-agents). An A-agent receives capital $k_i$ incurs cost $\alpha_i k_i$ and produces returns $R_i$
from an investment strategy $\pi(S_1)$, where $S_1 \subset S$ is a subset of the market $S$. A strategy
$\pi(S_1)$ is a mapping from observed signals at dates $\{t, \ldots, t + k\}$ to a vector of weights.
We can think of the proportional cost $\alpha_i$ as an overall maintenance and development cost
of the A-agent including programing, infrastructure, and energy cost.

The human principal (H-principal) allocates capital across the A-agents following
an allocation criteria $C(R_1, R_2)$ which is a mapping from observed returns at dates
$\{s, \ldots, s + jk\}$ to a 2-dimensional vector summing to $K$, the total amount of capital.
The A-agent is not subject to any perception bias and follows bayesian updating procedures, while the H-principal may be subject to perception bias of intensity $\lambda$ while applying the allocation criteria.

We first note that because of the cost, adding unlimited layers of A-agents to eliminate the perception bias (for example an A-agents can implement the allocation criteria), reduces the available capital to arbitrary low amounts. We can interpret the current framework as the outcome of a preliminary tradeoff decision. As long as a human decision takes place at some point of the allocation problem, we can think of it as a simplified description of the problem.

Another important aspects of the delegation framework is that the H-principal and the A-agents operate at different frequencies. For example, the A-agent may be an algorithmic trading strategy operating at ultra high frequency and the H-principal the management of a financial firm deciding on the allocation of personnel and ressources across investment strategies on a quarterly basis.

### 3.1 A simple example: model

We now put some structure on the mapping functions to quantify the bias transmission mechanism. We assume that $S_1$ and $S_2$ are two independent markets, in the sense that any portfolio in $S_1$ is uncorrelated to a portfolio in $S_2$. The asset return follow multivariate normal distribution and therefore the strategies implemented generate normally distributed returns. We assume that based on the unbiased estimates of the A-agent the returns follow independent normal distribution with mean $\mu_i$ and variance $\sigma_i^2$. The H-principal wants to allocate resources such that the marginal contribution to the overall risk of the invested wealth is evenly distributed across A-agents. In other words the criteria is a risk parity allocation between the two A-agents. The H-principal observes the return and has NIG (normal Inverse gamma) prior over the mean and variance of
the distribution. After a return sequence the estimates $\mu_i, \sigma_i^2 \mid R_{it}, \ldots, R_{nt}$ also have an NIG distribution. The predictive variance is denoted $\hat{\sigma}_{i,t}^2$ and to implement the criteria of risk parity the H-principal sets the capital allocation such that

$$\frac{k_1}{k_2} = \frac{\hat{\sigma}_{2,t}^2}{\hat{\sigma}_{1,t}^2}.$$  

Under the assumption of NIG prior, the predictive variance is given by

$$\hat{\sigma}_{i,t}^2 = \frac{\beta_{i,t}}{\alpha_{i,t} - 1}$$

where

$$\beta_{i,t+1} = \beta_t + \frac{\left[\sum (R_{it} - \bar{R}_i)^2\right]_t}{2} + \frac{(\bar{R}_i - \mu_{i,0})^2}{2} \frac{n\kappa}{n + \kappa}$$

has an additional term compared to the model used in the previous sections to account for uncertainty about the mean. Here again, following the previously proposed procedure, we introduce the risk after-effect bias through an adjustment factor $\Theta_t$

$$\beta_{i,t+1}^{ae} = \beta_{i,t}^{ae} + \Theta_t \cdot \left[\sum (R_{it} - \bar{R}_i)^2\right]_t + \frac{(\bar{R}_i - \mu_{i,0})^2}{2} \frac{n\kappa}{n + \kappa} \quad (5)$$

where $\Theta_t$ is given by equation (2) and corresponds to the ratio of successively observed sum of squared returns deviations.

### 3.2 A simple example: simulation

We set the model coefficients as follow $\mu_1 = 0.2, \sigma_1 = 0.1, \mu_2 = 0.2, \sigma_2 = 0.1, n = 20, \text{ and } T = 12$. Following the risk parity allocation, the principal should optimally allocate the fraction to both A-agents $k_{1t} = k_{2t}$. To measure the distance from the optimal allocation we introduce the cumulative absolute allocation distortion defined as the sum, over all $T$

---

\(^3\)This is a mild generalization of the setup used in the previous sections which requires 2 additional hyperparameters related to uncertainty about the mean of the distribution.
trading periods, of the absolute differences between the optimal allocation to A-agent 1, 
k_{1t} = 0.5, and the chosen allocation based on the H-principal’s estimate of the variances 
\hat{\sigma}_{i,t}^2. We vary the after-effect intensity \( \lambda \) from 0 to 2.

Figure 7 displays the cumulative absolute allocation distortion which ranges from 15% to nearly 50% as the after-effect intensity increases. The distortion is not zero even when \( \lambda = 0 \) since the purely bayesian posteriors are also noisy. The variance after-effect however, significantly increases the distortion.

An example of sample path for the standard deviation estimates and the allocation to \( k_1 \) are provided in figures 8 and 9 for a variance after-effect intensity \( \lambda = 1.5 \). We can see that the AE-bayesian estimates are far more volatile and as a consequence the deviations from the optimal allocation are quite significant.

4 Conclusion

Variance is a key input in the vast majority of financial models. Taking into account the experimentally grounded variance after-effect bias has a significant impact on portfolio decision in a context of partial information, and as a consequence can explain the large portfolio turnover observed in retail investors behavior. In equilibrium, these effects yield a significant increase in return volatility, and can contribute to our understanding of the excess volatility puzzle. There are numerous behavioral biases, such as for example over-confidence and over/under reaction, which can explain similar patterns, but the variance after-effect is a simple mechanism which operates at a basic perception and cognitive level, and as such may be an interesting candidate to consider.

The variance after-effect bias can be modeled by a simple departure from the standard bayesian updating procedure, and intensity of the effect can be parametrized using a single coefficient. The examples provided in this paper are based on univariate distribution\(^4\) but can be easily extended to multivariate settings.

\(^4\)or bivariate but independent distribution in the delegation framework example
References


Appendix A: Equilibrium Price

The state price density at time $T$ denoted $\xi_T$ is proportional to the marginal utility

$$\xi_T = \eta e^{-AC_T}$$

for some $\eta > 0$. In equilibrium, market clearing imposes $C_T = D_T$. The time $t$ equilibrium 0-coupon bond price with maturity $T$, denoted $B_t$, is given by

$$B_t = E_t [\eta e^{-AD_T}]$$

where $E_t$ denotes expectation conditional on the information available at time $t$. Any claim with measurable payoff $Z_T$ at time $T$ has an equilibrium price of

$$Z_t = E_t [\eta e^{-AD_T} Z_T].$$

Since we use the bond price as numeraire we can write

$$\frac{Z_t}{B_t} = \frac{E_t [e^{-AD_T} Z_T]}{E_t [e^{-AD_T}]}.$$

For the stock price $Z_T = D_T$, and equation (3) and (4) follow.
Figure 1: Portfolio turnover and performance based on 50000 simulations. The model coefficients are $\mu = 0.1$, $\sigma = 0.2$, $n = 20$, $T = 12$, $A = 2$. The variance after-effect intensity $\lambda$ ranges from 0 to 1.5.
Figure 2: Portfolio turnover and performance based on 50000 simulations. The model coefficients are $\mu = 0.1$, $\sigma = 0.2$, $n = 20$, $T = 12$, $A = 2$. The variance after-effect intensity $\lambda$ ranges from 0 to 1.5.
Figure 3: Sample path of variance estimates for bayesian (red) and AE-bayesian (blue), and adjustment factor. The model coefficients are $\lambda = 1$, $\mu = 0.1$, $\sigma = 0.2$, $n = 20$, $T = 30$, $A = 2$. 
Figure 4: Sample path of portfolio and turnover for bayesian (red) and AE-bayesian (blue). The model coefficients are $\lambda = 1$, $\mu = 0.1$, $\sigma = 0.2$, $n = 20$, $T = 30$, $A = 2$. 
Figure 5: Equilibrium stock return volatility based on 10000 simulations. The model coefficients are $\mu = 0.2$, $\sigma = 0.1$, $n = 10$, $T = 20$, $A = 2$. The variance after-effect intensity $\lambda$ ranges from 0 to 1.5.
Figure 6: Equilibrium stock return and variance estimate. The model coefficients are $\mu = 0.2$, $\sigma = 0.1$, $n = 10$, $T = 20$, $A = 2$. The variance after-effect intensity is set at $\lambda = 1$. 
Figure 7: Cumulative absolute allocation distortion in the delegation model with risk parity criteria. The model coefficients are $\mu_1 = 0.2$, $\sigma_1 = 0.1$, $\mu_2 = 0.2$, $\sigma_2 = 0.1$, $n = 20$, and $T = 12$. The variance after-effect intensity $\lambda$ ranges from 0 to 2.
Figure 8: Sample paths of standard deviation estimates in the delegation model with risk parity criteria. The model coefficients are $\mu_1 = 0.2$, $\sigma_1 = 0.1$, $\mu_2 = 0.2$, $\sigma_2 = 0.1$, $n = 20$, and $T = 12$. The variance after-effect intensity is $\lambda = 1.5$. 
Figure 9: Sample path of the allocation to $k_1$ in the delegation model with risk parity criteria. The model coefficients are $\mu_1 = 0.2$, $\sigma_1 = 0.1$, $\mu_2 = 0.2$, $\sigma_2 = 0.1$, $n = 20$, and $T = 12$. The variance after-effect intensity is $\lambda = 1.5$. 