



On the role of non-equilibrium focal points as coordination devices[☆]



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ABSTRACT

Considering a pure coordination game with a large number of equivalent equilibria, we argue that a focal point that is itself *not* a Nash equilibrium, and is Pareto dominated by all Nash equilibria, may enhance coordination substantially. Besides attracting the players' choices to itself, such a non-equilibrium focal point may act as an equilibrium selection device that the players use to coordinate on a small subset of Nash equilibria. We present experimental support for these two roles of non-equilibrium focal points as coordination devices, and suggest a theoretical explanation for this.

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1. Introduction

Many social interactions have to be modeled as a coordination game. Multiplicity of equilibria in such games implies that the players do not just need to find a solution to the game, but must also coordinate on the same solution. That is, they face strategic uncertainty. Schelling (1960) observed that, in everyday life, individuals who are confronted with coordination problems frequently seem to do surprisingly well, and that focal points play an important role by providing a point of convergence for individual expectations.¹ As Schelling put it: “*Most situations – perhaps every situation for people who are practiced at this kind of game – provide some clue for coordinating behavior, some focal point for each person's expectation of*

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¹ For a discussion of Schelling's analysis, see Sugden and Zamarrón (2006).

what the other expects him to expect to be expected to do. Finding the key, or rather finding **a** key – any key that is mutually recognized as the key becomes **the** key – may depend on imagination more than on logic; it may depend on analogy, precedent, accidental arrangement, symmetry, esthetic or geometric configuration, casuistic reasoning, and who the parties are and what they know about each other” (p. 57).

Following early work on focal points presenting the idea as such,² a considerable focal point literature (experimental as well as theoretical) has developed in the last few decades,³ essentially documenting and analyzing how “agents focus their attention on one equilibrium because it is more prominent or conspicuous than the others” (Young, 1993, p. 58). Thus, Peyton Young recognizes focal points as one of three broad equilibrium selection theories. The other two distinguished by Young are *introspection*, selecting some equilibria as a priori more reasonable than others (see, e.g., Harsanyi and Selten, 1988), and *dynamics* leading to the convergence of expectations through precedents (see, e.g., Crawford and Haller, 1990). With the literature centered on equilibria, the possibility that *non-equilibria* might be focal too, and might thus facilitate coordination, needs to be recognized and investigated. This paper’s contribution is a step to fill this gap, thus extending the literature on focal points.⁴

Our starting point will be a two-player matrix game with common interest and a multiplicity of equivalent Nash equilibria (NE). As explained in more detail in Section 2, the usual equilibrium selection criteria are not helpful in this game. We create, next, a salient payoff in this game by *reducing* one of the equilibrium payoffs, thus *eliminating* its pure strategy equilibrium property. Our goal is to test whether this alteration of the incentive structure, which confers salience to one payoff, helps individuals to coordinate, even if this payoff does *not* correspond to an equilibrium and it is Pareto *dominated* by the equilibrium payoffs.

In a number of laboratory experiments, we show that the introduction of such a non-equilibrium focal point (FP) may have a powerful effect on coordination success. Eventually the amount of coordination success increases by up to 251% compared to the benchmark game without FP. Our experimental evidence also shows that there are two ways in which the non-equilibrium FP helps to improve coordination. First, the non-equilibrium FP in itself becomes an attractor. We show that *removing* the pure strategy equilibrium property of an outcome can actually make it *more likely* to be played. Second, the FP acts as an equilibrium selection device, becoming a ‘stepping stone’ from which players jump to coordinate on a small subset of related NE that we will call the Associated Nash Equilibria (ANE). In addition, the experimental evidence indicates that the relative importance of the FP and the ANE depends on the amount of the focal payoff, and the effect of the FP becoming substantially stronger as the players are given more opportunities to reconsider and adjust their actions.

Following an analysis of the experimental data, we argue that in particular Variable Frame Theory (Bacharach, 1993; Bacharach and Bernasconi, 1997; Bacharach, 2006) may be helpful to understand the role that non-equilibrium focal points can play.

In summary, like Crawford et al. (2008), we explore in this paper the limits of focal points to enhance coordination and report the findings concerning the role of non-equilibrium focal points. Our results should help to recognize that non-equilibrium focal points may enhance coordination.

The remainder of this paper is organized as follows. In Section 2 we present the coordination game that we study. Section 3 explains the design of our experiments and proposes a set of hypotheses, while the experimental results are analyzed in Section 4. In Section 5 we make some theoretical observations, and Section 6 concludes.

2. Coordination game

In order to isolate the effect of a salient non-equilibrium payoff, we present a two-player game based on the payoff matrix in Fig. 1. The game is represented in normal form with player 1 choosing from the set of rows (r_1, \dots, r_n) and player 2 from the set of columns (c_1, \dots, c_n). To each pair (r_i, c_j) corresponds a payoff that is *equal for both players*, as indicated in the corresponding cells of Fig. 1. The fact that, no matter which outcome results, the players receive the same payoff means that we have a game of common interest, which allows us to focus on pure coordination problems. The game has thirty equivalent pure NE leading to a payoff of 100 for each player. Any miscoordination leads to a payoff of 0.

Notice that no equilibrium in this game is more reasonable than others, and neither payoff nor risk considerations distinguish any of these NE. All equilibria are efficient with the same payoffs, and all are equally risky. Hence, all players want to reach an equilibrium, but they are indifferent about which equilibrium they reach. Precedents are of little help since we consider essentially a sequence of one-shot games (random re-matching at each stage game) with very limited information feedback. We also made an effort to avoid any of the equilibria becoming more conspicuous through ‘label salience’.⁵ In fact, we strive to make all equilibria “nondescript” (in the sense of Bacharach, 1993) by eliminating any “labels”

² Besides Schelling (1960), see, e.g., Lewis (1969), Gauthier (1975) and Gilbert (1989).

³ See, e.g., Bacharach (1993), Mehta et al. (1994), Sugden (1995), Bacharach and Bernasconi (1997), Casajus (2001), Janssen (2001), Crawford and Iriberrri (2007b), Crawford et al. (2008), Bardsley et al. (2010), Isoni et al. (2010) and the references therein.

⁴ Brandts and MacLeod (1995), focussing on the issue of equilibrium refinements and without mentioning focal points, study the effect of public recommendations on equilibrium selection in a number of games. In the few cases that non-equilibrium play was recommended they observed no effect or little effect (if the recommended play led to the only fair and Pareto efficient outcome). See also Fehr et al. (2011) on sunspot-driven behavior.

⁵ ‘Label salience’ has received some attention in the literature as it has been observed that in pure coordination games with n equivalent equilibria, environmental signals, “labels” or “frames” (see Mehta et al., 1994, Binmore and Samuelson, 2006, or Crawford and Iriberrri, 2007b) become strategically

Our purpose in the paper is to analyze whether and how such a salient payoff that does not correspond to a pure-strategy equilibrium of the underlying static game can actually enhance coordination. To investigate these questions, we organized some laboratory experiments, which are explained in detail in the next section.

3. Experimental design and hypotheses

The basis for our experimental design is the game with a large number of equivalent NE presented in Fig. 1. To help avoid any of these NE being more prominent (as this would imply the occurrence of spurious focal points), we *add* a row and a column with only 0 payoffs at each side of the matrix.⁸ The largely empty center of the matrix is intended to serve the same purpose. Notice also that no equilibria lie on the diagonal, and that there are no labels distinguishing the actions. Once the instructions were read, subjects faced the payoff matrix on the screen for the remaining of the session. No natural language was used on the screen, thus limiting its possible confounding effect (see, e.g., Harrison, 2005). We use a number of variations of this game that consist basically of rotating or mirroring the payoff matrix, which implies that subjects in different sessions may face slightly differently configured payoff matrices.

As mentioned, we shave one of the 100 payoffs to make it focal, and use this focal payoff as a treatment variable to create four different treatments. The values for the focal payoffs used are 46, 87 and 99 points, respectively.⁹ In addition, we run a control treatment in which this payoff is 100 points, effectively eliminating the FP. Notice that making the focal payoff equal to 100 takes the game back to its original format as described in Fig. 1. This control treatment is meant to alert us of the existence of any nuisance attributes on which subjects could coordinate. Apart from the focal payoffs there is no difference between the treatments. The treatment variable allows us to investigate whether there is more or less coordination when the amount of the focal payoff approaches the amount of the equilibrium payoffs, and also to what extent this coordination takes place on the FP or on one of the NE.

We run eight sessions with the 46 treatment, and six sessions with each of the treatments 87, 99 and 100. At the start of each session, the subjects are split into two separate groups: row and column players. Each session consists of two parts, a one-shot game and a 50-period game, which are carried out sequentially.¹⁰ In the first part subjects play, with no time restriction, a one-shot version of the game by simply choosing one row (column). This is for a high payoff of €1.00 per 10 experimental payoff points. Once their decision has been made, they do not receive any feedback, and proceed straight to the second part. Notice that since we did not provide any feedback on the first part of the experiment, there is no endowment or learning effect related to the outcomes of that first part. The purpose of the first part is to test whether deductive selection principles would suffice for the players to coordinate successfully (cf. Schelling, 1960, p. 58). Moreover, the first part prepares the stage for the second one. Paying up to €10.00 for a single decision in a two-player coordination game, and giving the players all the time they wanted, is done to motivate the players to examine the characteristics of the game carefully right at the beginning.

Although one-shot games as such allow studying the role of focal points, in games with some complexity subjects may need more than one round to learn how to coordinate. That is where the second part comes in. In it, the subjects play 50 rounds of the same game, randomly re-matched in each round to one of the players in the other group. We choose a scheme with random re-matching in every round to abstract from reputation or signaling effects, allowing subjects to acquire experience while essentially mitigating the effect of creating a supergame. After each round in the second part, each player is told his payoff, but not the action chosen by the other player. Notice that this implies that a player can discover that the other player has chosen the focal action *only* if he has chosen the focal action himself as well. This implies that while the game has a history for each player, this history does not make some equilibria focal by precedent. It is a weak history that does not “de-symmetrize” the game. In this sense, our paper differs from a strand of literature where the repetition of the game with the same players – its history, so to speak – allows some equilibria to become focal points on which players can learn to coordinate (see Crawford and Haller, 1990; Blume and Gneezy, 2000; or Kuzmics et al., 2012).

All sessions took place in the computerized experimental laboratory (LeeX) at the Universitat Pompeu Fabra in Barcelona, were programmed using z-Tree (Fischbacher, 2007), and lasted about one hour each. The subjects were volunteer undergraduate students in Humanities, Commercial Studies, Business Administration, Law, Economics and Political Science. In each session there were 18 players simultaneously in the laboratory. Participants sat in front of personal computers, and could neither observe the screens of other players nor communicate with them. They were given written instructions, which contained information about the mechanics of the session and about the payoffs that depended on their decisions and the decisions of the subjects they were matched with.¹¹ In all treatments, the monetary reward was €1.00 per 10 experimental

⁸ Christenfeld (1995) studies the salience conferred to choices by placing them in first or last positions. Notice that the additional rows and columns imply that each of the four corners of the matrix constitutes a Nash equilibrium as well, with payoffs as low as those of the worst possible miscoordination, and that all rows (columns) including these empty ones are rationalizable. We will largely neglect these rows and columns with only 0 payoffs throughout the paper.

⁹ In one session (46.4), by mistake we actually used a focal payoff of 47 rather than 46.

¹⁰ Due to a computer glitch, one session (46.2) lasted only 45 periods. All analyses have been adjusted for this. For example, when considering the last ten periods we use periods 36–45 rather than periods 41–50 for this session.

¹¹ The instructions (in Spanish for the experiments) as well as some sample screenshots can be found in Appendix A. The complete set of matrices used is available from the authors.

Table 1
Earnings per player across treatments.

Treatment	Average earnings (€)	St. dev. earnings (€) ^a
46	8.39	3.09
87	12.27	2.82
99	14.82	8.10
100	5.32	1.82

^a St.dev. of session averages.

payoff points in the one-shot game, and €0.05 per 10 experimental payoff points in the 50-period game in the second part. On average, subjects made about €12 in addition to a show-up fee of €3.

We state the following hypotheses

Hypothesis 1. The behavior in treatment 100 (presented in Fig. 1) does not differ from what would be expected from random behavior. That is, the underlying game without the focal payoff does not contain any spurious focal points facilitating successful coordination, and hence it presents the players with a non-trivial coordination problem.

But when we create a non-equilibrium FP by shaving one of the 100 payoffs, this will enhance coordination success. Therefore:

Hypothesis 2. In the 46, 87 and 99 treatments we will observe more coordination success than in the 100 treatment.

As the attraction of the FP as well as the temptation to deviate from it may depend on the amount of the focal payoff:

Hypothesis 2a. The larger the focal payoff, the higher the coordination success.

However, since the FP is not an equilibrium, the row (column) player could deviate unilaterally to a specific row (column) to obtain (for herself and the other player) a payoff of 100. The two strategy profiles in which one player goes for the FP while the other chooses the best-reply to it both constitute a NE, and we call these two the Associated Nash Equilibria (ANE). Note that neither the FP nor the ANE would be favored over the other NE by standard payoff or risk considerations, and that the ANE are nonstrict just as the other NE.

Hypothesis 3. The coordination enhancement in the focal treatments mentioned in hypothesis 2 is achieved by coordinating either on the FP or on one of the ANE, and not by improved coordination on the other NE.

Hypothesis 3a. The larger the focal payoff, the more subjects make use of the FP, choosing either the focal action itself or its associated action.

Hypothesis 3b. In particular, the higher is the focal payoff the smaller is the proportion of players coordinating on the ANE rather than the FP.

However, if both column and row player were to deviate simultaneously from the FP, miscoordination with a payoff of zero would result. To decide whether to deviate or let the other player deviate, subjects may find inspiration in the history of the game as, e.g., a precedent of one ANE may break the symmetry between the two. Therefore:

Hypothesis 4. In each session, to the extent that the ANE are used successfully to coordinate, the relative shares of coordination success of the two ANE will not be 50–50, but one of the two will become prevalent.

The experimental evidence that follows will help to substantiate or dismiss these hypotheses.

4. Experimental results

In this section we present the experimental evidence supporting the two roles of non-equilibrium focal points, quantifying their effect, and thus providing a test of the hypotheses discussed above.

A first summary of the results appears in Table 1, which presents the average earnings (excluding the show-up fee of €3.00) per player for each treatment in both parts combined (one-shot plus 50 periods). In broad terms, Table 1 shows that coordination is enhanced by the introduction of a non-equilibrium FP, and that it increases with the amount of the focal payoff.

We now consider each of the hypotheses of Section 3, making a number of experimental observations concerning the actions and outcomes of the subjects in the one-shot as well as 50-period game. The experimental data at the level of the individual sessions are presented in Appendix B.

Hypothesis 1, claiming that one should expect random behavior in the 100 treatment (see Fig. 1) and that the underlying game poses a non-trivial coordination problem, is largely confirmed. The last entry of Fig. 2 shows the relative frequency of players choosing the most-frequently-chosen action in the 100 treatment, indicating that no action stood out. On average, the number of times the most chosen action was selected is 4.3 (24%), which is lower than the maximum to be expected

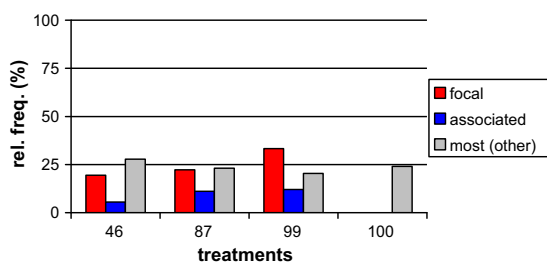


Fig. 2. One-shot: relative frequency of choices.

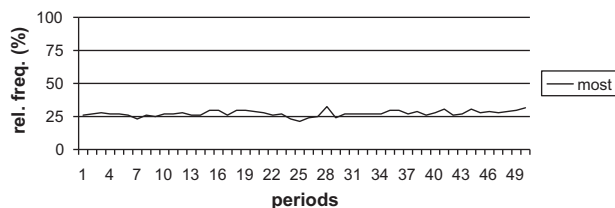


Fig. 3. 50 periods: most chosen action in "100" treatment.

if the players choose uniform randomly among the available actions.^{12,13} Hence, there appears to be little, if any, spurious focality attracting the players' choices. This is confirmed by the fact that the coordination success in the 100 treatment was not higher than what would be expected from random behavior.¹⁴ If players choose their actions uniform randomly, the expected payoff per player would be 13.3 in the 100 treatment,¹⁵ whereas the actual expected payoff is only slightly (but not significantly) higher, namely 14.2.¹⁶ Thus, in the one-shot game without a FP there is little coordination success, and there appears to be no spurious focality, due to nuisance attributes, facilitating coordination.

That there is little coordination success without a FP is confirmed in the 50-period game as well. Fig. 3 shows the relative frequency of the most chosen action (combining row and column players) in each period of the benchmark 100 treatment. We observe that the average frequency (4.9, or 27%, for periods 1–50) stays above the expected value in case of uniform random selection of actions,¹⁷ but this frequency did not increase significantly over time,¹⁸ and even in periods 41–50 it is just 5.2 (29%). Thus, it does not appear to be the case that there is one row/column that stands out in the underlying game without FP.

This can also be seen by the fact that the degree of success in achieving coordination remains relatively low in the 100 treatment. Fig. 4 shows the expected payoffs in the 50 periods of the 100 treatment (averaged over all players), which stays well below 20, increasing from 13.42 in periods 1–10 to 18.09 in periods 41–50 (see also Table 5 in Appendix B). Thus, the expected payoffs increase somewhat over time,¹⁹ and become higher than would be expected from random behavior,²⁰ but

¹² To find the most frequent (other) action, for each session we take the sum of the most frequently chosen column and the most frequently chosen row (in the focal treatments this is done excluding the focal and associated actions). Note that if nine row (column) players choose uniform randomly among 15 actions, then the probability that they will all choose a different action is less than 0.05. Hence, there will almost always be some action that, just by chance, attracts two or more choices. The expected maximum frequency observed with uniform random choices is 2.25 (25%). We approximated this expected maximum numerically, simulating the uniform random choice of players over rows and columns one million times.

¹³ We cannot reject the null hypothesis that the median of the most frequently chosen action equals the theoretical value implied by random choices (one-tailed binomial test). Here, as for almost all tests, we use the sessions as independent observations. We will indicate explicitly any deviation from this.

¹⁴ One-tailed binomial test. As mentioned in Section 2, we made an effort to eliminate any "labels" that could suggest "analogy", "geometric configuration" or any of the rules that subjects may apply to coordinate. That the effort was successful is confirmed by this result.

¹⁵ As mentioned above, we largely neglect the rows and columns with only 0 payoffs throughout the paper as those were only rarely chosen by the subjects. The fraction of empty border choices went down from 0.021 in the one-shot game to 0.001 in periods 26–50.

¹⁶ We compute these expected payoffs as follows. Consider a given period in a given session, and take, for example, one column player. The payoff for this column player depends on her action as well as the action of her opponent. Her expected payoff, then, is calculated by taking into account the actions of all the row players in the session in that period and the fact that she is equally likely to be matched to any of these row players. These expected payoffs can be averaged over players, periods and sessions. Our measure essentially corresponds to the coordination index used in [Bardsley et al. \(2010\)](#). We use these expected payoffs, rather than the actually realized payoffs, as a measure of coordination success, because the latter is sensitive to the actual realization of the random matching.

¹⁷ One-tailed binomial test, significant at 3.1% for periods 1–50.

¹⁸ Wilcoxon signed ranks test, comparing periods 41–50 with periods 1–10.

¹⁹ One-tailed Wilcoxon signed ranks test, comparing periods 41–50 with periods 1–10, significant at 3.1%.

²⁰ One-Endnote Texttailed binomial test, not significant for periods 1–10, but significant at 1.6% for periods 41–50 as well as periods 1–50.

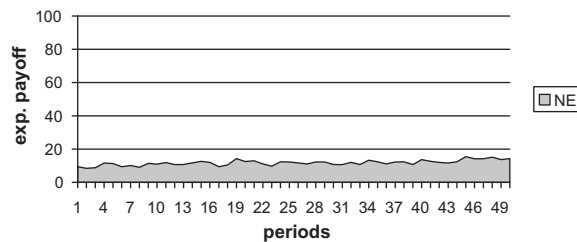


Fig. 4. 50 periods: expected payoffs in “100” treatment.

not substantially so.²¹ Thus, without a FP there is *little* coordination success, and there appears to be no spurious focality, due to nuisance attributes, facilitating coordination.

That there is no spurious FP was also confirmed in the focal treatments (46, 87 and 99), as there is no “other action” (excluding the focal and associated actions) that stands out,²² and the frequency of the most-frequently-chosen other action does not increase over time in any of the focal treatments.²³ Fig. 2 (see also Table 3 in Appendix B) shows the average number of times the most chosen other action was selected in the 46, 87 and 99 treatments of the one-shot game: 5.0, 4.2, and 3.7 respectively. Uniform random behavior would have led to an expected maximum of 4.3 (24%) in these treatments.²⁴ Fig. 5 shows the 50-period actions in all three focal treatments (see also Table 6 in Appendix B). As we see, the averages of the most frequently chosen other action *decrease* from period 1–10 to period 41–50 from 3.9 (22%) to 3.6 (20%), from 3.5 (19%) to 2.8 (16%), and from 3.2 (18%) to 1.9 (11%) for the 46, 87 and 99 treatments respectively. Thus, they remain at average values well below what would be expected from random behavior. In other words, apart from the focal and associated actions, in the focal treatments there is no single row or column that stands out more than what would be expected for uniform random behavior.

Hypothesis 2, maintaining that the FP would substantially enhance coordination success, is fully confirmed. Table 4 in Appendix B shows the expected payoffs for the one-shot game. We find somewhat higher expected payoffs in the focal treatments than in the 100 treatment,²⁵ but in the 46 and 87 treatments this is only marginally so: 16.9, 16.5 and 29.1 for the 46, 87 and 99 treatments respectively. In the 50-period game the coordination success is improved much more substantially. Fig. 6 shows the expected payoffs in the three focal treatments (averaged over all players), indicating with various colors the parts realized through the FP, the two ANE and any of the other NE. We distinguish the first and second ANE as follows. For each individual session we arbitrarily label the ANE that leads to the highest expected payoffs as “first”, and the other as “second”. We do this independently for the one-shot and the 50-period parts of the experiment. Notice that this implies that these labels do not necessarily refer to the same ANE in the one-shot and the 50-period part. Recall that if the players choose their actions randomly, the expected payoff is about 13, and that in the 100 treatment the average expected payoffs in periods 41–50 stay below 20 (18.1). In periods 41–50, in the 46 treatment the average expected payoffs reach 32.4, in the 87 treatment 55.2 and in the 99 treatment 63.5. In other words, in comparison with treatment 100, the expected payoffs in periods 41 to 50 were 1.8 times higher in the 46 treatment, 3.1 times higher in the 87 treatment, and 3.5 times higher in the 99 treatment. Thus, the expected payoffs in the three focal treatments were well *above* those in the non-focal treatment,²⁶ and these expected payoffs were increasing over time.²⁷

Hypothesis 2a, stating that the coordination success would increase with the amount of the focal payoff, was largely confirmed, as can be inferred from Fig. 6. Strictly speaking, on average this was correct in both the one-shot game (see Table 4 in Appendix

²¹ One-tailed binomial test. The one-sided 95% confidence interval for the median value of the expected payoffs in periods 41–50 ranges from 0 up to (but not incl.) 24.82. As Table 5 also shows, there are only two sessions where the average expected payoff in periods 41–50 exceeds 20, with the highest value being only 24.8. That these payoffs remain relatively low even after 50 periods seems related to the fact that, unlike the repeated coordination games in Crawford and Haller (1990), this multi-stage game is a complicated one as in each round the players are randomly and anonymously matched, and the only feedback a player gets concerns her own payoffs, without getting any direct information about the action of the other player.

²² In the one-shot game, we cannot reject the null hypothesis that the median of the most frequently chosen other action equals the theoretical value implied by random choices (one-tailed binomial test, using the individual actions as independent observations). In the 50-period game, in none of the focal treatments is the frequency of the most chosen other action higher than what would be expected from uniform random behavior (one-tailed binomial test, considering periods 1–10, 41–50 and 1–50).

²³ One-tailed Wilcoxon signed ranks test, comparing periods 41–50 with periods 1–10. In fact, in the 99 treatment the frequency over the most chosen other action decreases (significant at 1.6%).

²⁴ This is lower than for the 100 treatment, as the maximum is taken ignoring the frequencies for the focal and associated actions.

²⁵ One-tailed robust rank-order test, significant at 10%, 1% and 5% for the 46, 87, and 99 treatments respectively.

²⁶ One-tailed robust rank-order test, significant at 10, 1 and less than 1% for the 46, 87 and 99 treatments respectively for periods 1–50.

²⁷ One-tailed Wilcoxon signed ranks test, significant at 0.4, 1.6, and 1.6% for the 46, 87 and 99 treatments respectively for periods 41–50 against periods 1–10.

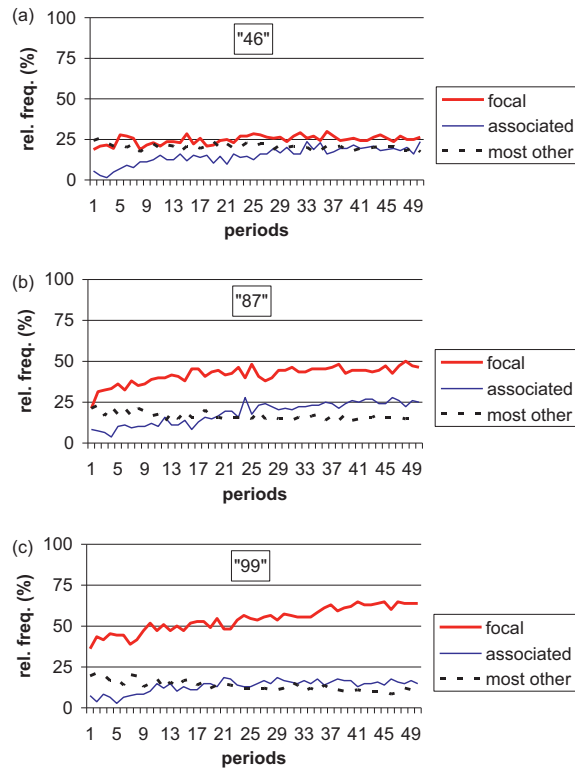


Fig. 5. (a) 50 periods: actions in “46” treatment, (b) 50 periods: actions in “87” treatment, and (c) 50 periods: actions in “99” treatment.

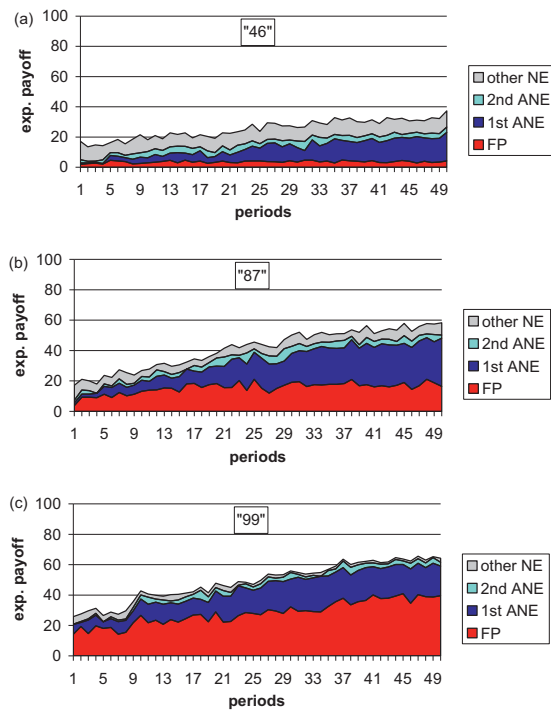


Fig. 6. (a) 50 periods: expected payoffs in “46” treatment, (b) 50 periods: expected payoffs in “87” treatment and (c) 50 periods: expected payoffs in “99” treatment.

B)²⁸ and the 50-period game (see Table 7 in Appendix B and Fig. 6),²⁹ but some of the differences were not significant. Perhaps Schelling's example of the painted line on the road is a good description of what is happening. Drivers have reasons to obey the line even if the state does not provide enforcement of it. But if the state draws the line too far over, drivers will ignore the line. One could perhaps argue that the probability that a certain convention helps coordination increases with "the strength of its case", or, in our terms, the amount of the FP's payoff.

Hypothesis 3, asserting that the coordination enhancement in the focal treatments is due to the FP, and not to improved coordination on the other NE, is essentially confirmed.

In each of the focal treatments subjects make use of the FP (choosing either the focal or its associated action) more than what should be expected from random behavior, in the one-shot game³⁰ as well as the 50-period game,³¹ and the frequency is increasing over time in all focal treatments.³² Aggregating focal and associated choices, the average number of times that the subjects made use of the FP in the one-shot game is 8.2 (46%) in the 99 treatment (see Fig. 2), which falls to 6.0 (33%) in the 87 treatment, and to 4.5 (25%) in the 46 treatment. Fig. 5 shows the frequencies of the actions in the 50-period game. As we see, the frequency with which the focal action itself is chosen increases from periods 1–10 to periods 41–50 from 4.0 to 4.6, from 6.0 to 8.2, and from 7.8 to 11.5 in the 46, 87 and 99 treatments respectively. Similarly, these frequencies for the associated actions increase from 1.3 to 3.5, from 1.6 to 4.6, and from 1.3 to 2.7. As a result, the number of subjects making use of the FP (choosing either the focal or associated action) becomes eventually 45%, 71% and 79% in periods 41–50 in the 46, 87 and 99 treatments respectively.

Related to this, the expected payoffs due to the FP as well as the ANE increase over time. Fig. 6 (see also Table 7 in Appendix B) shows the expected payoffs for the 50-period game. Comparing periods 1–10 with periods 41–50, the expected payoffs realized through the FP increase from 2.9 to 3.6 in the 46 treatment, from 9.9 to 17.3 in the 87 treatment, and from 18.3 to 38.8 in the 99 treatment. Similarly, the payoffs generated by the ANE increase from 4.6 to 18.8, from 5.7 to 28.7, and from 6.9 to 21.4 in the 46, 87 and 99 treatments respectively.³³ This leads to an increase in the share in the expected payoffs realized through the ANE alone from 28 to 58% in the 46 treatment, from 28 to 57% in the 87 treatment, and from 26 to 37% in the 99 treatment from periods 1–10 to periods 41–50. The combined effect of this increased importance of the FP and the ANE is that the share in the average expected payoffs obtained through either the FP or the ANE reaches 69%, 88% and 98% for the 46, 87 and 99 treatments respectively in periods 41–50.

What is more, the expected payoffs realized through any of the other NE are not higher in the focal treatments than in the non-focal 100 treatment, both in the one-shot game³⁴ and the 50-period game,³⁵ and these expected payoffs do not increase over time.³⁶ Fig. 6 (see also Table 7 in Appendix B) shows that the average expected payoffs per period generated by the other NE over the 50 periods are 9.4 (evolving from 9.0 in periods 1–10 to 10.2 in periods 41–50), 5.9 (from 6.3 to 6.7) and 2.4 (from 4.2 to 1.5) for the 46, 87 and 99 treatments respectively, whereas they are 15.7 in the 100 treatment (see Fig. 4).

Hypotheses 3a and 3b, claiming that the FP is more frequently used the larger the focal payoff, and that the ANE become more important relative to the FP the lower the focal payoff, are confirmed. As we can see in Figs. 2 and 5, subjects tend to use the FP more (choosing either the focal or associated action) the larger the focal payoff, in the one-shot game³⁷ as well as the 50-period game.³⁸ As to the relative importance of the ANE versus the FP in achieving coordination success, in the

²⁸ The expected payoffs are higher in the 99 treatment than in the 87 treatment and than in the 46 treatment (one-tailed robust rank-order test, significant at 5 and 10% respectively), while there is no significant difference between the 87 and the 46 treatments.

²⁹ The expected payoffs in the 87 and 99 treatments are higher than in the 46 treatment (one-tailed robust rank-order test, significant at 5 and 1% respectively for periods 1–50), while there is no significant difference between the 87 and 99 treatments.

³⁰ One-tailed binomial test based on individual actions, significant at any conventional level for each treatment.

³¹ One-tailed Binomial test. In the 87 and 99 treatments this is significant at 1.6% for periods 1–10, 41–50 as well as 1–50. In the 46 treatment, this is true as well for periods 1–10 (significant at 3.5%), but not for periods 41–50, or 1–50.

³² One-tailed Wilcoxon signed ranks test, comparing periods 41–50 with periods 1–10, significant at 5.5, 1.6 and 3.1% for the 46, 87 and 99 treatments respectively. Since 18 players per session choose among 15 actions (ignoring the empty rows and columns), uniform random behavior would lead to an expected frequency of 1.2 players choosing any specific action.

³³ One-tailed Wilcoxon signed ranks test comparing periods 41–50 with periods 1–10. For the expected payoffs realized through the FP, in the 87 and 99 treatments this is significant at 3.1 and 4.7% respectively, while there is no significant difference in the 46 treatment. For the expected payoffs realized through the ANE this is significant at 5.5, 1.6 and 1.6% for the 46, 87 and 99 treatments respectively. And for the expected payoffs realized through either the FP or the ANE this is significant at 5.5, 1.6 and 1.6% for the 46, 87 and 99 treatments respectively.

³⁴ One-tailed robust rank-order test. In fact, in the 87 and 99 treatments the expected payoff realized through the other NE is significantly lower than in the 100 treatment (both at 1%). Similar observations can be made if we average over all the (other) NE.

³⁵ The expected payoffs realized through any of the other NE are lower in the focal treatments than in the non-focal 100 treatment. One-tailed robust rank-order test, significant at 2.5% for the 46 treatment and less than 1% for the 87 and 99 treatments for periods 1–50. Similar observations can be made if we average over all the (other) NE.

³⁶ One-tailed Wilcoxon signed ranks test, comparing periods 41–50 with periods 1–10. In fact, in the 99 treatment the expected payoffs generated by the other NE decrease over time (significant at 6.3%).

³⁷ The one-tailed χ^2 test based on individual actions shows that this is significantly more in the 99 treatment than in the 87 or 46 treatment (at 10 and 1% respectively), while there is no significant difference between the 87 and 46 treatments. Essentially the same applies to the focal action as such (with the significance levels at 10 and 2% respectively), while there is no significant difference between the treatments as far as the associated actions are concerned.

³⁸ The one-tailed robust rank-order test shows that this is significantly more in the 87 and in the 99 treatments than in the 46 treatment (at 10 and 5% respectively) for periods 1–50, while there is no significant difference between the 99 and 87 treatments.

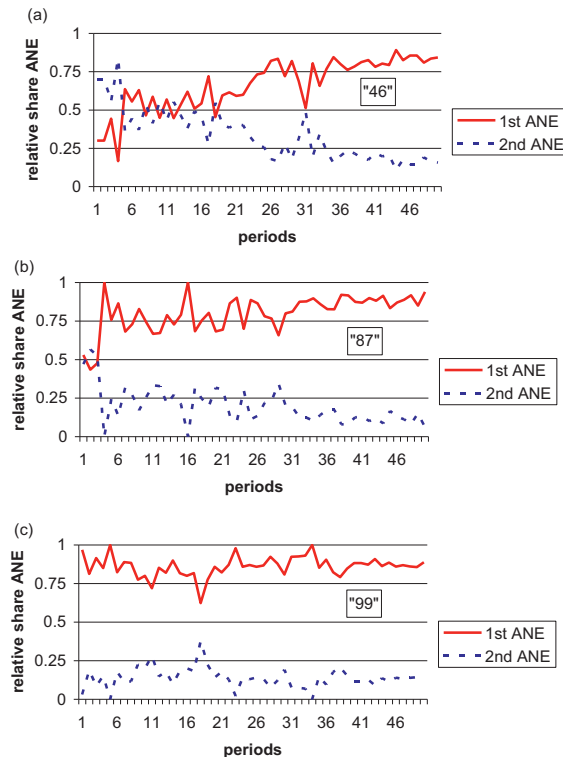


Fig. 7. (a) 50 periods: relative shares of expected payoffs for each ANE in “46” treatment, (b) 50 periods: relative shares of expected payoffs for each ANE in “87” treatment and (c) 50 periods: relative shares of expected payoffs for each ANE in “99” treatment.

one-shot game, there is no significant difference between the treatments.³⁹ But in the 50-period game, the importance of the ANE increases over time in the 46 and 87 treatments,⁴⁰ and related to this, the lower the focal payoff, the more important the ANE become relative to the FP.⁴¹ As we can see in Fig. 6, in the 46 treatment the expected payoffs obtained through the other NE and through the FP stay relatively constant. But building on the platform provided by the FP, the success of the ANE increases dramatically in the 46 treatment. Eventually, in periods 41–50, the ANE account for 84% of the payoffs related to the FP. A similar pattern can be observed in the 87 treatment, where eventually the ANE lead to 64% of the expected payoffs related to the FP. Finally, in the 99 treatment, it is the FP that leads early on to considerable payoffs, while the ANE become more and more important, providing eventually 37% of the expected payoffs related to the FP.

Hypothesis 4, stating that, to the extent that the ANE are used successfully to coordinate, one of the two ANE will become dominant, is clearly confirmed on average, although statistically the effect appears relatively weak.⁴² Fig. 7 shows the relative share in the expected payoffs of each of the ANE for each treatment, where the relative share is the expected payoffs realized through one of the ANE divided by the expected payoffs from the two ANE combined (see also Table 8 in Appendix B). As we see, in all focal treatments one ANE ends up being chosen much more often than the other. Of the expected payoffs realized through the two ANE, the relative shares realized through the first ANE increase over time from 51 to 83%, from 72 to 89%, and from 86 to 87% in the 46, 87 and 99 treatments respectively, although statistically the increase is only significant in the 87 treatment.⁴³ In other words, on average it seems that row players learn not to deviate from the FP when column players tend to deviate, and vice versa.

³⁹ One-tailed robust rank-order test.

⁴⁰ One-tailed Wilcoxon signed ranks test, significant at 9.8 and 1.6% respectively for periods 41–50 against periods 1–10, while there is no significant difference in the 99 treatment.

⁴¹ The higher the focal payoff, the more often the focal action itself is chosen (one-tailed robust rank-order test for periods 1–50, significant at 5% for “87” against “46”, 1% for “99” against “46”, and 2.5% for “99” against “87”), while there is no such significant difference between the various focal treatments as far as the frequency of the associated actions is concerned. In fact, in periods 41–50, the associated action is chosen significantly less in the 99 than in the 87 treatment (one-tailed robust rank-order at 5%). The lower the focal payoff, the higher the share of the expected payoffs realized through the ANE relative to the FP (one-tailed robust rank-order test, significant in periods 1–50 at 1, less than 1 and 1% for “87” against “46”, “99” against “46” and “99” against “87” respectively).

⁴² The one-sided 95% confidence interval for the median value of the relative share of the expected payoffs realized through the first ANE in periods 41–50 ranges from 1.00 down to 0.48, from 1.00 down to 0.55, and from 1.00 down to 0.43 in the 46, 87 and 99 treatments respectively. One-tailed binomial test.

⁴³ One-tailed Wilcoxon signed ranks test, comparing periods 41–50 with periods 1–10, significant at 7.8% in the 87 treatment.

Table 2

50 periods: conditional relative transition frequencies in “46”, “87” and “99” treatments.

Treatment	Having played focal action in previous period, they choose action				Having played associated action in previous period, they choose action				Having played other action in previous period, they choose action				
	Outcome previous period	Focal	Assoc.	Other	Outcome previous period	Focal	Assoc.	Other	Outcome previous period	Focal	Assoc.	Other (same)	Other (different)
46	FP	0.80	0.18	0.02									
	ANE	0.96	0.03	0.01	(A)NE	0.05	0.89	0.06	NE	0.01	0.03	0.68	0.28
	0	0.68	0.10	0.22	0	0.26	0.48	0.26	0	0.06	0.05	0.30	0.60
87	FP	0.90	0.09	0.01									
	ANE	0.97	0.02	0.01	(A)NE	0.04	0.93	0.04	NE	0.02	0.02	0.63	0.33
	0	0.85	0.07	0.08	0	0.26	0.56	0.18	0	0.07	0.04	0.27	0.62
99	FP	0.95	0.04	0.01									
	ANE	0.97	0.02	0.01	(A)NE	0.07	0.85	0.08	NE	0.01	0.02	0.61	0.35
	0	0.90	0.03	0.07	0	0.30	0.53	0.17	0	0.07	0.05	0.26	0.62

Some further insights into the role of the FP can be derived from an analysis of the dynamics of the subjects' choice behavior. We analyze all 17,550 instances at which a subject, having chosen either the focal, associated or some other action in the previous period, and having observed the outcome (either the FP, an ANE, some other NE, or a payoff of 0), decides whether to go for a focal action, an associated action or some other action (and, where relevant, whether this was the same or a different other action). Table 2 gives for each focal treatment the conditional relative transition frequencies. The three main rows concern the three focal treatments (46, 87 and 99). The three main groups of columns concern the action chosen in the previous period (focal, associated or other). Let us focus first on the first main row and first main column, i.e., subjects in treatment 46 who had chosen the focal action in the previous period. For these subjects there are three possible outcomes in the previous period: the FP, an ANE or a payoff of zero. For each of these outcomes, Table 2 specifies the relative frequencies with which these subjects choose the focal, associated or some other action in the next period. Similarly, we can find the corresponding conditional transition frequencies for the other two possible actions of the previous period (i.e., having played the associated or some other action) in the other two main groups of columns. And the second and third main row give all these probabilities for the 87 and 99 treatments.

Note that subjects who had chosen the focal action in the previous period did not have any reason to choose a different action if the outcome had been the ANE, whereas if the outcome had been the FP, they could have done better by deviating to the associated action. But deviating to the associated action is risky because subjects do not want to deviate both at the same time, as this would lead to a zero payoff. This suggests one should expect many subjects to stay put when they experienced a positive payoff after having chosen the focal action, and especially so after experiencing the ANE outcome. Table 2 shows that this is indeed the case. Subjects who had chosen the focal action in the previous period are most likely to do so again after any possible outcome (at least 68% of the times), and the probability that they switch to some other action is close to zero (at most 2%) if they experienced a FP or ANE outcome in the previous period. Note that, in line with Hypothesis 3b, having experienced the FP outcome, the chance that subjects choose once more the focal action increases with the amount of the focal payoff.⁴⁴ And subjects who played the focal action and experienced an ANE are more likely to repeat the focal action than those who had experienced the FP outcome.⁴⁵

Subjects who had chosen the associated action in the previous period are most likely to do so again after any possible outcome, but especially if they had been successful in the previous period.

Subjects who had chosen some other action in the previous period are very unlikely to choose the focal or associated action, but are slightly more likely to do so if the outcome in the previous period was unsuccessful. Most likely they will choose again some other action. If they had been successful in the previous period, they are most likely to stick to the same other action, otherwise they are most likely to switch to a different other action.

In summary, to conclude our analysis of the experimental results, the non-equilibrium focal points turn out to have a powerful effect on the amount of coordination success. This effect increases over time, as players get more and more opportunities to reconsider their actions. And the effect depends in a clear (positive) way on the level of the focal payoff. The improved coordination success is achieved in two ways. First, coordination success on the FP itself increases. Subsequently we see increased play of the associated actions, as the players seem to use coordination on the FP as a platform to deviate to one of the ANE, with the relative importance of the focal and associated actions depending on the level of the focal payoff. Thus, effectively the non-equilibrium FP acts both as an attractor and as an equilibrium selection device, selecting the ANE out of a much larger set of NE.

⁴⁴ The conditional frequencies of staying put at the focal action in the 87 and 99 treatments are higher than in the 46 treatment (one-tailed robust rank-order test, significant at 1%), while they are higher in treatment 99 than in treatment 87 (at 5% significance).

⁴⁵ One-tailed Wilcoxon signed ranks test, significant at 8.6 and 7.8% in the 46 and 99 treatments, while not significant in the 87 treatment.

	column with focal point	any column with 2 NE	any column
row with focal point	87	7.14	12.47
any row with 2 NE	7.14	13.78	13.33
any row	12.47	13.33	13.28

Fig. 8. Transformed game, Description 1.

5. Some theoretical observations

In this section we will consider a number of theories that appear compatible with the experimental data and that help to understand the observed roles of non-equilibrium focal points in enhancing coordination. Clearly, what follows is not a test of the theories described using our experimental results. Had we successfully done this, it would have helped to export the theories to novel situations. But recent attempts to investigate the robustness of focal points to variations of coordination games, see in particular [Bardsley et al. \(2010\)](#) and [Crawford et al. \(2008\)](#), cast a pessimistic view on the possibility of finding any simple, unified theory of focal points. What we hope to provide with this brief survey are some elements that may help to approximate the various modes of reasoning of the many subjects who managed to coordinate using the non-equilibrium FP.

5.1. Variable frame theory

As we mentioned in the introduction, the literature on focal points consists of modeling the coordination on a focal equilibrium. The idea of players choosing one equilibrium that stands out has been formalized in various ways (see, e.g., [Crawford and Haller, 1990](#); [Sugden, 1995](#); [Bacharach and Bernasconi, 1997](#); [Janssen, 2001](#); [Casajus, 2001](#); [Alós-Ferrer and Kuzmics, 2013](#)). As we will show, although thus far it has been applied only to equilibrium focal points, Variable Frame Theory ([Bacharach, 1993](#); [Bacharach and Bernasconi, 1997](#); [Bacharach, 2006](#); [Blume and Gneezy, 2010](#)), in particular, helps to elucidate how a FP may play an important role in achieving coordination, *even when this FP is not itself an equilibrium*.

The idea of VFT is that players perceive the game to be played within a certain frame, where a frame is a set of filters or schema of interpretation to make sense of their environment. Thus, players do not choose actions in the underlying game as such but instead, depending on the frame in which they perceive the game, classify actions into categories (partitioning the strategy set), and then choose a category. If there is more than one option within a certain category, then, following the Principle of Insufficient Reason, a player randomly picks any of these options. Note that this Principle does not mean that a player cannot distinguish these options as such, but rather that she has insufficient reason to pick one rather than another. If we apply the principles of VFT to our coordination game of [Fig. 1](#), where one of the 100 payoffs is replaced by an 87 payoff, the game can be transformed into the description of the game presented in [Fig. 8](#) (Description 1). Recognizing the characteristic ‘focality’, the players’ strategies can be categorized into ‘choose a row (column) with the FP’ and ‘choose any of the other rows (columns) that include two NE’. In addition, a player can decide to ignore the characteristic ‘focality’, leading to the category ‘choose just any of the rows (columns)’. [Fig. 8](#) shows two NE in the transformed game, and the one with both players choosing the focal action is the Pareto superior one, and hence VFT solution of the game. VFT uses this kind of transformation to formalize the fact that one NE is more focal than other NE. But note that there is a noteworthy new element here. The FP was *not* a NE in the underlying game, yet it emerges as the unique Pareto superior NE when looked at from the perspective of VFT.

However, once the FP presents itself as a solution of the transformed game in Description 1, of all the rows (columns) with two NE, one particular row (column) becomes a different category. That is, once a player recognizes the importance of the FP (as in Description 1), a player has sufficient reason to distinguish the row (column) that contains the Associated Nash Equilibrium (ANE) from the other rows (columns) with two NE, because these ANE can be reached by a unilateral deviation from the FP. This further transformation of the underlying game leads to Description 2 in [Fig. 9](#). We distinguish two types of NE in Description 2. First, the superior NE of both players going for the FP in Description 1 is replaced by two equilibria, with one player sticking to the focal action while the other deviates to the associated action, leading to the ANE outcome. There is also a NE in Description 2 in which both players going for any of the other NE of the underlying game, but this NE is inferior to the two ANE. Thus, by this procedure, we have formalized the equilibrium selection, since two of the twenty-nine NE of the underlying game (i.e., the two ANE) are singled out as the solutions for this game. From a semantic point of view, one could argue that these ANE become actually focal themselves through the introduction of the non-equilibrium focal point. Note that this focality would not be intrinsic but rather *derived* salience, as these ANE are prominent *only* through their property of being a better-response to the focal action, which in turn is related to the non-equilibrium character of the FP.

	column with focal point	associated column with 2 NE	other column with 2 NE	any column with 2 NE	any column
row with focal point	87	100	0	7.14	12.47
associated row with 2 NE	100	0	7.69	7.14	13.33
other row with 2 NE	0	7.69	14.79	14.29	13.33
any row with 2 NE	7.14	7.14	14.29	13.78	13.33
any row	12.47	13.33	13.33	13.33	13.28

Fig. 9. Transformed game, Description 2.

The application of the VFT principles can be summarized as follows:

Theoretical observation 1: A first step of following through the principles of VFT implies that a non-equilibrium FP can become the unique Pareto superior Nash equilibrium.

Theoretical observation 2: The principles of VFT imply that a non-equilibrium FP can transform a small subset of NE, the ANE, into Pareto superior Nash equilibria.

Thus, VFT seems a parsimonious theory. Based on the Principle of Insufficient Reason, it first explains the role played by the non-equilibrium FP, and then singles out the ANE as solutions to the game because they are Pareto superior equilibria of the transformed game.⁴⁶ As explained, Description 2 pre-requires Description 1. That is, the distinction of the ANE builds upon the distinction of the FP. We stop after two steps because there is no strict reason to deviate from an NE of the underlying game imply the same payoff. This leaves the question of which is the most relevant VFT description level for the players. As we saw in the previous section, this is an *empirical* question that depends in part on the relative payoffs involved. The players' behavior over time seems consistent with learning or adapting to the various description levels of VFT. Initially, in the one-shot game (see Fig. 2), the effect of the FP was very modest, and behavior was not much different from random behavior. However, over time we saw an increased attention for the FP, followed by a move toward the ANE (see Figs. 5 and 6). Clearly, the players' perception of the game as well as their beliefs about other players' perception change as they keep playing and learning the game.

5.2. Mixed-strategies equilibrium play

Instead of pure strategies, players might use mixed strategies. If we consider the game in Fig. 1 (Description 0), we see that there exists an infinite number of mixed NE, involving any number of rows (columns), and in particular, some involving the focal action with strictly positive probability. Hence, observing play of the focal action could be compatible with Nash equilibrium play.

But there are limits to how much mixed NE might help to understand the observed behavior and actually explain the role of non-equilibrium focal points. A first limitation with mixed NE is the large number of such equilibria. This raises questions about the plausibility for players to coordinate successfully on any of them. A second limitation with the mixed NE is that they do not single out the FP or the ANE. In fact, most of the mixed equilibria involve neither FP nor ANE, and the mixed NE involving the FP tend to be Pareto inferior. That is, one would still need some explanation for the role of the FP and the ANE as such.

If we assume bounded rationality by allowing the players to make mistakes, essentially the same arguments (existence as well as limitations) apply to the possibility of Quantal Response Equilibria (see, e.g., McKelvey and Palfrey, 1995; Goeree et al., 2008). And the same arguments also apply if we consider subgame perfect Nash equilibria of the multi-stage game of our 50-round experiment.

Interestingly, one of the mixed NE of the game in Fig. 1 (Description 0) involves only the focal and associated actions with strictly positive probability. In this mixed NE, the probability of playing the focal action is 0.65, 0.89, and 0.99 for the 46, 87 and 99 treatments respectively.

⁴⁶ The organization of the strategy space into frames seems reminiscent of the categorization studied by Fryer and Jackson (2008). See also Mullainathan (2002), and Gilboa and Schmeidler (1995). Note that although the analysis of 'attainable strategies' in Crawford and Haller (1990) seems to be very much in the same spirit as VFT, it does not allow to classify the strategies into the categories discussed as none of the strategies are 'symmetric', essentially because each of the strategies in our game is related in its own special way to the focal strategy.

Theoretical observation 3: There exists a mixed equilibrium in which only the focal and associated action are played with strictly positive probability, with the probability of the focal action increasing with the amount of the focal payoff.

Thus, if players choose the mixed NE involving only focal and associated actions, then according to this equilibrium theory, it would be more likely to observe the FP rather than the ANE as the focal payoff increases. Considering only the choices of focal and associated actions, the relative frequencies of the focal actions in periods 41–50 are 45%, 71% and 79% for the 46, 87 and 99 treatments respectively (see Table 5 in Appendix B). Notice that the differences between the treatments have the sign predicted by the mixed NE. What remains to be explained, then, is why players would focus their attention on the focal and associated actions only. This is exactly what is explained by VFT.

5.3. Team reasoning

Another possibility, also relying on bounded rationality, is team reasoning (Sugden, 1993; Bacharach, 2006). Suppose a player considers each action, presuming that his opponent will choose the same action, and then takes the one with the highest payoff. One difficulty for the players to apply this kind of team reasoning is that our game is presented in a highly asymmetric way (see Fig. 1), even though the underlying game is symmetric. The players would need (independently) to carry out some extensive re-ordering of rows and of columns to see which actions for the row and column player can be considered to be the same. They would find that the highest payoff resulting from both players choosing the same action would be corresponding to one of the NE. However, once we assume the frames of VFT, we get a symmetric description of the game (see Figs. 8 and 9), and team reasoning can be applied more effectively. In fact it provides a justification for the choice of the payoff dominant NE in the transformed games of Descriptions 1 and 2. At both Description levels, this team reasoning would explain the FP as the outcome. But note that, to do so, it needs to assume already the effect of focality being perceived through the frames of VFT. What is more, strict team reasoning cannot explain the ANE as an outcome of this game.

5.4. Level- k reasoning

Non-equilibrium models of behavior based on limited levels of reasoning of the players have been presented and used by Nagel (1995), Stahl and Wilson (1995), Bosch-Domènech et al. (2002), Camerer et al. (2004), and Crawford and Iriberry (2007a), among others. These level-of-reasoning models are primarily about the players' initial responses. Level- k reasoning requires a starting point, a Level-0 play. A player of type L0 is the anchoring type, representing L1 players' beliefs about other players' instinctive reaction to the game, and is as such the key to the explanatory power of level- k models. The first question, then, is whether this hypothetical non-strategic type is indifferent or either positively or negatively influenced by the FP. As our focal payoff is lower than any of the NE, let us first assume this would imply a *negative* effect for the focal row (column), albeit diminishing as the payoff differential decreases. With one FP payoff or one payoff of 100 as possible outcomes for the focal action, and two possibilities of a payoff of 100 for any other action, in the absence of any strategic reasoning about an opponent one should assume that the probability p for the focal action is smaller than $1/15$, with the remaining probabilities spread evenly over the other fourteen available actions. Given these probabilities for L0 players, an L1 player would exclude two actions: First, the row (column) with the FP (because of the lower payoff for the FP and because of the smaller probability that his L0 opponent chooses her focal action), and second, the associated row (because his L0 opponent has a smaller probability for choosing her focal action). Excluding those two actions, all remaining actions have the same expected payoff. Hence, anything goes for those remaining thirteen actions. Subsequent levels of reasoning, at each step excluding one more action for each player, will eventually single out one of the NE. This may look like an equilibrium selection theory, but usually level- k reasoning is not used to justify equilibrium behavior as such and, empirically, levels of reasoning above 2 or 3 are uncommon. For lower levels of reasoning, using the degrees of freedom provided by indifference, it seems that almost anything goes. But the theory provides one clear prediction: both the focal and the associated action are *excluded* for any reasoning level beyond L0.

If, on the other hand, we were to assume a *positive* effect of the lower focal payoff, then L0 players would choose the focal action with some probability p greater than that for each of the remaining options. This means that L1 players would choose the associated row (column) with probability one. L2 players optimizing against L1 players, then, would go either for the focal action, or for a further best-response deviation from the ANE. This means that L3 players choosing a best-response to L2 players will return with probability one to the associated action as long as L2 players are seen to choose the focal action with any strictly positive probability. Further levels of reasoning follow this pattern of oscillation between focal and associated actions. Hence, if we assumed a positive instinctive effect of the *lower* focal payoff and we have a mixture of players at various levels, we could observe a mixture of focal and associated actions as well as other actions.

Note that in both cases (with either a negative or a positive effect of the FP), what is not explained by level- k reasoning models is the role of the FP, as the effect of the FP is simply taken as an arbitrary starting point.

5.5. In summary

VFT seems appealing as it explains the role of focality as such, and it can explain FP as well as ANE outcomes. What is more, the clear evolution of our experimental data seems consistent with the idea that players learn descriptions of higher

VFT levels as they play. That said, the other theories might be helpful to understand the role of non-equilibrium focal points too, in particular in combination with VFT, with the description levels 0, 1 and 2 yielded by VFT modeling the narrowing down of the players' attention to the focal and associated actions.

6. Conclusion

In this paper we consider a pure coordination game with a large number of equivalent NE, in which players have a difficult time coordinating. We show that if one of the equilibrium payoffs is made salient by shaving its value, thus eliminating its (pure-strategy) equilibrium properties in the underlying static game, it induces a larger degree of coordination. This is achieved in two steps. Players are first attracted to the focal action and, subsequently, may move to the associated actions, leading to the selection of a small subset of equilibria (ANE) out of a much larger set of equilibria. We observe that whether individuals stay put at the FP or deviate to one of the ANE depends on the payoff difference between the FP and the ANE.

The primary goal of this paper was to extend the FP literature by demonstrating the coordinating power of non-equilibrium focal points, both in attracting choices to themselves and in helping with the selection of an equilibrium. Reporting the stylized facts of our laboratory experiment should create awareness of a wider role for focal points, and help to identify the coordinating role of non-equilibrium focal points also in other settings.

Appendix A. Supplementary data

Instructions used in the experiment and supplementary data associated with this article can be found in Appendices A and B, in the online version, at <http://dx.doi.org/10.1016/j.jebo.2013.07.014>.

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Appendix A: Instructions and screenshots

We first present the English version of the instructions. In the experiments we actually used a Spanish version (available from the authors upon request).

Introduction

- We will do a series of 2 experiments. The specific instructions for the second experiment will follow later, once the first experiment is over. The instructions for these experiments are simple, and if you pay attention, you can gain some money that will be paid to you at the end of the series of experiments. From now on till the end of this experimental session you are not allowed to communicate with each other. If you have a question, please raise your hand.

Experiment I

- There will be two equally sized groups of players (“*yellow*” and “*blue*”). Allocation to these groups will take place randomly and anonymously. Therefore, apart from knowing your own color, you will never be told who was in which group.
- The first experiment involves playing a game with another participant. To make pairs of players, each “*yellow*” player will be randomly and anonymously matched to a “*blue*” player. Note that you will never know the identity of the person you are matched with, nor will (s)he be aware of yours. Nor will you be told the payoffs of any other person.
- You will get a payoff table showing the various payoff levels that you and the other player can each obtain depending upon the action chosen by you and the other player. All players in this experiment will face the same payoff table.
- The actions for the “*blue*” players are shown on the rows, and for the “*yellow*” players on the columns.
- In each round, all players simultaneously choose an action by clicking on the row (for “*blue*” players) or column (“*yellow*” players) button of their choice.
- The resulting payoffs for both players are shown within the body of the table by the intersection of the actions chosen. The numbers in the table represent the payoffs in points.
- The points earned in this experiment I will be exchanged into Euros at the end of the session using the following exchange rate: Euro 1.00 per 10 points.
- There is no time limit for your decisions.
- We will do, next, a simple exercise with a fake payoff table to verify that you understand how the payoff table works.

Exercise with fake payoff table

Look at the payoff table below, and complete the following sentence:

If the white player chooses row X and the grey player chooses column Y,
then the payoff to the white player is: ,
and the payoff to the grey player is:

	Y			
	1	2	3	4
	5	6	7	8
X	9	10	11	12
	13	14	15	16

Experiment II

- We will continue with the same “yellow” and “blue” groups as in experiment I. The colors assigned will remain the same throughout the experiment.
- In this second experiment, you will play 50 rounds of the same basic two-player game as in experiment I. This time you will be randomly and anonymously matched to a player in the other group in each round *anew*.
- As in the previous experiment, you will get a payoff table showing the various payoff levels that you and the other player can each obtain depending upon the action chosen by you and the other player. All players in this experiment will face the same payoff table.
- As in the previous experiment, the actions for the “blue” players are shown on the rows, while the actions for the “yellow” players are shown on the columns.
- As in the previous experiment, in each round, all players simultaneously choose an action by clicking on the row (for “blue” players) or column (“yellow” players) button of their choice.
- As in the previous experiment, the resulting payoffs for both players are shown within the body of the table by the intersection of the actions chosen. The numbers in the table represent payoffs in points.
- The points earned in experiment II will be exchanged into Euros at the end of the session using the following exchange rate: Euro 0.05 per 10 points. Note that the exchange rate is different in this experiment.
- There is no time limit for your decisions.
- After each round, you will be told the points you earned in that round, as well as the total amount of points earned in experiment II up to that round.

Next, we show two sample screenshots for the one-shot game. Figure 10.a shows the screen of a (blue) row player, while Figure 10.b shows the screen of the corresponding (yellow) column player.

Appendix B: Experimental data at session level

Table 3 shows the numbers of choices in the individual sessions of each treatment in the one-shot game.

Table 3. one-shot: numbers of choices in individual sessions

session	frequencies *		
	focal	associated	most other
46.1	1	1	7
46.2	2	1	5
46.3	10	2	3
46.4	4	1	6
46.5	3	1	4
46.6	1	1	4
46.7	3	0	6
46.8	4	1	5
<i>avg.</i>	<i>3.5</i>	<i>1.0</i>	<i>5.0</i>
87.1	4	3	4
87.2	2	2	6
87.3	6	0	3
87.4	4	4	4
87.5	5	3	3
87.6	3	0	5
<i>avg.</i>	<i>4.0</i>	<i>2.0</i>	<i>4.2</i>
99.1	2	2	4
99.2	6	2	4
99.3	11	5	1
99.4	6	1	3
99.5	8	2	4
99.6	3	1	6
<i>avg.</i>	<i>6.0</i>	<i>2.2</i>	<i>3.7</i>
100.1	n.a.	n.a.	4
100.2	n.a.	n.a.	5
100.3	n.a.	n.a.	3
100.4	n.a.	n.a.	6
100.5	n.a.	n.a.	3
100.6	n.a.	n.a.	5
<i>avg.</i>	<i>n.a.</i>	<i>n.a.</i>	<i>4.3</i>

* There are 18 subjects in each session.

To show what do these observed choices imply in terms of expected coordination success, in Table 4, we present the expected payoffs for the individual sessions for each treatment in the one-shot game, and how these are attributed to the FP, the two ANE, and the other NE.

Table 4. one-shot: expected payoffs of individual sessions

session	avg. expected payoffs (points)				
	FP	1st ANE	2nd ANE	other NE	sum
46.1	0.00	1.23	0.00	14.81	16.05
46.2	0.00	2.47	0.00	6.17	8.64
46.3	13.63	7.41	4.94	1.23	27.21
46.4	1.74	3.70	0.00	16.05	21.49
46.5	1.14	2.47	0.00	17.28	20.89
46.6	0.00	1.23	0.00	16.05	17.28
46.7	1.14	0.00	0.00	6.17	7.31
46.8	1.70	1.23	0.00	13.58	16.52
<i>avg.</i>	<i>2.42</i>	<i>2.47</i>	<i>0.62</i>	<i>11.42</i>	<i>16.92</i>
87.1	4.30	7.41	0.00	6.17	17.88
87.2	0.00	4.94	0.00	9.88	14.81
87.3	9.67	0.00	0.00	7.41	17.07
87.4	4.30	7.41	2.47	3.70	17.88
87.5	6.44	4.94	3.70	2.47	17.56
87.6	0.00	0.00	0.00	13.58	13.58
<i>avg.</i>	<i>4.12</i>	<i>4.12</i>	<i>1.03</i>	<i>7.20</i>	<i>16.46</i>
99.1	1.22	2.47	0.00	14.81	18.51
99.2	11.00	7.41	0.00	7.41	25.81
99.3	34.22	43.21	0.00	0.00	77.43
99.4	11.00	3.70	0.00	6.17	20.88
99.5	0.00	19.75	0.00	2.47	22.22
99.6	2.44	2.47	0.00	4.94	9.85
<i>avg.</i>	<i>9.98</i>	<i>13.17</i>	<i>0.00</i>	<i>5.97</i>	<i>29.12</i>
100.1	n.a.	n.a.	n.a.	16.05	16.05
100.2	n.a.	n.a.	n.a.	13.58	13.58
100.3	n.a.	n.a.	n.a.	13.58	13.58
100.4	n.a.	n.a.	n.a.	12.35	12.35
100.5	n.a.	n.a.	n.a.	14.81	14.81
100.6	n.a.	n.a.	n.a.	14.81	14.81
<i>avg.</i>	<i>n.a.</i>	<i>n.a.</i>	<i>n.a.</i>	<i>14.20</i>	<i>14.20</i>

Table 5 concerns the individual "100" sessions in the 50-period game. On the left-hand side it presents the average frequency of the most chosen action (summed over row and column players) in each period for each session. On the right-hand side it shows the average expected payoffs in each of the individual sessions in the 100 treatment.

Table 5. 50 periods: frequencies of most chosen actions and expected payoffs in individual "100" sessions

session	avg. freq. of most chosen action *			avg. expected payoff (points)		
	period 1-10	period 41-50	period 1-50	period 1-10	period 41-50	period 1-50
100.1	5.0	6.4	5.6	15.56	24.82	20.35
100.2	4.5	5.2	4.9	12.96	16.54	15.04
100.3	4.8	4.7	4.9	12.47	16.54	14.86
100.4	4.6	4.5	4.5	13.21	14.44	14.22
100.5	4.6	4.5	4.5	15.93	15.31	15.41
100.6	4.7	5.8	4.9	10.37	20.86	14.25
avg.	4.7	5.2	4.9	13.42	18.09	15.69

* There are 18 subjects in each session.

Table 6 presents the average number of choices of the focal, associated, and most chosen other actions in the focal treatments for each of the individual sessions in the 50-period game.

Table 6. 50 periods: frequencies of actions in individual "46", "87" and "99" sessions

session	average frequencies *								
	focal choices			associated actions			most chosen other action		
	period 1-10	period 41-50	period 1-50	period 1-10	period 41-50	period 1-50	period 1-10	period 41-50	period 1-50
46.1	0.6	0.8	1.1	0.9	0.3	0.7	5.1	6.8	5.7
46.2	3.1	4.0	3.4	0.3	1.7	1.3	3.8	3.7	3.9
46.3	8.1	7.3	7.8	2.9	5.8	4.3	2.9	2.0	2.7
46.4	7.3	8.5	8.6	1.5	8.9	6.0	3.4	0.6	1.8
46.5	2.8	1.2	1.9	0.9	0.5	0.8	4.0	4.3	4.1
46.6	1.7	0.7	1.0	0.9	0.5	1.0	4.4	4.5	4.4
46.7	4.6	7.0	6.2	1.9	4.8	3.5	3.9	3.9	3.8
46.8	4.1	7.3	5.6	1.2	5.3	3.9	3.9	2.6	3.2
avg.	4.0	4.6	4.5	1.3	3.5	2.7	3.9	3.6	3.7
87.1	6.2	10.2	9.2	1.8	4.3	3.3	3.6	2.3	2.6
87.2	5.5	8.9	8.0	0.7	6.5	3.9	3.8	2.0	2.5
87.3	6.6	9.5	8.1	3.9	5.5	4.9	2.9	3.0	2.9
87.4	5.5	9.4	8.2	2.5	4.9	3.5	3.6	2.2	2.6
87.5	8.5	8.4	8.5	0.4	4.3	3.1	3.0	3.1	2.8
87.6	3.9	3.0	3.1	0.3	1.9	1.1	4.3	4.3	4.5
avg.	6.0	8.2	7.5	1.6	4.6	3.3	3.5	2.8	3.0
99.1	6.1	13.4	9.4	0.4	0.2	0.5	3.8	2.1	3.1
99.2	8.5	12.4	11.4	1.4	4.6	2.9	3.0	0.9	1.8
99.3	13.1	11.5	11.7	3.1	5.4	5.1	1.3	1.0	1.0
99.4	7.8	11.2	10.2	1.1	3.8	2.9	3.0	2.1	2.4
99.5	7.8	11.7	9.4	0.7	1.0	1.3	3.4	2.2	2.7
99.6	3.7	8.5	6.3	0.8	1.4	1.7	4.6	2.9	3.5
avg.	7.8	11.5	9.7	1.3	2.7	2.4	3.2	1.9	2.4

* There are 18 subjects in each session.

In Table 7 we look again at the average expected payoffs as a measure of the coordination success, allocating them, as above, to the FP, the first and second ANE, and any other NE, and distinguishing each of the individual sessions in the 46, 87, and 99 treatments in the 50-period game.

Table 7. 50 periods: expected payoffs in individual "46", "87" and "99" sessions

session	average expected payoffs (points)														
	FP			1st ANE			2nd ANE			other NE			total		
	per. 1-10	per. 41-50	per. 1-50	per. 1-10	per. 41-50	per. 1-50	per. 1-10	per. 41-50	per. 1-50	per. 1-10	per. 41-50	per. 1-50	per. 1-10	per. 41-50	per. 1-50
46.1	0.00	0.06	0.16	0.25	0.00	0.25	0.12	0.25	0.20	14.69	27.04	20.47	15.06	27.34	21.07
46.2	1.31	2.27	1.60	0.00	3.33	1.98	0.49	0.86	0.90	12.59	9.01	10.40	14.39	15.48	14.88
46.3	8.86	7.55	8.25	7.41	18.15	10.17	7.16	7.66	10.12	3.33	4.57	3.33	26.76	37.92	31.88
46.4	7.77	6.38	9.11	0.37	62.35	34.07	8.03	3.21	6.57	5.43	2.59	2.49	21.60	74.53	52.25
46.5	0.34	0.00	0.38	2.96	0.37	1.26	0.00	0.00	0.12	7.66	15.56	11.56	10.96	15.93	13.31
46.6	0.11	0.17	0.09	0.86	0.25	0.39	0.00	0.12	0.15	10.62	13.70	14.17	11.60	14.24	14.81
46.7	2.27	3.98	4.71	5.19	28.64	13.46	0.86	1.23	3.16	8.27	7.04	7.31	16.59	40.89	28.64
46.8	2.27	7.44	4.25	1.85	11.61	10.17	1.24	12.35	5.33	9.14	1.73	5.21	14.49	33.12	24.96
avg.	2.87	3.48	3.57	2.36	15.59	8.97	2.24	3.21	3.32	8.97	10.16	9.37	16.43	32.43	25.23
87.1	9.99	27.92	23.59	5.43	14.69	10.47	1.60	11.98	9.58	5.43	0.25	2.10	22.46	54.84	45.74
87.2	8.49	14.29	15.17	1.24	52.71	26.17	0.86	0.99	2.00	7.28	6.30	4.32	17.87	74.29	47.66
87.3	11.71	22.88	17.23	13.09	31.98	22.74	4.44	4.82	4.44	5.18	10.87	8.82	34.42	70.53	53.23
87.4	8.70	18.05	16.13	6.05	38.89	21.19	2.59	1.23	2.07	7.41	0.86	3.33	24.75	59.03	42.73
87.5	17.08	18.26	18.52	1.11	25.18	17.51	0.86	1.23	1.11	3.21	9.14	5.41	22.26	53.82	42.54
87.6	3.33	2.36	2.49	0.37	2.22	1.06	0.37	0.99	0.74	9.51	12.96	11.23	13.58	18.54	15.53
avg.	9.88	17.29	15.52	4.55	27.61	16.52	1.79	3.54	3.33	6.34	6.73	5.87	22.56	55.18	41.24
99.1	10.63	54.27	27.77	1.23	0.74	2.22	0.12	0.99	0.81	5.68	1.23	3.60	17.67	57.23	34.41
99.2	22.12	46.32	39.82	8.02	21.23	16.27	0.00	14.32	6.12	3.58	0.49	1.33	33.73	82.37	63.55
99.3	52.07	34.83	39.21	22.84	52.59	40.81	3.33	0.00	3.61	0.12	0.74	0.17	78.36	88.16	83.80
99.4	16.87	35.20	29.85	4.94	33.58	22.03	1.11	0.00	1.16	4.07	0.99	1.63	26.99	69.77	54.66
99.5	4.89	40.21	20.88	4.45	8.64	7.28	0.00	0.00	0.99	1.73	1.73	2.15	11.06	50.58	31.29
99.6	3.30	22.00	12.49	0.25	4.94	5.26	2.10	2.22	2.20	9.75	3.70	5.41	15.40	32.86	25.36
avg.	18.31	38.81	28.34	6.96	20.29	15.65	1.11	2.92	2.48	4.16	1.48	2.38	30.54	63.50	48.85

Table 8 concerns exclusively the expected payoffs realized by the ANE, showing the relative share of the expected payoffs realized by each of the ANE in the individual sessions in the 50-period game.

Table 8. 50 periods: relative shares of expected payoffs for each ANE in individual "46", "87" and "99" sessions

session	relative shares in expected payoffs of each ANE					
	period 1-10		period 41-50		period 1-50	
	1st ANE	2nd ANE	1st ANE	2nd ANE	1st ANE	2nd ANE
46.1	0.67	0.33	0.00	1.00	0.56	0.44
46.2	0.00	1.00	0.79	0.21	0.69	0.31
46.3	0.51	0.49	0.70	0.30	0.50	0.50
46.4	0.04	0.96	0.95	0.05	0.84	0.16
46.5	1.00	0.00	1.00	0.00	0.91	0.09
46.6	1.00	0.00	0.67	0.33	0.73	0.27
46.7	0.86	0.14	0.96	0.04	0.81	0.19
46.8	0.60	0.40	0.48	0.52	0.66	0.34
avg.*	0.51	0.49	0.83	0.17	0.73	0.27
87.1	0.77	0.23	0.55	0.45	0.52	0.48
87.2	0.59	0.41	0.98	0.02	0.93	0.07
87.3	0.75	0.25	0.87	0.13	0.84	0.16
87.4	0.70	0.30	0.97	0.03	0.91	0.09
87.5	0.56	0.44	0.95	0.05	0.94	0.06
87.6	0.50	0.50	0.69	0.31	0.59	0.41
avg.*	0.72	0.28	0.89	0.11	0.83	0.17
99.1	0.91	0.09	0.43	0.57	0.73	0.27
99.2	1.00	0.00	0.60	0.40	0.73	0.27
99.3	0.87	0.13	1.00	0.00	0.92	0.08
99.4	0.82	0.18	1.00	0.00	0.95	0.05
99.5	1.00	0.00	1.00	0.00	0.88	0.12
99.6	0.10	0.90	0.69	0.31	0.71	0.29
avg.*	0.86	0.14	0.87	0.13	0.86	0.14

* The weighted average for a treatment is computed as follows: $\Sigma(1st\ ANE)/\Sigma(1st\ ANE+2nd\ ANE)$